Implications of Nonlinear Dynamics for Financial Risk Management

by

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Abstract

This paper demonstrates that when log price changes are not IID, their conditional density may be more accurate than their unconditional density for describing short term behavior. Using the BDS test of independence and identical distribution, daily log price changes in four currency futures contracts are found to be not IID. While there appears to be no predictable conditional mean changes, conditional variances are predictable, and can be described by an autoregressive volatility model. Furthermore, this autoregressive volatility model seems to capture all the departures from independence and identical distribution.

Based on this model, daily log price changes are decomposed into a predictable part and an unpredictable part. The predictable part is described parametrically by the autoregressive volatility model. The unpredictable part can be modeled by an empirical density, either parametrically or nonparametrically. This two-step semi-nonparametric method yields a conditional density for daily log price changes, which has a number of uses in financial risk management.

In particular, one can directly calculate the capital requirement needed to cover losses of a futures position over one trading day. One can also use simulation methods to calculate the capital requirement over longer holding periods. This conditional density provides different, and probably more accurate, capital requirements than the unconditional density.

I. Introduction

A number of recent papers in the economics and finance literature have found strong evidence of nonlinear dynamics in short term movements of asset returns.[1] The next logical question is: What is the relevance of this finding? In the presence of any dynamics (whether linear or nonlinear), conditional densities can provide a better description of short term asset price movements than unconditional densities. This may be important for financial risk management, especially when highly leveraged instruments, such as futures contracts, are involved. For example, hedge ratios and the amount of capital needed to cover possible losses during the time a futures position is held depend critically on the probability distribution of changes in futures prices.

Traditional methods of estimating a probability density use a smoothed histogram of past price changes. This corresponds to the unconditional density. A conditional density can provide a more accurate assessment of price changes, since it uses more information. If the dynamics of asset prices were linear in nature, their conditional densities could be obtained in a straight forward manner. The empirical finding that the dynamics of asset prices are nonlinear, however, complicates substantially the estimation of their conditional densities. This paper illustrates how the conditional density can be estimated in a computationally efficient manner, and applies it to foreign currency futures.

The outline of the paper is as follows. Section II discusses the difference between conditional and unconditional densities. If price changes are independent and identically distributed (IID), then the two densities are the same. It is therefore important to test for independence and identical distribution. Section III introduces the Brock, Dechert, and Scheinkman (1987) test for independence and identical distribution, which is applied to four currency futures contracts traded in the Chicago Mercantile Exchange. It finds that price changes are not IID. In particular, there is strong evidence of conditional heteroskedasticity. Section IV describes a simple two-step semi-nonparametric method for approximating the conditional densities. Step one extracts the predictable parts of price changes parametrically. For the futures data, the conditional mean is approximately zero, but the conditional variance can be modelled by an autoregressive process. Step two describes the remaining unpredictable movements of price changes nonparametrically. Applications to futures trading are then provided in Section V. We show how to determine the capital needed to cover a given probability of losses over the next trading day. Using the estimated conditional density, the capital requirement changes with the conditional variance of futures price changes, while that based on the unconditional density is constant over time. We also show how to use simulation to determine the capital needed to cover a given probability of cumulative losses over a longer horizon. We find that the unconditional density can give time varying capital requirements, which may be more accurate than those from the unconditional density. Concluding remarks are offered in Section VI.

II.2. Conditional and Unconditional Densities

This section describes the difference between conditional and unconditional densities. To facilitate discussion, let P_t be the price of an asset at time t. Define $x_t = \log[P_t/P_{t-1}]$ as its continuously compounded rate of change. The unconditional density of x_t is obtained by fitting a density to the histogram of x_t, using either parametric or nonparametric methods. The conditional density of x_t given its own past values is obtained by postulating and estimating a complete probability model for the law of motion of x_t over time.[2] Usually the unconditional density is much simpler to estimate, particularly in the case of the parametric method, which explains its popularity in finance.[3] There are situations, however, when the conditional density gives a more accurate probability model of the short term behavior of x_t.

To highlight the differences between conditional and unconditional densities, consider the following example. Let x_t be a first order autoregressive process:

$$x_t = \alpha + \beta\ x_{t-1} + u_t,$$

where u_t is IID, normal, with mean 0 and variance σ, and $|\beta|<1$. The conditional distribution of x_t is normal, with mean $\alpha/[1-\beta]$ and variance $\sigma/[1-\beta^2]$, while the distribution of x_t conditional on x_{t-1} is normal, with mean $[\alpha+\beta x_{t-1}]$ and variance σ. The conditional and unconditional distributions are the same whenever $\beta=0$, i.e., x_t is IID. They are different whenever $\beta\neq0$, i.e., x_t is not IID.

The two distributions are related as follows. Suppose we only observe x_{t-2}. Then the conditional distribution of x_t given x_{t-2} is normal, with mean $[\alpha(1+\beta)+\beta^2 x_{t-2}]$ and variance $\sigma[1+\beta^2]$. By repeated substitution, we can show that the conditional distribution of x_t given x_{t-k} converges to the unconditional distribution as $k\to\infty$. In other words, the unconditional distribution describes the long run behavior of x_t, while the conditional distribution describes its short run behavior. In the first order autoregression, the conditional variance is always smaller than the unconditional variance. In general, however, the conditional variance can be larger or smaller than the unconditional variance.[4]

III. Test of Independence and Identical Distribution

We use the Brock, Dechert, and Scheinkman (1987) (BDS) test for independence and identical distribution. This test is chosen because it can detect many types of departures from independence and identical distribution, such as nonstationarity, nonlinearity, and deterministic chaos. Any of these departures from the IID case implies that the conditional distribution is different from the unconditional distribution. Furthermore, the BDS test can serve as a general model specification test, especially in the presence of nonlinear dynamics.[5]

The BDS test has been discussed in detail elsewhere.[6] We provide only a brief review. Let $\{x_t, t=1,...,T\}$ be a time series, and denote $x = (x_t, x_{t-1},...,x_{t-m+1})$ a point in the m-dimensional Euclidean space. Define the correlation integral $C_m(\delta)$ to be the fraction of pairs of points, x and x, which are within a distance δ of each other:

$$C_m(\delta) = \text{plim}_{T\to\infty} \#\{ (t,s), 0<t<T, 0<s<T: \max_{i=0,...,m-1} |x_{t-i}-x_{s-i}| < \delta \} / T^2.$$

For our purposes, we shall use the maximum norm, although the standard Euclidean norm is perfectly acceptable. If $\{x_t\}$ were IID, then $C_m(\delta) = [C_1(\delta)]^m$. Brock, Dechert, and Scheinkman (1987) construct a statistic for testing the null hypothesis that $C_m(\delta) = [C_1(\delta)]^m$.[7] They show that the test statistic is asymptotically a standard normal distribution. Brock, Hsieh, and LeBaron (1991) and Hsieh and LeBaron (1988) report extensive simulations and show that the asymptotic distribution is a good approximation of the finite sample distribution, when there are more than 500 observations. They recommend using δ between one-half to two times the standard deviation of the raw data. Also, the accuracy of the asymptotic distribution deteriorates for high imbedding dimensions, particularly when m is 10 or above.

Our data consist of daily settlement prices for four currency futures contracts traded on the Chicago Mercantile Exchange (CME): the British Pound (BP), Deutsche Mark (DM), Japanese Yen (JY), and Swiss Franc (SF), from February 22, 1985 to March 9, 1990, totaling 1275 observations per contract. The starting date corresponds to the time when daily price limits were removed. Currency futures expire four times per year. In order to obtain a continuous time series, the contracts were rolled over to the next expiration cycle 1 week prior to expiration.

It is appropriate to discuss why we choose to analyze currency futures prices instead of forward exchange rates, even though the forward exchange market is many times the size of the currency futures markets. The reason is that financial risk management is generally concerned with the market value of a futures or forward contract over its entire life. Unfortunately, daily forward exchange rates are typically given in fixed maturities of 1 month, 3 months, ... etc., which do not provide sufficient information. For this reason, we turn to currency futures, because futures exchanges provide information on daily movements of the futures price throughout the life of a futures contract. Cornell and Reinganum (1981) find that there is practically no difference between forwards and futures in the foreign exchange market. We can therefore use futures prices to construct a probability model, which can be applied to forward contracts as well.

Table 1 provides a statistical description of log price changes. The means are not statistically different from zero. The annualized standard deviations are 12.96%, 12.47%, 11.26%, and 13.82%, respectively, for the BP, DM, JY, and SF, assuming that each calendar year consists of 253 trading days. All four series have strong departures from normality, as the coefficients of skewness and kurtosis are statistically different from those of a normal distribution. The BDS statistic for testing independence and identical distribution are provided in Table 1, for imbedding dimensions (m) 2 through 5, and distances (δ) 0.5, 1.0, 1.5, and 2.0 times the standard deviation of the raw data. If we use the 1% marginal significance level, we will reject independence and identical distribution in all 16 statistics for the BP and the JY, 11 of the 16 for the DM, and 4 of the 16 for the SF. Even though the BDS statistics for each currency futures are not independent, they show strong evidence of departure of independence and identical distribution for at least 3 currency futures. This is consistent with similar findings in the spot currency markets, as in Hsieh (1989).

As the BDS test is sensitive to any departure from independence and identical distribution, it is useful to know the cause of the rejection. Table 2 provides some information. It shows that the autocorrelation coefficients of log price changes are not statistically different from zero, either individually or jointly (using the Box-Pierce statistic for the first 15 lags).[8] On the other hand, the autocorrelation coefficients of the absolute values of log price changes are much larger. More than half of them are statistically different from zero, and the joint test using the Box-Pierce statistic rejects the hypothesis that the first 15 lags are zero. This evidence is consistent with the hypothesis that the rejection of independence and identical distribution is not due to linear, but rather nonlinear, dependence in exchange rates.[9]

The rejection of independence and identical distribution implies that the conditional density differs from the unconditional density in describing short term dynamics of futures prices. Furthermore, the presence of nonlinear dependence implies that linear (e.g., Box-Jenkins) methods cannot be used to model the conditional density. This motivates the goal in the rest of this paper, namely, to obtain a useable form of the conditional density which takes into account the nonlinear dependence, and to provide some interesting applications.

IV. A Two-step Method for Estimating Conditional Densities

In theory, the conditional density can be estimated nonparametrically, for example, using kernels, splines, neural networks, or series expansions. In practice, nonparametric methods have two drawbacks. They require substantial computational time, and little is known about the sample sizes required for accurate estimation. In this paper, we try a different approach.

Our approach to estimating conditional density is essentially a two-step semi-nonparametric approach. Step one estimates the predictable part of the data parametrically. Step two estimates the remaining unpredictable part nonparametrically. Speed of computation is the primary motivation for doing this in two steps, rather than jointly estimating the parametric and nonparametric parts.[10]

The parametric part in step one deals with the conditional mean and conditional variance of x_t, defined as:

$$\mu_t = E[x_t | x_{t-1}, x_{t-2}, \ldots],$$

$$\sigma_t = V[x_t | x_{t-1}, x_{t-2}, \ldots] .$$

For the four currency futures, we will show, below, that the conditional mean is zero and that the conditional variance is time-varying and depends nonlinearly on past realizations of x_t. In addition, we demonstrate that the conditional variance captures most of the predictability of price changes, using the BDS statistic. Therefore, we model as IID the unpredictable part:

$$z_t = [x_t - \mu_t]/\sigma_t .$$

The nonparametric part in step two deals with the density of z_t.

It is important to note that not all nonlinear dependence can be modelled in this way. This method is not appropriate when, for example, there is dependence in higher order moments. Careful diagnostics are therefore needed.

IV.A Estimating the Conditional Mean Function

We now proceed to characterize the conditional mean function of price changes given its own past, which is defined as:

$$\mu_t = E[x_t | x_{t-1}, x_{t-2}, \ldots] = f(x_{t-1}, x_{t-2}, \ldots).$$

Operationally, this means that:

$$E[\, x_t - f(x_{t-1}, x_{t-2}, \ldots) \mid x_{t-1}, x_{t-2}, \ldots \,] = 0.$$

Based on the findings on spot currencies that the conditional mean is zero, we argue that the same is true for currency futures.

Hsieh (1989) proposes a test of the null hypothesis that the conditional mean function is zero. The test makes use of the fact that, if the conditional mean of x_t is zero, then its bicorrelation coefficients, $E[x_t x_{t-i} x_{t-j}]/V[x_t]^{3/2}$, are zero for $i,j \geq 1$.[11] Table 3 provides the estimated bicorrelation coefficients up to the fifth lag. None of them, either individually or jointly, are statistically different from zero. While the bicorrelation test results are consistent with the null hypothesis of a zero conditional mean function, Pemberton and Tong (1981) point out that there exist nonlinear models with zero bicorrelation coefficients and nonzero conditional means. To deal with these types of models, we turn to a second approach, using nonparametric methods to directly estimate the conditional mean function.

Suppose x_t is generated by the following model:

$$x_t = g(x_{t-1}, x_{t-2}, ...) + \varepsilon_t,$$

where ε_t is IID.[12] If g() is sufficiently well behaved, Stone (1977) shows that nonparametric regression methods can be used to estimate g() consistently. There are many ways to implement nonparametric regressions.

Diebold and Nason (1990) and Meese and Rose (1990) use the method of locally weighted regression (LWR) in Cleveland and Devlin (1988).[13] Briefly, LWR can be illustrated in the following way. Suppose we believe that the conditional mean function g() includes only x_{t-1}.[14] LWR looks at the history of x_t, finds those instances when x_{t-i-1} is close to x_{t-i} by choosing the nearest k neighbors of x_{t-1}, and runs a weighted regression of x_{t-i} on x_{t-i-1} by giving more weights to closer neighbors. This gives a local estimate of g() around the point x_{t-1}. We can use this local estimate to forecast x_t by evaluating it at x_{t-1}.

There are a number of choices to make in this forecasting exercise. One, the number of nearest neighbors k. We try 10% of all available history, up to 90%, in steps of 10%. Two, the number of lags of x_{t-1} to include as arguments of g(). We use lags 1 through 5. Three, the weighting scheme of the local regression. We use the tricubic weights proposed in Cleveland and Devlin (1988). Four, the length of the out-of-sample forecast. We choose the last third of our sample.

Table 4 provides the ratio of the root mean squared errors (RMSE) of the LWR forecasts to that of a random walk model of futures prices (where the predicted x_t is zero). For each currency, there are 45 different RMSEs, corresponding to the 5 choices of lag lengths and 9 sizes of nearest neighborhoods. A ratio larger than 1 indicates that the RMSE of the LWR is higher than that of the random walk model. In the BP, JY, and SF, LWR performed worse than the random walk model. In the DM, 3 of the 45 LWR forecasts beat the random walk, but the improvement is less than half a percent. These results are consistent with those in Diebold and Nason (1990) and Meese and Rose (1990), and indicate that there is little evidence of a nonzero conditional mean in price changes in currency futures. [15]

IV.B Estimating the Conditional Variance Function

While the conditional mean is statistically not different from zero, the large autocorrelations of the absolute values of price changes suggest that the conditional variance is time varying. The difficulty in modeling the conditional variance is that it is never observed directly. In this paper, we take two different approaches.

The first approach is motivated by the popularity of the autoregressive conditional heteroskedasticity models of Engle (1982), Bollerslev (1986), and Nelson (1991). See Bollerslev, Chow, and Kroner (1990) for a survey. We have selected Nelson's (1991) EGARCH model, which is given by:

$$X_t = \mu + h\eta_t,$$

$$\eta_t | \Omega_{t-1} \sim N(0,1),$$

$$\log h_t = \alpha + \beta \log h_{t-1} + \phi \left[|\eta_{t-1}| - (2/\pi) \right] + \gamma \eta_{t-1},$$

where Ω_{t-1} is the information set at time t-1.[16] Since h_t is known at t-1, it is included in Ω_{t-1}. The EGARCH model is chosen over Engle's (1982) ARCH or Bollerslev's (1986) GARCH models for two reasons: (a) EGARCH allows the conditional variance to respond differently to a decline versus an advance (by allowing γ to be different from zero) while ARCH and GARCH impose a symmetric response; and (b) unlike ARCH and GARCH, EGARCH does not need to impose any constraints on the coefficients of the variance equation to enforce nonnegativity of the variance. This makes estimation much simpler.

We use the Berndt, Hall, Hall, and Hausman (1974) estimation procedure, and the results are given in Table 5. Firstly, the estimates of β are all statistically greater than zero. In fact, those for the BP, DM, and SF are very close to 1, which indicates that volatility is highly persistent in currency futures. The much smaller value of β for the JY indicates that its volatility is less persistent. However, the estimates of β for all 4 currency futures are smaller than 1, which means that the distribution is strictly stationary. Secondly, there appears to be no asymmetry in the variance equation, since the estimates of γ are not statistically different from zero. These results are similar to those found in spot exchange rates.

While the EGARCH model can be justified on the grounds that it can approximate variance changes,[17] our main interest is to see if it can capture all the nonlinear dependence in price changes. If it does, we can proceed to the second step of our semi-nonparametric procedure. This can be tested as follows. Let $\hat{h}_t$ and denote the fitted values of h_t and μ in the EGARCH model. We want to test whether the remaining movements in price changes, called standardized residuals,

$$\hat{\ }_t = [x_t -]/\hat{h},$$

are IID. This can be done by running the BDS tests on the standardized residuals.

The results are reported in Table 6. There is one important caveat here. We cannot use the asymptotic distribution of the BDS test, as Brock, Hsieh, and LeBaron (1991) show that the BDS test is biased in favor of the null hypothesis of independence and identical distribution when applied to standardized residuals of EGARCH models. Therefore, we use simulated critical values of the BDS test which are provided in Table 6. They indicate that the standardized residuals still reject independence and identical distribution for the DM and the SF, which means that the EGARCH model cannot capture all the nonlinear dependence in those two currency futures.

We now turn to the second approach to modeling volatility. The idea is to construct a daily measure of volatility using intraday futures data, which then allows us to fit a time series model of volatility. As in Kupiec (1990), our daily measure of volatility is the Parkinson (1980) range estimator of the standard deviation:

$$\sigma_{P,t} = [.361 \times 1440 / M] \log[High_t / Low_t],$$

where $High_t$ and Low_t are the high and low transaction prices during each trading day, and M is the number of minutes during a trading day.[18] We should point out that the standard deviation of the trading day in the CME, which is approximately 400 minutes at the end of our sample, is scaled up to a trading day of 24 hours (i.e. 1440 minutes). While this particular scaling is motivated by the fact that the foreign exchange market is open around-the-clock, any scaling factor is innocuous, as the second step of our semi-nonparametric method will provide the appropriate scaling factor.

It is important to stress here that $\sigma_{P,t}$ is the *ex post* measure of volatility, while we are mainly interested in the *ex ante*, i.e., conditional, forecast of volatility. To obtain a conditional forecast of volatility, we posit the following model for log price changes, x_t:

$$x_t = \sigma_{P,t} u_t,$$

$$\log \sigma_{P,t} = \alpha + \Sigma \beta_i \log \sigma_{P,t-i} + \nu_t,$$

where ν_t is IID. We call this the autoregressive volatility model. It is motivated by the fact that $\sigma_{P,t}$ is autocorrelated.[19] We can recover the *ex ante* volatility, as follows. Regress log $\sigma_{P,t}$ on its own lags and a constant term using ordinary least squares. For simplicity, we call this the "autoregressive volatility" model. The number of lags of log $\sigma_{P,t}$ is determined by the Schwarz (1978) criterion: 8 for the BP, 8 for the DM, 5 for the JY, and 8 for the SF. The estimates are given in Table 7. The persistence of volatility is measured by the sum of the β coefficients, which are .782 for the BP, .760 for the DM, .624 for the JY, and .736 for the SF.

They are statistically less than 1 in all 4 cases, indicating that log volatility is strictly stationary. When compared to the EGARCH model, the autoregressive volatility model has much less persistence for the BP, DM, and SF. This will have an impact on our simulations in Section V.

As the issue of volatility persistence is important in the distinction between the autoregressive volatility and the EGARCH model, we perform several tests of sensitivity and misspecification of the autoregressive volatility model. Firstly, we include the lagged values of log volatility up until the 20th lag. This does not change our results substantially. In particular, the sums of the β parameters increase slightly, to .844 (BP), .793 (DM), .675 (JY), and .779 (SF). But they are still statistically less than 1, as the F(1,1234) statistics are 15.36 (BP), 21.69 (DM), 38.27 (JY), and 27.43 (SF).

Secondly, we add day-of-the-week dummy variables to the model, since the literature has found these to be statistically important in variance changes.[20] While most of the day-of-the-week dummies are statistically different from zero, they add little to the explanatory power. The ^{2}'s improve only marginally, rising to .284 (from .274), .242 from (.227), .181 (from .170), and .208 (from .193), respectively, in the BP, DM, JY, and SF. In addition, the Schwarz criterion worsens in three of the four currencies. These dummies also did not change the amount of volatility persistence. The sum of the β coefficients are .785 (BP), .765 (DM), .627 (JY), and .740 (SF), which are essentially the same as those without the dummies and remain statistically different from unity in all 4 cases. Thus, day-of-the-week dummies are excluded from the final model. Thirdly, we include lags of the log volatilities of the other currencies. They did not appear to be statistically significant, and the Schwarz criterion worsens in all cases. Hence we kept the specification as reported in Table 7.

To ensure that the autoregressive volatility model can capture all the predictability in currency futures, we run the BDS test on the standardized residuals:

$$z_t = x_t/{}_{P,t},$$

where ${}_{P,t}$ is the fitted value from the autoregressive volatility model. The results are reported in Table 8. Critical values of the BDS statistics are obtained through simulation, as done in Table 6. We find little evidence against the hypothesis that the standardized residuals are IID. For the BP, DM and JY, there are no rejections of the null, while for the SF, 3 of the 16 statistics reject the null. Note that this rate of rejection is much lower than that of the EGARCH model in Table 6.

IV.C Estimating the Density of the Unpredictable Part of Futures Price Changes

The tests indicate that the autoregressive volatility model is appropriately specified, and appears to have captured the predictable movements in exchange rates. We now proceed to the second step of our semi-nonparametric method, which involves modeling $z_t = x_t/{}_{P,t}$, the unpredictable part of log price changes.

Table 8 provides some information about z_t. The mean is close to zero. The standard deviation is close to unity. There is little evidence of skewness, but strong evidence of leptokurtosis. Using the BDS test, we have already shown that z_t is IID, so we can estimate its unconditional density using standard methods. For example, we can fit a parametric density function, such as the Student-t. Or we can fit a nonparametric density, using kernels or series expansions.

For the purposes of our applications in Section V, we actually do not need to estimate the density of z_t at all. Section V.A requires only the quantiles of z_t, which are provided in Table 8. In Section V.B, we simulate future values of x_t by "bootstrapping" from z_t, as per Efron (1982). We now turn to these applications.

V. Application to Risk Management: Minimum Capital Requirements

There are many uses of the conditional density of price changes. In this section, we calculate the minimum capital requirement of a futures position. First, we show that there is a direct method to obtain daily minimum capital requirements. Next, we show that we can obtain longer term minimum capital requirements, via simulation.

V.A Daily Minimum Capital Requirements

Suppose a firm holds a long position of L_t units of a foreign currency futures contract. An important question in risk management is: What is the minimum capital, K_t, needed to cover losses of this long position with a 99.5% probability? The minimum capital is the sum of prearranged lines of credit and short term liquid instruments which can be converted to cash almost instantaneously, e.g., Treasury bills, negotiable certificates of deposits, money market funds, interest bearing checking accounts, etc. We note that we have selected 99.5% as the "coverage probability" purely for illustrative purposes.[21]

The capital requirement, K_t, is determined as follows. Let P_{t+1} be the settlement price in the following trading day. The losses of the long position are given by $(P_t-P_{t+1})L_t$. Thus, we want K_t to solve the following equation:

$$\Pr\{ (P_t-P_{t+1}) L_t > K_t \} = .005.$$

The left hand side can be rewritten as follows:

$$\Pr\{ \log[1-\kappa_t]/\sigma_{t+1|t} > z_{t+1} \} = .005.$$

where $\kappa_t = K_t/[P_tL_t]$, $z_{t+1} = x_{t+1}/\sigma_{t+1|t}$, $x_{t+1} = \log[P_{t+1}/P_t]$, and $\sigma_{t+1|t}$ denotes exp{ $E_t[\log\sigma_{t+1}]$ }. In particular, we can use a rolling regression method to sequentially generate $\sigma_{t+1|t}$. The minimum capital is now expressed as a fraction of P_tL_t.

To solve for κ_t, we only need to know the quantiles of the distribution of z_{t+1}, which are provided in Table 8. The quantile z_ℓ is the point where:

$$\Pr\{ z_\ell > z_{t+1} \} = .005.$$

In particular, z_ℓ is -3.017 for the BP, -2.399 for the DM, -3.623 for the JY, and -2.306 for the SF. For each currency, then, the minimum capital as a fraction of the market value of the long position is:

$$\kappa_t = 1-\exp(\sigma_{t+1|t}\ z_\ell).$$

Since z_ℓ is a negative number, an increase in $\sigma_{t+1|t}$ will increase the capital requirement.

As $\sigma_{t+1|t}$ is time varying, so is κ_t. In contrast, the capital requirement using the unconditional density is constant over time. When the conditional variance is larger (smaller) than the unconditional variance, the capital requirement using the conditional density is higher (lower) than that of the unconditional density.

In the second example, suppose the firm is holding a short position of S_t units of currencies in futures contracts. [Shorts are represented by *negative* quantities, i.e., $S_t<0$.] The capital requirement, K_t, which can cover the losses of the short position with a 99.5% probability, is found in an analogous manner. Let $\zeta_t = K_t / [-P_t S_t]$ be the capital requirement as a fraction of $[-P_t S_t]$. Then ζ_t is given by the equation:

$$\zeta_t = \exp(\sigma_{t+1|t} z_h) - 1,$$

where z_h is the quantile of z_t such that:

$$\Pr\{ z_h > z_t \} = .995.$$

Based on Table 8, z_h equals 2.590 for the BP, 2.418 for the DM, 3.271 for the JY, and 2.572 for the SF. As z_h is positive, an increase in $\sigma_{t+1|t}$ will raise the capital requirement. In contrast, the capital requirement using the unconditional density is constant over time.

In the third example, we consider a futures exchange setting futures margin requirements to protect the capital of its clearing members from defaults by futures traders. There are two types of futures margins: initial margins and maintenance margins. For illustrative purposes, we concentrate on the maintenance margin. Suppose the futures exchange desires to set the maintenance margin to ensure that it is sufficient to cover possible losses of either long or short positions at least 99.5% of the time. In other words, the maintenance margin as a percent of the price times the size of the futures contract should be the maximum of the capital requirements for the long and the short sides, i.e., $\text{Max}\{\kappa_t, \zeta_t\}$. While there is a 0.5% chance that the maintenance margin cannot cover the losses of the futures contract, this should be interpreted as an upper bound of the default probability for a futures contract, since a trader can add funds to his account to cover losses exceeding the maintenance margin.

V.B Application to Risk Management: Longer Term Minimum Capital Requirements

So far, we have considered the capital requirements for holding a futures position for one trading day. It would be reasonable to ask how much capital is needed for holding a futures position for longer periods.

This consideration can arise in many contexts. For example, a firm is planning to use a currency futures contract to hedge the exchange rate risk of inflows of British Pounds three months from now. To goal of the hedge is to balance gains (losses) in the cash inflow with losses (gains) in the futures position as the exchange rate fluctuates. The problem facing the firm, however, is that a futures position is marked to market, so that gains and losses are settled at the end of each trading day, while the cash position is settled in entirety three months from now. If the exchange rate moves in such a way that the cash position is making profits while the futures position is sustaining losses, the firm may need additional funds to meet margin requirements on the futures position because it cannot use the gains in the cash position to offset these losses. If the firm is unable to meet margin requirements, it will be forced to liquidate the futures position prematurely, which defeats the purpose of hedging. Before the firm commits to the hedging strategy using futures, it must know how much capital (e.g., additional funds) may be needed to maintain this futures position for the next three months.

The answer to this question can be obtained via a simulation study. Start with the conditional density of price changes at the time when the firm initially opens the futures position. For the sake of illustration, take this to be the end of our data sample, March 9, 1990. Simulate the path of the futures price over the course of the next 3 months. At the end of each trading day, track the value of the futures position, and record its lowest value during the 3 month period. This is the maximum "draw down" for this simulated path, which represents the maximum loss sustained by the firm while holding the futures position. If the firm's additional funds is less than this maximum draw down, it would not be able to maintain its futures position. By repeating this for 10,000 simulated paths, we generate an empirical distribution of the maximum draw down. The capital requirement can then be set to that amount which is able to cover a given percentage of the simulated maximum draw downs. We use the 90% coverage probability, because 10,000 replications is not accurate enough to measure the extreme tails of a distribution.

The simulation can be done as follows. Recall that the semi-nonparametric model of futures price changes is given by:

$$x_t = \sigma_{P,t}\, u_t$$

$$\log \sigma_{P,t} = \alpha + \Sigma\, \beta_i \log \sigma_{P,t-i} + \nu_t,$$

where $x_t = \log[P_t/P_{t-1}]$. A simulated path of future x_t's is generated recursively, using the estimates of α and β's from our sample, and the values of $\sigma_{P,t}$ at the end of the sample. The u_t and ν_t are drawn randomly, with replacement, from the residuals in a "bootstrap" fashion, per Efron (1982).

Table 9 reports the results of the simulation experiment for the capital requirement needed to hold a futures position with 90% probability. We vary the holding period of the futures position over 1, 5, 10, 15, 20, 25, 30, 60, 90, and 180 trading days. The 95% central confidence intervals for these capital requirements are given in Table 10.[22] For comparison, we also report the simulations using the unconditional density, and the EGARCH model. In the case of the unconditional density, the x_t's are drawn randomly, with replacement, from the 1275 observed price changes. In the case of the EGARCH model, the simulated x_t's use the estimated values of α, β, ϕ, and γ, and the value of h_t at the end of the sample. The η_t's are drawn randomly, with replacement, from the standardized residuals, in a way analogous to the autoregressive volatility model.

To understand the results, keep in mind that we start the simulation on March 9, 1990, when the volatility is below the sample average.[23] Thus, the autoregressive volatility model predicts a lower volatility in the near future than the unconditional density.

Consider holding a long futures position in the BP. For a 1 day holding period, the capital requirement is 0.73% of the initial face value of the contract according to the autoregressive volatility model and 0.91% according to the unconditional density. [If we had started the simulation on a day which had a higher volatility than the sample average, the capital requirement based on the autoregressive volatility model would have been higher than that based on the unconditional density.] For a 5 day holding period, the capital requirements are, respectively, 1.9% and 2.3%.

These differences in capital requirements are both statistically and economically significant. In the case of the 1 day holding period for the BP, there is a 95% probability that the correct capital requirement using the autoregressive model is higher than 0.70% and lower than 0.74% of the initial face value of the contract. At the same time, there is a 95% probability that the correct capital requirement using the unconditional density is higher than 0.85% and lower than 0.95%. Furthermore, the difference between capital requirements of 0.73% versus 0.91% is economically significant, when transactions have face values of several hundred million dollars, such as the case when highly leverage instruments are involved.

Tables 9 and 10 also provide some information on the convergence behavior of the autoregressive volatility model to the unconditional density. Firstly, we observe that the capital requirements (and their associated confidence intervals) of the former approach those of the latter as the holding period lengthens. For the BP, this occurs in 90 (trading) days. The DM takes 60 trading days, while the JY takes only 30 days. But the SF takes more than 180 days.[24] However, the convergence is likely to be oscillatory rather than monotonic, as the autoregressive model of volatility has several lags.

In comparison, the EGARCH model produced dramatically different results. Over a 1-day holding period, the capital requirements based on the EGARCH model are similar to those based on the unconditional density. However, as we simulate into the future, the EGARCH model produces much larger capital requirements than both the autoregressive volatility model and the unconditional density. This phenomenon is due to the high degree of volatility persistence in the EGARCH model. During the simulation period, a large price change (either positive or negative) will cause the conditional variance of the EGARCH model to increase and to remain high for a long period of time.[25] In contrast, there is much less volatility persistence in the autoregressive volatility model and none in the unconditional model. This persistence in volatility also means that the convergence of the EGARCH model to the unconditional density is extremely slow. We have simulated as many as 500 trading days into the future. For the BP, DM, and SF, the capital requirements from the EGARCH model are still twice as high as those from the unconditional density. The exception is the JY. Its capital requirements from the EGARCH model are roughly 50% higher than those from the unconditional density. This demonstrates that, while the EGARCH model may produce satisfactory 1-day ahead volatility forecasts, it is may not be appropriate for multi-step ahead volatility forecasts.

Another interesting feature in Table 9 is that a short position requires more capital than a corresponding long position at any given coverage probability. This is due to the fact that the futures price is bounded below by zero, but unbounded above. Even when the logarithmic rate of change of the futures price is symmetric, the change in the futures price itself is asymmetric. Thus, the probability of a $1 decrease in futures price is less than that of a $1 increase. This accounts for the difference in the capital requirements between a long and a short position.

VI. Conclusions

In this paper, we demonstrate that when log price changes are not IID, their conditional density may be more accurate than their unconditional density for describing short term behavior. Using the BDS test of independence and identical distribution, we show that daily log price changes in four currency futures contracts are not IID. While there appears to be no predictable conditional mean changes, conditional variances are predictable, and can be described by an autoregressive volatility model. Furthermore, this autoregressive volatility model seems to capture all the departures from independence and identical distribution.

Based on this model, we can decompose daily log price changes into a predictable part and an unpredictable part. The predictable part is described parametrically by the autoregressive volatility model. The unpredictable part can be modeled by an empirical density, either parametrically or nonparametrically. This two-step semi-nonparametric method yields a conditional density for daily log price changes, which has a number of uses in financial risk management.

In particular, we show how to directly calculate the capital requirement needed to cover losses of a futures position over one trading day, and how to use simulation to obtain the capital requirement over longer holding periods. We find that the conditional density can provide different, and probably more accurate, capital requirements than the unconditional density.

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Notes:

See LeBaron (1988), Scheinkman and LeBaron (1989), and Hsieh (1991) for stock returns, and Hs (1989) for exchange rates.

This is more restrictive than the general notion, which allows conditioning on ot information. Our univariate approach is much simpler computationally than the multivari approach. When conditioning on other information, such as trading volume, we will need to mo these additional variables.

Parametric unconditional densities have been estimated by Fama (1965) and Blattberg and Gone (1974) for stock returns, and Westerfield (1977), Rogalski and Vinso (1978), Boothe and Glass (1988) for exchange rates.

Suppose x_t is given by the following process:

$x_t = \sigma_t u_t$,

$\log \sigma_t = \alpha + \beta \log \sigma_{t-1} + v_t$,

re u_t is IID, normal, with mean 0 and variance 1, and v_t is IID, normal, with mean 0 and varianc . Furthermore, u_t and v_s are independent for all t and s, and $|\beta| < 1$. It is easy to ver that the conditional and unconditional distribution of x_t is different, and that the conditio variance of x_t can be either larger or smaller than the unconditional variance.

See Brock, Hsieh, and LeBaron (1991) for a discussion of this point.

See Scheinkman and LeBaron (1989) and Hsieh (1989).

Note that IID implies that $C_m(\delta) = [C_1(\delta)]^m$, but the converse is not true. Dechert (1988) gi some pathological examples of non-IID data for which $C_m(\delta) = [C_1(\delta)]^m$.

This finding contradicts those of Hodrick and Srivastava (1987) and McCurdy and Morgan (198 who find strong autocorrelation in log price changes in currency futures prices using data which daily price limits were in effect but did not take them into account. Kodres (1988) u a limited dependent variable method, but fails to take account of the conditio heteroskedasticity, as pointed out by Harvey (1988).

The reader may be concerned with the role of maturity drift in these results using futures da It is possible that a fixed maturity futures price change is IID, but the distribution of an period maturity futures price change is different from that of an n-1 period maturity futu price change. This may induce "spurious" rejection of IID. To check that this is not the ca Appendix A and B (available upon request) provide the BDS statistics and autocorrelat coefficients of the absolute values of daily log price changes of spot currencies, collected the Board of Governors of the Federal Reserve System, for the same time period. The s exchange rates (which is a 2-day forward contract) have essentially the same statisti behavior as the currency futures.

The two-step procedure may suffer from some efficiency loss. However, with the sample size have, the longer computation time of joint estimation is a greater cost than any gain efficiency.

The proof relies on the law of iterated expectations. $E[x_t x_{t-i} x_{t-j}] =$
$E[x_t | x_{t-1}, ...] x_{t-i} x_{t-j}] = 0$.

Note that the restriction that ε_t is IID is needed to prove consistency of nonparametric meth to estimate the function g().

Diebold and Nason (1990) and Meese and Rose (1990) found that LWR does not outperform a ran walk model in forecasting spot exchange rates in terms of mean squared error or mean absol error. It is possible that LWR has low power in detecting conditional mean changes. But simulations in Hsieh (1991) show that LWR can detect all the nonlinear dynamics models m often cited in the time series literature.

The extension to the case with multiple lags is straight forward.

We have also tested for the significance of day-of-the-week dummies, which turn out not to b factor in the conditional mean. These results are available from the author upon request.

The $(2/\pi)$ is used to center the mean of $|\eta_{t-1}|$ at 0.

See Nelson (1990) for a discussion.

For the BP, the trading hours are 7:30 am to 1:24 pm from 85/2/22 until 85/10/14, and 7:20 am 1:24 pm from 85/10/15 to 88/10/04. For the DM, the corresponding trading hours are 7:30 am 1:20 pm and 7:20 am to 1:20 pm. For the JY, they are 7:30 am to 1:22 pm and 7:20 am to 1:22 For the SF, the corresponding trading hours are 7:30 am to 1:16 pm and 7:20 am to 1:16 p Since 88/10/05, the trading hours for all currency futures are 7:20 am to 2:00 pm.

A similar model was identified for the Standard and Poors 500 cash index in Hsieh (1991).

See Hsieh (1988) for a discussion of day-of-the-week effects in spot currencies.

We need a theory of hedging to determine whether "coverage probability" is the appropri concept for hedging, and what the optimal "coverage probability" should be.

Efron (1982) provides a nonparametric method to estimate the confidence interval for a quanti Let X be a random variable with distribution F. We want to estimate θ, defined as Prob$\{X<\theta\}$ Let x(1),...,x(n) be the ordered data from a sample of size n. A confidence interval for [x(j),x(k)], can be found as follows. Observe that Prob$\{x(j)<\theta\leq x(k)\}$ = Prob$\{j<Z\leq k\}$, where #$\{x(i)<\theta\}$ is a binomial distribution. Suppose we want a 90% confidence interval for θ. We determine j and k such that Prob$\{j<Z\leq k\}$ = 0.95. If n is small, we can use the exact binom distribution of Z. Since n is large in our case (i.e., 10,000), we approximate the binom distribution with a normal distribution.

On that day, the Parkinson volatilities are 12.48%, 9.51%, 7.90%, and 8.90%, respectively, the BP, DM, JY, and SF. Over the entire sample, the average of the Parkinson volatilities 16.13%, 16.06%, 12.68%, and 18.01%, respectively.

The SF actually takes about 250 trading days, according to our simulations which are reported in Tables 9 and 10.

This explanation is confirmed by the following experiment. We estimated the EGARCH mo subject to the constraint that β=0 in the variance equation. This allows much less volatil persistence. In the case of the BP, the restricted EGARCH model produced capital requireme of 12.26% at the 180 day holding period for long positions, and 31.00% for short positions These are much closer to the capital requirements in Table 9 for the autoregressive volatil model and the unconditional density, and very different from those from the unrestricted EGA model.

Table 1
Statistical Description of Daily Log Price Changes:
Feb. 22, 1985 - Mar. 9, 1990 (1275 observations)

	BP	DM	JY	SF
Mean	0.00045	0.00043	0.00032	0.00037
Median	0.00036	0.00000	0.00000	0.00016
Std dev	0.00815	0.00784	0.00708	0.00869
Skewness	0.36	0.28	0.34	0.18
Kurtosis	6.25	5.32	7.81	4.94
Maximum	0.04553	0.04832	0.05333	0.04967
Minimum	-0.02899	-0.03264	-0.04133	-0.03692

BDS statistics:

m	δ	BP	DM	JY	SF
2	0.5	2.39*	1.68	4.15*	1.01
3	0.5	2.76*	2.23	4.95*	1.08
4	0.5	3.58*	3.16*	6.39*	1.77
5	0.5	4.40*	3.91*	7.88*	2.57
2	1.0	3.34*	1.48	4.06*	0.46
3	1.0	4.00*	2.10	4.49*	0.85
4	1.0	4.86*	3.11*	5.69*	1.59
5	1.0	5.73*	3.85*	6.52*	2.40
2	1.5	3.96*	1.99	3.68*	0.81
3	1.5	4.84*	2.97*	4.29*	1.62
4	1.5	5.75*	3.95*	5.61*	2.57
5	1.5	6.54*	4.69*	6.32*	3.38*
2	2.0	3.88*	2.51*	3.16*	1.35
3	2.0	4.86*	3.79*	3.84*	2.37
4	2.0	5.77*	4.75*	5.14*	3.30*
5	2.0	6.54*	5.53*	5.73*	4.02*

* Significant at the 1% level using a two-tailed test.

Note:
m is the imbedding dimension.
δ is the distance between points, measured in terms of number of standard deviations of the raw data.
The critical values (marginal significance level) of the statistics for a two-tailed test are: 1.645 (10%), 1.960 (5%), 2.326 (2%), and 2.576 (1%).

Table 2
Autocorrelations of Price Changes and Their Absolute Values

	BP	DM	JY	SF
Autocorrelation Coefficients:				
ρ(1)	0.032	-0.019	0.024	-0.006
ρ(2)	-0.016	-0.009	0.000	-0.013
ρ(3)	-0.017	0.042	0.057	0.029
ρ(4)	-0.019	-0.043	-0.004	-0.032
ρ(5)	-0.005	0.014	0.012	0.009
ρ(6)	0.054	0.033	0.021	0.007
ρ(7)	-0.045	-0.021	-0.026	-0.017
ρ(8)	0.029	0.047	0.051	0.021
ρ(9)	-0.016	0.005	0.022	0.008
ρ(10)	-0.020	-0.037	-0.005	-0.033
ρ(11)	-0.039	-0.009	0.014	-0.011
ρ(12)	-0.015	-0.022	0.025	-0.008
ρ(13)	0.056	0.018	-0.002	0.034
ρ(14)	0.005	0.015	0.042	0.002
ρ(15)	0.052	0.056	0.022	0.066
Q(15)	20.09	17.12	14.25	12.26

	BP	DM	JY	SF
Autocorrelation Coefficients of absolute values:				
r(1)	0.107*	0.059	0.118*	0.027
r(2)	0.094*	0.038	0.058	0.025
r(3)	0.108*	0.079*	0.101*	0.052
r(4)	0.112*	0.055	0.041	0.040
r(5)	0.081*	0.088*	0.084*	0.084*
r(6)	0.096*	0.107*	0.087*	0.097*
r(7)	0.088*	0.099*	0.010	0.096*
r(8)	0.101*	0.087*	0.088*	0.061
r(9)	0.088*	0.063	0.069	0.054
r(10)	0.129*	0.128*	0.023	0.113*
r(11)	0.047	0.020	0.041	0.038
r(12)	0.078*	0.068	-0.005	0.075*
r(13)	0.092*	0.086*	0.023	0.088*
r(14)	0.116*	0.073*	0.055	0.048
r(15)	0.108*	0.115*	0.028	0.103*
Q(15)	182.41**	128.85**	79.50**	98.69**

* Significant at the 1% level using a two-tailed test.
** Significant at the 1% level using a one-tailed test.

Note:
Q(15) is the Box-Pierce statistic testing for the first 15 lags to be different from zero. The critical values (marginal significance levels) are:

22.31 (10%), 25.00 (5%), and 27.49 (1%).

Table 3
Bicorrelation Coefficients

Lags i	j	BP	DM	JY	SF
1	1	-0.119	-0.059	-0.072	-0.037
1	2	-0.041	-0.011	-0.030	-0.013
2	2	0.232	0.156	0.103	0.185
1	3	-0.024	0.009	0.073	-0.001
2	3	0.125	0.076	-0.003	0.060
3	3	-0.006	-0.097	0.231	-0.035
1	4	0.006	0.000	0.039	-0.012
2	4	-0.012	-0.013	0.034	-0.015
3	4	-0.030	0.021	0.068	0.027
4	4	0.008	-0.016	0.149	0.018
1	5	0.028	-0.020	0.039	-0.005
2	5	-0.007	-0.065	-0.023	-0.045
3	5	0.037	0.037	-0.033	-0.010
4	5	-0.097	-0.027	0.021	-0.005
5	5	0.026	0.015	0.051	0.061
$\chi^2(15)$		15.58	12.42	11.35	12.01

Table 4
Ratio of Root Mean Squared Forecast Errors

No. of Lags	Fraction of sample	BP	DM	JY	SF
1	0.1	1.0209	1.0081	1.0547	1.0285
1	0.2	1.0254	0.9953	1.0371	1.0325
1	0.3	1.0260	0.9971	1.0323	1.0250
1	0.4	1.0221	0.9996	1.0301	1.0214
1	0.5	1.0188	1.0020	1.0278	1.0196
1	0.6	1.0170	1.0037	1.0263	1.0186
1	0.7	1.0155	1.0051	1.0256	1.0172
1	0.8	1.0141	1.0064	1.0260	1.0154
1	0.9	1.0133	1.0068	1.0260	1.0135
2	0.1	1.0605	1.0312	1.1094	1.0342
2	0.2	1.0548	1.0211	1.0935	1.0121
2	0.3	1.0491	1.0162	1.0831	1.0080
2	0.4	1.0431	1.0102	1.0742	1.0044
2	0.5	1.0370	1.0067	1.0664	1.0019
2	0.6	1.0315	1.0048	1.0593	0.9997
2	0.7	1.0268	1.0041	1.0540	0.9982
2	0.8	1.0232	1.0038	1.0496	0.9973
2	0.9	1.0204	1.0040	1.0459	0.9976
3	0.1	1.1473	1.0777	1.2062	1.0744
3	0.2	1.1019	1.0655	1.1572	1.0305
3	0.3	1.0810	1.0567	1.1323	1.0190
3	0.4	1.0678	1.0477	1.1140	1.0148
3	0.5	1.0582	1.0404	1.0990	1.0112
3	0.6	1.0508	1.0347	1.0855	1.0089
3	0.7	1.0435	1.0303	1.0750	1.0067
3	0.8	1.0375	1.0252	1.0656	1.0048
3	0.9	1.0315	1.0196	1.0590	1.0038
4	0.1	1.3497	1.1755	1.3802	1.1763
4	0.2	1.2435	1.0841	1.2608	1.0698
4	0.3	1.1892	1.0587	1.2091	1.0373
4	0.4	1.1548	1.0472	1.1764	1.0208
4	0.5	1.1302	1.0402	1.1536	1.0125
4	0.6	1.1095	1.0348	1.1362	1.0081
4	0.7	1.0914	1.0291	1.1216	1.0053
4	0.8	1.0742	1.0241	1.1080	1.0049
4	0.9	1.0596	1.0194	1.0958	1.0056
5	0.1	1.4288	1.4386	1.7291	1.3289
5	0.2	1.2553	1.2397	1.4582	1.1802
5	0.3	1.1847	1.1638	1.3585	1.1271
5	0.4	1.1448	1.1182	1.2929	1.0958
5	0.5	1.1162	1.0875	1.2401	1.0739
5	0.6	1.0954	1.0661	1.1983	1.0583
5	0.7	1.0799	1.0507	1.1619	1.0468
5	0.8	1.0673	1.0404	1.1316	1.0382
5	0.9	1.0546	1.0338	1.1077	1.0314

Note: Underlined value represents the lowest ratio in a given currency.

Table 5
EGARCH Estimates

$$x_t = \mu + h\eta_t$$
$$\eta_t \sim N(0,1)$$
$$\log h_t = \alpha + \beta \log h_{t-1} + \phi \left[|\eta_{t-1}| - (2/\pi) \right] + \gamma \eta_{t-1}$$

	BP	DM	JY	SF
μ	0.000319	0.000377	0.000232	0.000239
	(0.000208)	(0.000214)	(0.000189)	(0.000235)
α	-0.688127	-1.072229	-4.438289	-0.993241
	(0.030088)	(0.041828)	(0.756704)	(0.032479)
β	0.928780	0.889511	0.550707	0.895527
	(0.002995)	(0.004386)	(0.075851)	(0.003508)
ϕ	0.135854	0.187005	0.282167	0.157669
	(0.019961)	(0.028388)	(0.093357)	(0.024013)
γ	-0.110718	0.084173	0.313274	0.129035
	(0.177458)	(0.147279)	(0.201531)	(0.166507)

Bollerslev-Woolridge robust standard errors are in parentheses.

Table 6
EGARCH Standardized Residuals
BDS Test for IID

m	δ	BP	DM	JY	SF
2	0.5	-0.61	-1.10	0.12	-1.34
3	0.5	-0.78	-1.35	0.17	-1.88*
4	0.5	-0.52	-1.08	0.95	-1.71*
5	0.5	-0.09	-0.99	1.90	-1.67
2	1.0	-0.50	-1.65	-0.62	-2.23*
3	1.0	-0.59	-1.77*	-0.69	-2.55*
4	1.0	-0.40	-1.45*	0.32	-2.35*
5	1.0	-0.14	-1.33*	0.89	-2.04*
2	1.5	-0.42	-1.54	-1.03	-2.41*
3	1.5	-0.56	-1.51*	-1.09	-2.59*
4	1.5	-0.50	-1.25*	0.16	-2.29*
5	1.5	-0.31	-1.19*	0.77	-2.08*
2	2.0	-0.42	-1.24	-1.01	-2.05*
3	2.0	-0.54	-1.18	-0.89	-2.07*
4	2.0	-0.41	-0.97	0.33	-1.72*
5	2.0	-0.28	-0.98	0.85	-1.58*

Note:
* Statistically significant at the 5% two-tailed test based on the simulated critical values of an EGARCH model for 1275 observations with 2000 replications:

	δ			
m	0.50	1.00	1.50	2.00
2.5% critical values				
2	-2.04	-1.95	-1.77	-1.62
3	-1.63	-1.39	-1.30	-1.31
4	-1.66	-1.22	-1.14	-1.15
5	-1.66	-1.22	-1.14	-1.15
97.5% critical values				
2	1.73	1.58	1.57	1.56
3	1.70	1.45	1.49	1.83
4	1.85	1.47	1.49	2.22
5	1.85	1.47	1.49	2.22

Table 7
Estimates of the Autoregressive Volatility Model
Using Parkinson's Standard Deviations

$$\log \sigma_{P,t} = \alpha + \Sigma \beta_i \log \sigma_{P,t-i} + \nu_t$$

	BP	DM	JY	SF
α	-1.037	-1.139	-1.874	-1.219
	(0.171)	(0.187)	(0.199)	(0.193)
$\log \sigma_{P,t-1}$	0.192	0.153	0.208	0.115
	(0.028)	(0.028)	(0.028)	(0.028)
$\log \sigma_{P,t-2}$	0.134	0.111	0.137	0.106
	(0.029)	(0.028)	(0.028)	(0.028)
$\log \sigma_{P,t-3}$	0.062	0.052	0.058	0.068
	(0.029)	(0.028)	(0.029)	(0.028)
$\log \sigma_{P,t-4}$	0.069	0.092	0.109	0.091
	(0.029)	(0.028)	(0.028)	(0.028)
$\log \sigma_{P,t-5}$	0.137	0.091	0.112	0.118
	(0.028)	(0.028)	(0.028)	(0.028)
$\log \sigma_{P,t-6}$	0.027	0.072		0.074
	(0.029)	(0.028)		(0.028)
$\log \sigma_{P,t-7}$	0.073	0.110		0.086
	(0.028)	(0.028)		(0.028)
$\log \sigma_{P,t-8}$	0.088	0.079		0.078
	(0.028)	(0.028)		(0.028)
^{2}	0.274	0.227	0.170	0.193
$\Sigma\beta_i$	0.782	0.760	0.624	0.736
	(0.129)	(0.124)	(0.165)	(0.099)
Test of				
$\Sigma\beta_i=1$	36.59	37.27	91.73	55.71
$F(n_1,n_2)$	1,1258	1,1258	1,1264	1,1260
	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Standard errors in parantheses, p-values in square brackets.

Note:
The standard errors and test of $\Sigma\beta_i=1$ do not change when using a

heteroskedasticity-consistent covariance matrix.

Table 8
Statistical Description of Standardized Residuals of
The Autoregressive Volatility Model

	BP	DM	JY	SF
Mean	0.042	0.036	0.037	0.024
Median	0.051	0.000	0.000	0.019
Std dev	0.880	0.853	1.031	0.842
Skewness	-0.035	0.053	0.196	0.025
Kurtosis	5.249	4.360	7.485	4.426
Maximum	5.078	3.389	6.897	3.513
Minimum	-3.560	-3.626	-5.205	-4.000
Quantiles:				
0.50%	-3.017	-2.399	-3.623	-2.306
1.00%	-2.474	-2.245	-2.821	-2.080
5.00%	-1.411	-1.319	-1.557	-1.322
10.00%	-0.970	-0.937	-1.046	-0.985
90.00%	1.067	1.135	1.228	1.096
95.00%	1.504	1.487	1.697	1.422
99.00%	2.304	2.220	2.611	2.137
99.50%	2.590	2.418	3.271	2.572
BDS statistics:				
m δ				
2 0.5	-0.61	-0.45	1.69	-1.27
3 0.5	-0.96	-1.09	1.45	-1.64
4 0.5	-0.97	-1.24	1.65	-1.20
5 0.5	-1.01	-1.36	2.04	-1.20
2 1.0	-0.24	-0.91	1.77	-1.80
3 1.0	-0.86	-1.28	1.33	-2.23*
4 1.0	-0.94	-1.17	1.69	-2.07*
5 1.0	-1.02	-1.25	1.77	-1.88
2 1.5	0.42	-0.68	1.48	-1.80
3 1.5	-0.23	-0.83	1.17	-2.04*
4 1.5	-0.32	-0.66	1.91	-1.82
5 1.5	-0.44	-0.73	1.99	-1.73
2 2.0	0.75	-0.40	0.84	-1.42
3 2.0	0.41	-0.32	0.88	-1.52
4 2.0	0.38	-0.11	1.60	-1.31
5 2.0	0.26	-0.20	1.66	-1.34

Table 8 (cont.)

Note:
* Statistically significant at the 5% two-tailed test based on the simulated critical values of an autoregressive volatility model for 1275 observations with 2000 replications:

	δ			
m	0.50	1.00	1.50	2.00
2.5% critical values				
2	-1.84	-1.86	-1.86	-1.96
3	-1.86	-1.88	-1.85	-1.96
4	-1.86	-1.87	-1.93	-1.96
5	-1.77	-1.89	-1.91	-1.96
97.5% critical values				
2	1.90	1.94	1.92	1.96
3	2.01	1.94	2.00	1.96
4	2.01	1.95	2.12	1.96
5	2.04	2.06	2.05	1.96

Table 9
Capital Requirement for 90% Coverage Probability
As a Percent of the Initial Value

	No. of Days	Long Position AR	Long Position Uncond	Long Position EGARCH	Short Position AR	Short Position Uncond	Short Position EGARCH
BP	1	0.73	0.91	0.93	0.80	0.98	1.05
	5	1.90	2.30	2.61	2.18	2.76	3.00
	10	2.83	3.27	4.19	3.38	4.22	4.88
	15	3.54	3.94	5.72	4.45	5.48	6.67
	20	4.10	4.61	6.96	5.24	6.33	8.43
	25	4.59	5.15	8.25	6.20	7.36	10.46
	30	5.02	5.58	9.08	7.11	8.33	12.06
	60	7.24	7.44	14.50	11.64	12.87	20.71
	90	8.74	8.70	17.91	15.45	16.90	28.03
	180	11.38	10.67	24.25	25.81	27.36	48.02
DM	1	0.72	0.87	0.83	0.89	1.00	0.95
	5	1.89	2.18	2.34	2.23	2.70	2.91
	10	2.77	3.14	3.93	3.40	4.12	5.03
	15	3.52	3.86	5.37	4.36	5.30	6.92
	20	4.05	4.45	6.54	5.19	6.14	8.91
	25	4.55	4.90	7.86	6.14	7.21	10.69
	30	4.93	5.37	8.75	7.02	7.88	12.36
	60	7.16	7.24	13.14	11.36	12.38	20.86
	90	8.87	8.39	16.06	14.68	16.16	27.75
	180	11.38	10.35	21.69	24.25	26.25	45.68
JY	1	0.56	0.74	0.72	0.68	0.87	0.86
	5	1.61	1.99	2.22	1.92	2.36	2.73
	10	2.59	2.82	3.46	3.06	3.53	4.41
	15	3.30	3.46	4.37	4.11	4.60	5.79
	20	3.95	4.10	5.09	5.13	5.45	6.77
	25	4.42	4.58	5.78	5.91	6.30	7.98
	30	4.95	4.92	6.34	6.58	6.85	8.81
	60	6.99	6.84	8.72	10.53	10.74	13.58
	90	8.43	8.00	10.51	13.61	14.00	17.63
	180	10.97	10.27	13.99	21.86	22.21	27.39
SF	1	0.82	0.97	0.89	0.93	1.12	0.98
	5	1.99	2.51	2.48	2.23	2.93	2.98
	10	2.87	3.60	4.12	3.37	4.53	5.09
	15	3.67	4.35	5.60	4.22	5.67	7.03
	20	4.24	5.10	6.82	5.09	6.69	8.86
	25	4.81	5.65	8.12	5.90	7.77	10.93
	30	5.23	6.20	9.12	6.70	8.47	12.50
	60	7.69	8.41	13.73	10.55	13.10	21.27
	90	9.23	9.93	16.89	13.60	17.06	27.80

180	12.18	12.57	22.92		21.72	27.45	45.47

Table 10
Approximate 95% Central Confidence Intervals for
Capital Requirement for 90% Coverage Probability
As a Percent of the Initial Value

	No. of Days	Long Position AR	Long Position Uncond	Long Position EGARCH	Short Position AR	Short Position Uncond	Short Position EGARCH
BP	1	[0.70, 0.74]	[0.86, 0.95]	[0.90, 0.96]	[0.78, 0.82]	[0.96, 0.99]	[1.01, 1.07]
	5	[1.87, 1.95]	[2.26, 2.37]	[2.54, 2.67]	[2.14, 2.25]	[2.70, 2.83]	[2.95, 3.06]
	10	[2.76, 2.91]	[3.19, 3.34]	[4.08, 4.30]	[3.30, 3.48]	[4.15, 4.30]	[4.77, 4.99]
	15	[3.47, 3.61]	[3.87, 4.02]	[5.57, 5.90]	[4.36, 4.53]	[5.38, 5.59]	[6.54, 6.84]
	20	[4.02, 4.20]	[4.52, 4.72]	[6.82, 7.17]	[5.12, 5.34]	[6.22, 6.44]	[8.31, 8.62]
	25	[4.49, 4.70]	[5.03, 5.29]	[8.06, 8.43]	[6.08, 6.33]	[7.22, 7.51]	[10.26,10.64]
	30	[4.94, 5.14]	[5.44, 5.72]	[8.87, 9.32]	[7.00, 7.25]	[8.15, 8.48]	[11.83,12.30]
	60	[7.10, 7.45]	[7.31, 7.58]	[14.21,14.78]	[11.37,11.89]	[12.66,13.08]	[20.41,21.11]
	90	[8.55, 8.94]	[8.57, 8.92]	[17.51,18.25]	[15.22,15.73]	[16.56,17.23]	[27.50,28.62]
	180	[11.15,11.66]	[10.45,10.92]	[23.93,24.74]	[25.33,26.29]	[27.00,27.76]	[47.00,49.05]
DM	1	[0.68, 0.75]	[0.81, 0.90]	[0.78, 0.85]	[0.85, 0.91]	[0.96, 1.08]	[0.93, 0.99]
	5	[1.85, 1.94]	[2.15, 2.23]	[2.28, 2.40]	[2.19, 2.28]	[2.64, 2.75]	[2.86, 2.97]
	10	[2.70, 2.84]	[3.08, 3.22]	[3.86, 4.02]	[3.34, 3.47]	[4.04, 4.22]	[4.92, 5.16]
	15	[3.44, 3.58]	[3.77, 3.93]	[5.28, 5.51]	[4.28, 4.44]	[5.21, 5.40]	[6.80, 7.08]
	20	[3.96, 4.14]	[4.35, 4.55]	[6.41, 6.69]	[5.10, 5.28]	[6.02, 6.26]	[8.70, 9.07]
	25	[4.47, 4.65]	[4.80, 5.00]	[7.71, 8.00]	[6.00, 6.26]	[7.07, 7.38]	[10.47,10.90]
	30	[4.84, 5.06]	[5.24, 5.47]	[8.57, 8.95]	[6.91, 7.16]	[7.73, 8.02]	[12.10,12.65]
	60	[7.03, 7.33]	[7.12, 7.42]	[12.94,13.43]	[11.15,11.59]	[12.18,12.57]	[20.58,21.32]
	90	[8.66, 9.06]	[8.22, 8.60]	[15.77,16.44]	[14.48,15.03]	[15.89,16.46]	[27.17,28.27]
	180	[11.13,11.63]	[10.11,10.59]	[21.31,22.14]	[23.88,24.61]	[25.83,26.64]	[44.81,46.62]
JY	1	[0.56, 0.60]	[0.72, 0.76]	[0.70, 0.75]	[0.68, 0.72]	[0.86, 0.92]	[0.81, 0.89]
	5	[1.60, 1.67]	[1.94, 2.04]	[2.16, 2.28]	[1.89, 1.98]	[2.33, 2.41]	[2.67, 2.81]
	10	[2.45, 2.58]	[2.77, 2.88]	[3.38, 3.53]	[3.06, 3.20]	[3.46, 3.61]	[4.30, 4.51]
	15	[3.23, 3.39]	[3.41, 3.54]	[4.26, 4.46]	[4.03, 4.21]	[4.50, 4.69]	[5.65, 5.93]
	20	[3.84, 4.01]	[4.02, 4.19]	[4.99, 5.18]	[4.93, 5.15]	[5.36, 5.58]	[6.65, 6.93]
	25	[4.33, 4.52]	[4.47, 4.67]	[5.66, 5.91]	[5.75, 6.00]	[6.17, 6.41]	[7.81, 8.13]
	30	[4.79, 4.97]	[4.84, 5.03]	[6.24, 6.46]	[6.50, 6.77]	[6.74, 6.99]	[8.61, 9.05]
	60	[6.90, 7.19]	[6.69, 6.97]	[8.61, 8.92]	[10.33,10.70]	[10.57,10.95]	[13.35,13.87]
	90	[8.29, 8.63]	[7.87, 8.16]	[10.32,10.69]	[13.36,13.91]	[13.81,14.28]	[17.31,17.93]
	180	[10.76,11.23]	[10.09,10.49]	[13.74,14.25]	[21.47,22.26]	[21.79,22.47]	[26.97,27.93]
SF	1	[0.79, 0.84]	[0.95, 1.01]	[0.86, 0.92]	[0.89, 0.96]	[1.10, 1.16]	[0.95, 1.02]
	5	[1.94, 2.04]	[2.45, 2.57]	[2.44, 2.54]	[2.19, 2.29]	[2.86, 3.00]	[2.91, 3.04]
	10	[2.80, 2.94]	[3.51, 3.67]	[4.06, 4.18]	[3.30, 3.45]	[4.43, 4.62]	[4.98, 5.21]
	15	[3.59, 3.75]	[4.28, 4.45]	[5.46, 5.74]	[4.16, 4.32]	[5.57, 5.75]	[6.91, 7.17]
	20	[4.15, 4.33]	[4.99, 5.20]	[6.68, 6.97]	[4.98, 5.18]	[6.57, 6.83]	[8.68, 9.06]
	25	[4.71, 4.93]	[5.54, 5.77]	[7.96, 8.27]	[5.80, 6.02]	[7.60, 7.93]	[10.63,11.12]
	30	[5.11, 5.34]	[6.11, 6.31]	[8.97, 9.29]	[6.58, 6.84]	[8.33, 8.65]	[12.16,12.80]
	60	[7.55, 7.80]	[8.23, 8.57]	[13.48,14.00]	[10.37,10.80]	[12.92,13.37]	[20.88,21.64]
	90	[9.06, 9.42]	[9.72,10.17]	[16.69,17.17]	[13.32,13.89]	[16.83,17.34]	[27.36,28.37]

180	[11.98,12.43]	[12.27,12.87]	[22.62,23.29]	[21.34,22.09]	[27.01,27.88]	[44.72,46.46]

Table 10 (cont.)

Note: The first number in the square bracket is the left side of the confidence interval. The second number is the right side of the confidence interval.

Appendix A
BDS Statistics of Log Spot Price Changes
Feb. 22, 1985 - Mar. 9, 1990 (1267 observations)

m	δ	BP	DM	JY	SF
2	0.5	4.30*	2.71*	4.85*	2.22
3	0.5	4.41*	4.37*	6.10*	2.70*
4	0.5	4.70*	5.61*	7.40*	3.84*
5	0.5	4.62*	6.44*	8.66*	4.51*
2	1.0	4.50*	1.90	3.91*	1.85
3	1.0	4.87*	3.18*	4.24*	2.69*
4	1.0	5.53*	4.41*	5.26*	3.96*
5	1.0	6.12*	5.04*	6.04*	4.53*
2	1.5	4.87*	1.75	3.97*	1.79
3	1.5	5.43*	2.84*	4.16*	2.71*
4	1.5	6.19*	3.92*	4.82*	4.01*
5	1.5	6.86*	4.70*	5.16*	4.70*
2	2.0	4.82*	1.79	3.87*	1.94
3	2.0	5.56*	2.78*	4.22*	2.92*
4	2.0	6.27*	3.56*	4.69*	3.96*
5	2.0	6.90*	4.36*	4.96*	4.62*

* Significant at the 1% level using a two-tailed test.

Note:
m is the imbedding dimension.
δ is the distance between points, measured in terms of number of standard deviations of the raw data.
The critical values (marginal significance level) of the statistics for a two-tailed test are: 1.645 (10%), 1.960 (5%), 2.326 (2%), and 2.576 (1%).

Appendix B
Autocorrelation Coefficients of Absolute Values of Log Spot Price Changes
Feb. 22, 1985 - Mar. 9, 1990 (1267 observations)

	BP	DM	JY	SF
r(1)	0.119*	0.059	0.115*	0.044
r(2)	0.094*	0.070	0.079*	0.062
r(3)	0.103*	0.095*	0.100*	0.081*
r(4)	0.116*	0.085*	0.039	0.042
r(5)	0.124*	0.094*	0.120*	0.072
r(6)	0.118*	0.120*	0.103*	0.096*
r(7)	0.080*	0.133*	0.040	0.114*
r(8)	0.089*	0.078*	0.062	0.084*
r(9)	0.096*	0.049	0.045	0.050
r(10)	0.155*	0.113*	0.090*	0.107*
r(11)	0.100*	0.090*	0.103*	0.097*
r(12)	0.060	0.035	0.003	0.032
r(13)	0.053	0.106*	0.048	0.082*
r(14)	0.083*	0.068	0.072	0.068
r(15)	0.147*	0.029	-0.021	0.020
Q(15)	211.58**	141.87**	112.95**	106.28**

* Significant at the 1% level using a two-tailed test.
** Significant at the 1% level using a one-tailed test.

Note:
Q(15) is the Box-Pierce statistic testing for the first 15 lags to be different from zero. The critical values (marginal significance levels) are: 22.31 (10%), 25.00 (5%), and 27.49 (1%).

Table 7 (old)
Statistical Description of the Log of Parkinson's Standard Deviations:
Feb. 22, 1985 - Mar. 9, 1990 (1275 observations)

	BP	DM	JY	SF
Mean	-4.73	-4.73	-4.99	-4.60
Median	-4.78	-4.75	-5.02	-4.62
Std dev	0.529	0.521	0.549	0.495
Skewness	0.28	0.12	0.33	0.20
Kurtosis	3.17	2.90	3.19	3.08
Maximum	-2.35	-3.00	-2.63	-2.66
Minimum	-6.52	-6.28	-6.64	-6.17
Autocorrelation Coefficients:				
ρ(1)	0.401	0.340	0.322	0.285
ρ(2)	0.370	0.312	0.280	0.277
ρ(3)	0.329	0.273	0.227	0.255
ρ(4)	0.324	0.293	0.250	0.265
ρ(5)	0.363	0.295	0.247	0.282
ρ(6)	0.307	0.289	0.201	0.259
ρ(7)	0.322	0.309	0.204	0.264
ρ(8)	0.321	0.286	0.224	0.255
ρ(9)	0.315	0.266	0.176	0.231
ρ(10)	0.307	0.250	0.140	0.217
ρ(11)	0.289	0.200	0.130	0.190
ρ(12)	0.302	0.253	0.150	0.239
ρ(13)	0.306	0.231	0.107	0.211
ρ(14)	0.312	0.234	0.112	0.194
ρ(15)	0.319	0.241	0.119	0.203
Q(15)	2022**	1416**	777**	1124**

** Significant at the 1% level using a one-tailed test.

Note:
Q(15) is the Box-Pierce statistic testing for the first 15 lags to be different from zero. The critical values (marginal significance levels) are: 22.31 (10%), 25.00 (5%), and 27.49 (1%).