Government Guarantees and Self-Fulfilling Speculative Attacks

Technical Appendix

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A The Baseline Model

A.1 The Lemma and Other Preliminary Results

We start by considering the household's optimization problem. The household solves the following dynamic programming problem for $t \geq T+1$:

$$V^F(a_t, M_t) = \max_{c_t, a_{t+1}, M_{t+1}} \left[\log c_t + \phi \log \frac{M_t}{S_t} + \beta V^F(a_{t+1}, M_{t+1}) \right]$$

subject to

$$a_{t+1} = Ra_t + w_t + \pi_t - \tau_t - c_t - (M_{t+1} - M_t)/S_t. \tag{48}$$

The first order and envelope conditions are: $1/c_t = \theta_t$

$$\beta V_1^F(a_{t+1}, M_{t+1}) = \theta_t \tag{49}$$

$$\beta V_2^F(a_{t+1}, M_{t+1}) = \theta_t / S_t \tag{50}$$

$$V_1^F(a_t, M_t) = \theta_t R \tag{51}$$

$$V_2^F(a_t, M_t) = \phi / M_t + \theta_t / S_t, \tag{52}$$

where θ_t is the Lagrange multiplier on the budget constraint.

Substituting (49) into (51) and noting that $\beta = 1/R$, we have $\theta_t = \theta_{t+1}$. This implies that $c_t = c_{t+1} = c^F$ for $t \geq T+1$. Using this fact and substituting (52) into (50) we have

$$\frac{M_{t+1}}{S_{t+1}} = \frac{\beta \phi c^F}{S_{t+1}/S_t - \beta}, \text{ for } t \geq T+1. \tag{53}$$

In period T households face the following dynamic programming problem

$$V^D(a_T, M_T, x_T^h) = \max_{c_T, \chi^D, a_{T+1}, M_{T+1}} \left[\log c_T + \phi \log \frac{M_T}{S_T} + \beta V^F(a_{T+1}, M_{T+1}) \right]$$

subject to

$$a_{T+1} = Ra_T + w_T + \pi_T - \tau_T - c_T - \frac{M_{T+1} - M_T}{S_T} + \chi^D \left(1 - \frac{S_{T-1}}{S_T} \right) + x_T^h \left(\frac{1}{F_T} - \frac{1}{S_T} \right).$$

As long as $S_T > S_{T-1}$ the household will want to make χ^D infinite if it can. Since it is constrained by the fact that the government will only supply χ dollars we replace the household's problem with

$$V^D(a_T, M_T, x_T^h) = \max_{c_T, a_{T+1}, M_{T+1}} \left[\log c_T + \phi \log \frac{M_T}{S_T} + \beta V^F(a_{T+1}, M_{T+1})\right]$$

subject to

$$a_{T+1} = Ra_T + w_T + \pi_T - \tau_T - c_T - \frac{M_{T+1} - M_T}{S_T} + \chi\left(1 - \frac{S_{T-1}}{S_T}\right) + x_T^h\left(\frac{1}{F_T} - \frac{1}{S_T}\right). \quad (54)$$

The first order and envelope conditions are

$$c_T^{-1} = \theta_T \quad (55)$$

$$\beta V_1^F(a_{T+1}, M_{T+1}) = \theta_T \quad (56)$$

$$\beta V_2^F(a_{T+1}, M_{T+1}) = \theta_T / S_T \quad (57)$$

$$V_1^D(a_T, M_T, x_T^h) = R\theta_T \quad (58)$$

$$V_2^D(a_T, M_T, x_T^h) = \phi/M_T + \theta_T/S_T \quad (59)$$

$$V_3^D(a_T, M_T, x_T^h) = \theta_T\left(1/F_T - 1/S_T\right). \quad (60)$$

Notice that (51) implies that $V_1^F(a_{T+1}, M_{T+1}) = R/c^F$. Since $\beta = 1/R$ combining (55) and (56) we then obtain $c_T = c^F$. From (52) we have $V_2^F(a_{T+1}, M_{T+1}) = \phi/M_{T+1} + 1/(c^F S_{T+1})$. Hence from (57) we have

$$\frac{M_{T+1}}{S_{T+1}} = \frac{\beta\phi c^F}{S_{T+1}/S_T - \beta}. \quad (61)$$

To solve for c^F we iterate on (48) and combine it with (54) to obtain

$$\begin{aligned} a_T \;\; = \;\; & R^{-1}\sum_{j=0}^{\infty} R^{-j}(c_{T+j} - w_{T+j} - \pi_{T+j} + \tau_{T+j}) + R^{-1}\sum_{j=0}^{\infty} R^{-j}\frac{M_{T+1+j} - M_{T+j}}{S_{T+j}} - \\ & R^{-1}[\chi(1 - S_{T-1}/S_T) + x_T^h(1/F_T - 1/S_T)]. \end{aligned} \quad (62)$$

where we have imposed $\lim_{j\to\infty} R^{-j}a_{t+j} = 0$. Using $c_t = c^F$, for $t \geq T$, (53) and (61):

$$\begin{aligned} a_T \;\; = \;\; & (1+\beta\phi)c^F/(R-1) - R^{-1}\sum_{j=0}^{\infty} R^{-j}(w_{T+j} + \pi_{T+j} - \tau_{T+j}) - R^{-1}M_T/S_T - \\ & R^{-1}[\chi(1 - S_{T-1}/S_T) + x_T^h(1/F_T - 1/S_T)] \end{aligned} \quad (63)$$

For $t < T$, households solve the following dynamic programming problem

$$\begin{aligned} V^I(a_t, M_t, x_t^h) \;\; = \;\; & \max_{c_t, a_{t+1}, M_{t+1}, x_{t+1}^h} \{\log c_t + \phi\log(M_t/S_t) + \\ & \beta[(1-q)V^I(a_{t+1}, M_{t+1}, x_{t+1}^h) + qV^D(a_{t+1}, M_{t+1}, x_{t+1}^h)]\} \end{aligned}$$

subject to

$$a_{t+1} = Ra_t + w_t + \pi_t - \tau - c_t - (M_{t+1} - M_t)/S_t + x_t^h\left(1/F_t - 1/S_t\right). \tag{64}$$

The first order and envelope conditions are

$$1/c_t = \theta_t \tag{65}$$

$$\beta(1-q)V_1^I(a_{t+1}, M_{t+1}, x_{t+1}^h) + \beta q V_1^D(a_{t+1}, M_{t+1}, x_{t+1}^h) = \theta_t \tag{66}$$

$$\beta(1-q)V_2^I(a_{t+1}, M_{t+1}, x_{t+1}^h) + \beta q V_2^D(a_{t+1}, M_{t+1}, x_{t+1}^h) = \theta_t/S_t \tag{67}$$

$$(1-q)V_3^I(a_{t+1}, M_{t+1}, x_{t+1}^h) + q V_3^D(a_{t+1}, M_{t+1}, x_{t+1}^h) = 0 \tag{68}$$

$$V_1^I(a_t, M_t, x_t^h) = \theta_t R \tag{69}$$

$$V_2^I(a_t, M_t, x_t^h) = \phi/M_t + \theta_t/S_t \tag{70}$$

$$V_3^I(a_t, M_t, x_t^h) = \theta_t\left(1/F_t - 1/S_t\right). \tag{71}$$

If we substitute (60) and (71) into (68) we obtain

$$(1-q)\theta_{t+1}^I\left(1/F_{t+1}^I - 1/S_{t+1}^I\right) + q\theta_{t+1}^D\left(1/F_{t+1}^D - 1/S_{t+1}^D\right) = 0. \tag{72}$$

Here F_{t+1}^I, S_{t+1}^I and θ_{t+1}^I represent the values taken on by F_{t+1}, S_{t+1} and θ_{t+1} if the exchange rate remains fixed at $t+1$, while F_{t+1}^D, S_{t+1}^D and θ_{t+1}^D represent the values taken on by F_{t+1}, S_{t+1} and θ_{t+1} if a devaluation occurs at date $t+1$. Since F_{t+1} is realized prior to S_{t+1}, it follows that $1/F_{t+1}^D = 1/F_{t+1}^I = 1/F_{t+1} = (1-q)/S_{t+1}^I + q/S_{t+1}^D$. Using this result (72) implies $\theta_{t+1}^D = \theta_{t+1}^I = \theta_{t+1}$. From (65) and (55) this implies that the value of $c_{t+1} = 1/\theta_{t+1}$ does not depend on whether a devaluation occurs or not at $t+1$.

Notice that (58) implies $V_1^D(a_{t+1}, M_{t+1}, x_{t+1}^h) = R/c_{t+1}$. Substituting this, (65) and (69) into (66) we get $c_t = c$ for all t. Next we substitute (59), (70), (55), (65), and our previous results into (67) to get

$$M_{t+1} = \frac{\beta\phi c}{\left(\frac{1}{S_t} - \beta\frac{1}{F_{t+1}}\right)}, \text{ for } t < T. \tag{73}$$

To solve for the equilibrium sequences of S_t we note that the government uses the money supply rule $M_t = M^I$ for $t \leq T$, and $M_{T+j} = \gamma^j(M_T - \chi S_{T-1})$ for $j \geq 1$. From (53) and (61) we have $S_t^{-1} = \beta S_{t+1}^{-1} + \beta\phi c^F M_{t+1}^{-1}$ for $t \geq T$. Iterating forward on this equation, using the money supply rule, and imposing $\lim_{j\to\infty}\beta^j S_{t+j}^{-1} = 0$,[1] we obtain $S_{T+j} = \gamma^j S_T$ for $j \geq 0$, and $S_T = R(\gamma-\beta)(M^I - \chi S_{T-1})/\phi c$. From (73) we have $S_t = S^I$ for $t < T$, and $S_T = S^D$ where $S^D = R(\gamma-\beta)(M^I - \chi S^I)/\phi c$ and $S^I = R[1-\beta(1-q)]/(\phi c/M^I + q/S^D)$. Thus, given $M^D = M^I - \chi S^I$, we have $M^D/S^D = \phi c/(R\gamma - 1)$. We have proven the Lemma.

We note that in equilibrium

$$w_t = \begin{cases} w^I & \text{for } t \leq T \\ w^F & \text{for } t > T \end{cases} \quad \pi_t = \begin{cases} \pi^I & \text{for } t < T \\ \pi^D & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \quad \tau_t = \begin{cases} \tau & \text{for } t \neq T \\ \tau + \tau^D & \text{for } t = T. \end{cases}$$

[1]This condition is implied by the transversality condition applying to real balances.

Since the household does not know when the devaluation will take place, in every period t in which the devaluation has not yet taken place, the household will set x^h_{t+1} and a_{t+1} so that (63) holds for $t+1$ rather than T. This implies that for all $t \leq T$,

$$x^h_t = \frac{-Ra_t + \frac{R+\phi}{R-1}c - \left(w^I + \frac{1}{R-1}w^F\right) - \pi^D + \frac{R}{R-1}\tau + \tau^D - \frac{M^I}{S^D} - \chi\left(1 - \frac{S^I}{S^D}\right)}{\left(\frac{1}{F} - \frac{1}{S^D}\right)} \tag{74}$$

Substituting this into (64) we get $a_t = (Ra_{t-1} - \kappa)/(1-q)$, $t \leq T$, where

$$\begin{aligned}\kappa \;=\; & \left(1 - q + q\frac{R+\phi}{R-1}\right)c + \left[(R-1+q)\frac{\tau}{R-1} + q\tau^D\right] - \left(w^I + \frac{q}{R-1}w^F\right) - \\ & [(1-q)\pi^I + q\pi^D] - q\left[\frac{M^I}{S^D} + \chi\left(1 - \frac{S^I}{S^D}\right)\right].\end{aligned}$$

If we use (48) to obtain the household's lifetime budget constraint at any $t > T$ we have $a_t = a^F$ where

$$a^F = \frac{(R+\phi)\gamma - (1+\phi)}{(R-1)(R\gamma - 1)}c + \frac{1}{R-1}(\tau - w^F).$$

Hence, under the fixed exchange rate regime, once the agent has set a_{t+1}, this implies that $E_t a_{t+j} = R^{j-1}a_{t+1} + (1+R+\cdots+R^{j-2})(qa^F - \kappa)$ for $j \geq 2$. If we impose $\lim_{j\to\infty} E_t R^{-j} a_{t+j} = 0$ we get $a_t = a^I = (\kappa - qa^F)/(R-1)$ for $t \leq T$. Given the law of motion above, this implies $a^I = a^F = a_0 = \kappa/(R-1+q)$. Furthermore $x^h_t = x^h$ for $t \leq T$, where x^h is obtained by substituting $a_t = a_0$ into (74).

The government's flow budget constraint for $t \neq T$ is $f_{t+1} = Rf_t + (M_{t+1} - M_t)/S_t + \tau_t - g_t$. For $t = T$, the government budget constraint is

$$f_{T+1} = Rf_T + (M_{T+1} - M_T)/S_T - \chi(1 - S_{T-1}/S_T) - \Gamma + \tau_T - g_T.$$

This implies that the government's lifetime budget constraint at date T is

$$f_T = R^{-1}\left[\chi\left(1 - \frac{S_{T-1}}{S_T}\right) + \Gamma + \sum_{j=0}^{\infty} R^{-j}(g_{T+j} - \tau_{T+j}) - \sum_{j=0}^{\infty} R^{-j}\frac{M_{T+1+j} - M_{T+j}}{S_{T+j}}\right]. \tag{75}$$

If we combine this with (62) we get

$$a_T + f_T = R^{-1}\left[\sum_{j=0}^{\infty} R^{-j}(c_{T+j} + g_{T+j} - w_{T+j} - \pi_{T+j}) + \Gamma - x^h_T\left(\frac{1}{F_T} - \frac{1}{S_T}\right)\right].$$

We assume that $f_t = f_0 = (g-\tau)/(R-1)$, for $t \leq T$, and that $g_t = g$, $\forall t$. We also use the facts that $a_T = a_0$, $c_t = c$, $\forall t$, $1/F_T = 1/F = (1-q)/S^I + q/S^D$, $S_T = S^D$ and $x^h_T = x^h$, as well as the sequences for w_t and π_t given above to obtain

$$a_0 + f_0 = \frac{1}{R-1}(c+g) - R^{-1}\left[w^I + \frac{1}{R-1}w^F + \pi^D - \Gamma + x^h\left(\frac{1}{F} - \frac{1}{S^D}\right)\right].$$

Since $\tau_t = \tau = g - (R-1)f_0$, for $t < T$, (64) implies

$$a_0 + f_0 = \frac{1}{R-1}(c+g) - \frac{1}{R-1}\left[w^I + \pi^I + x^h\left(\frac{1}{F} - \frac{1}{S^I}\right)\right]. \tag{76}$$

Combining these two equations we have

$$x^h = \left[w^I - w^F + R\pi^I - (R-1)\pi^D + (R-1)\Gamma\right] / \left[(R-1+q)\left(\frac{1}{S^I} - \frac{1}{S^D}\right)\right], \tag{77}$$

which is equivalent to (74).

Substituting (77) into (76) and noting that $(1-q)\pi^I + q\pi^D = 0$, we get

$$c = (R-1)(a_0+f_0) - g + (R-1+q)^{-1}\left[(R-1)w^I + qw^F - q(R-1)\Gamma\right], \tag{78}$$

which establishes (31). The expressions for S^D and S^I given above can be rewritten as

$$m^I = \frac{\phi c}{R\gamma - 1}\frac{S^D}{S^I} + \chi \tag{79}$$

$$m^I = \frac{\phi c}{R-1+q(1-S^I/S^D)}, \tag{80}$$

where $m^I = M^I/S^I$, and which establishes (39). Notice that our previous assumptions, (53) and (61) imply that (75) can be rewritten as

$$\tau^D + \frac{R}{R-1}\frac{\gamma-1}{R\gamma-1}\phi c = \chi + \Gamma. \tag{81}$$

A.2 Proofs of Propositions 3–5

We now establish Propositions 3–5, by solving equations (78)–(81). To prove Proposition 4 we note that we take q and γ as given and solve the 4 equations for 4 unknowns: c, m^I, S^I/S^D, and τ^D. To prove Proposition 3 we note that we take q as given and set $\tau^D = 0$, and solve the 4 equations for 4 unknowns: c, m^I, S^I/S^D, and γ. We start with Proposition 4 rather than Proposition 3 because we will use results from the proof of Proposition 4 in proving Proposition 3. Finally, when we turn to Proposition 5, we take q as given, and set $\tau^D = \Gamma = 0$. Then we solve for c, m^I, S^I/S^D, and γ.

Proposition 4. We prove Proposition 4 first as we can take the post-devaluation money growth rate, $\gamma > 1$, parametrically. Consider the four equations (78)–(81). Let $\sigma = S^I/S^D$. In the model with guarantees $w^I = A(1-q+q\sigma)/[(1-q)R+\delta+q\omega]$ and $w^F = A/[\gamma(R+\delta)]$. When there are government guarantees, banks will go bankrupt in the devaluation state and the government will pay $\Gamma = RL = R(d/S^I) = RFw^I/S^I$ to the banks' foreign creditors. Hence, we define the function $\Gamma(q) = RA/[(1-q)R+\delta+q\omega]$ for $q > 0$, and $\Gamma(q) = 0$ for $q = 0$. For convenience, we define the function $\bar{\Gamma}(q) = RA/[(1-q)R+\delta+q\omega]$ for $0 \leq q \leq 1$, so that $\bar{\Gamma}(0) = \lim_{0 \leftarrow q}\Gamma(q)$. This definition is useful because it equates the function at $q = 0$ to its limit as q approaches 0 from above. I.e. $\bar{\Gamma}(0) = \lim_{0 \leftarrow q}\bar{\Gamma}(q)$ whereas $\Gamma(0) \neq \lim_{0 \leftarrow q}\Gamma(q)$.

We rewrite (78) as $c = c^1(\sigma; q)$ where

$$c^1(\sigma; q) \equiv (R-1)(a_0 + f_0) - g + \frac{A}{R-1+q}\left[(R-1)\frac{1-q+q\sigma-qR}{(1-q)R+\delta+q\omega} + \frac{q}{\gamma(R+\delta)}\right] \quad (82)$$

Combining (80) and (79) we get $c = c^2(\sigma; q)$, where

$$c^2(\sigma; q) \equiv (\chi/\phi)\left[\frac{1}{R-1+q(1-\sigma)} - \frac{1}{(R\gamma-1)\sigma}\right]^{-1}. \quad (83)$$

Given a solution for c and σ the required fiscal reform is obtained from (81) as

$$\tau^D = \chi + \Gamma(q) - \frac{R}{R-1}\frac{\gamma-1}{R\gamma-1}\phi c. \quad (84)$$

To show that there is an equilibrium with self-fulfilling attacks, we will demonstrate that $c^1(\sigma; q) = c^2(\sigma; q) > 0$ for some $\sigma < 1$ and $q > 0$. To do this we make reference to Figure 1.

The first step in our proof is to characterize the curves $c^1(\sigma; q)$ and $c^2(\sigma; q)$. The straight line is $c^1(\sigma; q)$. It is clear from (82) that $c^1_\sigma(\sigma; q) > 0$ for $q > 0$. We have $\lim_{q\to 0} c^1(\sigma; q) = c_S$, $\forall\sigma$, where c_S is the level of consumption under the sustainable fixed exchange rate regime. For $\sigma \leq 1$

$$\frac{1-q+q\sigma-qR}{(1-q)R+\delta+q\omega} < \frac{1}{R+\delta} \text{ and } \frac{1}{\gamma(R+\delta)} < \frac{1}{R+\delta}.$$

It follows that $c^1(\sigma; q) < c_S$ for all $q > 0$. Furthermore it is straightforward to show that $c^1_q(\sigma; q) < 0$ for all σ.

The curve in Figure 1 is $c^2(\sigma; q)$. We note that $c^2(\sigma; q) > 0$ only for $\sigma > \underline{\sigma}(q) = (R-1+q)/(R\gamma-1+q)$. For $\sigma > \underline{\sigma}(q)$, $c^2_\sigma(\sigma; q) < 0$, and $\lim_{\underline{\sigma}(q)\leftarrow\sigma} c^2(\sigma; q) = \infty$. Also $c^2(1; q) = c^2(1) = (\chi/\phi)(R-1)(R\gamma-1)/[R(\gamma-1)] > 0$. We have

$$\lim_{q\to 0} c^2(\sigma; q) = (\chi/\phi)\left[\frac{1}{R-1} - \frac{1}{(R\gamma-1)\sigma}\right]^{-1} < c^2(\sigma; q)$$

for $\underline{\sigma}(0) < \underline{\sigma}(q) < \sigma < 1$. We also note that it is straightforward to show that $c^2_q(\sigma; q) > 0$ for $\sigma < 1$, while $c^2_q(\sigma; q) = 0$ for $\sigma = 1$.

Now that we have characterized c^1 and c^2 we conclude our proof. For fixed q, Figure 1 makes clear that a necessary and sufficient condition for a solution such that $c > 0$ and $\sigma < 1$ is

$$c^1(1; q) > c^2(1) > 0. \quad (85)$$

Since $\lim_{0\leftarrow q} c^1(1, q) = c_S$, (85) will be satisfied for sufficiently small q as long as $c_S > c^2(1)$ or, equivalently, as stated in the proposition, if $\chi < R(\gamma-1)m_S/(R\gamma-1)$, where $m_S = \phi c_S/(R-1)$ is the level of real balances under the sustainable fixed exchange rate regime.

Notice that since $c < c_S$ and $\chi < R(\gamma-1)m_S/(R\gamma-1)$, (84) implies that $\tau^D < \Gamma(q)$ in equilibrium.

Proposition 3. We rewrite (78)–(81) as $c = c^1(\gamma, \sigma; q)$, where

$$c^1(\gamma, \sigma; q) \equiv (R-1)(a_0+f_0) - g + \frac{A}{R-1+q}\left[(R-1)\frac{1-q+q\sigma-qR}{(1-q)R+\delta+q\omega} + \frac{q}{\gamma(R+\delta)}\right] \quad (86)$$

$$c = c^2(\gamma, \sigma; q) \equiv (\chi/\phi)\left[\frac{1}{R-1+q(1-\sigma)} - \frac{1}{(R\gamma-1)\sigma}\right]^{-1}, \quad (87)$$

$$c = c^3(\gamma; q) = \frac{1}{\phi}\frac{R-1}{R}\frac{R\gamma-1}{\gamma-1}[\chi + \bar{\Gamma}(q)]. \quad (88)$$

The additional complication of this proposition is that we now have 3 nonlinear equations in 3 unknowns, the additional unknown being γ. Our proof is structured as follows. First, we borrow the analysis from the proof of Proposition 4 to solve the equation $c^1(\gamma, \sigma; q) = c^2(\gamma, \sigma; q)$ for (c, σ) given $(\gamma; q)$. We will denote the implied solution for c as $C^1(\gamma; q)$. This is symmetric to the second part of our proof which examines $c^3(\gamma; q)$. During these two steps we characterize C^1 and c^3 using Figure 2. Our proof concludes by showing that there are pairs $(\gamma; q)$ with $\gamma > 1$, and $0 < q < 1$, such that $C^1(\gamma; q) = c^3(\gamma; q)$.

Step 1. Taking γ as given, we denote the value of σ for which $c^1(\gamma, \sigma; q) = c^2(\gamma, \sigma; q)$ as $\sigma = \sigma(\gamma; q)$. The value of c for which $c^1(\gamma, \sigma; q) = c^2(\gamma, \sigma; q)$ is given by $c = C^1(\gamma; q) = c^1(\gamma, \sigma(\gamma; q); q) = c^2(\gamma, \sigma(\gamma; q); q)$.

It is useful to characterize σ and C^1. First, we examine their derivatives with respect to q. Recall, from Proposition 4, that $c^1_q < 0$, for all $0 \le \sigma \le 1$. Furthermore, $c^2_q > 0$ for $\sigma < 1$, while $c^2_q = 0$ for $\sigma = 1$. We also have $c^1_\sigma = 0$ for $q = 0$, $c^1_\sigma > 0$ for $q > 0$ and $c^2_\sigma < 0$ for all q. By totally differentiating $c^1 = c^2$ with respect to σ and q, we can use these facts to show that $\sigma_q = (c^2_q - c^1_q)/(c^1_\sigma - c^2_\sigma) > 0$. We cannot unambiguously sign $C^1_q = c^1_q + c^1_\sigma \sigma_q = (c^1_q c^2_\sigma - c^1_\sigma c^2_q)/(c^2_\sigma - c^1_\sigma)$. Over some range, however, C^1 must be decreasing in q because $C^1 = c_S$ for $q = 0$, $C^1 < c_S$ for all $q > 0$, and $C^1_q|_{q=0} = c^1_q|_{q=0} < 0$.

Second, we characterize the range of q for which σ and C^1 are defined. Recall from the proof of Proposition 4, that (i) $c^2(\gamma, 1; q)$ does not depend on q so we denote it as $c^2(\gamma, 1)$ and (ii) σ and C^1 are defined for any q such that $c^1(\gamma, 1; q) > c^2(\gamma, 1)$. There are two possibilities implied by these facts: σ and C^1 are defined either (i) for all $0 \le q \le \bar{q}(\gamma) = 1$, or (ii) for all $0 \le q < \bar{q}(\gamma) < 1$ where $\bar{q}(\gamma)$ is the value of q for which $c^1(\gamma, 1; q) = c^2(\gamma, 1)$. In the latter case, when $\bar{q}(\gamma) < 1$, we also have the result that the lowest value of C^1 for that γ is $c^2(\gamma, 1)$.

Third, we characterize the derivatives of σ and C^1 with respect to γ. From (86), $c^1_\gamma = 0$ for $q = 0$ and $c^1_\gamma < 0$ for $q > 0$. From (87), $c^2_\gamma < 0$ for all q. Hence, it is not possible to sign $\sigma_\gamma = (c^2_\gamma - c^1_\gamma)/(c^1_\sigma - c^2_\sigma)$. However, it is possible to sign C^1_γ since $C^1_\gamma = c^1_\gamma + c^1_\sigma \sigma_\gamma = (c^1_\gamma c^2_\sigma - c^1_\sigma c^2_\gamma)/(c^2_\sigma - c^1_\sigma)$. This implies $C^1_\gamma = 0$ for $q = 0$ and $C^1_\gamma < 0$ for $q > 0$.

Finally, we characterize the range of γ for which σ and C^1 are defined. The lower bound on values of γ for which σ and C^1 are defined is

$$\underline{\gamma} = \frac{Rc_S - (\chi/\phi)(R-1)}{R[c_S - (\chi/\phi)(R-1)]},$$

because $c^2(\underline{\gamma}, 1) = c_S$. Hence $\bar{q}(\underline{\gamma}) = 0$. For $\gamma > \underline{\gamma}$, $0 < (R-1)\chi/\phi < c^2(\gamma, 1) < c_S$. This implies that there is no upper limit on γ for which σ and C^1 are defined, since $\lim_{0 \leftarrow q} c^1(\gamma, 1; q) = c_S > c^2(\gamma, 1)$ for all $\gamma > \underline{\gamma}$. So we have $\bar{q}(\gamma) > 0$ for all $\gamma > \underline{\gamma}$.

C^1 is illustrated as a function of q in Figure 2 using these results. Notice that $C^1(\underline{\gamma}; q)$ is only defined at $q = 0$ and equals c_S. For any $\gamma > \underline{\gamma}$, such as γ^*, $C^1(\gamma; q) = c_S$ for $q = 0$, and $C^1(\gamma; q) < c_S$ for $0 < q < \bar{q}(\gamma)$. The figure is consistent with $C^1_\gamma < 0$, but since we cannot sign C^1_q, we illustrate C^1 as a non-monotonic function of q, except in the neighborhood of $q = 0$ since $C^1_q(\gamma; q)|_{q=0} = c^1_q|_{q=0} < 0$. In drawing the figure we have also used the otherwise unimportant fact that $c^1_{qq}|_{q=0} < 0$.

Step 2. Now consider $c^3(\gamma; q)$. It is defined for all $0 \leq q \leq 1$ and $\gamma > 1$. From (88), $c^3_\gamma < 0$ and $c^3_q > 0$. Given the assumption in the statement of the proposition that $\phi c_S/(R-1) < \chi + \Gamma(q)$ for some $q > 0$, it follows that $\phi c_S/(R-1) < \chi + \bar{\Gamma}(0)$. Hence, we can evaluate c^3 at

$$\gamma^* = \frac{R\phi c_S - (R-1)[\chi + \bar{\Gamma}(0)]}{R\phi c_S - R(R-1)[\chi + \bar{\Gamma}(0)]} > \underline{\gamma} > 1.$$

We get $c^3(\gamma^*; q) = c_S[\chi + \bar{\Gamma}(q)]/[\chi + \bar{\Gamma}(0)]$. This implies $c^3(\gamma^*, 0) = c_S$ and that $c^3(\gamma^*, q) > c_S$ for all $q > 0$. For any $\gamma < \gamma^*$, $c^3(\gamma, q) > c_S$ for all q. For any $\gamma > \gamma^*$, $c^3(\gamma, 0) < c_S$.

c^3 is illustrated as a function of q in Figure 2 using these results. In drawing the figure we have used the otherwise unimportant facts that $c^3_{qq} > 0$ and $c^3_{q\gamma}|_{q=0} < 0$.

Conclusion. With reference to Figure 2 it is clear that no equilibria exist for $\gamma < \gamma^*$. Consider, instead, any $\gamma > \gamma^*$. Clearly $c^3(\gamma, 0) < C^1(\gamma, 0) = c_S$. Furthermore, since c^3 is a continuous, increasing function of q, and is defined for all $0 \leq q \leq 1$, and since C^1 is a continuous function of q, a *sufficient* condition for C^1 to cross c^3 *at least* once for $0 < q < 1$ is $c^3(\gamma, 0) \geq C^1(\gamma, \bar{q}(\gamma))$.

Notice that when $\bar{q}(\gamma) < 1$, $C^1(\gamma, \bar{q}(\gamma)) = c^2(\gamma, 1)$ and the sufficient condition for an intersection becomes $c^3(\gamma, 0) \geq c^2(\gamma, 1)$; some algebra shows that this is equivalent to $\bar{\Gamma}(0) \geq 0$ which always holds. So for any $\gamma > \gamma^*$ with $\bar{q}(\gamma) < 1$, there is at least one $0 < q < 1$ for which an equilibrium exists with $c < c_S$ and $\sigma < 1$.

The only situation that remains for us to consider is the possibility that there are no $\gamma > \gamma^*$ for which $\bar{q}(\gamma) < 1$. That is $\bar{q}(\gamma) = 1$ for all $\gamma > \gamma^*$. This implies that $C^1(\gamma, \bar{q}(\gamma)) = C^1(\gamma, 1) = c^1(\gamma, \sigma(\gamma, 1); 1)$. Notice that $\lim_{\gamma^* \leftarrow \gamma} c^3(\gamma, 0) = c_S$. On the other hand,

$$\lim_{\gamma^* \leftarrow \gamma} c^1(\gamma, \sigma(\gamma, 1); 1) = c_S - A\left[\frac{R\gamma^* - 1}{R\gamma^*(R+\delta)} + \frac{R-1}{R}\frac{R - \sigma(\gamma^*, 1)}{\delta + \omega}\right] < c_S.$$

This implies that the sufficient condition for an intersection is satisfied, at least, for γ sufficiently close to γ^*. For these γ, there is at least one $0 < q < 1$ for which an equilibrium exists with $c < c_S$ and $\sigma < 1$.

Proposition 5 *Suppose that there are no government guarantees to bank's foreign creditors. In addition, suppose that agents believe that in the event of a devaluation taxes and* *government spending remain constant, while the growth rate of money, γ, is constant. Then, self-fulfilling speculative attacks cannot occur.*

The basic idea used in our proof is that absent government guarantees Γ equals 0. So, if a self-fulfilling speculative attack succeeded, the government's liabilities would increase only by χ, which is the loss of reserves at the time of the attack. Since the government's intertemporal budget constraint held prior to the attack, this means that the value of seigniorage revenues associated with a particular value of γ would have to exactly equal χ in equilibrium. Under the assumptions of our model this is not possible unless $S^D/S^I = 1$.

Proposition 5 does not rule out the existence of self-fulfilling speculative attacks for more complicated paths of the money supply after the devaluation, or different money demand formulations.[1] Still, the proposition establishes the presumption that eliminating guarantees makes self-fulfilling speculative attacks less likely.

Proposition 5. We focus, again, on the four equations (78)–(81). Without guarantees $w^I = A(1-q+q\sigma)/(R+\delta)$, $w^F = A/[\gamma(R+\delta)]$ and $\Gamma = 0$. This means (78) becomes

$$c = (R-1)(a_0 + f_0) + \frac{A}{(R-1+q)(R+\delta)}\left[(R-1)(1-q+q\sigma) + q/\gamma\right] - g, \tag{89}$$

With $\tau^D = 0$, and $\Gamma = 0$, (81) is given by

$$\frac{R}{R-1}\frac{\gamma-1}{R\gamma-1}\phi c = \chi \tag{90}$$

Notice that (90) combined with (80) and (79) implies

$$\left[\frac{R}{R-1}(\gamma-1) + \frac{1}{\sigma}\right]\frac{\phi c}{R\gamma-1} = \frac{\phi c}{R-1+q(1-\sigma)} \tag{91}$$

It is easy to verify that for any $c > 0$, (91) implies $\sigma = 1$.[2]

B The Extended Model with Demand Deposits

B.1 The Banking Sector

B.1.1 Under the Fixed Exchange Rate Regime

In any period that begins under the fixed exchange rate we have

$$E(V^R) = E(\pi + R^b D^*) = \frac{R^a S^I L}{F} - \delta L + \frac{M + S^I D^*}{F} - R^d \frac{D^h + S^I L}{F}.$$

Notice that $E(V^R)$ does not depend on x so that the optimal hedging strategy can be found by minimizing $ECB \equiv E(\min\{V^R, R^b D^*\})$.

[1] Results in Obstfeld (1986) suggest that such self-fulfilling attacks are possible.

[2] Since (91) is a quadratic equation, there is another solution for σ, but it is negative.

No Government Guarantees

In the case where there are no government guarantees, the interest rate R^b will be set according to

$$RD^* = \sum_{s \in \{S^I, S^D\}} \Pr(S = s) \left\{ \begin{array}{ll} R^b D^* & \text{if } \pi(\cdot; s) \geq 0 \\ V^R(\cdot; s) - \omega L & \text{otherwise.} \end{array} \right\} \tag{92}$$

1. Given the rest of its portfolio, if the bank chooses x such that $\pi(\cdot; s) \geq 0$ for all s, then $R^b = R$. Given the definition of ECB we have $ECB = RD^*$.
2. If the bank chooses x such that $\pi(\cdot; S^D) \geq 0$ and $\pi(\cdot; S^I) < 0$ then

$$RD^* = (1 - q) \left[V^R(\cdot; S^I) - \omega L \right] + q R^b D^* \tag{93}$$

and

$$ECB = (1 - q) V^R(\cdot; S^I) + q R^b D^*.$$

It follows from (93) that

$$ECB = RD^* + (1 - q) \omega L.$$

3. If the bank chooses x such that $\pi(\cdot; S^I) \geq 0$ and $\pi(\cdot; S^D) < 0$ then

$$RD^* = (1 - q) R^b D^* + q \left[V^R(\cdot; S^D) - \omega L \right] \tag{94}$$

and

$$ECB = (1 - q) R^b D^* + q V^R(\cdot; S^D).$$

It follows from (94) that

$$ECB = RD^* + q \omega L.$$

4. If the bank chooses x such that $\pi(\cdot; s) < 0$ for all s then

$$RD^* = (1 - q) \left[V^R(\cdot; S^I) - \omega L \right] + q \left[V^R(\cdot; S^D) - \omega L \right] \tag{95}$$

and

$$ECB = (1 - q) V^R(\cdot; S^I) + q V^R(\cdot; S^D)$$

It follows from (95) that

$$ECB = RD^* + \omega L.$$

Obviously, the bank minimizes ECB by choosing x so that it is hedged in all states of the world. (We will establish that this is feasible below.) Hence

$$V = \frac{R^a S^I L}{F} - \delta L + \frac{M + S^I D^*}{F} - R^d \frac{D^h + S^I L}{F} - RD^*.$$

Now we use the fact that $D^h = M$ and $M + S^I D^* = \xi(D^h + S^I L)$ to substitute out M and D^*. We obtain

$$V = \left[\frac{(R^a - R^d + \xi) S^I}{F} - (\delta + \xi R) \right] L - \left[\frac{(R^d - \xi) S^I}{F} - R(1 - \xi) \right] \frac{D^h}{S^I}.$$

This means that the equilibrium interest rates, R^a and R^d must satisfy

$$\frac{(R^a - R^d + \xi)S^I}{F} = \delta + \xi R \qquad \frac{(R^d - \xi)S^I}{F} = R(1-\xi)$$

or

$$R^d = \xi + \frac{F}{S^I}R(1-\xi) \tag{96}$$

$$R^a = R^d + \frac{F}{S^I}\delta + \left(\frac{F}{S^I}R - 1\right)\xi = \frac{F}{S^I}(R+\delta). \tag{97}$$

Notice that at these interest rates

$$\begin{aligned}\pi^L &= \left[\frac{(R^a - R^d + \xi)S^I}{S} - (\delta + R\xi)\right]L - \left[\frac{\left(R^d - \xi\right)S^I}{S} - R(1-\xi)\right]\frac{D^h}{S^I} \\ &= (\frac{1}{S} - \frac{1}{F})F\left[(\delta + R\xi)L - R(1-\xi)\frac{D^h}{S^I}\right]\end{aligned}$$

The argument in square brackets is always positive since

$$(\delta + R\xi)L - R(1-\xi)D^h/S^I = (\delta + R\xi)L - R(\xi L - D^*) = \delta L + RD^*.$$

Thus, $\pi^L > 0$ if $S = S^I$, $\pi^L < 0$ if $S = S^D$. Profits from hedging are

$$\pi^H = -x\left(\frac{1}{S} - \frac{1}{F}\right).$$

Hence the bank can be fully hedged by setting

$$\begin{aligned}x &= F(\delta L + RD^*) \\ &= F[(\delta + R\xi)L - R(1-\xi)D^h/S^I] \\ &= F[(\delta + R\xi)L - R(1-\xi)M/S^I].\end{aligned}$$

With Government Guarantees

Under guarantees, if there is default when $S = S^D$, foreign creditors receive $\max\left\{V^R(\cdot; S^D) - \omega L, RD^*\right\}$.

1. As before, if the bank chooses x such that $\pi(\cdot; s) \geq 0$ for all s, then $R^b = R$ and $ECB = RD^*$.

2. As before, if the bank chooses x such that $\pi(\cdot; S^D) \geq 0$ and $\pi(\cdot; S^I) < 0$ then

$$RD^* = (1-q)\left[V^R(\cdot; S^I) - \omega L\right] + qR^bD^* \tag{98}$$

and

$$ECB = (1-q)V^R(\cdot; S^I) + qR^bD^*.$$

It follows from (98) that

$$ECB = RD^* + (1-q)\omega L.$$

3. If the bank chooses x such that $\pi(\cdot; S^I) \geq 0$ and $\pi(\cdot; S^D) < 0$ then

$$RD^* = (1-q)R^b D^* + q \max\left\{V^R(\cdot; S^D) - \omega L, RD^*\right\} \tag{99}$$

Notice that $V^R(\cdot; S^D) < RD^*$, since otherwise $\pi(\cdot; S^I) \geq 0$ and $\pi(\cdot; S^D) < 0$ would imply $V^R(\cdot; S^I) > V^R(\cdot; S^D) \geq RD^*$, and this would imply that the bank was fully-hedged. Hence (99) implies $R^b = R$ and

$$ECB = (1-q)RD^* + qV^R(\cdot; S^D).$$

4. If the bank chooses x such that $\pi(\cdot; s) < 0$ for all s then

$$RD^* = (1-q)\left[V^R(\cdot; S^I) - \omega L\right] + q \max\left\{V^R(\cdot; S^D) - \omega L, RD^*\right\}. \tag{100}$$

This means that either $RD^* = E(V^R) - \omega L$ (if $V^R(\cdot; S^D) - \omega L \geq RD^*$) or $RD^* = V^R(\cdot; S^I) - \omega L$ (if $V^R(\cdot; S^D) - \omega L < RD^*$). In the first case

$$ECB = E(V^R) = RD^* + \omega L.$$

In the second case,

$$\begin{aligned} ECB &= E(V^R) = (1-q)V^R(\cdot; S^I) + qV^R(\cdot; S^D) \\ &= (1-q)RD^* + (1-q)\omega L + qV^R(\cdot; S^D). \end{aligned}$$

These results imply that strategy 3 is dominant whenever it is feasible, because $V^R(\cdot; S^D) < RD^*$ and $ECB < RD^*$. In addition, ECB is minimized by minimizing $V^R(\cdot; S^D)$. This is achieved by setting x so that $V^R(\cdot; S^D) = \omega L$, i.e.

$$x = (1-q)^{-1}\left(\frac{1}{S^I} - \frac{1}{S^D}\right)^{-1}\left[(\omega + \delta)L - \frac{(R^a - R^d + \xi)S^I L + (\xi - R^d)D^h}{S^D}\right].$$

In this case, $ECB = (1-q)RD^* + q\omega L$ and

$$\begin{aligned} V &= \frac{(R^a - R^d + \xi)S^I L}{F} - \delta L + \frac{(\xi - R^d)D^h}{F} - (1-q)RD^* - q\omega L \\ &= \left[\frac{(R^a - R^d + \xi)S^I}{F} - \delta - (1-q)\xi R - q\omega\right] L - \left[\frac{(R^d - \xi)S^I}{F} - (1-q)(1-\xi)R\right]\frac{D^h}{S^I}. \end{aligned}$$

So the equilibrium interest rates satisfy

$$\frac{(R^a - R^d + \xi)S^I}{F} = \delta + (1-q)\xi R + q\omega \qquad \frac{(R^d - \xi)S^I}{F} = (1-q)(1-\xi)R$$

or

$$R^d = \xi + \frac{F}{S^I}(1-q)(1-\xi)R \tag{101}$$

$$\begin{aligned} R^a &= R^d - \xi + \frac{F}{S^I}[\delta + (1-q)\xi R + q\omega] \\ &= \frac{F}{S^I}[(1-q)R + \delta + q\omega]. \end{aligned} \tag{102}$$

B.1.2 Under the Floating Exchange Rate Regime

In this case there is no uncertainty. We have $1/S_t = 1/F_t = 1/(\gamma S_{t-1})$. Hedging is irrelevant as hedging profits are always zero regardless of the hedge position. We also have

$$\pi_t^L = \left[\frac{R^a - R^d + \xi}{\gamma} - (\delta + \xi R)\right] L - \left[\frac{R^d - \xi}{\gamma} - R(1-\xi)\right] \frac{D_t^h}{S_{t-1}}.$$

This means that

$$\frac{R^a - R^d + \xi}{\gamma} = \delta + \xi R \qquad \frac{R^d - \xi}{\gamma} = R(1-\xi)$$

or

$$R^d = \xi + \gamma(1-\xi)R \tag{103}$$

$$R^a = R^d - \xi + \gamma(\delta + \xi R) = \gamma(\delta + R). \tag{104}$$

B.2 Firms

Firms produce output Ah, using labor, h. At the beginning of each period they enter into contracts with workers at the real wage rate w, so their wage bill in dollar terms is wh. Firms borrow d pesos from banks at the net nominal interest rate $R^a - R^d$ (since banks pay interest on the deposits of the firms). So the firm's net liability at the end of the period from these loans is $(R^a - R^d)d/S$. Firms pay workers before they receive interest on their deposits loans.

B.2.1 Under the Fixed Exchange Rate

To ensure that they have sufficient funds on hand to pay workers wh dollars the firms hedge. We use x^f to denote the amount of local currency sold forward by the firm. Hedging profits are $x_f(1/F - 1/S)$.

The firm's net dollar profits at the end of the period are

$$\pi_f = Ah - wh - (R^a - R^d)d/S + x_f(1/F - 1/S).$$

Firms maximize $E(\pi_f) = Ah - wh - (R^a - R^d)d/F$ subject to the constraint that they have sufficient pesos on hand to pay their wage bill in advance: $Swh \leq d + Sx_f\,(1/F - 1/S)$, for all S. The two constraints imply that $d = x_f = Fwh$. So $E(\pi_f) = Ah - (1 + R^a - R^d)wh$. The firm's first order condition for labor is $A = (1 + R^a - R^d)w$. Realized profits are given by $\pi_f = Ah(1 - F/S)$ with $E(\pi_f) = 0$.

B.2.2 Under the Floating Exchange Rate Regime

There is no uncertainty under the float so firms simply set $d = Swh$, where S is the value of the exchange rate firms know will obtain at the end of the period. A firm's profits are given by

$$\pi_f = Ah - wh - (R^a - R^d)d/S = [A - (1 + R^a - R^d)w]h.$$

The firm's first order condition for labor is $A = (1 + R^a - R^d)w$. Realized profits are $\pi_f = 0$.

B.3 Households

B.3.1 Periods that Follow a Speculative Attack

We let T denote the time period in which the economy moves to a floating exchange rate regime. For $t \geq T+1$, the household solves the following dynamic programming problem:

$$V^F(a_t, D_t^h) = \max_{c_t, a_{t+1}, D_{t+1}^h} \left[\log c_t + \phi \log \frac{D_t^h}{S_t} + \beta V^F(a_{t+1}, D_{t+1}^h) \right]$$

subject to

$$a_{t+1} = Ra_t + w_t + \pi_t - \tau_t - c_t - (D_{t+1}^h - R_t^d D_t^h)/S_t. \tag{105}$$

The first order and envelope conditions are: $1/c_t = \theta_t$

$$\beta V_1^F(a_{t+1}, D_{t+1}^h) = \theta_t \tag{106}$$

$$\beta V_2^F(a_{t+1}, D_{t+1}^h) = \theta_t / S_t \tag{107}$$

$$V_1^F(a_t, D_t^h) = \theta_t R \tag{108}$$

$$V_2^F(a_t, D_t^h) = \phi / D_t^h + R_t^d \theta_t / S_t, \tag{109}$$

where θ_t is the Lagrange multiplier on the budget constraint.

Substituting (106) into (108) and noting that $\beta = 1/R$, we have $\theta_t = \theta_{t+1}$. This implies that $c_t = c_{t+1} = c^F$ for $t \geq T+1$. Using this fact and substituting (109) into (107) we have

$$\frac{D_{t+1}^h}{S_{t+1}} = \frac{\beta \phi c^F}{S_{t+1}/S_t - \beta R_{t+1}^d}, \text{ for } t \geq T+1. \tag{110}$$

B.3.2 The Period in which a Speculative Attack Occurs

In period T households face the following dynamic programming problem

$$V^D(a_T, D_T^h, x_T^h) = \max_{c_T, a_{T+1}, D_{T+1}^h} \left[\log c_T + \phi \log \frac{D_T^h}{S_T} + \beta V^F(a_{T+1}, D_{T+1}^h) \right]$$

subject to

$$a_{T+1} = Ra_T + w_T + \pi_T - \tau_T - c_T - \frac{D_{T+1}^h - R_T^d D_T^h}{S_T} + \chi \left(1 - \frac{S_{T-1}}{S_T}\right) + x_T^h \left(\frac{1}{F_T} - \frac{1}{S_T}\right). \tag{111}$$

The first order and envelope conditions are

$$c_T^{-1} = \theta_T \tag{112}$$

$$\beta V_1^F(a_{T+1}, D_{T+1}^h) = \theta_T \tag{113}$$

$$\beta V_2^F(a_{T+1}, D_{T+1}^h) = \theta_T / S_T \tag{114}$$

$$V_1^D(a_T, D_T^h, x_T^h) = R\theta_T \tag{115}$$

$$V_2^D(a_T, D_T^h, x_T^h) = \phi/D_T^h + \theta_T R_T^d/S_T \tag{116}$$

$$V_3^D(a_T, D_T^h, x_T^h) = \theta_T\left(1/F_T - 1/S_T\right). \tag{117}$$

Notice that (108) implies that $V_1^F(a_{T+1}, D_{T+1}^h) = R/c^F$. Since $\beta = 1/R$ combining (112) and (113) we then obtain $c_T = c^F$. >From (109) we have $V_2^F(a_{T+1}, D_{T+1}^h) = \phi/D_{T+1}^h + R_{T+1}^d/(c^F S_{T+1})$. Hence from (114) we have

$$\frac{D_{T+1}^h}{S_{T+1}} = \frac{\beta\phi c^F}{S_{T+1}/S_T - \beta R_{T+1}^d}. \tag{118}$$

To solve for c^F we iterate on (105) and combine it with (111) to obtain

$$\begin{aligned} a_T \;&=\; R^{-1}\sum_{j=0}^{\infty} R^{-j}(c_{T+j} - w_{T+j} - \pi_{T+j} + \tau_{T+j}) + R^{-1}\sum_{j=0}^{\infty} R^{-j}\frac{D_{T+1+j}^h - R_{T+j}^d D_{T+j}^h}{S_{T+j}} - \\ &\quad R^{-1}[\chi(1 - S_{T-1}/S_T) + x_T^h(1/F_T - 1/S_T)]. \end{aligned} \tag{119}$$

where we have imposed $\lim_{j\to\infty} R^{-j}a_{t+j} = 0$. Using $c_t = c^F$, for $t \geq T$, (110) and (118):

$$\begin{aligned} a_T \;&=\; (1+\beta\phi)c^F/(R-1) - R^{-1}\sum_{j=0}^{\infty} R^{-j}(w_{T+j} + \pi_{T+j} - \tau_{T+j}) - R^{-1}\frac{R_T^d D_T^h}{S_T} - \\ &\quad R^{-1}[\chi(1 - S_{T-1}/S_T) + x_T^h(1/F_T - 1/S_T)] \end{aligned} \tag{120}$$

B.3.3 Periods in which the Exchange Rate is Fixed

For $t < T$, households solve the following dynamic programming problem

$$\begin{aligned} V^I(a_t, D_t^h, x_t^h) \;&=\; \max_{c_t, a_{t+1}, D_{t+1}^h, x_{t+1}^h} \{\log c_t + \phi\log(D_t^h/S_t) + \\ &\qquad \beta[(1-q)V^I(a_{t+1}, D_{t+1}^h, x_{t+1}^h) + qV^D(a_{t+1}, D_{t+1}^h, x_{t+1}^h)]\} \end{aligned}$$

subject to

$$a_{t+1} = Ra_t + w_t + \pi_t - \tau_t - c_t - (D_{t+1}^h - R_t^d D_t^h)/S_t + x_t^h\left(1/F_t - 1/S_t\right). \tag{121}$$

The first order and envelope conditions are

$$1/c_t = \theta_t \tag{122}$$

$$\beta(1-q)V_1^I(a_{t+1}, D_{t+1}^h, x_{t+1}^h) + \beta q V_1^D(a_{t+1}, D_{t+1}^h, x_{t+1}^h) = \theta_t \tag{123}$$

$$\beta(1-q)V_2^I(a_{t+1}, D_{t+1}^h, x_{t+1}^h) + \beta q V_2^D(a_{t+1}, D_{t+1}^h, x_{t+1}^h) = \theta_t/S_t \tag{124}$$

$$(1-q)V_3^I(a_{t+1}, D_{t+1}^h, x_{t+1}^h) + qV_3^D(a_{t+1}, D_{t+1}^h, x_{t+1}^h) = 0 \tag{125}$$

$$V_1^I(a_t, D_t^h, x_t^h) = \theta_t R \tag{126}$$

$$V_2^I(a_t, D_t^h, x_t^h) = \phi/D_t^h + \theta_t R_t^d/S_t \tag{127}$$

$$V_3^I(a_t, D_t^h, x_t^h) = \theta_t \left(1/F_t - 1/S_t\right). \tag{128}$$

If we substitute (117) and (128) into (125) we obtain

$$(1-q)\theta_{t+1}^I \left(1/F_{t+1}^I - 1/S_{t+1}^I\right) + q\theta_{t+1}^D \left(1/F_{t+1}^D - 1/S_{t+1}^D\right) = 0. \tag{129}$$

Here F_{t+1}^I, S_{t+1}^I and θ_{t+1}^I represent the values taken on by F_{t+1}, S_{t+1} and θ_{t+1} if the exchange rate remains fixed at $t+1$, while F_{t+1}^D, S_{t+1}^D and θ_{t+1}^D represent the values taken on by F_{t+1}, S_{t+1} and θ_{t+1} if a devaluation occurs at date $t+1$. Since F_{t+1} is realized prior to S_{t+1}, it follows that $1/F_{t+1}^D = 1/F_{t+1}^I = 1/F_{t+1} = (1-q)/S_{t+1}^I + q/S_{t+1}^D$. Using this result (129) implies $\theta_{t+1}^D = \theta_{t+1}^I = \theta_{t+1}$. From (112) this implies that the value of $c_{t+1} = 1/\theta_{t+1}$ does not depend on whether a devaluation occurs or not at $t+1$.

Notice that (115) implies $V_1^D(a_{t+1}, D_{t+1}^h, x_{t+1}^h) = R/c_{t+1}$. Substituting this, (122) and (126) into (123) we get $c_t = c$ for all t. Next we substitute (116), (127), (112), (122), and our previous results into (124) (noting that R_{t+1}^d and D_{t+1}^h are determined at time t, so they cannot depend on the outcome for the exchange rate regime at date $t+1$) to get

$$D_{t+1}^h = \frac{\beta\phi c}{\left(1/S_t - \beta R_{t+1}^d/F_{t+1}\right)}, \text{ for } t < T. \tag{130}$$

B.4 Properties of the Equilibrium

B.4.1 Some Simple Results

To solve for the equilibrium sequences of S_t we note that the government uses the money supply rule $M_t = M^I$ for $t \leq T$, and $M_{T+j} = \gamma^j(M_T - \chi S_{T-1})$ for $j \geq 1$. We also have the equilibrium condition $M_t = D_t^h$ for all t. Using (103), we also have $R_{t+1}^d = R_D^d = \xi + \gamma(1-\xi)R$ for $t \geq T$. So, from (110) and (118) we have $S_t^{-1} = \beta R_D^d S_{t+1}^{-1} + \beta\phi c M_{t+1}^{-1}$ for $t \geq T$. Iterating forward on this equation, using the money supply rule, and imposing $\lim_{j\to\infty} \beta^j S_{t+j}^{-1} = 0$,[3] we obtain $S_{T+j} = \gamma^j S_T$ for $j \geq 0$, and

$$S_T = R(\gamma - \beta R_D^d)(M_T - \chi S_{T-1})/(\phi c). \tag{131}$$

Above, we used the notation S^I to denote S_t for $t < T$. We also used S^D to denote S_T. So (131) implies that

$$S^D = R(\gamma - \beta R_D^d)(M_T - \chi S^I)/(\phi c). \tag{132}$$

Then (130) implies that $S^I = [R - R_I^d(1-q)]/(\phi c/M^I + R_I^d q/S^D)$ where R_I^d is either given by (96) or (101) depending on whether or not there are government guarantees. It will also be convenient below to note that (118) can be rewritten as

$$M^D/S^D = \phi c/(R\gamma - R_D^d). \tag{133}$$

[3] This condition is implied by the transversality condition applying to real balances.

We note that in equilibrium

$$w_t = \begin{cases} w^I & \text{for } t \le T \\ w^F & \text{for } t > T \end{cases} \qquad \pi_t = \begin{cases} \pi^I & \text{for } t < T \\ \pi^D & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \qquad \tau_t = \begin{cases} \tau & \text{for } t \ne T \\ \tau + \tau^D & \text{for } t = T. \end{cases}$$

The firm's first order condition implies that $w_t = A/(1 + R_t^a - R_t^d)$. We also have $R_t^a = R_I^a$ for $t \le T$ [given by (97) or (102)], $R_t^a = R_D^a$ for $t > T$ [given by (104)], $R_t^d = R_D^a$ for $t \le T$ [given by (96) or (101)] and $R_t^d = R_D^d$ for $t > T$ [given by (103)]. So

$$\begin{aligned} w^I &= A/(1 + R_I^a - R_I^d) \\ w^F &= A/(1 + R_D^a - R_D^d). \end{aligned}$$

We have $h = 1$ so $\pi_f = A(1 - F/S)$. Hence

$$\begin{aligned} \pi^I &= A(1 - F/S^I) = -qA\frac{1 - S^I/S^D}{1 - q + qS^I/S^D} \\ \pi^D &= A(1 - F/S^D) = (1 - q)A\frac{1 - S^I/S^D}{1 - q + qS^I/S^D}. \end{aligned}$$

B.4.2 Solving for Consumption

In every period t in which the devaluation has not yet taken place (this will end up corresponding to $t < T$) the household will set x_{t+1}^h and a_{t+1} consistent with (120). That is, we can rewrite (120) as

$$\begin{aligned} a_{t+1} &= (1 + \beta\phi)c/(R - 1) - R^{-1}\sum_{j=0}^{\infty} R^{-j}(w_{t+1+j} + \pi_{t+1+j} - \tau_{t+1+j}) - R^{-1}\frac{R_{t+1}^d D_{t+1}^h}{S_{t+1}} - \\ &\qquad R^{-1}[\chi(1 - S_t/S_{t+1}) + x_{t+1}^h(1/F_{t+1} - 1/S_{t+1})] \end{aligned}$$

where we will substitute in the facts that conditional on a devaluation at $t + 1$, $w_{t+1} = w^I$, $w_{t+1+j} = w^F$ for $j \ge 1$, $\pi_{t+1} = \pi^D$, $\pi_{t+1+j} = 0$ for $j \ge 1$, $\tau_{t+1} = \tau + \tau^D$, $\tau_{t+1+j} = \tau$ for $j \ge 1$, $R_{t+1}^d = R_I^d$, $D_{t+1}^h = M^I$, $S_{t+1} = S^D$, $S_t = S^I$, and $1/F_{t+1} = 1/F$. We then get

$$\begin{aligned} a_{t+1} &= (1 + \beta\phi)c/(R - 1) - R^{-1}w^I - \frac{R^{-1}}{R - 1}w^F - R^{-1}\pi^D + \frac{1}{R - 1}\tau + R^{-1}\tau^D - \\ &\qquad R^{-1}\frac{R_I^d M^I}{S^D} - R^{-1}[\chi(1 - S^I/S^D) + x_{t+1}^h(1/F - 1/S^D)]. \end{aligned}$$

This equation applies for $t < T$. Moving the date subscripts back to t, it applies for $t \le T$, and we can solve for x_t^h:

$$x_t^h = \frac{-Ra_t + \frac{R+\phi}{R-1}c - w^I - \frac{1}{R-1}w^F - \pi^D + \frac{R}{R-1}\tau + \tau^D - \frac{R_I^d M^I}{S^D} - \chi(1 - S^I/S^D)}{(1/F - 1/S^D)} \tag{134}$$

for $t \le T$.

But, of course, for $t < T$, the household's choice of x_t^h (made at $t-1$) is also consistent with (121). Using $w_t = w^I$, $\pi_t = \pi^I$, $\tau_t = \tau$, $c_t = c$, $D_{t+1}^h = D_t^h = M^I$, $R_t^d = R_I^d$, $S_t = S^I$ and $1/F = (1-q)/S^I + q/S^D$ for $t < T$, and substituting (134) into (121) we obtain:

$$\begin{aligned} a_{t+1} \;=\; & \frac{1}{1-q} R a_t + \frac{1}{1-q} w^I + \frac{q}{1-q}\frac{1}{R-1} w^F + \pi^I + \frac{q}{1-q}\pi^D - \\ & \left(1 + \frac{q}{1-q}\frac{R}{R-1}\right)\tau - \frac{q}{1-q}\tau^D - \left[1 + \frac{q}{1-q}\frac{R+\phi}{R-1}\right] c + \\ & (R_I^d - 1)\frac{M^I}{S^I} + \frac{q}{1-q}\frac{R_I^d M^I}{S^D} + \frac{q}{1-q}\chi(1 - \frac{S^I}{S^D}), \end{aligned}$$

for $t < T$. Noting that the π^I and $q\pi^D/(1-q)$ terms cancel each other out, we can write this as

$$a_{t+1} = \frac{1}{1-q}(R a_t - \kappa), \qquad t < T, \tag{135}$$

where

$$\begin{aligned} \kappa \;=\; & \left(1 - q + q\frac{R+\phi}{R-1}\right) c + \left(1 - q + q\frac{R}{R-1}\right)\tau + q\tau^D - w^I - q\frac{1}{R-1} w^F - \\ & (1-q)(R_I^d - 1)\frac{M^I}{S^I} - q\frac{R_I^d M^I}{S^D} - q\chi(1 - \frac{S^I}{S^D}), \end{aligned}$$

So at any date t with $S_t = S^I$ the household knows that $a_{t+1} = (R a_t - \kappa)/(1-q)$. It also knows that there is a probability $1-q$ that $S_{t+1} = S^I$ and that $a_{t+2} = (R a_{t+1} - \kappa)/(1-q)$. But how is a_{t+2} determined if, with probability q, $S_{t+1} = S^D$? The expression for a_{t+2} should correspond to our expressions for a_{T+1}. If we iterate on (105) we obtain

$$a_{T+1} = a^F = \frac{(R+\phi)\gamma - R_D^d(1+\phi)}{(R-1)(R\gamma - R_D^d)} c + \frac{1}{R-1}(\tau - w^F).$$

So

$$E_t a_{t+2} = (1-q)\frac{R a_{t+1} - \kappa}{1-q} + q a^F = R a_{t+1} - \kappa + q a^F.$$

Now consider $E_t a_{t+3}$. Notice that

$$\begin{aligned} E_t a_{t+3} &= E_t E_{t+1} a_{t+3} = E_t(R a_{t+2} - \kappa + q a^F) \\ &= R^2 a_{t+1} - (1+R)\kappa + (1+R) q a^F. \end{aligned}$$

Iterating on this expression we obtain

$$E_t R^{-j} a_{t+j} = R^{-1}\left(a_{t+1} + \frac{1 - R^{1-j}}{1-R}\kappa - \frac{1 - R^{1-j}}{1-R} q a^F\right) \qquad j \geq 2$$

If we impose $\lim_{j\to\infty} E_t R^{-j} a_{t+j} = 0$ we get $a_t = a^I = (\kappa - q a^F)/(R-1)$ for $t \leq T$. But we also have $a^I = (R a^I - \kappa)/(1-q)$, from (135), or $a^I = \kappa/(R - 1 + q)$. But this, in turn, means that $a^I = a^F = a_0 = \kappa/(R-1+q)$. Furthermore $x^h_t = x^h$ for $t \leq T$, where x^h is obtained by substituting $a_t = a_0$ into (134).

The government's flow budget constraint for $t \neq T$ is $f_{t+1} = Rf_t + (M_{t+1} - M_t)/S_t + \tau_t - g_t$. For $t = T$, the government budget constraint is

$$f_{T+1} = Rf_T + (M_{T+1} - M_T)/S_T - \chi(1 - S_{T-1}/S_T) - \Gamma + \tau_T - g_T.$$

This implies that the government's lifetime budget constraint at date T is

$$f_T = \frac{1}{R}\left[\chi\left(1 - \frac{S_{T-1}}{S_T}\right) + \Gamma + \sum_{j=0}^{\infty} R^{-j}(g_{T+j} - \tau_{T+j}) - \sum_{j=0}^{\infty} R^{-j}\frac{M_{T+1+j} - M_{T+j}}{S_{T+j}}\right]. \quad (136)$$

If we combine (136) with (119) we get

$$a_T + f_T = R^{-1}\left\{\sum_{j=0}^{\infty} R^{-j}\left[c_{T+j} + g_{T+j} - w_{T+j} - \pi_{T+j} - \frac{(R^d_{T+j} - 1)M_{T+j}}{S_{T+j}}\right] + \Gamma - x^h_T\left(\frac{1}{F_T} - \frac{1}{S_T}\right)\right\}$$

We assume that $f_t = f_0 = (g - \tau)/(R-1)$, for $t \leq T$, and that $g_t = g$, $\forall t$. We also use the facts that $a_T = a_0$, $c_t = c$, $\forall t$, $1/F_T = 1/F = (1-q)/S^I + q/S^D$, $S_T = S^D$ and $x^h_T = x^h$, as well as the sequences for w_t and π_t given above to obtain

$$\begin{aligned} a_0 + f_0 &= \frac{1}{R-1}(c+g) - R^{-1}\left[w^I + \frac{1}{R-1}w^F + \pi^D - \Gamma + x^h\left(\frac{1}{F} - \frac{1}{S^D}\right)\right] \\ &\quad - R^{-1}\left[\frac{(R^d_I - 1)M^I}{S^D} + \frac{1}{R-1}\frac{(R^d_D - 1)M^D}{S^D}\right] \end{aligned} \quad (137)$$

Since $\tau_t = \tau = g - (R-1)f_0$, and $a_{t+1} = a_t = a_0$, for $t < T$, (121) implies

$$a_0 + f_0 = \frac{1}{R-1}\left\{c + g - w^I - \pi^I - x^h\left(\frac{1}{F} - \frac{1}{S^I}\right) - (R^d_I - 1)\frac{M^I}{S^I}\right\}. \quad (138)$$

We can solve (137) and (138) for x^h and c:

$$x^h = \frac{w^I - w^F + R\pi^I - (R-1)\pi^D + (R-1)\Gamma + \left(\frac{R}{S^I} - \frac{R-1}{S^D}\right)(R^d_I - 1)M^I - \frac{(R^d_D - 1)M^D}{S^D}}{(R-1+q)\left(\frac{1}{S^I} - \frac{1}{S^D}\right)}, \quad (139)$$

and

$$\begin{aligned} c &= (R-1)(a_0 + f_0) - g + \frac{1}{R-1+q}\left[(R-1)w^I + qw^F - q(R-1)\Gamma\right] + \\ &\quad \frac{1}{R-1+q}\left[(R-1)(R^d_I - 1)\frac{M^I}{F} + q(R^d_D - 1)\frac{M^D}{S^D}\right]. \end{aligned} \quad (140)$$

The expressions for S^D and S^I given above can be rewritten as

$$m^I = \frac{\phi c}{R\gamma - R^d_D}\frac{S^D}{S^I} + \chi \quad (141)$$

$$m^I = \frac{\phi c}{R - R^d_I(1 - q + qS^I/S^D)}, \quad (142)$$

where $m^I = M^I/S^I$. Notice that our previous assumptions, (110) and (118) imply that (136) can be rewritten as

$$\tau^D + \frac{R}{R-1}\frac{\gamma-1}{R\gamma - R_D^d}\phi c = \chi + \Gamma. \tag{143}$$

B.5 Existence of Equilibrium

We need to establish that there exists a $c > 0$, $S^D/S^I \geq 1$, $\gamma \geq 1$, $m^I > 0$ and values of $R_D^d \geq 1$ and $R_I^d \geq 1$ such that for $q \geq 0$, (140), (141), (142), (143) and the equations defining R_D^d and R_I^d hold.

B.5.1 Sustainable Fixed Exchange Rate

In the case of a sustainable fixed exchange rate regime, we assume $q = 0$ and let $S^D = S^I$. The equations (103) and (141) become irrelevant. So we have

$$\begin{aligned}
c &= (R-1)(a_0+f_0) - g + w + (R^d-1)m \\
m &= \frac{\phi c}{R - R^d} \\
R^d &= R - \xi(R-1) \\
R^a &= R + \delta \\
w &= A/(1+R^a-R^d).
\end{aligned}$$

The solution for consumption is

$$c = c_S \equiv \frac{\xi}{\xi-(1-\xi)\phi}\left[(R-1)(a_0+f_0) - g + \frac{A}{1+\delta+(R-1)\xi}\right]. \tag{144}$$

This is positive as long as

$$g < (R-1)(a_0+f_0) + \frac{A}{1+\delta+(R-1)\xi},$$

and $\xi > \phi/(1+\phi)$. The solution for real balances is

$$m^I = m_S = \frac{\phi c_s}{\xi(R-1)} \tag{145}$$

B.5.2 No Government Guarantees

When there are no government guarantees we assume that $\Gamma = \tau^D = 0$. We also use the notation $\sigma \equiv S^I/S^D$. The equations defining R_I^d and R_D^d, (96) and (103), can be rewritten as

$$\begin{aligned}
R_I^d &= \xi + \frac{1-\xi}{1-q+q\sigma}R \\
R_D^d &= \xi + (1-\xi)\gamma R.
\end{aligned}$$

Substituting these expressions into (141)—(143) we obtain

$$m^I = \frac{\phi c}{\xi(\gamma R-1)}\frac{1}{\sigma}+\chi \tag{146}$$

$$m^I = \frac{\phi c}{\xi[R-1+q(1-\sigma)]} \tag{147}$$

$$\chi = \frac{R}{R-1}(\gamma-1)\frac{\phi c}{\xi(\gamma R-1)}. \tag{148}$$

If we substitute (148) into (146) and then substitute the resulting equation into (147) we obtain

$$\left[\frac{1}{\sigma}+\frac{R(\gamma-1)}{R-1}\right]\frac{1}{(\gamma R-1)} = \frac{1}{R-1+q(1-\sigma)}$$

after cancelling the common factor $\phi c/\xi$, and combining terms. This equation has two solutions for σ: one is less than zero, the other is $\sigma = 1$. Since an equilibrium with speculative attacks requires $\sigma < 1$ we conclude that such equilibria do not exist when there are no government guarantees.

B.5.3 Government Guarantees

Again, using the notation $\sigma = S^I/S^D$, with government guarantees we have

$$R_I^d = \xi+(1-q)(1-\xi)R/(1-q+q\sigma) \tag{149}$$

$$R_D^d = \xi+\gamma(1-\xi)R \tag{150}$$

$$R_I^a = [(1-q)R+\delta+q\omega]/(1-q+q\sigma) \tag{151}$$

$$R_D^a = \gamma(\delta+R) \tag{152}$$

$$w^I = \frac{A}{1+R_I^a-R_I^d} = \frac{A}{1-\xi+[\xi(1-q)R+\delta+q\omega]/(1-q+q\sigma)} \tag{153}$$

$$w^F = \frac{A}{1+R_D^a-R_D^d} = \frac{A}{1+\gamma(\delta+\xi R)-\xi} \tag{154}$$

Hence (141) and (142) can be rewritten

$$m^I = \frac{\phi c}{\xi(R\gamma-1)}\frac{1}{\sigma}+\chi \tag{155}$$

$$m^I = \frac{\phi c}{(R-1)\xi(1-q)+q(R-\xi\sigma)} \tag{156}$$

To rewrite (143) and (140) we need an expression for Γ, the size of the government bailout of the banking system. If the state $S = S^D$ is realized the government repays banks' foreign creditors $\Gamma = RD_I^*$, where D_I^* represents the size of banks' foreign borrowing under the fixed exchange rate regime. Recall that banks are subject to the reserve requirement given in (44). Once we note that $D_t^h = M_t$ for all t, we can rewrite (44) as $D_I^* = \xi L^I - (1-\xi)m^I$, where

$L^I = d^I/S^I = Fw^I/S^I$ represents the dollar value of banks' loans to firms under the fixed exchange rate regime. Hence

$$\Gamma = RD_I^* = R[\xi Fw^I/S^I - (1-\xi)m^I]$$

We substitute out w^I using (153) and m^I using (155)

$$\Gamma = R\left[\frac{\xi A}{(1-\xi)(1-q+q\sigma)+\xi(1-q)R+\delta+q\omega} - \frac{1-\xi}{\sigma}\frac{\phi c}{\xi(R\gamma-1)} - (1-\xi)\chi\right]. \quad (157)$$

An alternative expression is obtained by using (156) to substitute out m^I:

$$\Gamma = R\left[\frac{\xi A}{(1-\xi)(1-q+q\sigma)+\xi(1-q)R+\delta+q\omega} - \frac{(1-\xi)\phi c}{(R-1)\xi(1-q)+q(R-\xi\sigma)}\right]. \quad (158)$$

Substituting (157) and (150) into (143) and rearranging terms we get

$$\begin{aligned} R\left(\frac{\gamma-1}{R-1}+\frac{1-\xi}{\sigma}\right)\frac{\phi c}{\xi(R\gamma-1)} &= \frac{R\xi A}{(1-\xi)(1-q+q\sigma)+\xi(1-q)R+\delta+q\omega} + \\ &\quad \chi[1-R(1-\xi)] - \tau^D \end{aligned} \quad (159)$$

Substituting (153), (154), (158), (149), (150), (155) and (156) into (140) we get

$$\left[1 - \frac{\varsigma(\sigma)(1-\xi)\phi}{R-1+q}\right]c = (R-1)(a_0+f_0) - g + \frac{\lambda(\gamma,\sigma)A}{R-1+q} \quad (160)$$

where

$$\begin{aligned} \lambda(\gamma,\sigma) &\equiv \frac{(R-1)(1-q+q\sigma-qR\xi)}{\xi(1-q)R+\delta+q\omega+(1-\xi)(1-q+q\sigma)} + \frac{q}{\gamma(\delta+\xi R)+1-\xi} \\ \varsigma(\sigma) &\equiv \frac{[R-1+q(1-\sigma)](R-1)}{(R-1)\xi(1-q)+q(R-\xi\sigma)} + \frac{q}{\xi}. \end{aligned}$$

The equations (155) and (156) can be combined to eliminate m^I as an unknown. Together they imply that

$$\left[\frac{1}{(R-1)\xi(1-q)+q(R-\xi\sigma)} - \frac{1}{\xi(R\gamma-1)\sigma}\right]\phi c = \chi \quad (161)$$

Equilibrium with Fiscal Reform

When we consider the case where the government chooses an arbitrary $\gamma > 1$, and satisfies its budget constraint by setting τ^D appropriately, we take q and γ as given and look for a pair (c, σ) that satisfies (160) and (161). We would then substitute these values of c and σ into (159) to determine the necessary fiscal reform τ^D.

Our strategy for finding a solution involves substituting c out of (161) using (160). This leaves us with a single equation in one unknown, σ:

$$\psi(\sigma; q) = 0 \tag{162}$$

where

$$\psi(\sigma; q) \equiv \left[\frac{1}{(R-1)\xi(1-q) + q(R-\xi\sigma)} - \frac{1}{\xi(R\gamma-1)\sigma}\right] \frac{(R-1)(a_0+f_0) - g + \frac{\lambda(\gamma,\sigma)A}{R-1+q}}{1 - \frac{\varsigma(\sigma)(1-\xi)\phi}{R-1+q}} - \frac{\chi}{\phi} \tag{163}$$

First we solve (162) for the case where $q = 0$. This simplifies (162) dramatically because

$$\psi(\sigma; 0) = \left[\frac{1}{(R-1)} - \frac{1}{(R\gamma-1)\sigma}\right] \frac{c_S}{\xi} - \frac{\chi}{\phi}$$

We note that $\psi(\sigma; 0)$ has the following properties. For $\sigma > 0$, it is strictly increasing in σ, since $\psi_\sigma(\sigma; 0) = c_S/[(R\gamma - 1)\xi\sigma^2]$; $\lim_{0\leftarrow\sigma} \psi(\sigma, 0) = -\infty$; and at $\sigma = 1$ we have

$$\psi(1, 0) = \frac{R(\gamma-1)}{(R-1)(R\gamma-1)} \frac{c_S}{\xi} - \frac{\chi}{\phi}.$$

Notice that if the condition

$$\chi < \frac{R(\gamma-1)}{R\gamma-1} \frac{\phi c_S}{\xi(R-1)} = \frac{R(\gamma-1)}{R\gamma-1} m_S$$

is satisfied,[4] then these properties imply that $\psi(\sigma, 0) = 0$ has a unique solution at

$$\sigma = \frac{m_S}{m_S - \chi} \frac{R-1}{R\gamma-1} < 1.$$

We also have $c = c_S$, and the solution for τ^D is obtained from (159):

$$\tau^D = \chi + \frac{R\xi A}{1 - \xi + \xi R + \delta} - \left(\frac{\gamma-1}{R\gamma-1} + 1 - \xi\right) R m_S. \tag{164}$$

For $q > 0$ we argue that, at least for small q equilibria with $\sigma < 1$ also exist. We make this argument using Figure 3, where we have plotted $\psi(\sigma; 0)$ according to the properties we described above. Since $\psi(\sigma; q)$ is uniformly continuous in σ and q in a neighborhood of $q = 0$ we argue that for sufficiently small $q > 0$, we know that $\psi(\sigma; q)$ lies within an arbitrarily small neighborhood of $\psi(\sigma; 0)$. This implies that for these small values of q an equilibrium also exists with $\sigma < 1$. Since the expressions in (160) and (159) are also uniformly continuous in c, σ, τ^D and q, we can argue, by extension, that the equilibrium value of c will be near c_S and the equilibrium value of τ^D will be near the value given in (164).

Equilibrium without Fiscal Reform

[4] Notice that this is the same condition we imposed in stating Proposition 4 for the baseline model.

Now we consider the case where $\tau^D = 0$, and the government must choose γ in order to satisfy its budget constraint. We again take q as given, but now look for a triple (c, σ, γ) that satisfies (160), (161) and (159).

Our strategy for finding a solution involves substituting c out of our equations. Again, we combine (161) and (160) to obtain the equation

$$\psi^1(\gamma, \sigma; q) = 0 \tag{165}$$

where $\psi^1(\gamma, \sigma; q)$ corresponds to the expression in (163). We also combine (160), (159) and $\tau^D = 0$ to obtain

$$\psi^2(\gamma, \sigma; q) = 0$$

where

$$\begin{aligned} \psi^2(\gamma, \sigma; q) \quad \equiv \quad & R\left(\frac{\gamma - 1}{R-1} + \frac{1-\xi}{\sigma}\right)\frac{\phi}{\xi(R\gamma - 1)}\frac{(R-1)(a_0 + f_0) - g + \frac{\lambda(\gamma,\sigma)A}{R-1+q}}{1 - \frac{\varsigma(\sigma)(1-\xi)\phi}{R-1+q}} - \\ & \chi[1 - R(1-\xi)] - \frac{R\xi A}{(1-\xi)(1-q+q\sigma) + \xi(1-q)R + \delta + q\omega} \end{aligned} \tag{166}$$

We first examine $\psi^1(\gamma, \sigma; q) = 0$ and $\psi^2(\gamma, \sigma; q) = 0$ under the assumption that $q = 0$. When $q = 0$ the equations $\psi^1(\gamma, \sigma; q) = 0$ and $\psi^2(\gamma, \sigma; q) = 0$ reduce to

$$\psi^1(\gamma, \sigma; 0) = \left[\frac{1}{(R-1)\xi} - \frac{1}{\xi(R\gamma - 1)\sigma}\right] c_S - \frac{\chi}{\phi} = 0 \tag{167}$$

and

$$\psi^2(\gamma, \sigma; 0) = R\left(\frac{\gamma - 1}{R-1} + \frac{1-\xi}{\sigma}\right)\frac{\phi}{\xi(R\gamma - 1)} c_S - B = 0 \tag{168}$$

where, using (157)

$$\begin{aligned} B \quad &= \quad \chi + \lim_{(q,\gamma)\to(0,\infty)} \Gamma \\ &= \quad \frac{R\xi A}{(1-\xi) + \xi R + \delta} + \chi[1 - R(1-\xi)] \end{aligned} \tag{169}$$

We plot $\psi^1(\gamma, \sigma; 0) = 0$ and $\psi^2(\gamma, \sigma; 0) = 0$ in Figure 4. This is made easier by solving (167) and (168) for γ to obtain:

$$\gamma \quad = \quad \gamma^1(\sigma) = \frac{m_S[1 + (R-1)/\sigma] - \chi}{(m_S - \chi)R} \tag{170}$$

$$\gamma \quad = \quad \gamma^2(\sigma) = \frac{m_S[1 - (R-1)(1-\xi)/\sigma] - B/R}{m_S - B} \tag{171}$$

$\psi^1(\gamma, \sigma; 0) = 0$ is represented by $\gamma = \gamma^1(\sigma)$ which has the following properties: when $\chi < c_S\phi/[\xi(R-1)] = m_S$,[5] γ^1 is strictly decreasing in σ, $\lim_{0\leftarrow\sigma} \gamma^1(\sigma) = \infty$, and when $\sigma = 1$ we have

$$\gamma^1(1) = \frac{Rm_S - \chi}{R(m_S - \chi)} > 1.$$

[5] An equivalent assumption is made in the statement of Proposition 3 for the baseline model.

$\psi^2(\gamma,\sigma;0)=0$ is represented by $\gamma=\gamma^2(\sigma)$ which, when

$$R(1-\xi)m_S < B < m_S \tag{172}$$

has the following properties: γ^2 is strictly increasing in σ, $\lim_{0\leftarrow\sigma}\gamma^2(\sigma)=-\infty$, and when $\sigma=1$ we have

$$\gamma^2(1)=\frac{m_S[1-(R-1)(1-\xi)]-B/R}{m_S-B}>1.$$

If $\gamma^2(1)>\gamma^1(1)$, then the curves intersect for $\gamma>1$ and $\sigma<1$. Notice that this is true whenever

$$B>\chi[1-R(1-\xi)]+(1-\xi)Rm_S$$

But, using the definition of B, (169), we can see that this is equivalent to

$$\frac{\xi A}{1-\xi+\xi R+\delta}-(1-\xi)m_S>0$$

or equivalently, that

$$D_S^*>0. \tag{173}$$

where D_S^* is the amount of foreign borrowing by banks in the sustainable fixed exchange rate regime. Notice that the left-hand inequality in (172) is satisfied whenever (173) is satisfied. Hence, the analog of Proposition 3 for the extended model would require us to impose only the following side conditions:

$$\chi+\lim_{(q,\gamma)\to(0,\infty)}\Gamma<m_S \quad \text{and} \quad D_S^*>0 \tag{174}$$

These conditions are analogous to our side conditions on Proposition 3.

The solution for c is $c=c_S$, the solution for σ is

$$\sigma=\frac{[R(1-\xi)+1]m_S-R\chi(1-\xi)-B}{m_S-\chi}=1-\frac{RD_S^*}{m_S-\chi}$$

and the solution for γ is

$$\gamma=\frac{Rm_S-\chi-RD_S^*}{R(m_S-\chi-RD_S^*)}.$$

So, for $q=0$ we have $\sigma<1$, $\gamma>1$ and $c=c_S$. For $q>0$ we argue that, at least for small q equilibria with $\gamma>1$ and $\sigma<1$ also exist. We make this argument using Figure 4. Since ψ^1 and ψ^2 are uniformly continuous in γ and σ and q in a neighborhood of $q=0$ we argue that for sufficiently small $q>0$, we know that $\psi^1(\gamma,\sigma;q)$ and $\psi^2(\gamma,\sigma;q)$ lie within arbitrarily small neighborhoods of $\psi^1(\gamma,\sigma;0)$ and $\psi^2(\gamma,\sigma;0)$. This implies that for these small values of q an equilibrium also exists with $\gamma>1$ and $\sigma<1$. Since the expression in (160) is also uniformly continuous in c, σ, γ and q, we can argue, by extension, that the equilibrium value of c will be near c_S.

FIGURE 1

DIAGRAM FOR PROPOSITION 4

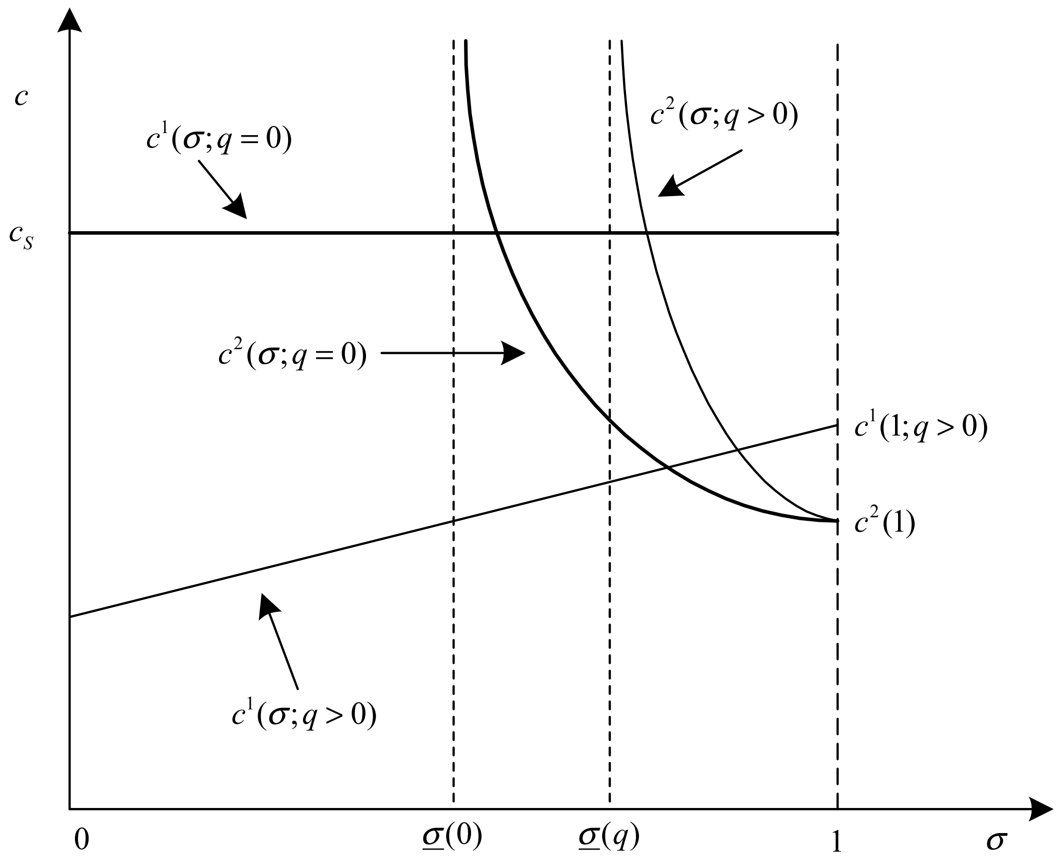

FIGURE 2

DIAGRAM FOR PROPOSITION 3

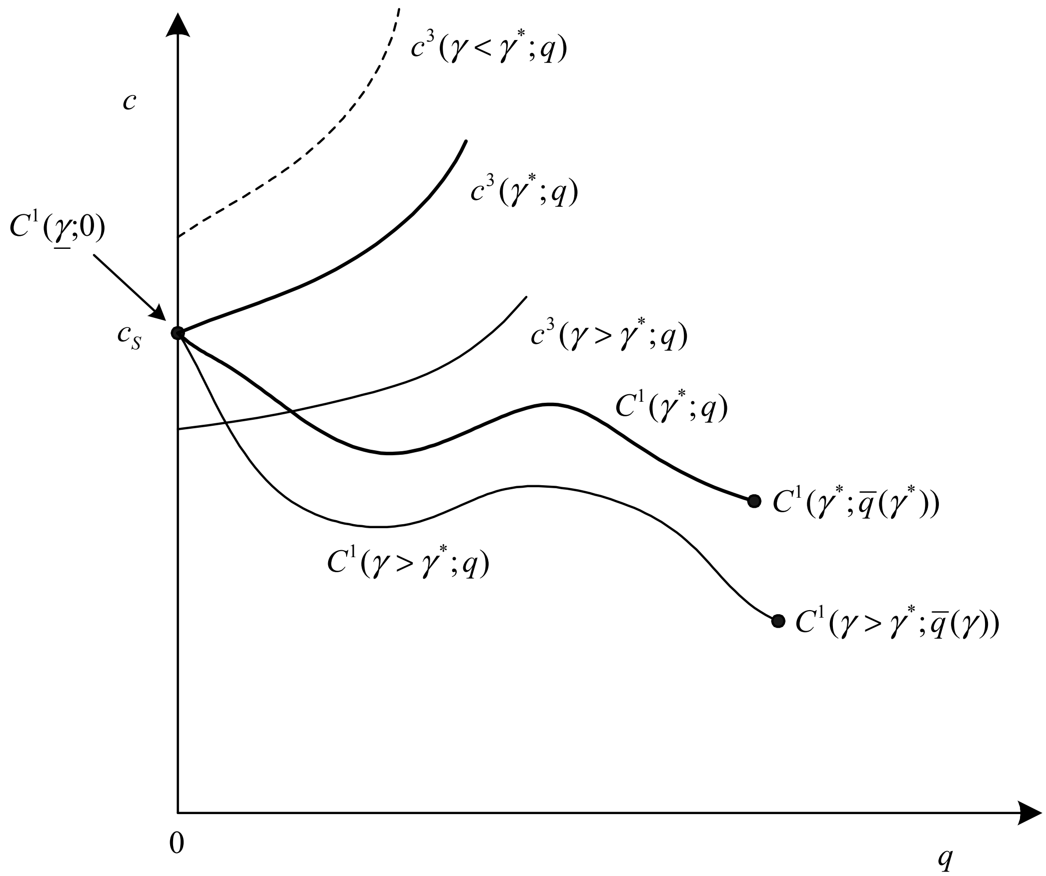

FIGURE 3

DIAGRAM FOR THE EXTENDED MODEL
Equilibrium with Arbitrary γ(Fiscal Reform)

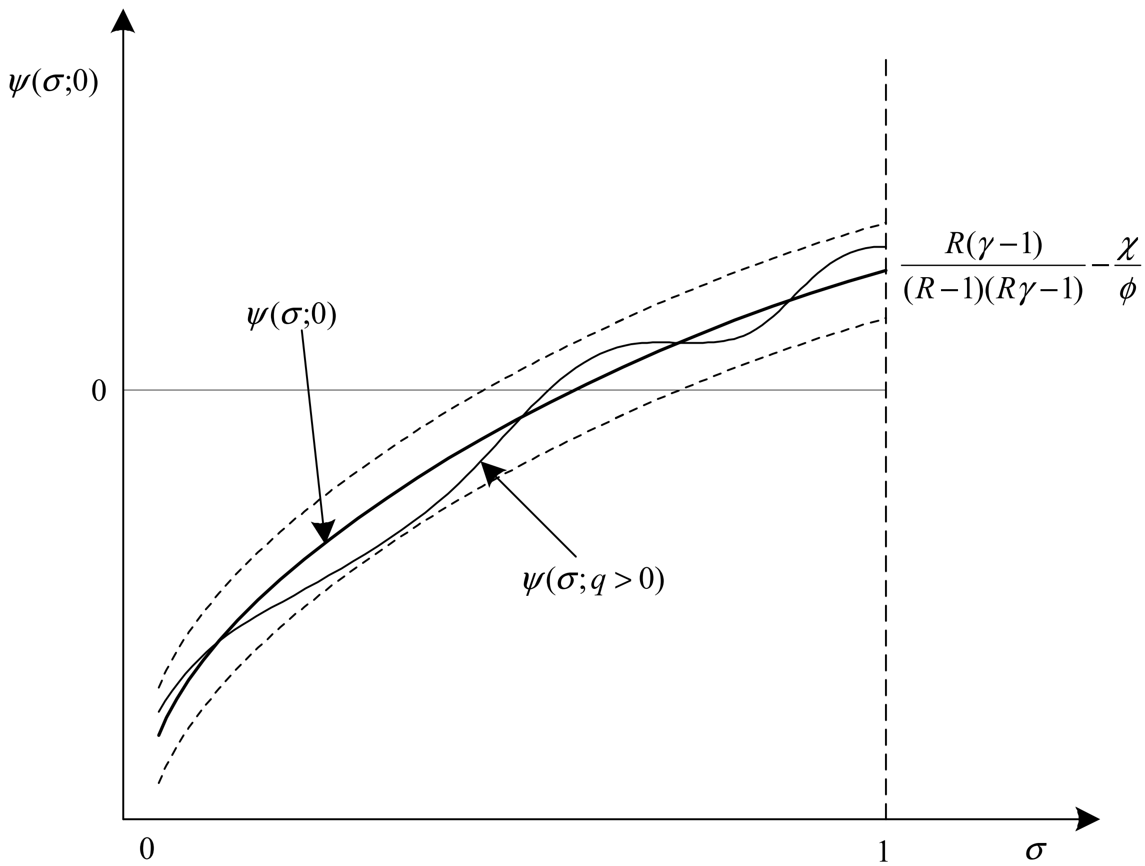

FIGURE 4

DIAGRAM FOR THE EXTENDED MODEL
Equilibrium with No Fiscal Reform