EMPIRICAL ANALYSIS OF IMPLIED VOLATILITY : STOCKS, BONDS AND CURRENCIES

by

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I. Introduction

Volatility, being one of the key variables in most models in modern finance, has deservedly attracted substantial amounts of research attention. Over the last decade, financial markets have richly rewarded empirical researchers with dramatic swings. This, coupled with the explosive growth in derivative markets and an abundance of price data, has resulted in a diverse collection of empirical studies that reflect the wide range of financial models where volatility assumes a central role. However, many fundamental questions remain unresolved. In this paper we focus on the modest task of documenting the statistical properties of "volatility" in the three major financial markets - stocks, bonds and currencies. The lack of precision in the way we use the term "volatility" is deliberate. It is deliberate because an immediate problem in a study of this type is the question of measurement.

At the conceptual level, there is the issue whether using standard measures of volatility based on historical prices is inferior to implied volatility derived from option prices; the latter of which should better reflect market expectations. At a more detailed level, there is the question of whether high frequency data, such as intra-day prices, offer additional insight to the traditional approach of using lower frequency (daily, weekly, or monthly) data over longer calendar spans. Clearly, the choice of data frequency and the technique for measuring volatility depends on the application to which it is put. Here, we attempt to provide empirical input to that decision. The volatility measure we focus on is the time series of "implied volatility" derived from observed option prices. The methodology is essentially based on Hsieh (1991) and Fung and Hsieh (1991) using high frequency (tick-by-tick) data.

The choice of markets covers the stock market, as proxied by the S&P futures contract at the CME, the bond market, proxied by the Bond futures contract at the CBT, and the currency market proxied by the Deutschemark futures contract at the IMM. This way, not only will we encompass most of the major financial instruments, but will also be able to contrast their empirical behaviour. The paper is organised as follows. In section II, we document some stylised facts conerning volatility behaviour in these markets. Section III defines the different ways of measuring volatility and analyse their information content. Section IV compares the our finding to the stylised facts in section II in the context of the inherent assumptions in option pricing models admitting stochastic volatility. Section V summarises our analyses and present our concluding remarks.

II. Some Stylised Facts

To motivate the subsequent analysis, we present in this section selected empirical observations. Figures 1, 2 and 3 plot the historical behaviour of implied volatility versus historical price volatility based on the standard deviation of daily closing prices of the lead futures contract on a rolling 20 days basis. Implied volatility is computed based on closing prices of the nearest to at-the-money series on the lead futures contract using the Barone-Adesi and Whaley (1987) approximation. Short term rates used are the T-bill rates of the closest matching maturity to the option. The actual plot in these figures is based on the average implied volatility of the puts and calls. Futures contracts are rolled over to the next in the standard March-June-Sept-Dec cycle at the beginning of the delivery month. Whilst there are other arguably more precise methods for adjusting contract rollovers in each of the instrument type, we adopted this rule for simplicity and for uniformaty across markets. Technically, the aging of the option contracts (as we move towards option expiration) may in itself cause implied volatility to shift, but the basic point here is that the changes over time is of substantial magnitude and not easily accounted for by the fine tuning of institutional factors. The more obvious conclusion is that these volatility time series are volatile and their time series behaviour merit more formal analysis.

Another interesting question is whether implied volatility is functionally related to the level of the underlying asset price. Various propositions have been put forward to substantiate such associations in different markets. We shall defer analysing this question after we have presented some formal statistics on the data in the next section. However, we would like to leave the reader with the following question. Are implied volatilities related to the level of the underlying asset price ? If so, is the relationship stable ? Figure 4 shows the usual graph one receives in the industry. Certainly, even casual observations will cast doubt on concluding a total lack of relationship between implied volatility and price level. Another striking feature is that should there be an association between implied volatility and prices, the form of the relationship is unlikely to be uniform across asset categories. We investigate this issue in section IV.

The combined effect of stochastic volatility and a possible correlation between volatility and price level would cause standard option models to imply volatilities of the same underlying asset to vary across strikes - an observation consistent with option models admitting stochastic volatility; see Hull and White (1987a, b, 1988), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), and Chesney and Scott (1989). Figures 5 to 9 provide a simple graphical illustration. In each of the figures, the x-axis gives the ratio of the closing futures price to the repsective option strike price in %. The y-axis expresses in % the ratio of the implied volatility of options at respective strikes to that of the one nearest to at-the-money. For example, figure 5 graphs this for the S&P futures options for two specific dates : Jan 2nd 1991 (for March options on the March contract) and May 1st 1991 (for September options on the September contract). It is clear from the figure that out-of-the-money puts and calls trade at substantially different imply volatilities than at-the-money options; that is if standard option models are used to compute implied volatilities. Granted that there are other institutional factors that could cause this 'strike-bias', certainly if empirical facts support stochastic volatilities it would be sufficient to induce standard models to imply this bias as the above authors have demonstrated theoretically. Adjusting for this bias calls for an option pricing model that explicitly accommodate this type of volatility behaviour. However, the literature has yet to reach an agreement on how this should be done. More specifically, how one determines the market price of volatility risk remains an open issue. Here we prefer to take an empirical route towards progress and report on these relationships in the next two sections.

Another important fact to bear in mind is that this 'strike bias' may well be time dependent. In figure 5, the shaded lines plot the implied volatility ratios observed at a different point in time. Granted that the options have slightly different maturities (Jan. observations on March options versus May observations on Sept. options) the "levelling" of the strike bias is hard to dismiss as simply due to a maturity difference in the options. Moving to other markets, figures 6, 7 and 8, reveal similar shifts and appear to display different degrees of bias and time shifts across markets. In fact in the case of currency futures options, figure 7 and 8, the strike bias appeared to have steepened even comparing Jan. observations on March options to May observations on June options. It is therefore important to bear this in mind before entering into more formal analysis.

Finally, Merville and Pieptea (1989) reported a mean reverting tendency of implied volatility in stock options. This is consistent with options of different maturities having different implied volatilities when computed via standard option models.

In other words, implied volatilities would appear to have a maturity (or term) sturcture. With the Black-Scholes model being widely accepted, it is now common place to have OTC options quoted in implied volatility terms where it is understood that the model used is the standard Black-Scholes model (sometimes with minor variations to adjust for the underlying instrument type) for European options. Whilst the hedging of such positions may well be based on an entirely different model, "Black-Scholes implied vol" is certainly the industry standard. To illustrate this point, figure 9 graphs the "quoted" volatility of Yen/Dollar options at maturities ranging from one month to one year obtained from currency brokers that regularly provide these quotes on Telerate and Reuters (the reported figures are averaged over several brokers). It is clear from the graph that options of different maturities are 'priced' at different volatilities. Furthermore, this maturity or term structure could assume different shapes at different points in time. Figure 10 gives a similar graph on DEM/Dollar options. As can be seen, the slope of the quoted term structure can range from being flat, downward sloping to upward sloping. Figure 11 displays two term structures for DEM/Dollar options and Yen/Dollar options observed at the same time to illustrate the point that the shape of the term structure could slope differently depending on the underlying asset. In fact, Figures 9 and 10 illustrate the movement of this structure where, over the first half of 1991, Yen options moved from being downward sloping to a flat structure, whereas DEM options went from being downward sloping to upward sloping and back to being downward sloping. It is entirely possible that with suitable adjustments to Black-Scholes, one could derive an option model that accounts for the time series behaviour of volatility such that the resultant "implied volatility" is flat across maturities at all times. Such a model will undoubtedly call for additional input parameters (compared to Balck-Scholes) that describes the time dependence of volatility; see for example Wiggins (1987), or Hull and White (1987a, b, 1988). Our empirical results offer further insight to this issue.

III Data Series and Basic Statistics

In order to keep the scope of the empirical work within reasonable limits without losing sight of the the cross-market contrast objective, we have chosen to proxy the stock market by the S&P index, the bond market by the longer end of the US Government Bond market, and the currencies market by a major currency such as the Deutschemark. A further consideration on the choice of data is the issue of liquidity and the associated data quality problem. As liquidity for both the US Government Bond market and the Foreign Exchange market resides with the over-the-counter markets (broker-dealer for bonds and inter-bank for currencies), options prices on these instruments for any reasonable continuous period are generally unobtainable. As a result, we prefer to use exchange traded futures contracts and options on futures contracts to proxy the behaviour of these asset categories. For stocks, we used the Chicago Mercantile Exchange's S&P Futures contract, the Chicago Board of Trade's Bond Futures contract for bonds, and the International Monetary Market's Deutschemarks Futures contract for Foreign Exchange. The data for the S&P futures and options contract and the DM futures and options contracts is from the Chicago Mercantile Exchange's (CME) tick-by-tick (quote capture) data. The data for the US Treasury Bond futures and options contract is from the Chicago Board of Trade (CBT)'s Education and Marketing Services. The Stocks and Bonds data span the period of March 1st, 1983 to July 31 st, 1989. The DM data begins later on Feb. 26, 1985 and ends on the same day. The shorter data period for the DM is to avoid potential distortion to the price dynamics due to a change in the dialy price limit prior to the 1985 start date.

Several measures of volatilities are computed. These are :

$\sigma_h(t)$ rolling standard deviation based on the daily Log relative of closing prices $\text{Log}_e(closing\,price_t / closing\,price_{t-1})$

from t-20 to t

$\sigma_k(t)$ standard deviaiton based on extreme values, as in Parkinson (1980); $(.361)^{0.5} \times \mathrm{Log}_e(Hi/Lo)$
Here the implicit assumption is that the true trading day can be approximated by the openning hours of the exchange

$\sigma_c(t)$ implied volatility of the nearest to at-the-the-money call options derived using parameters as described in section II

$\sigma_p(t)$ implied volatility of put options computed in the same way as for the calls

A few comments on the choice of these volatility measures. The 20-days rolling historical standard deviation, $\sigma_h(t)$, is choosen simply becasue it is a commonly used measure of volatility. The Parkinson (1980) measue, $\sigma_k(t)$, makes better use of intra-day price dynamics than the close-to-close type of measure like $\sigma_h(t)$. It therefore represents a useful first step towards the use of higher frequency price data than daily closes and is therefore included here for comparison.

The two implied volatility series $\sigma_c(t)$, $\sigma_p(t)$, have to be interpreted with some care. On the one hand they represent reasonable proxies of the market's expectation of future price volatility. However, an uncomfortable theoretical issue arises. These measures are computed using a model that assumes a stationary stochastic process in the underlying asset prices, it therefore would be internally inconsistent to subsequently fit these series to non-stationary models. This fortunately is not the way we choose to interpret these "implied volatilites" at this point. Initially, we prefer to concentrate on the question of information content in the implied volatilities. In this context, one can think of the Barone-Adesi and Whaley (1987) model purely as an algorithm that computes forecasts of price volatilities labelled here as "implied volatilities" for convenience. We then compare these measures to other more direct measures of volatility. These "implied volatilities" would be accepted or rejected for further analysis based solely on their information content. Viewed this way, we can defer the difficult theoretical issues until the statistical properties of $\sigma_c(t)$, and $\sigma_p(t)$ can be shown to merit further analysis. Note also that we only include near to at-the-money options for two reasons. Firstly, liquidity is generally greater for these options compare to the out-of-the-money series. Secondly, given the observed "strike bias" reported in section II, including out-of-the-money (or in-the-money) options would introduce further errors into the reported statistics.

Tables 1 to 3 report the statistics on these volatility measures for the S&P, DM and T Bonds respectively. In addition to the four measures, which are reported in logarithmic form, we also included a new measure of volatility which we use as our proxy for "realised volatility", $\sigma^*(t)$. The rationale for focusing on the transformed volatility series is as follows. Since by construct, these measures are positive definite and are therefore bounded below by zero but theoretically unbounded above which induces an asymmetric behaviour of the time series. Figures 12 to 29 plot these series in the three markets.

The method for assessing the information content of the four volatility measures, $\sigma_h(t)$, $\sigma_k(t)$, $\sigma_c(t)$, and $\sigma_p(t)$, is based on Hsieh (1991), and Fung and Hsieh (1991). The reasoning runs as follows. We regard a volatility measure as informative if it is indicative of future realised price movements or volatility. Since these empirical measures are on a daily basis, we need a proxy of realised daily price volatility. This is achieved by measuring the standard deviation of the series $\{r_j\}$:

$$r_j = \mathrm{Log}_e(Price_j / Price_{j-15minutes})$$

where j spans the daily trading hours (open to close of the market). The measure, $\sigma^*(t)$ is therefore the daily standard deviation of the series of 15 minutes rates of change for each trading day excluding the market close to market open (or overnight) effect. However, an adjustment needs to be made to bridge the computed $\sigma^*(t)$ to better approximate a 24-hours standard deviation measure. For the DM futures, we multiply the standard deviation of the 15 minutes data by $\sqrt{96}$, to reflect the 96 15-minutes interval in a 24 hours day, to obtain a daily standard deviation. Here the implicit assumption is that liquidity in the currency market does not diminish outside of US day time hours. For the S&P and T Bonds, we need to account for the nontrading hours differently.

Let O_t and C_t, denote the openning and closing prices for day t respectively. Let VT_t denote the "total daily variance" from the close of the market in one day to the next. We can decompose this total variance into two parts :

$$VT_t = V1_t + V2_t$$

where $V1_t$ is the variance of the rate of change between C_{t-1} and O_t, and $V2_t$ the variance of the rate of change between O_t and C_t. We estimate $V2_t$ by scaling up the standard deviation of the 15-minutes data. Specifically, suppose that there are n 15-minutes intervals during the trading day, then $V2_t$ is the standard deviation of the 15-minutes data multiplied by $\sqrt{n}$. To obtain VT_t we use the approximation :

$$VT_t = (1+\theta) \times V2_t$$

where $\theta = V1_t / V2_t$ estimated using :

$$\theta = \frac{[\mathrm{Log}_e(O_t / C_{t-1})]^2}{[\mathrm{Log}_e(C_t / O_t)]^2}$$

over the sample period. The same procedure is also used to scale the Parkinson Hi/Lo volatility measure $\sigma_k(t)$.

The 15 minutes time interval is an empirical choice. Going beyond 15 minutes to higher frequency data brings in empirical issues beyond the scope of this paper. For instance, there are significantly different autocorrelation characteristics for the three markets. At high data frequency, more work is needed to correct the autocorrelation effect that is due purely to bid/offer spreads. We opted for the 15 minutes interval, which is sufficiently long to mitigate a lot of the bid/offer spread induced autocorrelation and yet short enough to allow for a meaningful number of observations- approximately 26 observations per day- to compute a daily standard deviation. The time series of corresponding daily returns $\{x_{t+1}\}$ is then computed using the log relative of close-to-close prices and is based on information external to that used in computing $\sigma^*(t)$. This series of daily returns when transformed to be in absolute terms, $\{|x_{t+1}|\}$, displayed a high level of autocorrelation than the original series which is close to mean zero with varying degrees of autocorrelation. This is consistent with the significant levels of autocorrelation in all the volatitily series in tables 1 to 3. Next we define a series of standardised variables :

$$z_h = \left\{ \frac{x(t+1)}{\sigma_h(t)} \right\}$$

$$z_k = \left\{ \frac{x(t+1)}{\sigma_k(t)} \right\}$$

$$z^* = \left\{ \frac{x(t+1)}{\sigma^*(t)} \right\}$$

$$z_c = \left\{ \frac{x(t+1)}{\sigma_c(t)} \right\}$$

$$z_p = \left\{ \frac{x(t+1)}{\sigma_p(t)} \right\}$$

We say that a volatility measure has better information content if the corresponding standardised variable is closer to being iid; with mean zero and a standard deviation of 1. This is in many ways analogous to the structure of a GARCH model. Consider a GARCH type structure where :

$$x_{t+1} \mid I_t \sim D(0, h_t)$$

Here we informally approximate $h_t^{1/2}$ by the four volatility measures. Tables 4, 5 and 6 present the results for the S&P, DM and T-Bonds. An additional column of data labelled as Raw Data representing the statistics on $\{x_t\}$ is included for comparison.

Standard t-tests are reported against N(0,1) for the five volatility measures in each of the three markets. In all cases, the standardised variables $\{z_i\}$ for i = h, k, *, c, p showed substantial amount of reduced autocorrelation. The t-tests for $\{z_i\}$, i = *, c, p all failed to reject $\{z_i\} \sim N(0,1)$. There is however, some troublesome levels of kurtosis for the case of the S&P series. We suspect that this is due to the inclusion of the 87' stock market crash in the data series. The bottom of table 4 recompute the moments in two subperiods excluding the period of the crash. Generally the kurtosis levels are much reduced. Based on this iid test, we would conclude that the "implied volatility" series performed just as well as the realised volatility $\sigma^*(t)$, and could be loosely described as having superior information content to the two other measures $\sigma_h(t)$ and $\sigma_k(t)$. At this stage, the evidence would support further investigations on the information content of the implied volatilities.

The next stage of our test design is a straight forward one-step-ahead forecast of "realised volatility" as proxied by $\sigma^*(t)$. For completeness, we include the $\sigma^*(t)$ series itself in the test. Two simplistic time series forecasts are included, the first is simply a random walk model of $E_{t-1}(\sigma^*(t)) = \sigma^*(t-1)$. The second model is a simple autoregressive model based on the observed autocorrelation patterns in tables 1, 2 and 3. Table 7 shows the fitted model. Generally, it was necessary to go up to lag 15 for the S&P and T Bonds whereas it was sufficient to stop at lag 8 for the DM futures. The figures immediately below the coefficients in table 7 are the standard errors. Here we overfit somewhat by using the entire sample at our disposal to determine the order of the autoregressive models; AR(15) for stocks and bonds and AR(8) for the DM futures. The forecasts are based on $E_{t-1}(\sigma^*(t)) = E(.|AR(n,t-1))$. Clearly the latter forecast based on the AR models contains information going beyond that available at t-1, but only to the extent of the "order" of the AR models. The coefficients to the models are refitted period by period. Table 8 contains the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) for each of the forecast models.

It is hardly surprising that the "over-fitted" AR models did the best. In the case of the S&P, except for the over-fitted AR model, there is little to choose between the others. In other words, the implied volatilities add little to the straight forward rolling historical volatility series. A conclusion that is consistent with Canina and Figlewski (1990). In the case of DM, the implied volatilities performed almost as well as the over-fitted AR model. They show a reduction of RMSE, and MAE by almost 20% on average against the other three models. In the case of the T-Bonds, the results are similar to that of stocks with the implied volatilites showing only marginal improvements to the simple historcial rolling volatility.

The broad conclusion at this point is that a great deal of the observed non-stationarity and non-linearity in the underlying assets' returns come from the non-stationarity of their volatilities as shown in the iid tests. Although implied volatilities performed as well as other standard measures in the iid tests, they do not appear to forecast future realised price movement any better for stocks and bonds. Only in the case of the DM, did the implied volatilities add information. We can offer one pluasible explanation to this phenomena. Stocks and bonds are physical financial assets with natural demand to hold over long horizons. This cannot be said for currencies. Thereofore, if investors are net long of stocks and bonds, the insurance attribute of options in a world with transaction costs could be a significant contributing factor to the option premium. This in turn will afffect the computed implied volatilities based on a model casts in a frictionless market setting. In other words, the diverse clientele of investors in the stock and bond markets injects an additional dimension to our analysis of implied volatility.

Section IV Implications for Option Pricing Models

The stylised facts outlined in seciton II are, with specific assumptions, formally stated in the stochastic volatility models in Wiggins (1987) and Hull and White (1987a, b, 1988). In these models, the stochastic process for the underlying asset is:

$$dS(t) = \mu S(t)dt + \sigma(t)S(t)dz_s$$
$$d\sigma(t) = f(\sigma(t))dt + \theta\sigma(t)dz_\sigma$$
$$\rho\, dt = dz_s \times dz_\sigma$$

The function forms used for $f(\sigma(t))$, tended to be variations of a first order mean reversion model with a constant variance term θ. Given this specification, the above authors were able to simulate option prices that exhibit implied volatility different from the input volatility when derived from a standard option pricing model assuming constant variance. The "stylised facts" reported in section II on the stochastic nature of implied volatilities and the "strike-bais" are, although informal but, consistent empirical evidence in favor of the above models. Specifically, the asymmetric "strike bias" in the S&P futures options is consistent with a negative ρ. The assumption of mean reversion in $\sigma(t)$, is consistent with the observed maturity structure to the implied volatilities reported in section II. Whilst these observations support an underlying stochastic process similar to the one specified above, a few words of caution is in order.

Firstly, the term structure we observed in section II and its time variations appeared to be more complex than just a simple first order model. Secondly, the intertemporal behaviour of the "strike bias" is of a magnitude that may not be consistent with a stationary θ, and ρ.

In terms of the empirically results in section III, the AR models reported in Table 7 would be consistent with mean reverting behaviour, since the coefficients of the lag terms sum to less than unity. However, the order of the AR model is inconsistent with the structure suggested in the stochastic volatility options literature. Although one could investigate alternatives such as ARMA models of a reduced lag structure, preliminary results suggest a nonstationary mean term which brings in further complications to the existing option models both in terms of misspecification of the stochastic processes as well as adding to the problem of estimating the market price of volatility risk. The implied volatility series display similar autocorrelation behaviour to that of the 15-minutes volatility series. Until a more robust structure capturing the time series behaviour of these volatility measures can be established, reliable estimates of θ, and ρ continue to elude us. From the evidence we have thus far, models that depend on the stationarity of θ, and ρ appear to be poor propositions.

V Summary and Concluding Remarks

We began the paper by reporting some stylised facts on the behaviour of implied volatilities in three options markets - S&P futures, DM futures, and T Bond futures. At first glance, the observations look promising in that they are broadly consistent with the predictions of stochastic volatility option models of Hull and White (1987a, b, 1988) and Wiggins (1987). Although there are some tell tale signs that the stochastics process may be more complex than the ones used in the literature thus far, they are sufficiently encouraging to warrant further investigation. Realising the high demand for data quantity in analysing time series behaviour of volatility and the attendant measurement problems, we adopted a different approach. Here we chose to interpret the observable implied volatilities from option prices as one of several volatility forecasts we examine without reference to the theoretical assumptions underlying the option model. It is important to note that we based our observations on the nearest to at-the-money-options rather than averaging over options of different strikes as in other studies. This is becasue, given the strike bias we observe in section II, out-of-the-money (or in-the-money) options bring additional measurement errors. Also, a quick reference to the reported volume on out(in)-the-money options reveals a wide disparity in liquidity compared to at-the-money options. Since we are concerned with the information content of the implied volatilities, it makes more sense to avoid the out(in)-of-the-money series. In terms of other, more historical price based, volatility measures, we added to the standard daily price change type of measure two additional alternatives influenced by our belief that intra-day price movements play a crucial role in volatility measures. The first additional measure is based on the extreme value method as proposed in Parkinson (1980) using daily Hi/Lo prices to estimate standard deviations. The second measure is based on 15-minutes price changes extracted from tick-by-tick observations of the market. We use this latter measure to proxy "relaised volatility".

In order to proceed we need to define a form of information criterion. In the spirit of GARCH models we used the alternative measures of volatilities, including the implied volatilities, in place of a volatility equation in the GARCH structure. We say that these measures contain information if they reduce observed autocorrelations in the underlying daily returns and can standardise the returns to be closer to iid. Under this criterion, the implied volatilities performed satisfactorily. As a result, we proceeded to perform a on-step-ahead forecast test on the predictive ability of these volatility measures using our proxy of realised volatility as the benchmark. Here, the results are mixed. The implied volatilities performed better than the historical volatilities in the case of DM futures but added little by way of smaller forecast errors for stocks and bonds. In Fung and Hsieh (1991), we also investigated whether implied volatilities are better regarded as forecasts of volatility realised over the remaining life of the option rather than a one-day ahead forecast we used here. Our results in that paper suggest that forecast horizon is not an issue. We therefore ended our analysis at this point and chose not to proceed along the lines of Canina and Figlewski (1990).

Overall, our impression of the empirical evidence is that implied volatilities derived from frictionless market models are affected by institutional factors distorting the time series analysis. This is particularly the case in stock and bond markets, but less so with currencies. The more promising route appears to be using high frequency data (tick-by-tick) to model volatility directly. Judging from the results thus far, the stochastic process that best describe volatilites of asset prices may well be more complex than the typical first order mean revertin models used in stochastic volatility option models to date.

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Table 1 Measures of S&P Log Volatility

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol
Mean	-4.666	-4.641	-4.585	-4.522	-4.503
Std Dev	0.477	0.527	0.454	0.326	0.341
Skewness	2.118	0.958	1.414	0.926	1.043
Kurtosis	10.477	6.591	9.811	5.402	5.961
Maximum	-2.370	-1.422	-1.177	-2.803	-2.705
Minimum	-5.521	-6.244	-6.168	-5.123	-5.183
Autocorrelation coefficients					
Lag 1	0.982	0.471	0.671	0.980	0.976
2	0.963	0.448	0.627	0.965	0.961
3	0.942	0.422	0.600	0.954	0.952
4	0.921	0.469	0.592	0.943	0.942
5	0.899	0.435	0.548	0.932	0.929
6	0.877	0.402	0.503	0.918	0.915
7	0.854	0.339	0.480	0.908	0.901
8	0.830	0.360	0.476	0.901	0.892
9	0.807	0.376	0.466	0.892	0.885
10	0.781	0.344	0.433	0.883	0.874
11	0.755	0.343	0.431	0.876	0.866
12	0.730	0.301	0.398	0.870	0.858
13	0.705	0.311	0.387	0.864	0.850
14	0.681	0.322	0.406	0.859	0.846
15	0.657	0.313	0.423	0.853	0.839
16	0.633	0.311	0.418	0.846	0.831
17	0.608	0.265	0.381	0.838	0.825
18	0.583	0.261	0.354	0.830	0.820
19	0.557	0.262	0.351	0.824	0.812
20	0.530	0.311	0.369	0.815	0.805

Table 2 Measures of DM Log Volatility

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol
Mean	-4.977	-4.721	-4.742	-4.872	-4.869
Std dev	0.337	0.530	0.469	0.213	0.214
Skewness	0.023	0.091	0.065	-0.040	-0.033
Kurtosis	2.847	2.845	3.014	2.650	2.653
Maximum	-4.210	-3.000	-2.960	-4.405	-4.412
Minimum	-6.092	-6.277	-6.293	-5.719	-5.660
Autocorrelation coefficients					
Lag 1	0.974	0.353	0.464	0.968	0.967
2	0.946	0.332	0.417	0.944	0.945
3	0.918	0.296	0.390	0.925	0.926
4	0.891	0.322	0.416	0.905	0.907
5	0.864	0.319	0.410	0.890	0.892
6	0.835	0.311	0.386	0.876	0.876
7	0.802	0.328	0.385	0.859	0.860
8	0.768	0.307	0.356	0.843	0.843
9	0.733	0.277	0.340	0.829	0.827
10	0.697	0.251	0.320	0.813	0.812
11	0.659	0.220	0.310	0.796	0.796
12	0.620	0.253	0.310	0.780	0.780
13	0.580	0.244	0.291	0.762	0.763
14	0.541	0.257	0.294	0.749	0.748
15	0.500	0.259	0.304	0.732	0.732
16	0.459	0.207	0.280	0.716	0.716
17	0.417	0.170	0.266	0.699	0.700
18	0.378	0.144	0.197	0.684	0.685
19	0.338	0.209	0.279	0.669	0.669
20	0.299	0.185	0.248	0.655	0.655

Table 3 Measures of T Bond Log Volatility

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol
Mean	-4.924	-4.739	-4.812	-4.843	-4.832
Std dev	0.305	0.487	0.428	0.212	0.209
Skewness	0.351	0.145	0.076	0.545	0.460
Kurtosis	2.707	2.941	3.047	3.442	3.073
Maximum	-4.095	-3.208	-3.234	-3.785	-4.116
Minimum	-5.666	-6.319	-6.185	-5.383	-5.364
Autocorrelation coefficients					
Lag 1	0.975	0.325	0.557	0.966	0.970
2	0.952	0.332	0.497	0.946	0.949
3	0.928	0.335	0.472	0.926	0.932
4	0.903	0.341	0.513	0.907	0.912
5	0.876	0.315	0.492	0.887	0.895
6	0.850	0.256	0.437	0.868	0.874
7	0.824	0.287	0.421	0.848	0.856
8	0.799	0.296	0.424	0.829	0.838
9	0.773	0.272	0.427	0.806	0.818
10	0.745	0.259	0.405	0.790	0.799
11	0.716	0.264	0.386	0.773	0.781
12	0.688	0.251	0.365	0.755	0.764
13	0.660	0.223	0.387	0.740	0.748
14	0.631	0.226	0.360	0.723	0.731
15	0.602	0.277	0.394	0.707	0.715
16	0.576	0.207	0.364	0.688	0.698
17	0.551	0.229	0.348	0.671	0.682
18	0.528	0.190	0.344	0.657	0.668
19	0.503	0.219	0.362	0.639	0.654
20	0.477	0.176	0.309	0.623	0.639

Table 4 Standardized Data for S&P

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol	Raw Data
Mean	0.064	0.062	0.048	0.042	0.044	0.000
Std dev	1.234	1.212	1.112	1.090	1.085	0.016
Skewness	-2.740	-0.526	-0.855	-4.862	-5.153	-7.064
Kurtosis	39.384	7.916	10.398	80.992	88.926	179.348
Maximum	4.359	6.311	6.042	3.199	3.841	0.177
Minimum	-17.321	-7.781	-7.681	-18.647	-19.018	-0.337
t(Mean=0)	1.736	1.712	1.445	1.290	1.357	
t(SD=1)	1.024	2.226	1.100	0.309	0.280	
Autocorrelation coefficients of absolute values						
Lag 1	0.044	-0.179	-0.050	0.092	0.096	0.299
2	0.011	-0.023	-0.057	0.064	0.089	0.355
3	0.017	-0.020	-0.022	0.058	0.066	0.265
4	0.014	0.039	0.060	0.037	0.038	0.158
5	-0.002	-0.015	-0.010	0.031	0.021	0.207
6	0.018	0.011	-0.009	0.045	0.041	0.178
7	-0.020	-0.030	-0.040	-0.007	-0.005	0.106
8	-0.015	-0.009	-0.010	0.025	0.018	0.146
9	0.029	0.006	0.032	0.072	0.077	0.177
10	-0.034	-0.042	-0.055	-0.023	-0.024	0.044
11	-0.026	0.034	-0.004	0.003	0.004	0.093
12	-0.052	-0.054	-0.036	-0.013	-0.010	0.057
13	-0.025	-0.004	-0.013	-0.004	-0.007	0.062
14	-0.010	0.007	0.014	0.025	0.022	0.103
15	0.004	0.048	0.036	0.033	0.033	0.065
16	-0.043	-0.042	-0.020	-0.004	-0.003	0.068
17	-0.019	0.017	0.009	0.006	0.005	0.060
18	-0.043	0.008	0.009	-0.004	-0.001	0.068
19	0.001	-0.013	0.009	0.042	0.048	0.084
20	-0.062	0.008	-0.012	-0.025	-0.024	0.050
Subperiod: Feb 26, 1985 - Jun 30, 1987 (592 observations)						
Mean	0.079	0.081	0.070	0.072	0.072	0.001
Std Dev	1.138	1.187	1.045	0.980	0.977	0.010
Skewness	-0.396	-0.920	-0.802	-0.447	-0.446	-0.623
Kurtosis	5.983	9.093	8.380	5.147	5.187	6.022
Maximum	4.240	4.258	3.811	3.199	3.196	0.033
Minimum	-6.305	-7.781	-6.790	-5.116	-5.160	-0.057
Subperiod: Jan 4, 1988 - Jul 31, 1989 (399 observations)						
Mean	0.094	0.080	0.070	0.062	0.065	0.001
Std Dev	1.095	1.232	1.150	0.841	0.805	0.011
Skewness	-0.326	0.100	-0.372	-1.056	-0.936	-1.541
Kurtosis	6.158	7.379	11.045	9.629	9.238	16.890
Maximum	4.359	6.311	6.042	2.908	2.855	0.042
Minimum	-4.979	-5.657	-7.218	-5.508	-5.271	-0.088

Table 5 Standardized Data for DM

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol	Raw Data
Mean	0.047	0.034	0.021	0.042	0.042	0.000
Std dev	1.143	1.025	1.000	0.973	0.971	0.008
Skewness	-0.099	0.148	-0.400	0.064	0.058	0.270
Kurtosis	5.189	7.036	7.983	4.170	4.229	5.426
Maximum	4.893	5.780	4.035	4.666	4.677	0.048
Minimum	-5.800	-4.769	-7.507	-3.360	-3.419	-0.033
t(Mean=0)	1.376	1.110	0.703	1.445	1.448	
t(SD=1)	2.046	0.332	0.000	-0.522	-0.556	
Autocorrelation coefficients of absolute values						
Lag 1	0.030	-0.099	-0.066	-0.003	-0.002	0.066
2	-0.022	-0.042	-0.080	-0.037	-0.036	0.035
3	0.015	-0.048	-0.053	0.012	0.013	0.083
4	0.021	-0.018	-0.035	0.016	0.014	0.078
5	0.036	-0.030	-0.044	0.029	0.028	0.090
6	0.033	-0.004	0.007	0.043	0.042	0.122
7	0.043	0.024	0.037	0.044	0.045	0.106
8	0.006	0.003	0.008	0.023	0.022	0.095
9	0.017	0.011	0.026	0.021	0.021	0.066
10	0.040	0.028	0.053	0.066	0.066	0.131
11	-0.028	-0.034	-0.023	-0.015	-0.017	0.032
12	-0.020	-0.001	-0.002	0.004	0.003	0.062
13	0.014	0.003	-0.012	0.034	0.033	0.097
14	-0.006	-0.008	-0.002	0.018	0.014	0.083
15	0.017	0.038	0.056	0.047	0.047	0.108
16	-0.006	0.021	0.025	0.042	0.039	0.074
17	-0.064	-0.019	-0.024	-0.027	-0.025	0.018
18	-0.037	0.007	-0.016	0.011	0.011	0.056
19	-0.044	0.007	-0.013	-0.017	-0.017	0.028
20	-0.070	-0.015	0.022	-0.014	-0.013	0.036

Table 6 Standardized Data for T bond

		Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol	Raw Data
Mean		0.060	0.066	0.055	0.056	0.057	0.000
Std dev		1.108	1.099	1.104	0.992	0.984	0.008
Skewness		-0.087	0.033	0.110	-0.060	-0.025	0.075
Kurtosis		4.830	7.902	7.955	3.974	4.035	4.348
Maximum		4.574	5.840	6.983	3.737	3.823	0.039
Minimum		-5.168	-7.317	-6.270	-3.899	-3.831	-0.025
t(Mean=0)		1.812	2.010	1.667	1.889	1.939	
t(SD=1)		1.667	1.148	1.195	-0.157	-0.312	
Autocorrelation coefficients of absolute values							
Lag	1	-0.017	-0.155	-0.089	-0.019	-0.004	0.103
	2	-0.006	-0.054	-0.070	-0.010	-0.015	0.080
	3	0.019	-0.021	-0.033	0.031	0.030	0.127
	4	0.032	0.036	0.017	0.069	0.067	0.176
	5	0.012	0.019	0.030	0.029	0.037	0.146
	6	-0.002	-0.041	-0.040	0.024	0.026	0.136
	7	-0.042	-0.038	-0.028	-0.024	-0.023	0.065
	8	0.009	0.011	0.003	0.029	0.027	0.110
	9	0.023	-0.011	0.017	0.041	0.047	0.119
	10	-0.016	-0.056	-0.051	0.003	-0.001	0.095
	11	0.010	-0.008	-0.015	0.037	0.031	0.106
	12	-0.022	-0.016	-0.021	-0.007	-0.005	0.076
	13	-0.013	-0.005	-0.001	0.036	0.031	0.111
	14	-0.014	-0.002	-0.021	0.028	0.020	0.073
	15	-0.045	0.020	0.002	0.009	0.002	0.068
	16	-0.027	-0.021	-0.014	0.015	0.013	0.088
	17	-0.060	-0.030	-0.032	-0.009	-0.016	0.045
	18	-0.015	0.029	0.032	0.046	0.041	0.078
	19	0.023	0.061	0.059	0.074	0.076	0.114
	20	0.013	0.027	0.038	0.061	0.058	0.105

Table 7 Autoregressive Model of Log 15 Minute Volatility

	S&P	DM	T Bond
Constant	-0.453	-0.853	0.001
	0.141	0.168	0.000
Lag 1	0.337	0.213	0.326
	0.049	0.032	0.041
Lag 2	0.187	0.112	0.083
	0.036	0.030	0.045
Lag 3	0.120	0.069	0.071
	0.031	0.031	0.045
Lag 4	0.133	0.127	0.200
	0.032	0.030	0.036
Lag 5	0.036	0.111	0.052
	0.036	0.032	0.042
Lag 6	-0.034	0.058	0.005
	0.032	0.028	0.035
Lag 7	-0.018	0.090	0.005
	0.033	0.031	0.037
Lag 8	0.035	0.041	0.032
	0.034	0.030	0.036
Lag 9	0.048		0.013
	0.033		0.040
Lag 10	-0.024		-0.007
	0.035		0.034
Lag 11	0.028		0.013
	0.032		0.037
Lag 12	-0.051		-0.016
	0.032		0.039
Lag 13	-0.047		0.044
	0.032		0.036
Lag 14	0.050		-0.012
	0.031		0.046
Lag 15	0.101		0.068
	0.030		0.035
Rbar-sq	0.548	0.340	0.448

Table 8 Forecast Errors of Log 15 Minute Volatility

	Rolling Historical Vol	High/ Low Vol	15 Min Tick Vol	Call Implied Vol	Put Implied Vol	Forecasted 15 Min Tick Vol
S&P						
RMSE	0.378	0.435	0.368	0.368	0.379	0.326
MAE	0.286	0.350	0.286	0.265	0.274	0.241
DM						
RMSE	0.492	0.520	0.485	0.404	0.404	0.396
MAE	0.380	0.398	0.363	0.314	0.313	0.304
T Bonds						
RMSE	0.376	0.471	0.403	0.374	0.373	0.336
MAE	0.302	0.367	0.313	0.283	0.281	0.264

FIGURE 1

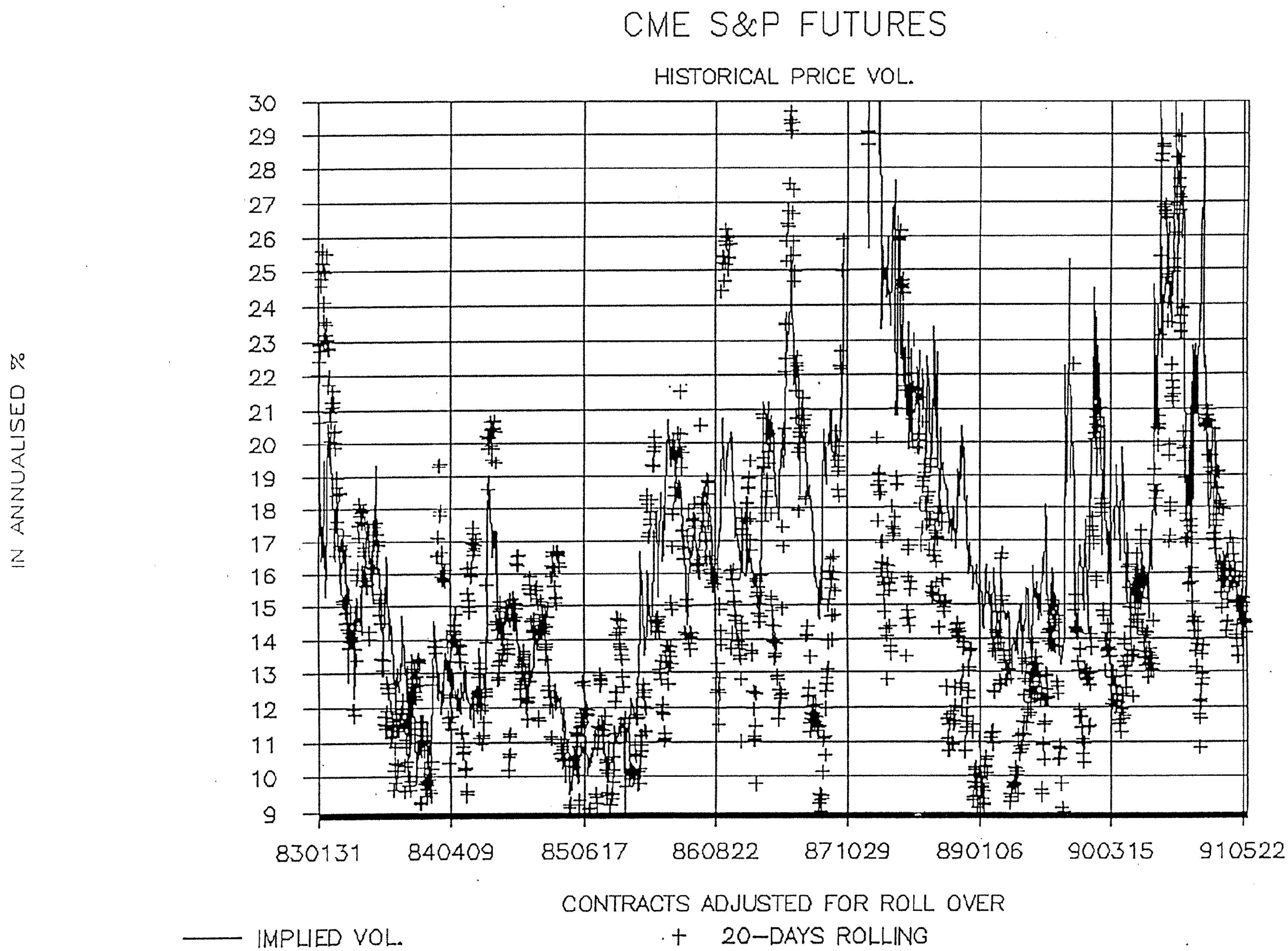
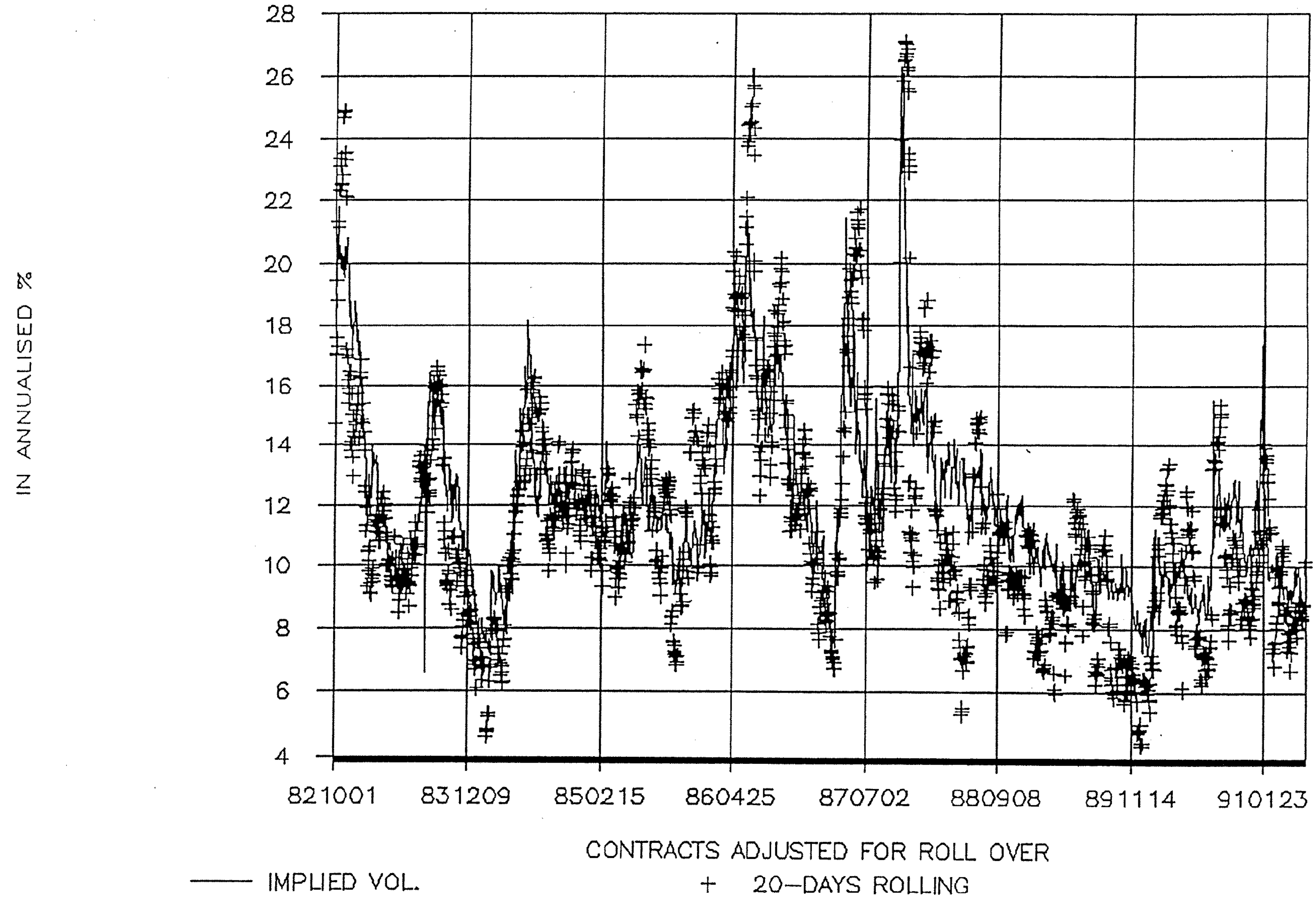

CME S&P FUTURES
HISTORICAL PRICE VOL.
IN ANNUALISED %
30
29
28
27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
830131
840409
850617
860822
871029
890106
900315
910522
CONTRACTS ADJUSTED FOR ROLL OVER
IMPLIED VOL.
+ 20–DAYS ROLLING

FIGURE 2

CBT BOND FUTURES

HISTORICAL PRICE VOL.



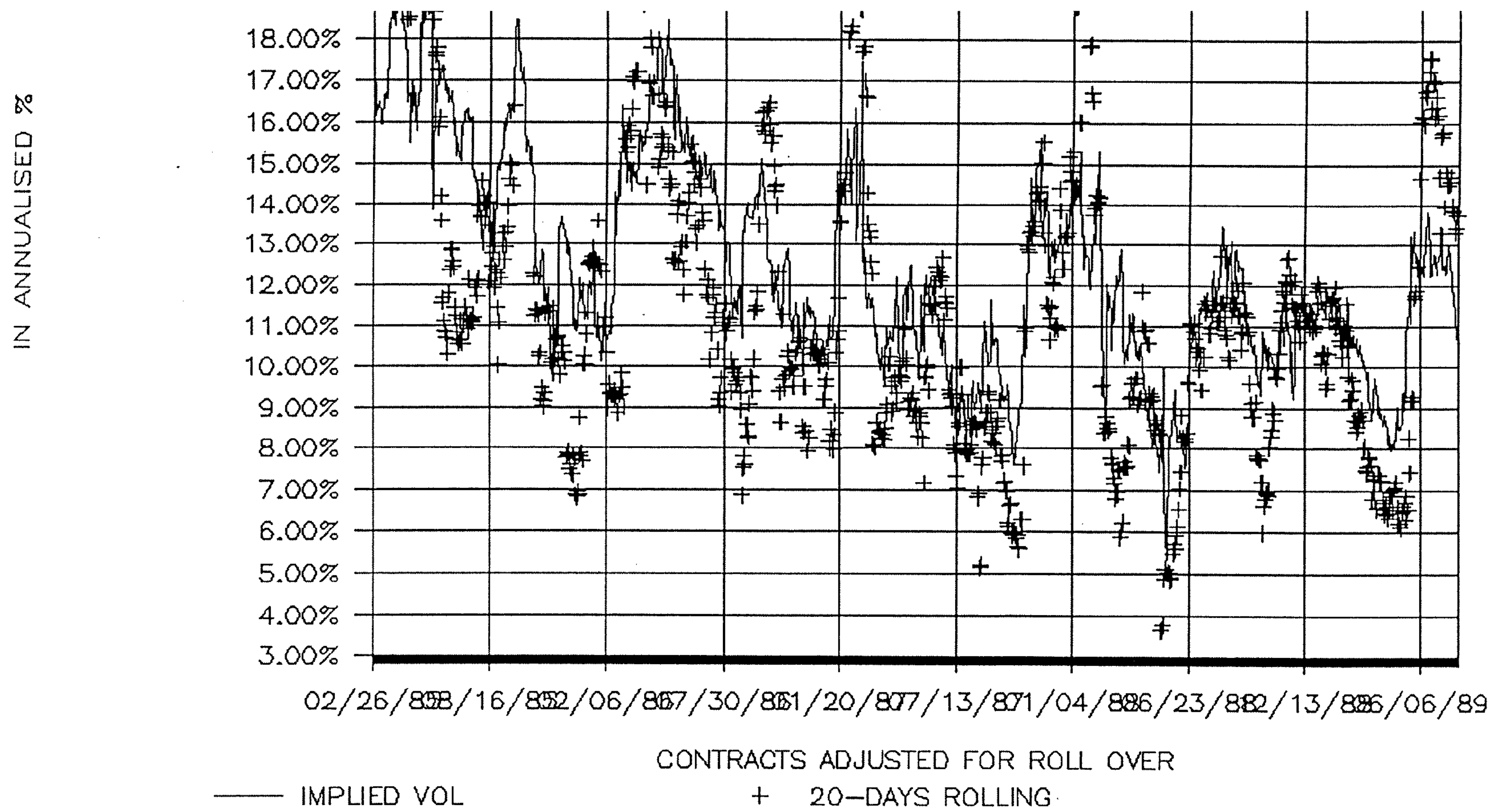

IN ANNUALISED %
18.00%
17.00%
16.00%
15.00%
14.00%
13.00%
12.00%
11.00%
10.00%
9.00%
8.00%
7.00%
6.00%
5.00%
4.00%
3.00%
02/26/8503/16/8502/06/8607/30/8601/20/8707/13/8701/04/8806/23/8812/13/8806/06/89
CONTRACTS ADJUSTED FOR ROLL OVER
IMPLIED VOL
+ 20-DAYS ROLLING

FIGURE 4

Discount Corporation of New York Futures

Futures Price vs. Implied Volatility*

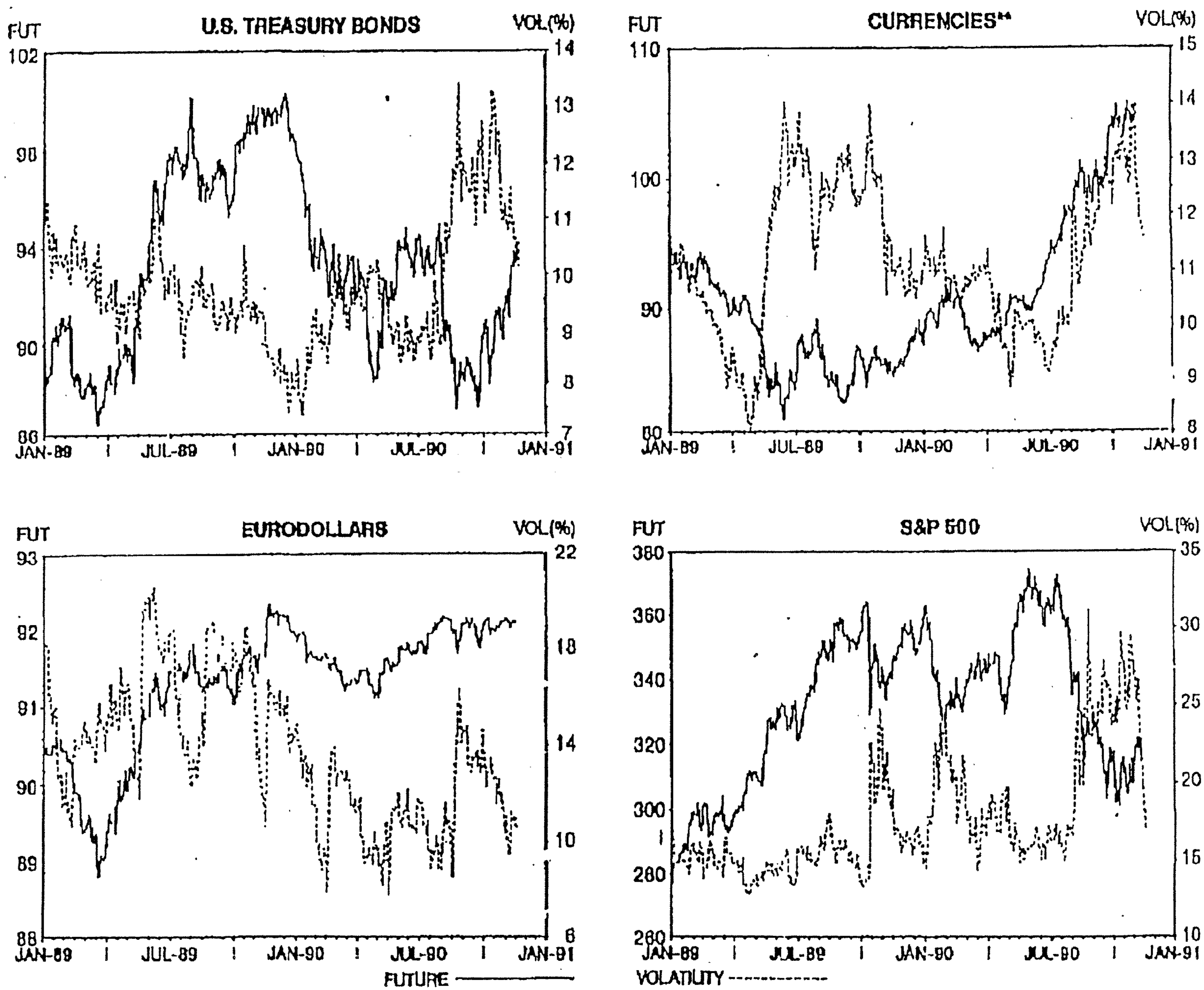


FIGURE 5

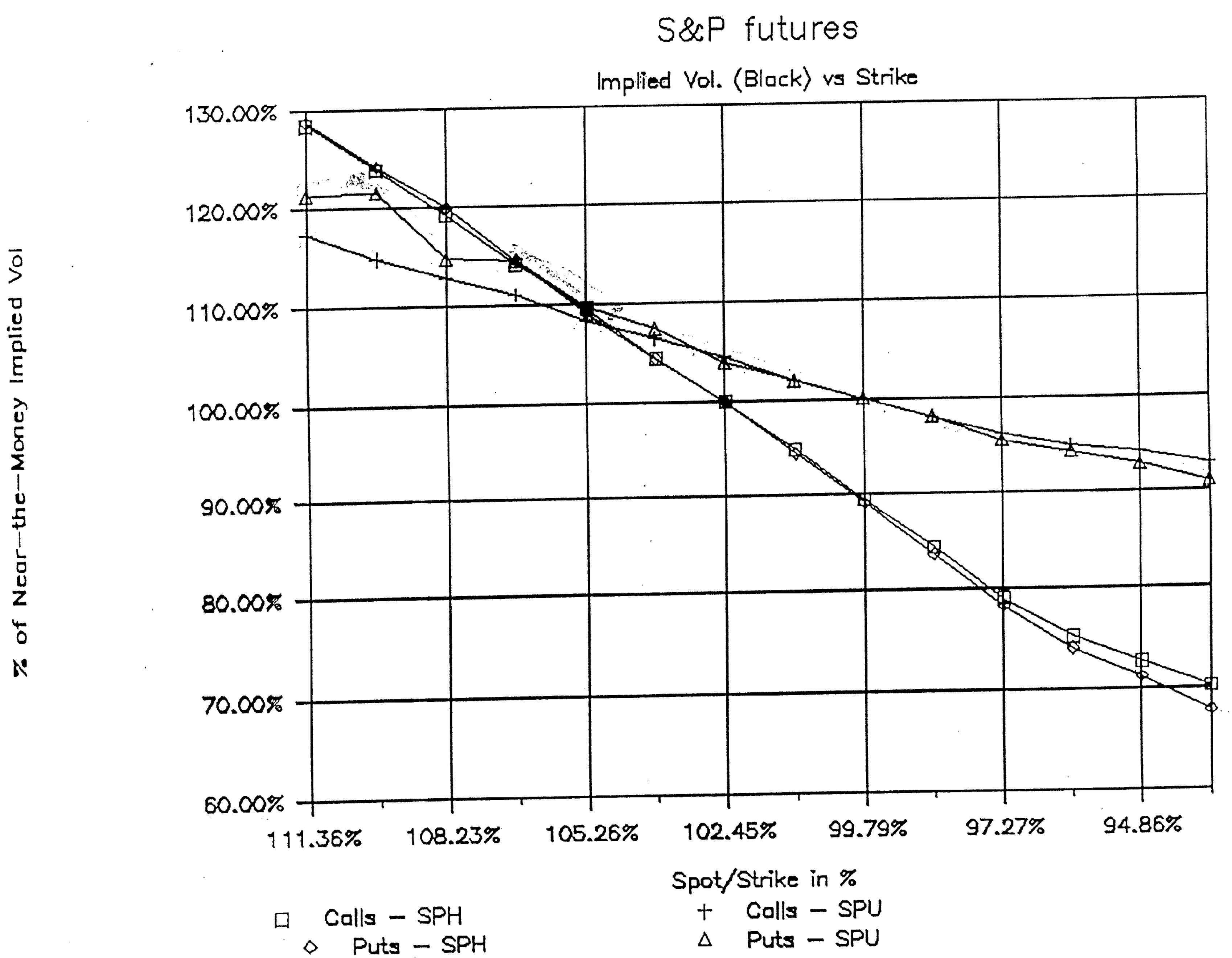

S&P futures
Implied Vol. (Black) vs Strike
% of Near-the-Money Implied Vol
130.00%
120.00%
110.00%
100.00%
90.00%
80.00%
70.00%
60.00%
111.36%
108.23%
105.26%
102.45%
99.79%
97.27%
94.86%
Spot/Strike in %
□ Calls — SPH
◇ Puts — SPH
+ Calls — SPU
△ Puts — SPU

FIGURE 6

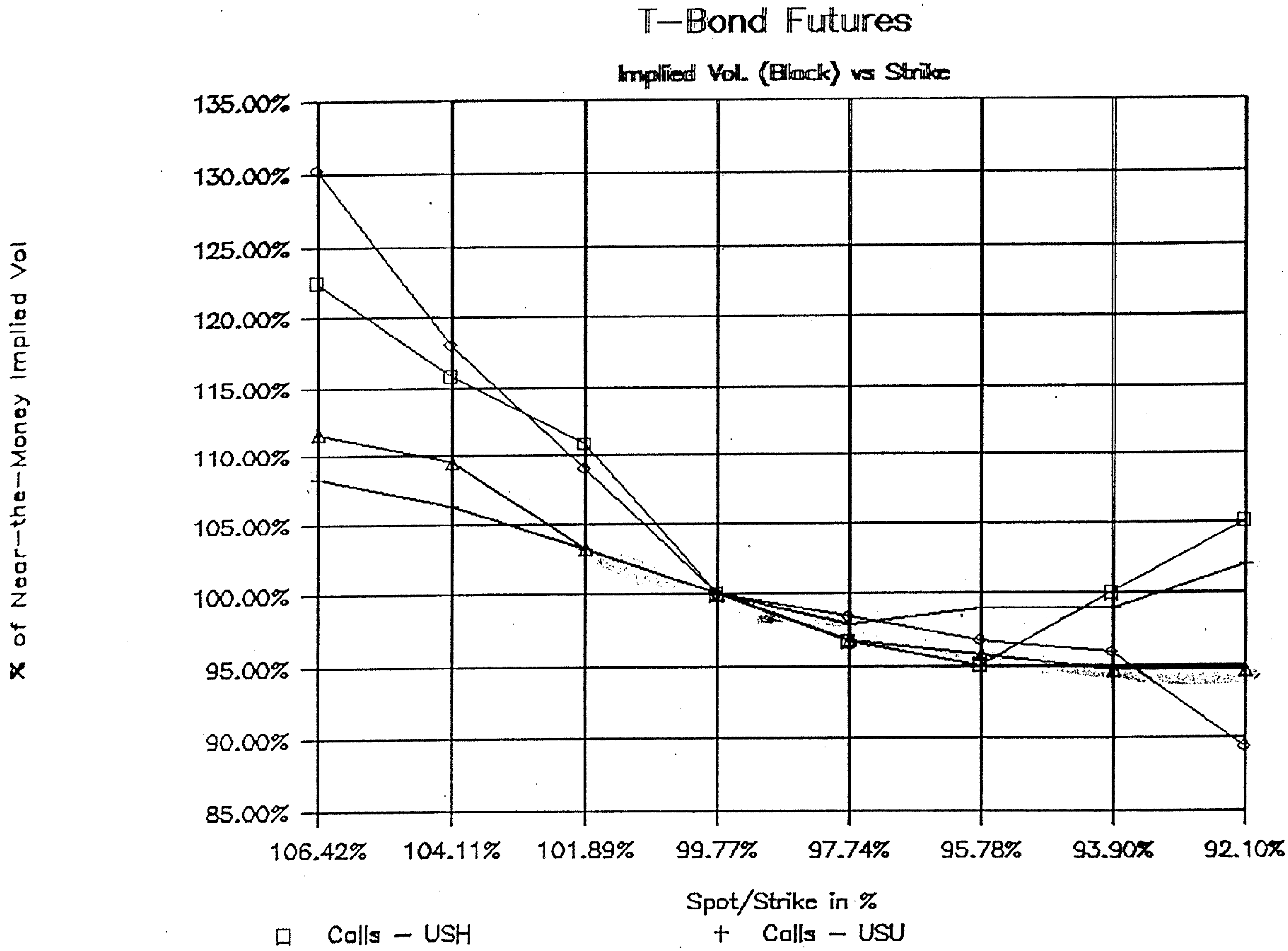

T—Bond Futures
Implied Vol. (Black) vs Strike
135.00%
130.00%
125.00%
120.00%
115.00%
110.00%
105.00%
100.00%
95.00%
90.00%
85.00%
% of Near—the—Money Implied Vol
106.42%
104.11%
101.89%
99.77%
97.74%
95.78%
93.90%
92.10%
Spot/Strike in %
□ Calls — USH
+ Calls — USU

FIGURE 7

(CME) DM FUTURES

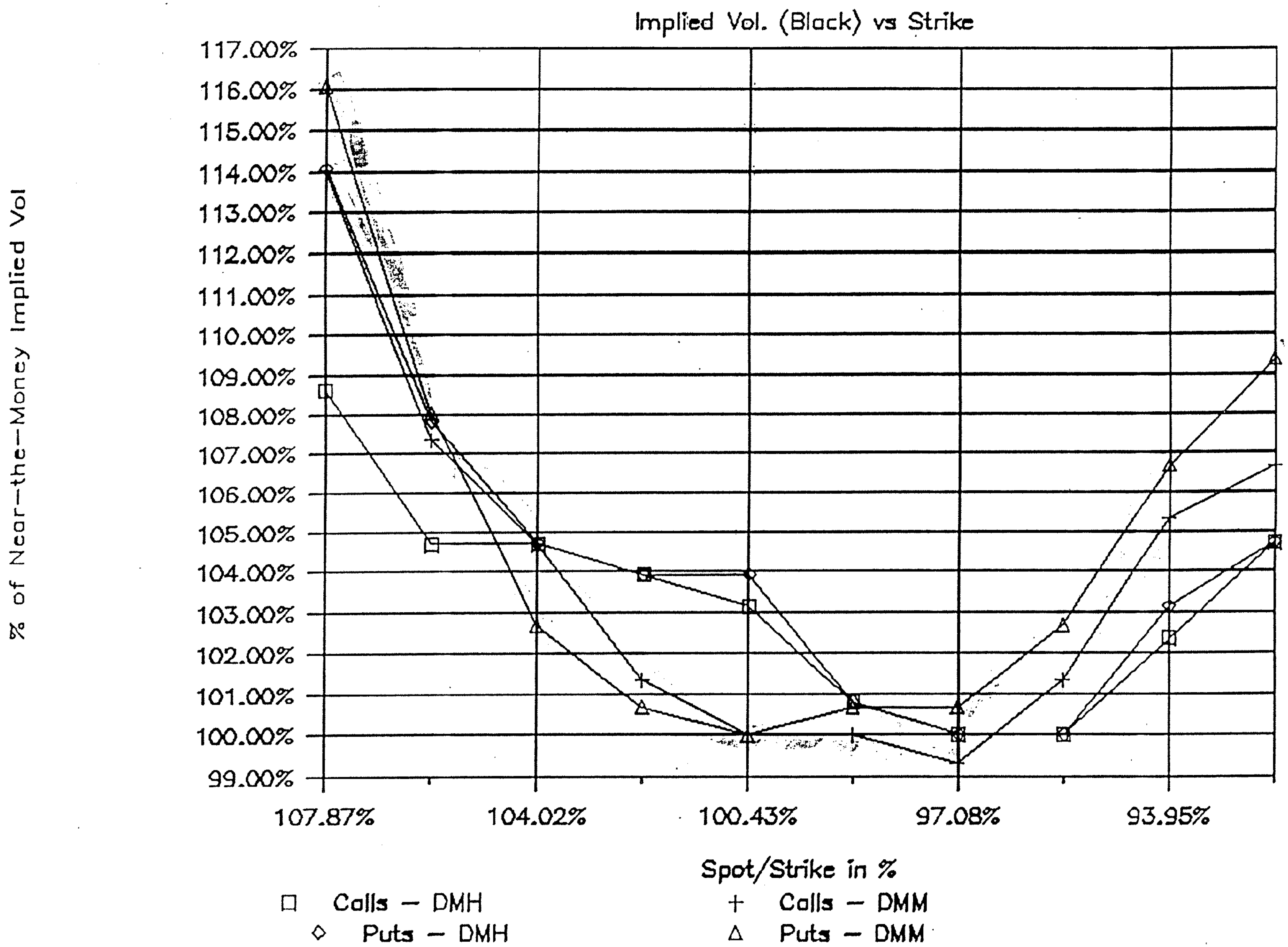


FIGURE 8

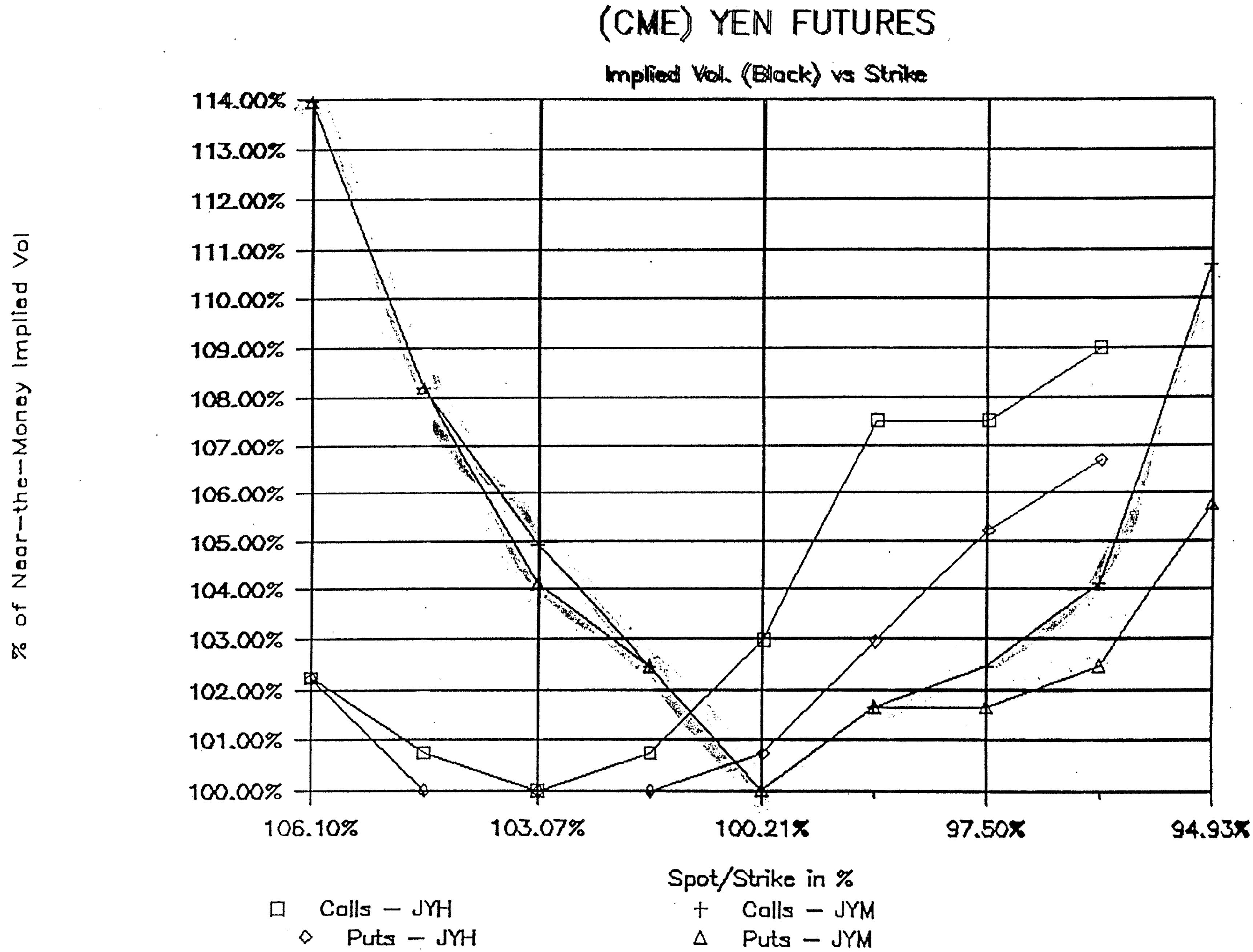

(CME) YEN FUTURES
Implied Vol. (Black) vs Strike
114.00%
113.00%
112.00%
111.00%
110.00%
109.00%
108.00%
107.00%
106.00%
105.00%
104.00%
103.00%
102.00%
101.00%
100.00%
% of Near-the-Money Implied Vol
106.10%
103.07%
100.21%
97.50%
94.93%
Spot/Strike in %
Calls — JYH
Puts — JYH
Calls — JYM
Puts — JYM

FIGURE 9

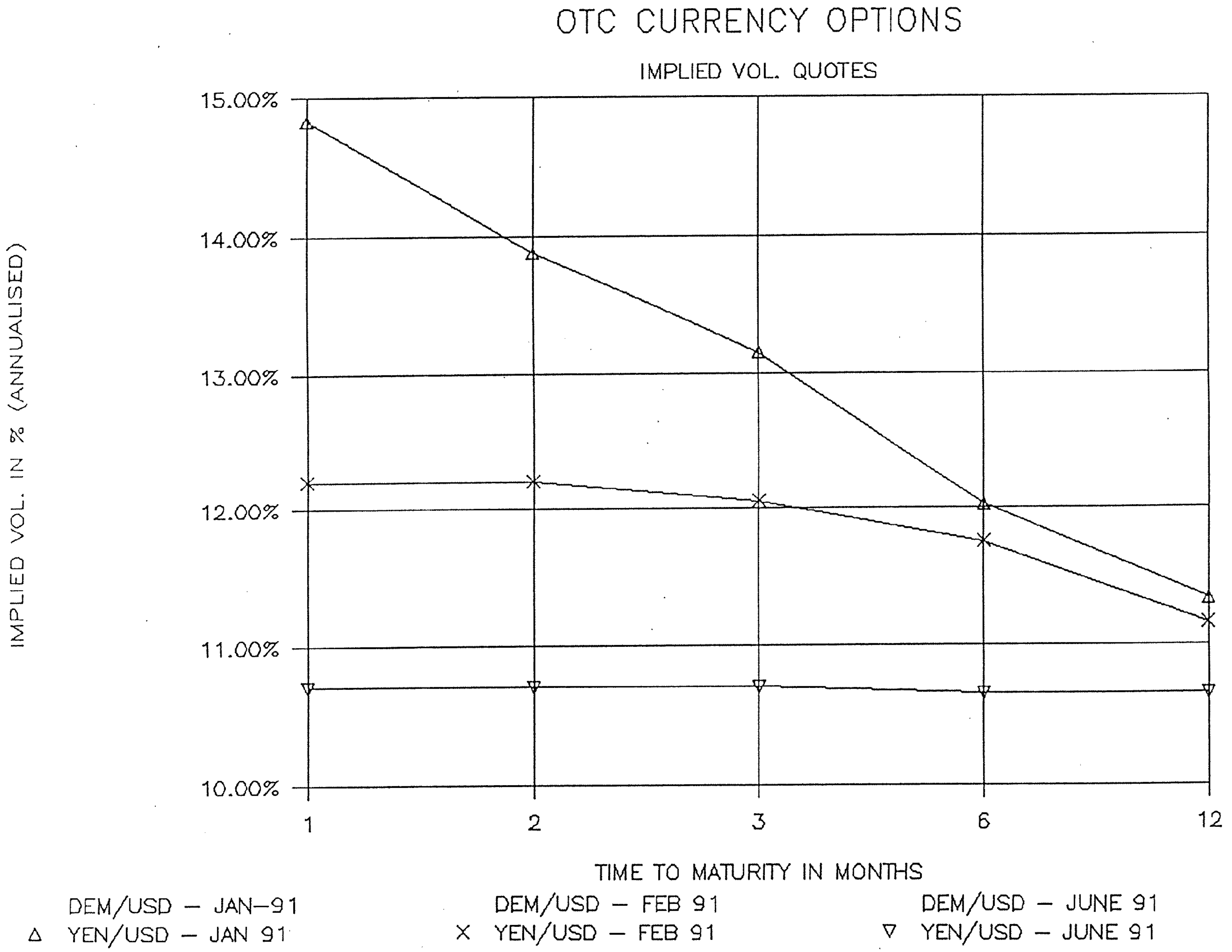

OTC CURRENCY OPTIONS
IMPLIED VOL. QUOTES
IMPLIED VOL. IN % (ANNUALISED)
15.00%
14.00%
13.00%
12.00%
11.00%
10.00%
1
2
3
6
12
TIME TO MATURITY IN MONTHS
DEM/USD – JAN–91
YEN/USD – JAN 91
DEM/USD – FEB 91
YEN/USD – FEB 91
DEM/USD – JUNE 91
YEN/USD – JUNE 91

FIGURE 10

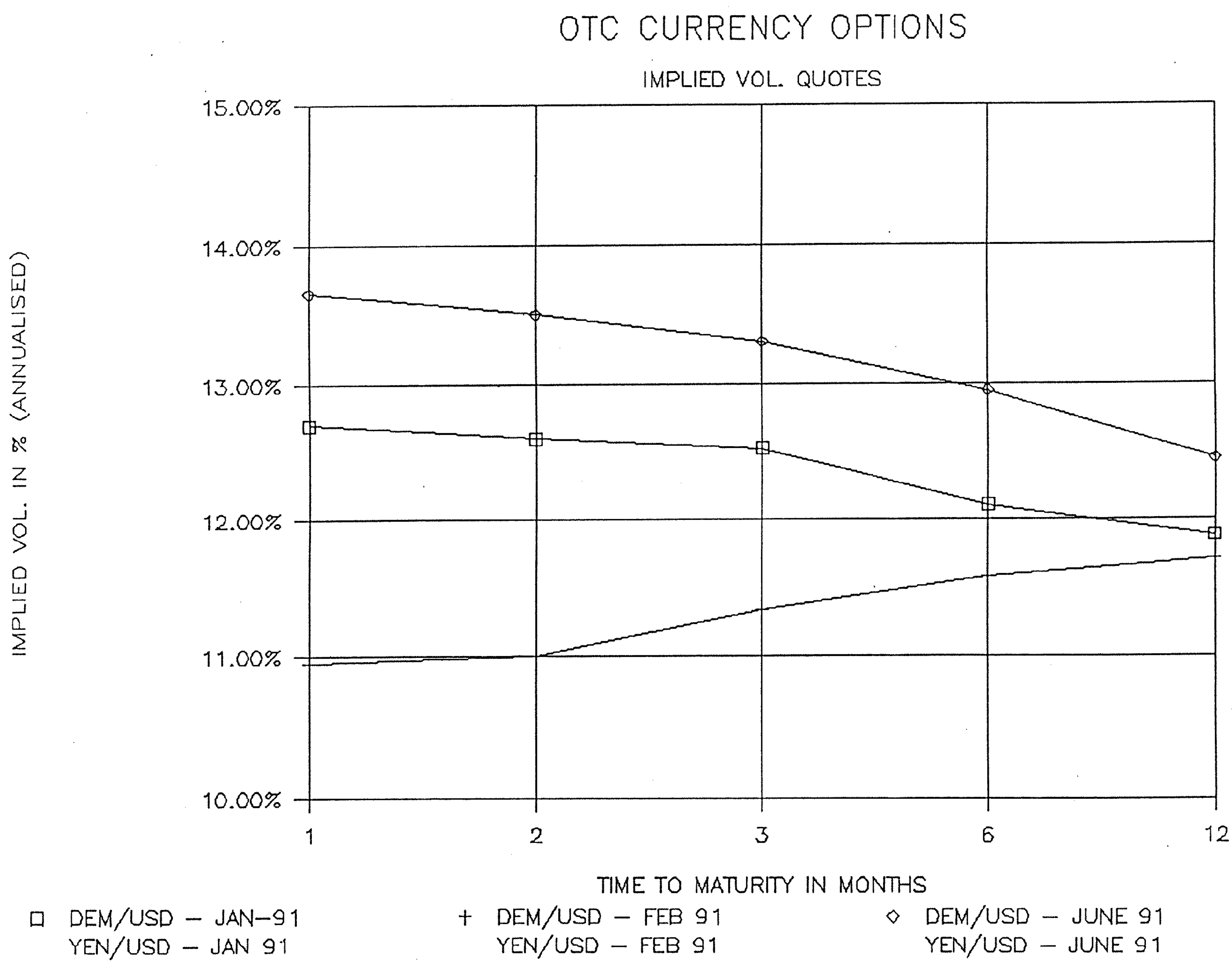
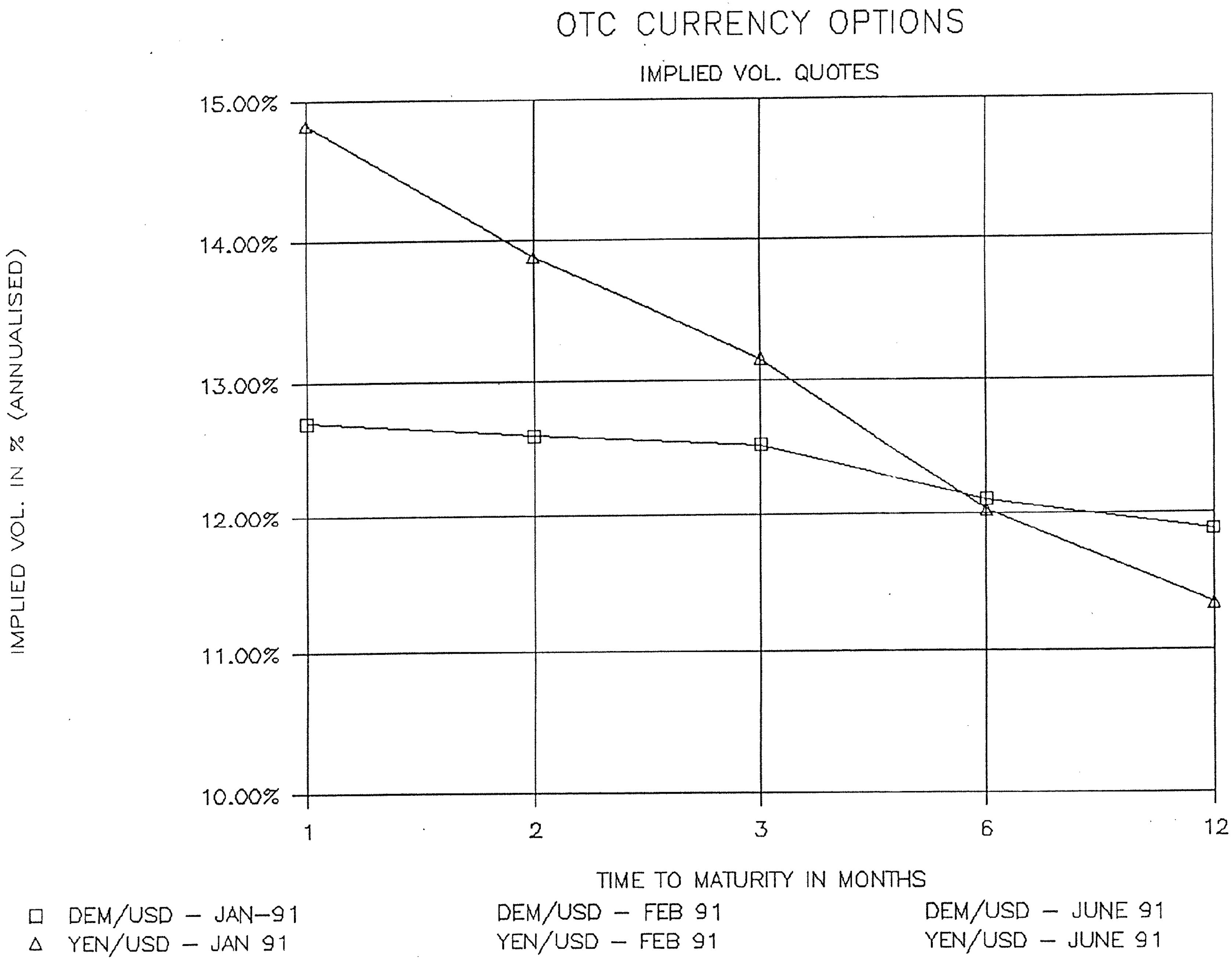

OTC CURRENCY OPTIONS
IMPLIED VOL. QUOTES
IMPLIED VOL. IN % (ANNUALISED)
15.00%
14.00%
13.00%
12.00%
11.00%
10.00%
1
2
3
6
12
TIME TO MATURITY IN MONTHS
□ DEM/USD – JAN–91
YEN/USD – JAN 91
+ DEM/USD – FEB 91
YEN/USD – FEB 91
◇ DEM/USD – JUNE 91
YEN/USD – JUNE 91

FIGURE 11


OTC CURRENCY OPTIONS
IMPLIED VOL. QUOTES
IMPLIED VOL. IN % (ANNUALISED)
15.00%
14.00%
13.00%
12.00%
11.00%
10.00%
1
2
3
6
12
TIME TO MATURITY IN MONTHS
DEM/USD – JAN–91
YEN/USD – JAN 91
DEM/USD – FEB 91
YEN/USD – FEB 91
DEM/USD – JUNE 91
YEN/USD – JUNE 91

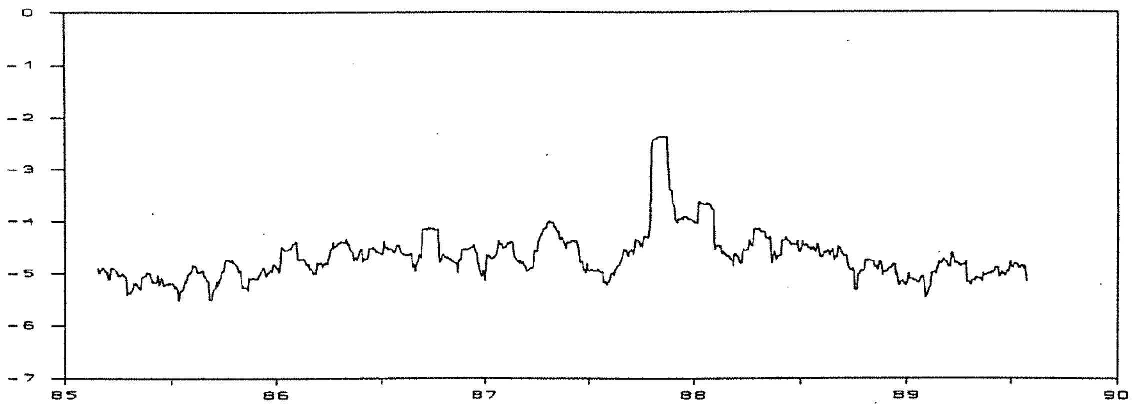


Figure 12 S&P: Log of Historical Volatility

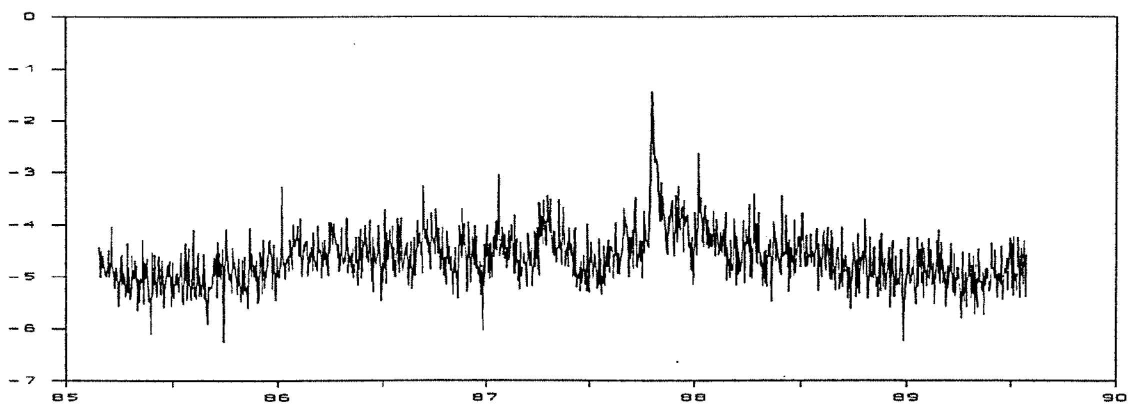


Figure 13 S&P: Log of High/Low Volatility

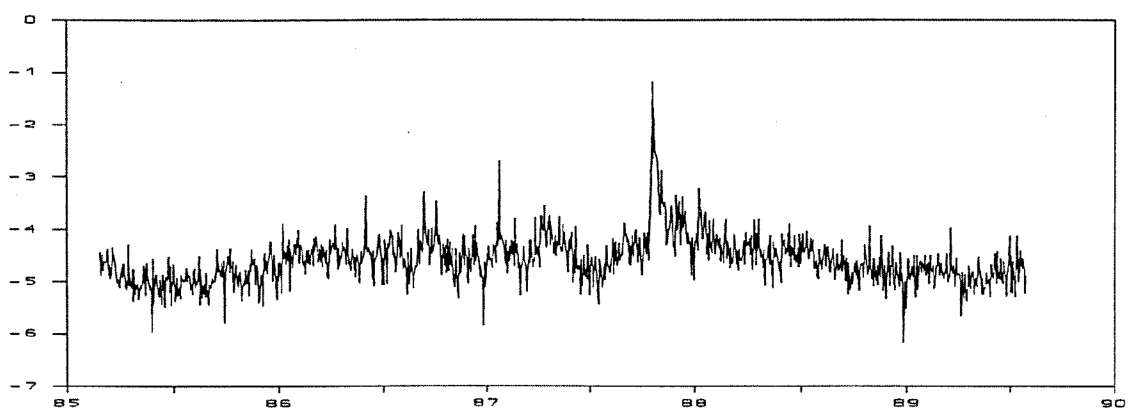


Figure 14 S&P: Log of 15 Minute Volatility

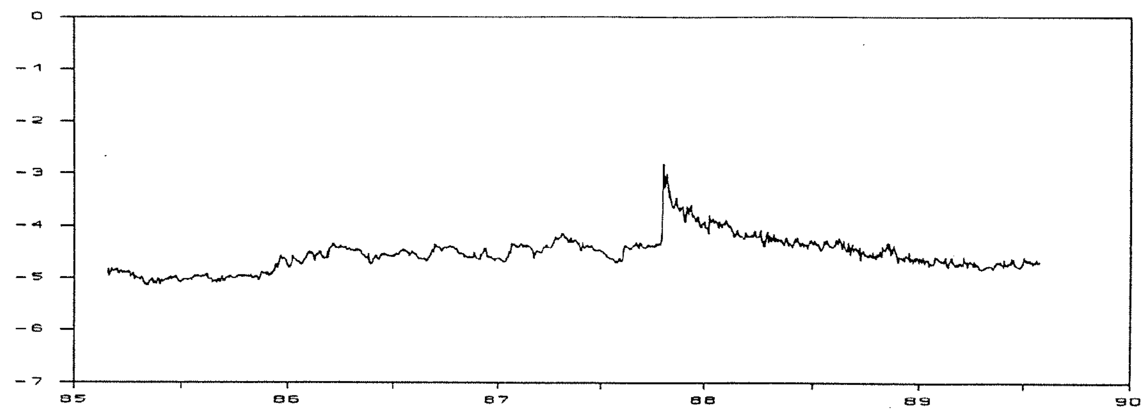


Figure 15 S&P: Log of Call Implied Volatility

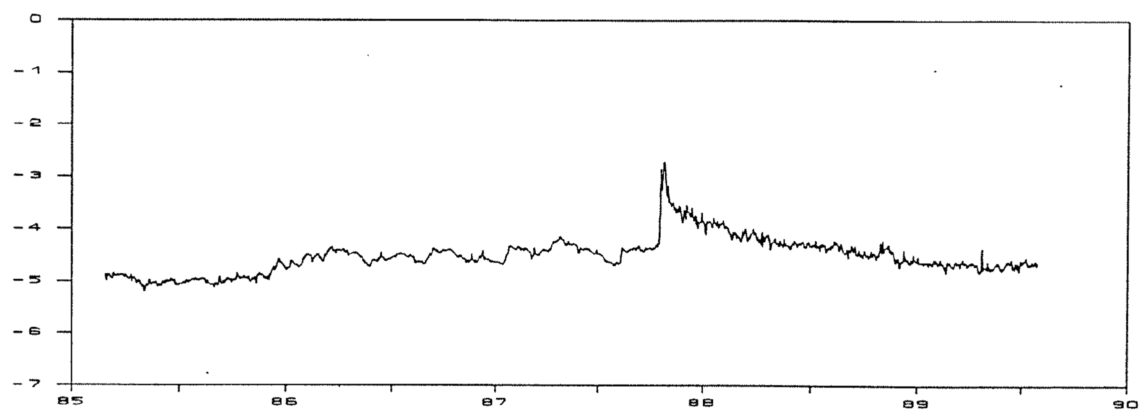


Figure 16 S&P: Log of Put Implied Volatility

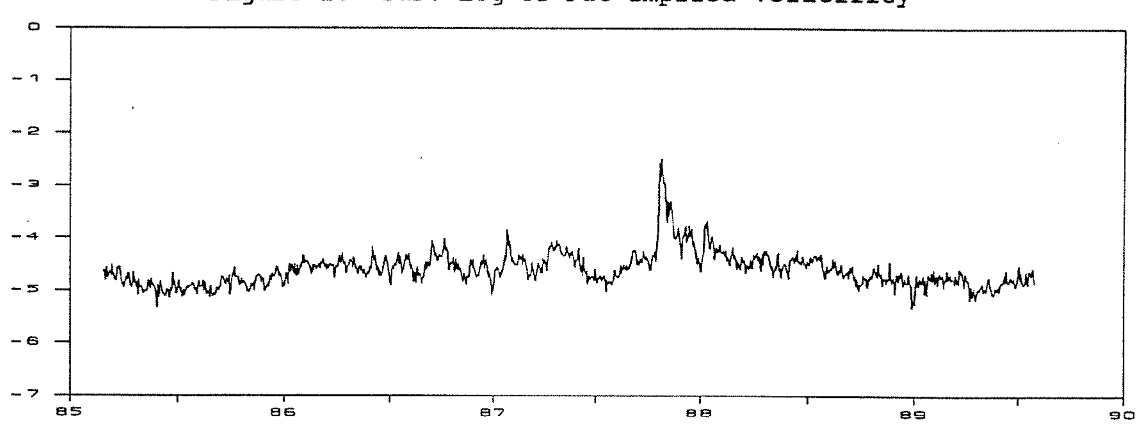


Figure 17 S&P: Prediction of Log 5 Min Volatility

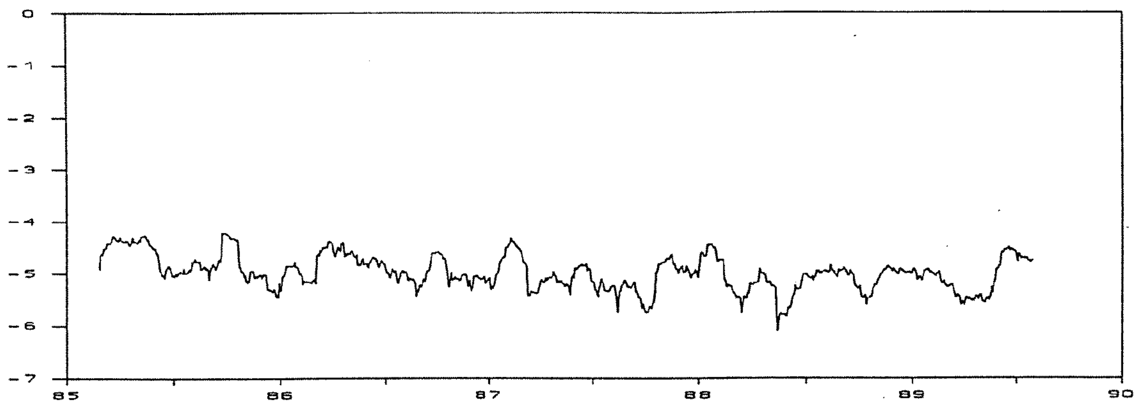


Figure 18 DM: Log of Historical Volatility

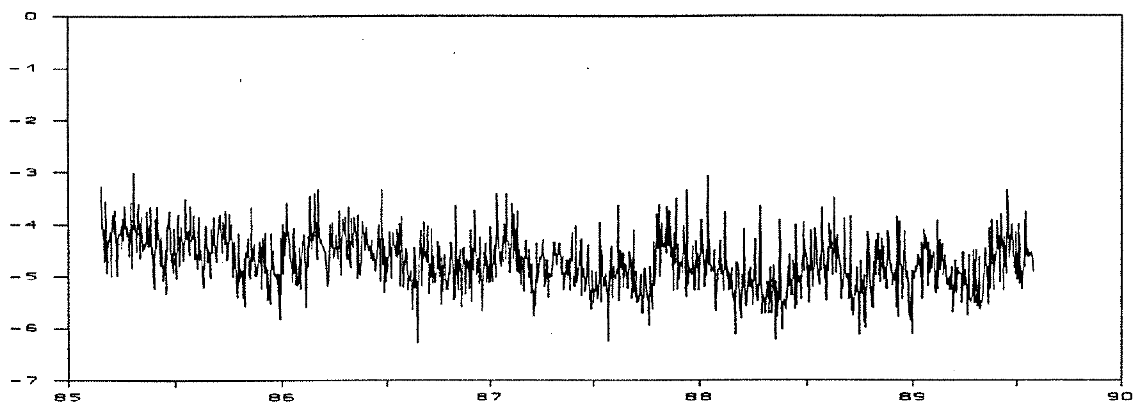


Figure 19 DM: Log of High/Low Volatility

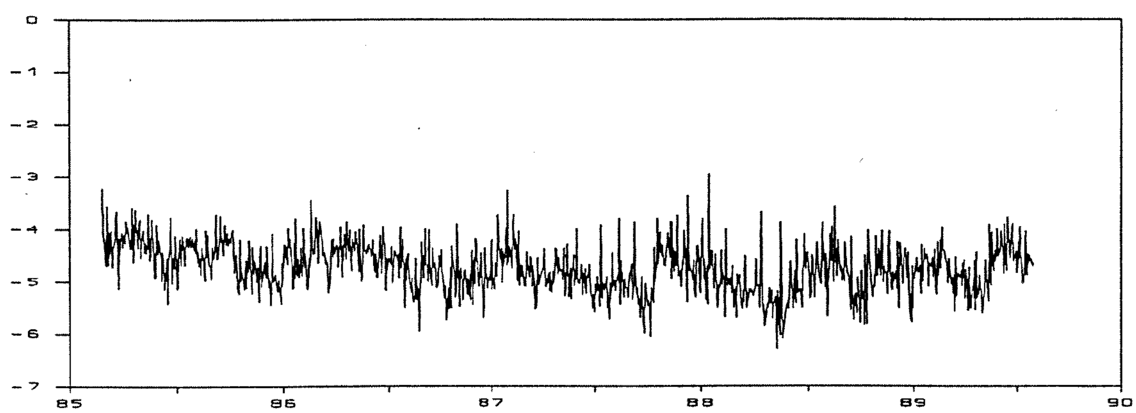


Figure 20 DM: Log of 15 Minute Volatility

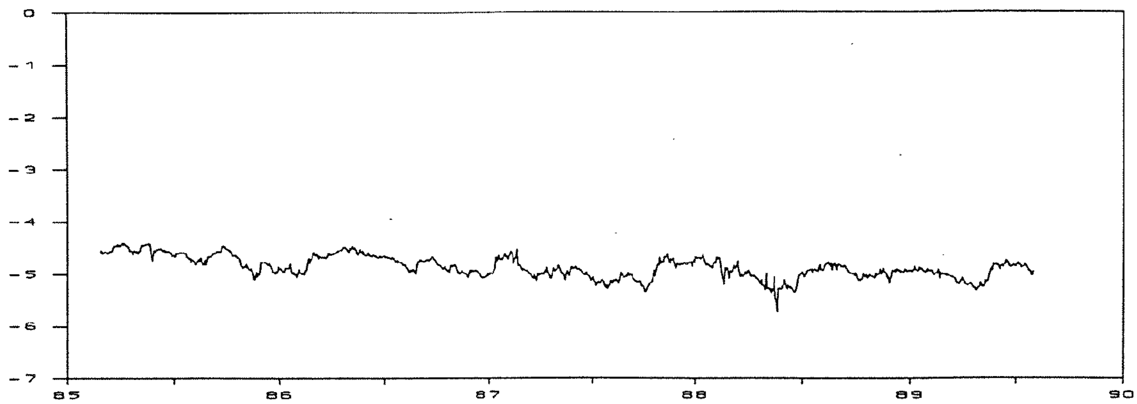


Figure 21 DM: Log of Call Implied Volatility

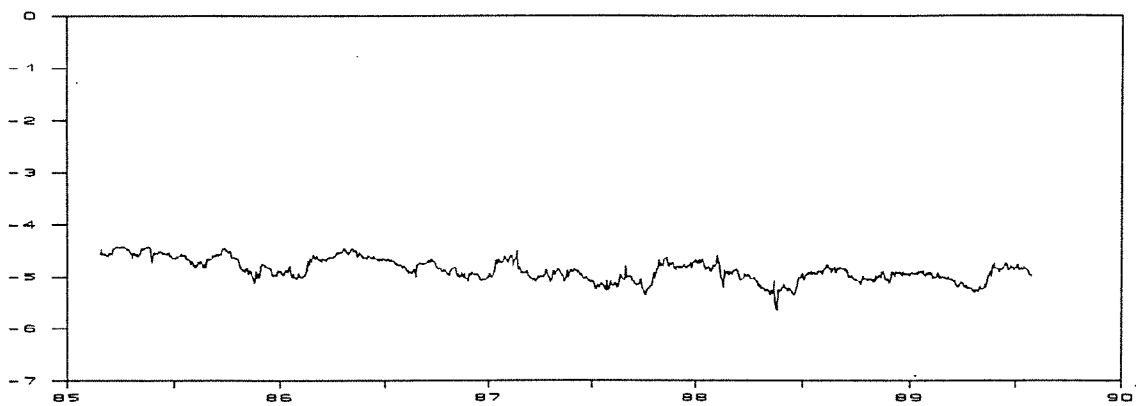


Figure 22 DM: Log of Put Implied Volatility

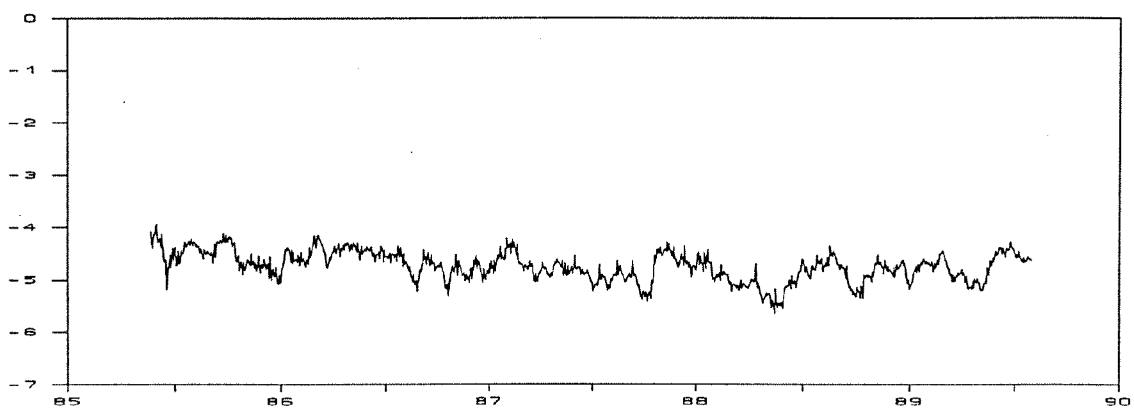


Figure 23 DM: Prediction of Log 5 Min Volatility

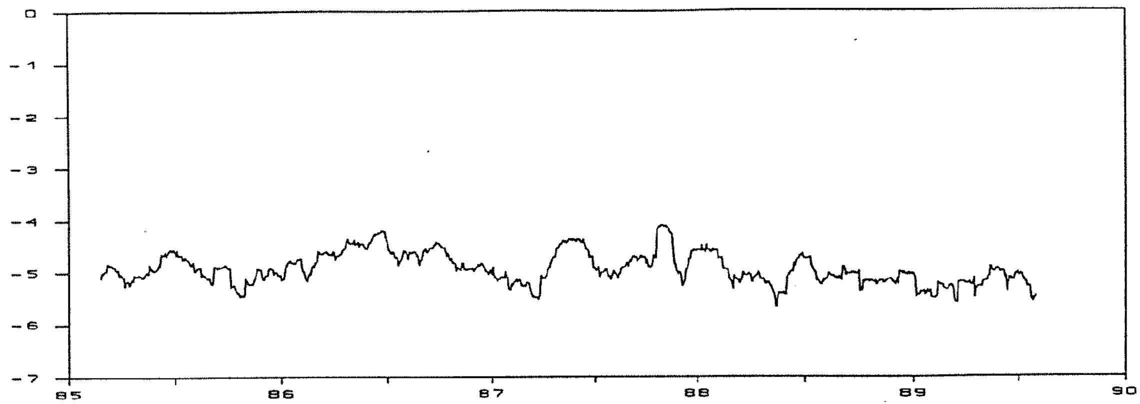


Figure 24 TBond: Log of Historical Volatility

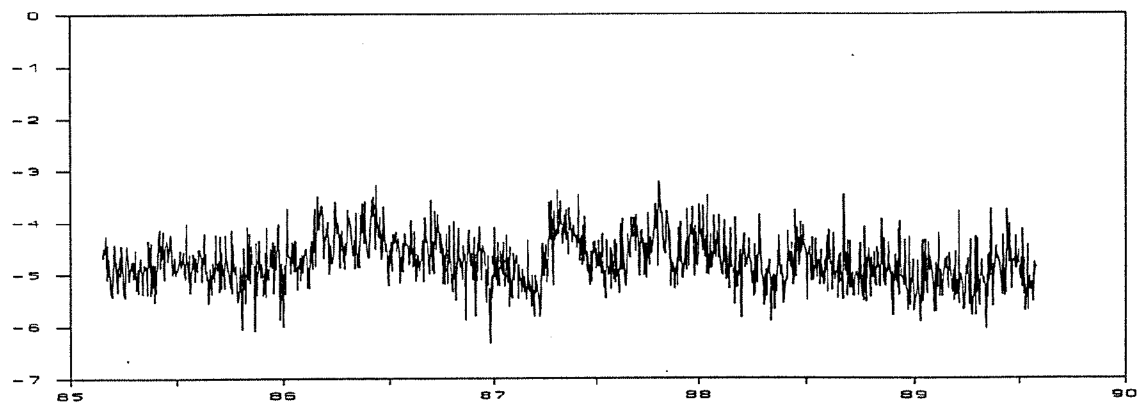


Figure 25 Tbond: Log of High/Low Volatility

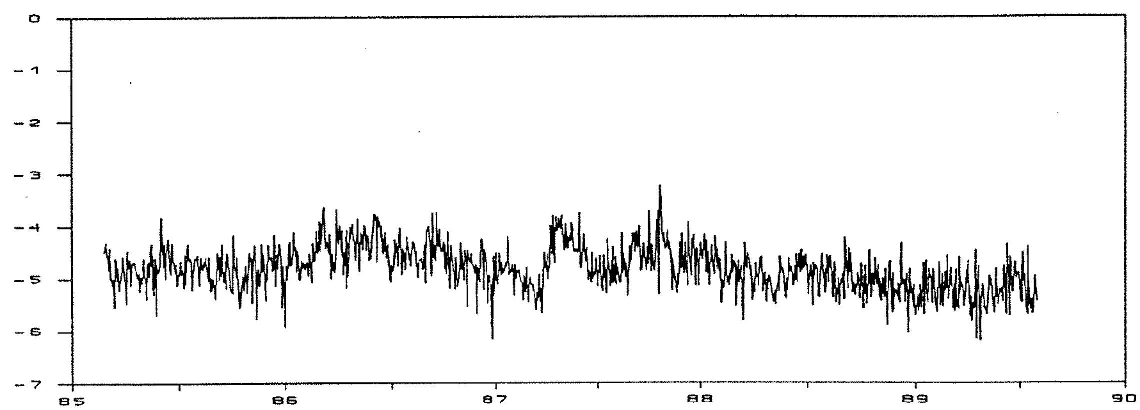


Figure 26 Tbond: Log of 15 Minute Volatility

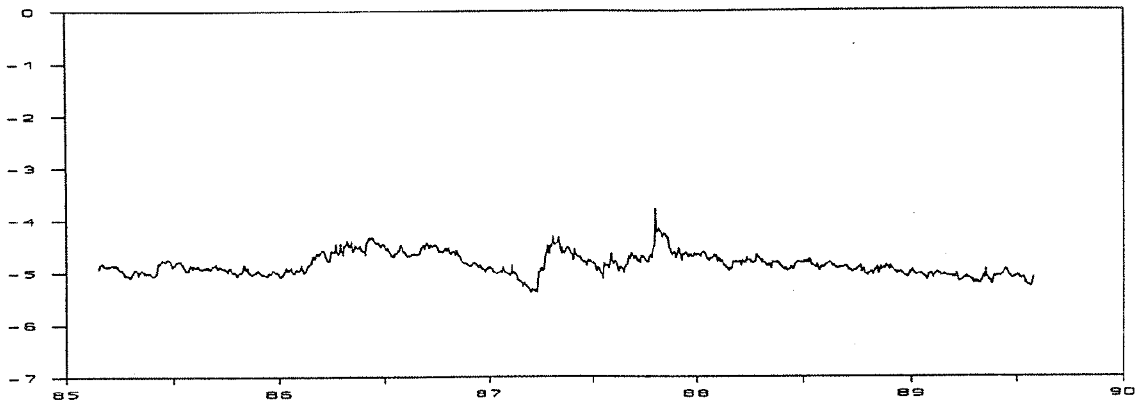


Figure 27 Tbond: Log of Call Implied Volatility

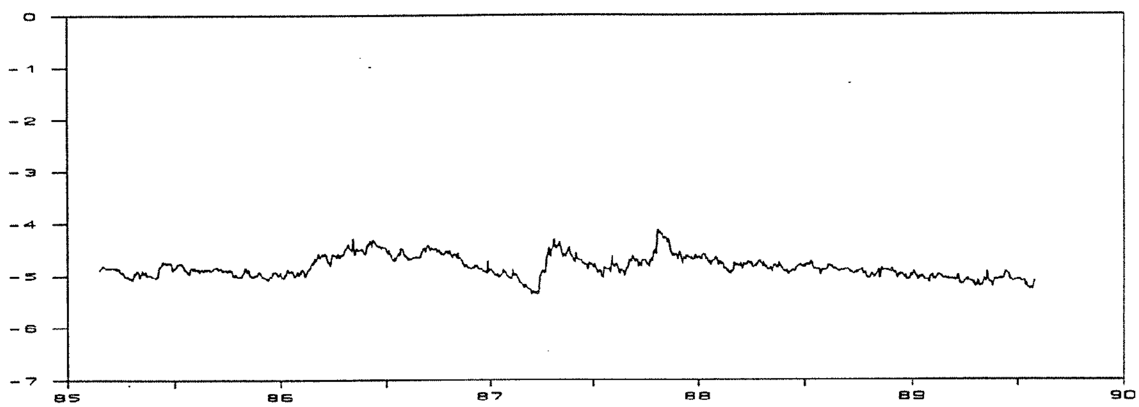


Figure 28 Tbond: Log of Put Implied Volatility

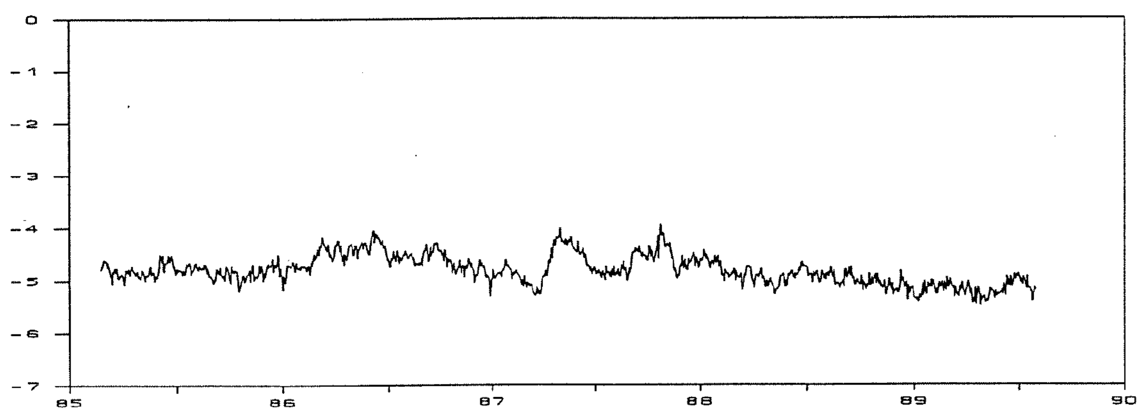


Figure 29 Tbond: Prediction of Log 5 Min Volatility