The Use of Sanctions with Adapting Targets

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June 2012

Abstract

A major argument against the use of economic sanctions is that the sanctioned state has time to adapt to economic hardships. I examine the impact of sanctions when the sanctioned state has the ability to adapt in an otherwise standard bargaining model. I show that bargaining leverage gained through constant sanctioning depends on the adaptability of the sanctioned state. In contrast, alternate sanctioning is immune to adaptability and provides more bargaining leverage for the sanctioning state. This finding is robust to introduction of informational asymmetry. Moreover, alternate sanctioning may eliminate informational asymmetry without actual use of sanctions.

1 Introduction

In his September 27, 2009 New York Times article on the US - Iran relations in regard to nuclear non-proliferation, Roger Cohen argues that "sanctions won’t work" partly because "Iran is inured to sanctions after years of living with them." However, economic sanctions remain preferred foreign policy tools despite several criticisms (Galtung 1967, Wallensteen 1968, Pape 1997, Morgan and Schwebach 1997, Tsebelis 1990). A major argument against sanctions is that the sanctioned state has time to adapt to economic hardships, finding alternative suppliers of markets for the goods that are scarce, forging new relationships with states that are not sides to the sanctions, or

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utilizing its own production capacity and reducing its dependence on the outside world (Galtung 1967).\footnote{Other counter-arguments exist in the literature as well: For example, Pape (1997) argues that sanctions will generate a "rally around the flag" effect in the sanctioned state and will not lead to the policy changes the sanctioning state was hoping to achieve. Ang and Peksen (2007) examine issue salience and show that if the sanctioning state and the sanctioned state do not value equally the issue about which the sanctions are implemented, then it is unlikely that sanctions will be effective. McGillivray and Stam (2004) find that if domestic public opinion in the sanctioned state favors continuing a set of actions despite the presence of sanctions, or if the benefits of these actions exceed the costs of sanctions, then it is unlikely that the sanctions will work effectively. On the other hand, McGillivray and Smith (2000) show that agent-specific punishment where sanctioning side punishes the leaders in the target state may work better than broad-based economic sanctions where the sanctions are imposed on the population at large.}

Examples abound. When the US implemented a food embargo in Iraq in 1990, Iraq was a major importer of food and was nowhere close to having self-sufficiency in food production. The embargo was accordingly expected to reduce the living standards of the Iraqis to the extent that would force Saddam Hussein to back down in his standoff with the United Nations. However, unexpectedly, Iraq was able to adapt remarkably well, and found its own potential to produce food and reached some certain level of agricultural self-sufficiency that left the country much less vulnerable to international pressure of this type (Selden, 1999). Similarly, a Newsweek article on January 28, 2010 reports on Iran that, "While sanctions have slowed the development of Iran’s energy sector and stifled economic growth, the regime has become adept at shipping banned goods through third countries, funding its activities in currencies other than the US dollar, and inviting non-Western entities to step in on commercially attractive terms in key sectors of the economy such as infrastructure, energy and telecommunications."

In this paper, I study the impact of sanctions when the sanctioned country has the ability to adapt in an otherwise standard bargaining model (Rubinstein 1982).\footnote{See Osborne and Rubinstein (1990) for an extensive review of the literature on bargaining models.} I develop an infinite horizon game with two players, a sender (he) and a target state (she). The players collectively receive a flow of \( v \) units of consumption good over an infinite time-horizon. They can consume the good only if they mutually agree to share it. The players discount their future payoffs. They make offers in an alternating fashion in the beginning of every period. If a player’s proposal is accepted, the two players share \( v \) ac-
cordingly forever. If a proposal is rejected, $v$ of that period perishes. Unlike in the standard model, the sender can affect disagreement payoffs (Haller and Holden 1990, Fernandez and Glazer 1991), that is, when an offer is rejected, the sender can impose sanctions on the target state to reduce her consumption from other resources at a cost to himself. The target state can adapt to sanctions. She suffers to the full extent from newly imposed sanctions but to a lesser extent from sanctions that prevail from previous periods.

In contrast to the standard model, multiple subgame perfect equilibria may exist when players can endogenously determine disagreement payoffs. Finding all equilibria is beyond the scope of this paper. However, multiplicity of equilibria allows us to study and compare various sanctioning strategies and thereby address the impact of adaptability to sanctions on bargaining outcomes. Consequently, I study three types of equilibrium.

The first type of equilibrium does not involve sanctions. The outcomes of this equilibrium coincide with the unique equilibrium outcome of the standard Rubinstein bargaining game.

In the second type of equilibrium, the sender keeps sanctioning the target at the same level as long as they continue to disagree, which happens only off the equilibrium path. When the cost of sanctions to the sender is not prohibitively high, such equilibria provide the sender with a better equilibrium outcome and the target with a worse one. That is, the threat of sanctions in case of disagreement improves the sender’s bargaining leverage in negotiations.

Finally, I study a third class of equilibria, in which the sender alternates between more restrictive and less restrictive sanctions as long as they continue to disagree. Surprisingly, this equilibrium provides better outcomes for the sender than constant sanctioning. Since sanctions are costly for the sender, one may be tempted to think that the sender is better off because he avoids some cost during less restrictive sanctions. However, alternate sanctioning is also imposed only off the equilibrium path that the cost of sanctions is not the driving force behind this result. In fact, this gain is present even when the sender incurs no cost from sanctions. The intuition behind this result is that alternate sanctioning prevents the target from adapting to restrictive sanctions, making sanctions more effective in case of disagreement and thereby improving the sender’s bargaining leverage further. Since sanctions are imposed only off the equilibrium path in both types of equilibria, the gain in bargaining leverage comes to the sender at no actual cost. These findings are robustly predicted as well when players are randomly selected to make
an offer every period, therefore they are not by-products of the alternating offers bargaining protocol.

Complete information games of sanctions predict that the threat of sanctions off the equilibrium path is sufficient to generate bargaining leverage, and costly sanctions are observed in equilibrium when incomplete information is introduced (Hovi, Huseby and Sprinz 2005, Lacy and Niou 2004, Morgan and Schwebach 1997, Smith 1996). In order to study sanctions that may be observed in equilibrium, I introduce incomplete information to the model.

I study three types of informational asymmetry: (i) the sender may be uncertain about the extent of sanctions he can impose, (ii) the sender may be uncertain about how much the target values the issue in dispute – issue salience –, (iii) the sender may be uncertain about the target’s ability to adapt. Regardless of the type of informational asymmetry, there exist separating perfect Bayesian equilibrium with constant sanctioning and separating perfect Bayesian equilibrium with alternate sanctioning. In all cases, I study the most efficient separating equilibrium among the ones that provides the sender with the highest payoff.

The first two cases are fundamentally different than the last one. In the former, the sender knows about the target’s adaptability. Then informational asymmetry regarding other aspects of the game is eliminated differently under alternative sanction policies. An equilibrium delay emerges in both cases. However, sanctions are observed with positive probability in equilibrium with constant sanctioning, but not in equilibrium with alternate sanctioning.

Case (iii) provides a very surprising prediction. When the sender adopts alternate sanctioning, neither delay nor sanctions are observed in equilibrium. This finding contrasts sharply with the findings of incomplete information bargaining games in the literature, which predict delay and inefficiency. The intuition behind this result is that alternate sanctioning frees the sender from target’s adaptability, therefore uncertainty about target adaptability becomes irrelevant in the equilibrium. In contrast, when the sender adopts constant sanctioning, delay and sanctions are observed in equilibrium.

This paper is closely related to the literature on bargaining games in which players can endogenously determine disagreement payoffs. Haller and Holden (1990) and Fernandez and Glazer (1991) are the first to point out potential multiplicity of equilibria in Rubinstein bargaining games when disagreement payoffs are endogenous. Unlike in the standard Rubinstein game, some of these equilibria may be inefficient, explaining wasteful phenomena such as strikes, delay in bargaining and wars. Avery and Zemsky (1994) identify
a general principal of money burning to explain the underlying mechanism of inefficiency in complete information bargaining games. Busch and Wen (1995) extend earlier findings to a more general setting in which disagreement periods are determined by a general normal form game. Similar to these works, sanctions in my model allow the sender to endogenously determine payoffs when players cannot settle. However, the target adapts to sanctions. The level of sanctions in previous periods determines payoffs and the impact of sanctions in the following period. In other words, unlike in the earlier works, previous sanctions constitute a payoff relevant state for the continuation game, which is determined endogenously by the sender’s acts.

I introduce the complete information bargaining model in the next section and discuss equilibria in Section 3. I show in the appendix that the predictions are robust to changes in the bargaining protocol. In Section 4, I analyze sanctions under three types of informational asymmetry. I defer the technical analysis to the appendix.

2 Model

The benchmark model is an extension of Rubinstein (1982) bargaining model: There are two players, Sender (he) and Target (she). The players bargain over a flow of \( v \) units of a perishable consumption good that they can share and consume only if they mutually agree to do so. \( v \) models the stakes on the negotiation table. For example, it may represent the total surplus from forgoing activities to develop nuclear weapons. In addition to \( v \), player \( i \in \{S, T\} \) receives a flow of \( v_i \) every period. This flow is generated by players’ outside economic activities, such as income generated from exporting crude oil. Since \( S \) is not subject to sanctions, I normalize his outside flow to \( v_S = 0 \). I also normalize \( T \)’s outside flow to \( v_T = 1 \).

The players bargain over \( v \) as follows: They make offers in an alternating and deterministic order. The player that is selected to make an offer makes a proposal \((v-x, x)\) where \( x \) is the sender’s share and \( v-x \) is the target’s share. If an agreement of \((v-x, x)\) is reached, the sender consumes his share of the agreement, \( v-x \), and the target consumes \( x \) along with her outside flow of 1 thereafter. If an offer is rejected, the sender takes a unilateral action to decrease the target’s momentary consumption from her outside flow by \( y \in [0, 1] \). I will refer to this as sanctioning the target and \( y \) as the level of sanctions. \( y = 0 \) corresponds to no sanctions and any level of sanction
$y \in [0, 1]$ is feasible. When a player rejects a proposal, s/he makes the next proposal. The game continues until one player accepts the other’s proposal. Player $i$ discounts future payoffs by $\delta_i \in [0, 1)$.

Sanctions are costly. The sender pays a cost $\psi y$, $\psi \geq 0$, to impose sanctions, which may include costs generated by the international and domestic political climate.

Let $z_t$ denote player $i$’s consumption in period $t$ and $y_t$ be the level of sanctions in period $t$. The sender’s per-period payoff is $z_{St} - \psi y_t$, which is his consumption minus the cost of sanctions in the given period. Then his lifetime utility is given by

$$\sum_t \delta_t^t (z_{St} - \psi y_t)$$

Since $v_S = 0$, $z_{St} = 0$ if an agreement is not reached.

The target has the ability to adapt to sanctions. Let $\phi \in [0, 1]$ measure her ability to adapt. The target suffers fully from additional sanctions only, she adapts and and suffers a portion $(1 - \phi)$ of sanctions that prevail from previous period. Let the level of sanctions be $y_{t-1}$ and $y_t$ in the previous and current periods, respectively. If $y_t \leq y_{t-1}$, then the target has already adapted and incurs a cost of $(1 - \phi)y_t$. If $y_t > y_{t-1}$, then the target pays a cost of $(1 - \phi)y_{t-1}$ for sanctions that prevail from the previous period and pays the full cost of $y_t - y_{t-1}$ for the additional sanctions, taking the total cost to $y_t - \phi y_{t-1}$. This can be written succinctly as

$$(1 - \phi)y_t + \phi[y_t - y_{t-1}]_+$$

where

$$[y_t - y_{t-1}]_+ = \begin{cases} y_t - y_{t-1} \text{ if } y_t \geq y_{t-1} \\ 0 \text{ otherwise} \end{cases}$$

The higher $\phi$ is, the more adaptable the target is and hence the smaller are the costs that the target pays. The target’s lifetime utility is given by

$$\sum_t \delta_T^t(z_{Tt} - (1 - \phi)y_t - \phi[y_t - y_{t-1}]_+)$$

\textsuperscript{3}The sender can aim for broader sanctions of $y > v_T$. Since sanctions lower the target’s outside flow of $v_T$, any sanction beyond $v_T$ is not effective. This implies $y \in [0, v_T]$ in the complete information game. When the sender is uncertain about the value of $v_T$, he may impose $y > v_T$ in equilibrium, as in the incomplete information game in Section 4.
I assume that negotiations start at $t = 1$ with the target’s offer and the sender has already been sanctioning the target fully, i.e. $y_0 = 1$. Sanctions are aimed to impose economic deprivation, which will trigger political anger and protests, and will eventually lead to changes in the elites’ behavior in the sanctioned country or their removal from power. In other words, sanctions are expected to impose “audience costs” on the leader. Throughout the text, I assume that $T$ represents the leader of the target country and $(1 - \phi)y_t + \phi[y_t - y_{t-1}]_+$ is the audience cost imposed by her citizens on the leader. The only part of the cost that is exogenous is $\phi$. Later I discuss how this parameter can vary, for example by consumption type, regime type or sanction type.

It is well-known that, in contrast to the standard Rubinstein game, multiple equilibria may exist when players can determine disagreement payoffs endogenously (Haller and Holden 1990, Fernandez and Glazer 1991, Avery and Zemsky 1994, Busch and Wen 1995). As I mentioned earlier, finding all equilibria is beyond the scope of this paper. The main goal in this paper is to study and compare various sanctioning strategies to address the impact of ability to adapt to sanctions on bargaining. In the next section, I discuss three types of equilibrium. The first type of equilibrium does not involve any sanctions. The outcomes of this equilibrium coincide with the unique equilibrium outcome of the standard Rubinstein game. In the second type of equilibrium, which I refer to as “constant sanctioning” the sender keeps sanctioning the target at the same level as long as they continue to disagree. Finally, in the third type of equilibrium, which I refer to as “alternate sanctioning,” the sender alternates between more restrictive and less restrictive sanctions as players continue to disagree.

## 3 Equilibrium

A strategy profile of offers, sanctions and acceptance/rejection decisions is a subgame perfect equilibrium if it is a Nash equilibrium in every subgame.

Formally, $x_{it} \in [0, v]$ is player $i$’s offer for the target’s share in period $t$; $y_{it} \geq 0$ is the level of sanctions when player $i$’s offer of $x_{it}$ is rejected; $a_{it}(x) \in \{accept, reject\}$ is player $i$’s decision to accept or reject an offer of $x$ made by the other player in period $t$.

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4Since cost of sanctions is determined by the level of past sanctions, I need to initialize $y_0$. This initialization avoids unnecessary technical details in the analysis of the first period.
I use the following convention: \( t \) represents a period that the sender makes an offer. I refer to the sender’s offer periods as \(... t - 2, t, t + 2, ...\) and the target’s offer periods as \(..., t - 1, t + 1, t + 3...\)

The sender’s strategy is denoted by \( \sigma_S = (x_{St}, y_{St}, y_{Tt}, a_{St})_{t=1,2,...} \), and the target’s strategy is denoted by \( \sigma_T = (x_{Tt}, a_{Tt})_{t=1,2,...} \). A player makes an offer or an acceptance/rejection decision every other period. One can set \( x_{it} \) and \( a_{it} \) arbitrarily in periods that are not relevant for player \( i \). A strategy profile \((\sigma_S, \sigma_T)\) is a subgame perfect equilibrium (SPE) if the continuation of \((\sigma_S, \sigma_T)\) forms a Nash equilibrium at every subgame.

The standard Rubinstein bargaining game has a unique SPE, in which the following offers are made and accepted immediately:

\[
x^*_S = \frac{\delta_T (1 - \delta_S)}{1 - \delta_S \delta_T} v \quad \text{and} \quad x^*_T = \frac{1 - \delta_S}{1 - \delta_S \delta_T} v
\]

As the following proposition asserts, these offers can be sustained with no sanctions in an SPE of the extended game despite the game starts at the state of full sanctions.

**Proposition 1** The following is an SPE with no sanctions: The sender offers \( x_{St} = x^*_S \), accepts any offer \( x_{Tt} \leq x^*_T \), rejects all other offers, and does not sanction the target after a rejection. The target offers \( x_{Tt} = x^*_T \), accepts any offer \( x_{St} \geq x^*_S \), and rejects all other offers.

The players will use this SPE to enforce sanctions in equilibrium below. Define

\[
\bar{\psi} = \frac{\delta_S (1 - \delta_T) (1 - \phi)}{(1 - \delta_S) (1 - \delta_S \delta_T + \delta_S)}
\]

The following proposition characterizes a class of SPE with constant sanctioning:

**Proposition 2** If \( \psi \leq \bar{\psi} \), the following is an SPE with constant sanctioning for any \( y \in [0, 1] \). Define

\[
\bar{x}_S(y) = x^*_S - \frac{1}{1 - \delta_S \delta_T} \left[ (1 - \delta_T) (1 - \phi) - \delta_T (1 - \delta_S) \psi \right] y,
\]

\[
\bar{x}_T(y) = x^*_T - \frac{1}{1 - \delta_S \delta_T} \left[ \delta_S (1 - \delta_T) (1 - \phi) - (1 - \delta_S) \psi \right] y
\]
The sender offers \( x_{St} = \bar{x}_S(y) \), accepts any offer \( x_{Tt} \leq \bar{x}_T(y) \), rejects all other offers, and imposes a sanction of \( y \) on the target after a rejection, i.e. \( y_{St} = y_{Tt,t+1} = y \). The target offers \( x_{Tt} = \bar{x}_T(y) \), accepts any offer \( x_{St} \geq \bar{x}_S(y) \), and rejects all other offers. If the sender deviates by choosing another sanction level, then the players play the SPE with no sanctions (Proposition 1) in the continuation game.

Next, define

\[
\tilde{\psi} = \frac{\delta_S(1 - \delta_T)}{(1 - \delta_S)(1 - \delta_S \delta_T)} > \psi
\]

The following proposition characterizes a class of SPE with alternate sanctioning:

**Proposition 3** If \( \psi \leq \tilde{\psi} \), the following is an SPE with alternate sanctioning for any \( y \in [0, 1] \). Define

\[
\bar{x}_S(y) = x_S^* - \frac{(1 - \delta_T)y}{1 - \delta_S \delta_T},
\]

\[
\bar{x}_T(y) = x_T^* - \frac{\delta_S(1 - \delta_T)y}{1 - \delta_S \delta_T}
\]

The sender offers \( x_{St} = \bar{x}_S(y) \), accepts any offer \( x_{Tt} \leq \bar{x}_T(y) \), rejects all other offers, and imposes a sanction of \( y \) on the target after a rejection, i.e. \( y_{St} = y \) and \( y_{Tt,t+1} = 0 \). The target offers \( x_{Tt} = \bar{x}_T(y) \), accepts any offer \( x_{St} \geq \bar{x}_S(y) \), and rejects all other offers. If the sender deviates by choosing another sanction level, then the players play the SPE with no sanctions (Proposition 1) in the continuation game.

When \( \psi \leq \tilde{\psi} \), i.e. sanctions are not prohibitively costly for the sender, the sender can utilize sanctions off the equilibrium path to his advantage. The equilibrium shares of the target decreases with the level of sanctions in an SPE with sanctions: \( \bar{x}_S(y) < x_S^* \) and \( \bar{x}_T(y) < x_T^* \).

Similarly, an SPE with alternate sanctioning provides the sender with better equilibrium outcomes: \( \bar{x}_S(y) < x_S^* \) and \( \bar{x}_T(y) < x_T^* \). More importantly, alternate sanctioning provides two more benefits. First, the sender can utilize sanctions for a larger range of the cost parameter \( \psi \). In particular, when \( \psi < \tilde{\psi} \leq \tilde{\psi} \), SPE with constant sanctioning does not exist, but SPE with alternate sanctioning continues to exist. Second, for every level of
sanctions $y$, the sender is better off in the SPE with alternate sanctioning: $\bar{x}_S(y) < \bar{x}_S(y)$ and $\bar{x}_T(y) < \bar{x}_T(y)$. In other words, when the target can adapt, alternate sanctioning is more effective in convincing the target with a lower offer than constant sanctioning. The basic intuition behind this striking result is that alternate sanctioning prevents the target from adapting to restrictive sanctions, making sanctions more effective in case of disagreement and thereby improving the sender’s bargaining leverage further.

In an SPE with alternate sanctioning, the sender sanctions the target only when the target rejects his offer but not when the sender rejects the target’s offer. In contrast, in an SPE with constant sanctioning, the sender sanctions the target whenever an offer is rejected, by himself or by the target. Therefore one may be tempted to think that the sender is better off with alternate sanctioning mainly because he avoids some cost during periods of no sanctions. However, sanctions are imposed only off the equilibrium path so the sender does not pay any actual cost under either sanctioning policy. On the other hand, costs may generate some advantage for the target off the equilibrium path. The target’s offer of $\bar{x}_T(y)$ in an SPE with constant sanctioning is an increasing function of $\psi$. This advantage is also reflected in the target’s offer of $\bar{x}_S(y)$. In order to abstract from this effect and to better isolate the consequences of adapting to sanctions, suppose that sanctions are costless, i.e. $\psi = 0$. It continues to be the case that $\bar{x}_S(y) < \bar{x}_S(y)$ and $\bar{x}_T(y) < \bar{x}_T(y)$ for all $\phi > 0$. That is, even when the target cannot take advantage of the cost that the sender pays to impose sanctions, she is less vulnerable under constant sanctioning.

$\bar{\psi}$ is a decreasing function of $\phi$, where as $\bar{\psi}$ is independent of $\phi$. This implies that the range of $\psi$ for which an SPE with constant sanctioning exists shrinks as $\phi$ gets larger (i.e. the target becomes more adaptable), and that region for an SPE with alternate sanctioning is not affected by a change in target’s adaptability. Finally, equilibrium shares of the sender in an SPE with constant sanctioning $(v - \bar{x}_i(y))$ are decreasing functions of $\phi$, implying that the constant sanctioning become less effective as the target becomes more adaptable. In contrast, equilibrium shares of the sender in an SPE with alternate sanctioning $(v - \bar{x}_i(y))$ are independent of $\phi$, so the impact of adaptability vanishes with alternate sanctioning.

These predictions are novel to the model. In the appendix, I analyze the same complete information game under the assumption that every period the sender is chosen with probability $\pi$ and the target is chosen with probability $1 - \pi$ to make an offer in that period. $\pi$ may represent the sender’s
bargaining power, which may be determined by economic and international climate. The prediction that alternate sanctioning provides the sender with more bargaining leverage than constant sanctioning is robust to this change in the bargaining protocol.

Sanctions are aimed to impose economic deprivation, which will trigger political anger and protests, and will eventually lead to changes in the elites’ behavior in the sanctioned country or their removal from power. Therefore, the cost that $T$ incurs can be interpreted as “audience costs” imposed on the leader by citizens of the target state. Then when would a target leader be more likely to adapt to sanctions, i.e. have a higher $\phi$?

One interpretation regards the regime type in the target state. It is argued that democratic leaders generally pay greater audience costs than their autocratic counterparts who are less accountable to the public (Fearon 1994). A democratic leader will have a significantly lower $\phi$ than does an autocratic leader with ability to repress domestic opposition over time. In line with the recent arguments in the economic sanctions literature, that sanctions are more effective with democracies (McGillivray and Smith 2000, McGillivray and Stam 2004, Lektzian and Souva 2007, Marinov 2005), my model predicts that constant sanctioning is less effective when negotiating with autocracies, since both the range for an equilibrium with constant sanctioning and the sender’s share in such equilibria shrink with target’s adaptability.

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5 An independent body of work studies audience costs as an important mechanism that links domestic and international politics. These models generally argue that a political leader can establish credibility for a threat that he makes when he goes public with it, and ties his hands in front of their domestic audiences. If the leader backs down from his public commitment, then he pays some domestic political costs (Fearon 1994, 1997, also see Schelling 1966). The literature offers several explanations for why a political leader would pay an audience cost for backing down from a public threat: (i) he violates the national honor (Fearon 1994), (ii) he signals incompetence to voters (Fearon 1994, Smith 1998), (iii) he loses international credibility (Guisinger and Smith 2002; Sartori 2002), and (iv) voters use the punishment mechanism to allow the leader to generate international bargaining leverage (Leventoglu and Tarar 2005). For further discussions about the microfoundations of audience costs, see Gowa (1999), Schultz (1999) and Slantchev (2006). For experimental evidence about existence of audience costs, see Tomz (2007). The effects of such audience costs have been explored in both non-crisis (Leventoglu and Tarar 2005) and crisis bargaining situations (Fearon 1994, Schultz 1999, Smith 1998, Kurizaki 2007, Tarar and Leventoglu 2009). A typical non-crisis bargaining is a Rubinstein bargaining game where if the bargaining breaks down, the players receive their status quo payoffs (Rubinstein 1982). In a typical crisis bargaining, the players fight if bargaining breaks down (Fearon 1995, Powell 1996, 1999, Leventoglu and Tarar 2008).
On another note, the cost of losing power for authoritarian leaders is often much higher than for democratic leaders, including the possibility of death, among others (Gowa, 1995). Weeks (2008) examines different types of autocracies and presents statistical evidence showing that certain types of autocratic regimes face the same level of audience costs as democratic leaders. For example, in autocracies where opposition groups have solved their coordination problem of challenging the government, autocrats should face significant audience costs. Moreover, in a democracy, it seems that audience costs could be especially high just before new elections are held, and this would in particular be the case if the leader is also politically vulnerable, for example, if he is facing a weak economy or some other domestic crisis. One might argue that this type of leader has a very low $\phi$, because elections are fast approaching, and he is politically vulnerable. A leader who is domestically secure and not facing an election has a relatively higher $\phi$. Leaders that are facing elections, but domestically secure and leaders that are not facing elections but domestically vulnerable have intermediate $\phi$.

Another interpretation of $\phi$ concerns the type of consumption goods that are sanctioned. Sanctions on goods that are thought to be essential (e.g., food, medicine) affect a larger population, where as sanctions on goods that are thought to be luxurious (e.g., high end cars) affect a smaller population. It would be more difficult for the target leader to provide a larger citizen base with their needs, since constant sanctioning becomes even less effective when they are imposed on luxury goods.

A third interpretation of $\phi$ regards the type of sanctions. Financial sanctions may impose higher audience costs on the political leader than trade sanctions since they are easier to enforce and harder to adapt. Financial sanctions can simply stop the aid or loans flowing to the target country, and it is difficult for the target state to replace financial relationships quickly in particular when they concern direct aid to the country (Hufbauer, et al. 2007). Trade sanctions, however, may deflect trade but not cut it off completely. That is, the target state may identify alternative markets for the goods that are scarce, and the audience costs will be lower. Even if the target state cannot locate an alternative supplier, it may possess domestic production capacity (Hufbauer, et al. 2007). For example, Stedman (1998) argues that in former Yugoslavia, an arms embargo did not have much effect on Serbia since it had its own weapons production capability.

This completes the discussion of the benchmark model. I study sanctions with incomplete information in the next section.
4 Sanctions with Incomplete Information

In the previous section, sanctions are imposed only off the equilibrium path in all of the SPE presented in Proposition 2 and 3. In general, complete information games of sanctions predict that the threat of a sanction is sufficient to generate bargaining leverage, and costly sanctions are observed in equilibrium when incomplete information is introduced (Hovi, Huseby and Sprinz 2005, Lacy and Niou 2004, Morgan and Schwebach 1997, Smith 1996). In this section, I introduce asymmetric information to the model in order to study inefficiency from delay and sanctions that may be observed in equilibrium.

Informational asymmetry may relate to several aspects of the game. In the following, I focus on the following cases: (i) sender may be uncertain about the extent of sanctions he can impose, (ii) sender may be uncertain about how target values the issue at stake – issue salience –, (iii) sender may be uncertain about target’s ability to adapt.

Cases (i) and (ii) are fundamentally different than case (iii). In the former, the sender knows about the target’s adaptability. Then informational asymmetry regarding other aspects of the game is eliminated differently under alternative sanction policies. In cases (i) and (ii), there exist separating perfect Bayesian equilibrium with constant sanctioning and separating perfect Bayesian equilibrium with alternate sanctioning. In such separating equilibrium, informational asymmetry is eliminated by subjecting different types of targets to different risks of delay and sanctions. In each case, I focus on the most efficient separating equilibrium among the ones that provide the sender with the largest payoff. Same predictions obtain in these cases: An equilibrium delay emerges. That is, the sender rejects the target’s high offers with positive probability. When the sender adopts constant sanctioning, he sanctions the target after rejecting an offer. Therefore sanctions occur with positive probability in such equilibrium. When the sender adopts alternate sanctioning, he rejects high offers with a higher probability. That is, expected delay is longer. However, sanctions do not occur in equilibrium despite that the sender rejects the target’s offer with positive probability. If the players are patient enough, the sender is better off with alternate sanctioning in case (i). He is always better off with alternate sanctioning in case (ii).

Case (iii) provides a surprising prediction. When the sender adopts alternate sanctioning, no delay and no sanctions occur in equilibrium. This finding contrasts sharply with the predictions of incomplete information bargaining games in the literature, which predict delay and inefficiency. The
intuition behind this result is that alternate sanctioning frees the sender from target’s adaptability, therefore uncertainty about target’s adaptability becomes irrelevant in equilibrium. In contrast, when the sender adopts constant sanctioning, delay and sanctions are observed in equilibrium.

To simplify the analysis, I also assume that $\delta_S = \delta_T = \delta$ below.

### 4.1 Sanction effectiveness

I assume that the sender knows $\phi$, the target’s adaptability, but is uncertain about whether sanctions will be effective or not. I model this type of uncertainty as follows. Suppose that $v_T$ is either high or low. Let $v_T = v_h$ with probability $\theta$ and $v_T = v_l < v_h$ with probability $1 - \theta$. Any sanction of $y > v_T$ is effectively equivalent to a sanction of $v_T$ because the sender cannot impose sanctions on nonexistent resources. Thus, a sanction of $y \in (v_l, v_h]$ will be fully effective if $v_T = v_h$, but will be wasteful for the sender’s purpose if $v_T = v_l$.

Denote the target’s type as $\tau = \{h, l\}$. The target’s type is her private information, otherwise the structure of the game is common knowledge.

I show in the appendix that there always exists a separating Perfect Bayesian Equilibrium (PBE) in which different types of targets make different offers so that the types are revealed and informational asymmetry is eliminated in the equilibrium. In such an equilibrium, type $h$, who is vulnerable to sanctions, asks for a small share of the pie for herself. Type $l$, on the other hand, separates herself from type $h$ by asking for a larger share for herself. Upon observing the offers, the sender updates his beliefs accordingly. He accepts type $h$’s offer right away. He rejects type $l$’s higher offer and sanctions her with positive probability, then in the next period, he offers type $l$ a share that is larger than what he would offer to type $h$, which type $l$ accepts.

Both an SPE with constant sanctioning (Proposition 2) and an SPE with alternate sanctioning (Proposition 3) may be a part of such equilibrium.

There are multiple separating equilibria. I focus on the most efficient one among the ones that provide the sender with the highest expected payoff. Define $\bar{y}_l = v_l$, $\bar{y}_h = \min\{v_h, v_l + \frac{(1 - \delta^2)(v + (1 - \phi + \psi)v_l)}{\delta(1 - \phi) - \psi}\}$.
and
\[ \tilde{\alpha} = \frac{(\delta(1 - \phi) - \psi)(\bar{y}_h - v_l)}{(1 - \delta^2)(v + (1 - \phi + \psi)v_l)} \]
then the following proposition summarizes a separating equilibrium with an SPE with constant sanctioning on and off the equilibrium path.

**Proposition 4** The following is a PBE: Type \( \tau \) target offers \( \bar{x}_T(\bar{y}_r) \). If the sender receives an offer of \( x \leq \bar{x}_T(\bar{y}_h) \), he believes that the target is of type \( h \) and accepts her offer with probability 1. If the sender receives an offer of \( \bar{x}_T(\bar{y}_h) \), he believes that the target is of type \( l \), accepts her offer with probability \( 1 - \tilde{\alpha} \), rejects it with probability \( \tilde{\alpha} \) and the players play according to the SPE with constant sanctions of \( \bar{y}_h \) of Proposition 2 thereafter. In particular, the sender imposes a sanction of \( \bar{y}_l = v_l \) and offers \( \bar{x}_S(\bar{y}_l) \) in the next period, which the target accepts. If the sender receives any other offer, he believes that the target is of type \( h \), rejects the offer, and the players play according to the SPE with constant sanctions of \( \bar{y}_h \) of Proposition 2.

Since the sender is indifferent between accepting and rejecting type \( l \)'s offer, his expected equilibrium payoff is
\[ \bar{u} = v - \theta \bar{x}_T(\bar{y}_h) - (1 - \theta)\bar{x}_T(v_l) \]
Next define \( \bar{y}_l = v_l \),
\[ \bar{y}_h = \min\{v_h, v_l + \frac{(1 - \delta^2)v}{\delta}\} \leq \bar{y}_h \]
and
\[ \tilde{\alpha} = \frac{\delta(\bar{y}_h - v_l)}{(1 - \delta^2)v} \]
then the following proposition summarizes a separating equilibrium with an SPE with alternate sanctioning on and off the equilibrium path.

**Proposition 5** The following is a PBE: Type \( \tau \) target offers \( \bar{x}_T(\bar{y}_r) \). If the sender receives an offer of \( x \leq \bar{x}_T(\bar{y}_h) \), he believes that the target is of type \( h \) and accepts her offer with probability 1. If the sender receives an offer of \( \bar{x}_T(\bar{y}_h) \), he believes that the target is of type \( l \), accepts her offer with probability \( 1 - \tilde{\alpha} \), rejects it with probability \( \tilde{\alpha} \) and the players play according to the SPE of Proposition 3, alternating between sanctions of \( \bar{y}_h \) and no sanctions.
In particular, the sender offers \( \tilde{x}_S(\tilde{y}_l) \) in the next period, which the target accepts. If the sender receives any other offer, he believes that the target is of type \( h \), rejects the offer, and plays according to the SPE of Proposition 3, alternating between sanctions of \( \tilde{y}_h \) and no sanctions. Since he does not sanction the target after rejecting the target’s offer, no sanction is observed in the first period of the game.

Since the sender is indifferent between accepting and rejecting type \( l \)’s offer, his expected equilibrium payoff is

\[
\bar{u} = v - \theta \tilde{x}_T(\tilde{y}_h) - (1 - \theta) \tilde{x}_T(v_l)
\]

The analysis yields several interesting and novel findings. First, \( \tilde{y}_h \leq \tilde{y}_l \). That is, equilibrium sanctions are less restrictive in the PBE of Proposition 5, which utilizes a scheme of alternating sanctions. More importantly, while sanctions are observed in the equilibrium of Proposition 4, which calls for sanctions whenever an offer is rejected, sanctions are not observed in the equilibrium of Proposition 5. \( \tilde{\alpha} \geq \bar{\alpha} \) so that expected delay is potentially longer in the equilibrium of Proposition 5. However, a delay in this equilibrium does not induce sanctions. If \( v_h \) is large enough, then \( \tilde{\alpha} = \bar{\alpha} = 1 \) so that a delay occurs with probability \( 1 - \theta \) in both equilibria.

To summarize, when the sender is uncertain about the extent of sanctions he can impose, an equilibrium delay may emerge. The target can credibly signal her type to the sender in a separating equilibrium by facing a risk of delay. Such a separating equilibrium is possible with constant as well as alternate sanctioning. More importantly, in addition to the risk of delay, the target also faces the risk of sanctions in equilibrium that involves constant sanctioning, but sanctions are not observed after delay in equilibrium with alternate sanctioning.

4.2 Issue Salience

To model issue salience - that the target may attach a different value to the issue at dispute (e.g. see Ang and Peksen 2007) -, I assume that the target achieves a payoff of \( \gamma x \) from a share of \( x \). Suppose that \( \gamma \) is either high or low. Let \( \gamma = \gamma_h \) with probability \( \theta \) and \( \gamma = \gamma_l < \gamma_h \) with probability \( 1 - \theta \). Denote the target’s type as \( \tau = \{h, l\} \). The target’s type is her private information, otherwise the structure of the game is common knowledge.
I show in the appendix that equilibrium offers are increasing functions of $\gamma$ in the complete information game. In particular, the SPE without sanctions coincides with the unique SPE of the standard Rubinstein game without sanctions. Equilibrium offers in an SPE with constant sanctioning are given by

$$
\tilde{x}_S(y; \gamma) = x^*_S - \frac{1}{1 + \delta} \left[ \frac{1 - \phi}{\gamma} - \delta \psi \right] y,
$$

$$
\tilde{x}_T(y; \gamma) = x^*_T - \frac{1}{1 + \delta} \left[ \frac{\delta(1 - \phi)}{\gamma} - \psi \right] y,
$$

and equilibrium offers in an SPE with alternate sanctioning are given by

$$
\tilde{x}_S(y; \gamma) = x^*_S - \frac{y}{\gamma(1 + \delta)},
$$

$$
\tilde{x}_T(y; \gamma) = x^*_T - \frac{\delta y}{\gamma(1 + \delta)}.
$$

The SPE without sanctions is independent of $\gamma$, offers in SPE with constant or alternate sanctioning are increasing functions of $\gamma$.

I also show in the appendix there always exists a separating Perfect Bayesian Equilibrium (PBE) in which different types of targets make different offers so that the types are revealed and informational asymmetry is eliminated in the equilibrium. In such an equilibrium, type $l$, who cares less about the issue, asks for a small share of the pie for herself. Type $h$, on the other hand, separates herself from type $l$ by asking for a larger share for herself. Upon observing the offers, the sender updates his beliefs accordingly. He accepts type $l$’s offer right away. He rejects type $h$’s higher offer and sanctions her with positive probability, then in the next period, he offers type $h$ a share that is larger than what he would offer to type $l$, which type $h$ accepts.

Both an SPE with constant sanctioning (Proposition 2) and an SPE with alternate sanctioning (Proposition 3) may be a part of such equilibrium.

There are multiple separating equilibria. I focus on the most efficient one among the ones that provide the sender with the highest expected payoff. Redefine

$$
\tilde{\alpha} = \frac{\delta(1 - \phi)(\gamma_h - \gamma_l)}{\gamma_l \gamma_h (1 - \delta)(v + \psi) + \delta(1 - \phi)(\delta \gamma_h - \gamma_l) + (1 - \delta^2) \gamma_h (1 - \phi)} \in (0, 1)
$$

then the following proposition summarizes a separating equilibrium with an SPE with constant sanctioning on and off the equilibrium path.
Proposition 6 The following is a PBE: Type $\tau$ target offers $\bar{x}_T(1; \gamma_{\tau})$. If the sender receives an offer of $x \leq \bar{x}_T(1; \gamma_l)$, he believes that the target is of type $l$ and accepts her offer with probability 1. If the sender receives an offer of $\bar{x}_T(1; \gamma_h)$, he believes that the target is of type $h$, accepts her offer with probability $1 - \bar{\alpha}$, rejects it with probability $\bar{\alpha}$ and the players play according to the SPE with constant sanctions of $y = 1$ of Proposition 2 thereafter. In particular, the sender imposes a sanction of $y = 1$ and offers $\bar{x}_S(1; \gamma_h)$ in the next period, which the target accepts. If the sender receives any other offer, he believes that the target is of type $l$, rejects the offer, and the players play according to the SPE with constant sanctions of $y = 1$ of Proposition 2 with the belief that $\gamma = \gamma_l$.

Next redefine
\[
\bar{\alpha} = \frac{\delta(\gamma_h - \gamma_l)}{\gamma_l\gamma_h(1 - \delta)v + \delta(\gamma_h - \gamma_l)} \in (\bar{\alpha}, 1)
\]
then the following proposition summarizes a separating equilibrium with an SPE with alternate sanctioning on and off the equilibrium path.

Proposition 7 The following is a PBE: Type $\tau$ target offers $\bar{x}_T(1; \gamma_{\tau})$. If the sender receives an offer of $x \leq \bar{x}_T(1; \gamma_l)$, he believes that the target is of type $l$ and accepts her offer with probability 1. If the sender receives an offer of $\bar{x}_T(1; \gamma_h)$, he believes that the target is of type $h$, accepts her offer with probability $1 - \bar{\alpha}$, rejects it with probability $\bar{\alpha}$ and the players play according to the SPE of Proposition 3, alternating between sanctions of $y = 1$ and no sanctions. In particular, the sender offers $\bar{x}_S(1; \gamma_h)$ in the next period, which the target accepts. If the sender receives any other offer, he believes that the target is of type $l$, rejects the offer, and plays according to the SPE of Proposition 3 with the belief that $\gamma = \gamma_l$, alternating between sanctions of $y = 1$ and no sanctions. Since he does not sanction the target after rejecting the target’s offer, no sanction is observed in the first period of the game.

In this case, equilibrium sanctions are independent of target’s type and equally restrictive in both equilibria. Similar to the previous case, while sanctions are observed in the equilibrium of Proposition 6, which calls for sanctions whenever an offer is rejected, sanctions are not observed in the equilibrium of Proposition 7. $\bar{\alpha} > \bar{\alpha}$ so that expected delay is longer in the equilibrium of Proposition 7. However, a delay in this equilibrium does not induce sanctions.
To summarize, when the sender is uncertain about how the target values the issue in dispute, an equilibrium delay may emerge. The target can credibly signal her type to the sender in a separating equilibrium by facing a risk of delay. Such a separating equilibrium is possible with constant as well as alternate sanctioning. More importantly, in addition to the risk of delay, the target also faces the risk of sanctions in equilibrium that involves constant sanctioning, but sanctions are not observed after delay in equilibrium with alternate sanctioning.

4.3 Uncertainty on Adaptability

To model uncertainty about the target’s ability to adapt, suppose that $\phi$ is either high or low. Let $\phi = \phi_h$ with probability $\theta$ and $\phi = \phi_l < \phi_h$ with probability $1 - \theta$. Denote the target’s type as $\tau = \{h, l\}$. The target’s type is her private information, otherwise the structure of the game is common knowledge.

Since this case only adds uncertainty to the benchmark model, its complete information analysis is the same. Introducing $\phi$ as an argument to the offer functions, equilibrium offers in an SPE with constant sanctioning are given by

$$\bar{x}_S(y; \phi) = x_S^* - \frac{1}{1 + \delta} [(1 - \phi) - \delta \psi] y$$

and

$$\bar{x}_T(y; \phi) = x_T^* - \frac{1}{1 + \delta} [\delta (1 - \phi) - \psi] y,$$

and equilibrium offers in an SPE with alternate sanctioning are given by

$$\bar{x}_S(y; \phi) = x_S^* - \frac{y}{1 + \delta}$$

and

$$\bar{x}_T(y; \phi) = x_T^* - \frac{\delta y}{1 + \delta}.$$

The SPE without sanctions is independent of $\phi$, offers in SPE with constant or alternate sanctioning are increasing functions of $\phi$.

I show in the appendix that there always exists a separating Perfect Bayesian Equilibrium (PBE) in which different types of targets make different offers so that the types are revealed and informational asymmetry is eliminated in the equilibrium. In such an equilibrium, type $l$, who adapts
less, asks for a small share of the pie for herself. Type \( h \), on the other hand, separates herself from type \( l \) by asking for a larger share for herself. Upon observing the offers, the sender updates his beliefs accordingly. He accepts type \( l \)'s offer right away. He rejects type \( h \)'s higher offer and sanctions her with positive probability, then in the next period, he offers type \( h \) a share that is larger than what he would offer to type \( l \), which type \( h \) accepts. Both an SPE with constant sanctioning (Proposition 2) and an SPE with alternate sanctioning (Proposition 3) may be a part of such equilibrium.

As before, there are multiple separating equilibria and I focus on the most efficient one among the ones that provide the sender with the highest expected payoff. Redefine

\[
\bar{\alpha} = \frac{\delta(\phi_h - \gamma)}{(1 - \delta)(v + \psi + 1) + (\delta \phi_h - \phi_l)} \in (0, 1)
\]

then the following proposition summarizes a separating equilibrium with an SPE with constant sanctioning on and off the equilibrium path.

**Proposition 8** The following is a PBE: Type \( \tau \) target offers \( \bar{x}_T(1; \phi_\tau) \). If the sender receives an offer of \( x \leq \bar{x}_T(1; \phi_l) \), he believes that the target is of type \( l \) and accepts her offer with probability 1. If the sender receives an offer of \( \bar{x}_T(1; \phi_h) \), he believes that the target is of type \( h \), accepts her offer with probability \( 1 - \bar{\alpha} \), rejects it with probability \( \bar{\alpha} \) and the players play according to the SPE with constant sanctions of \( y = 1 \) of Proposition 2 thereafter. In particular, the sender imposes a sanction of \( y = 1 \) and offers \( \bar{x}_S(1; \phi_h) \) in the next period, which the target accepts. If the sender receives any other offer, he believes that the target is of type \( l \), rejects the offer, and the players play according to the SPE with constant sanctions of \( y = 1 \) of Proposition 2 with the belief that \( \phi = \phi_l \).

More importantly, when the sender adopts alternate sanctioning, equilibrium offers become independent of \( \phi \). In other words, uncertainty about \( \phi \) has no impact on equilibrium behavior. The SPE with alternate sanctioning with \( y = 1 \) (Proposition 3) provides the highest payoff for the sender within equilibria with alternate sanctioning. Neither delay nor sanctions occur in this equilibrium.

To summarize, when the sender is uncertain about the target’s ability to adapt, delay and sanctions emerge in equilibrium with constant sanctioning but not in equilibrium with alternate sanctioning.
5 Conclusion

In this paper, I develop a bargaining model with sanctions and examine the impact of adaptability to sanctions on bargaining outcomes. The analysis provides novel and interesting predictions. First, as expected, sanctions enhance sender’s bargaining leverage in negotiations. However, these gains diminish as target’s adaptability to sanctions improves. More importantly, temporary reliefs from sanctions provide more bargaining leverage for the sender in comparison to constantly sanctioning a country. Introduction of informational asymmetry may cause delay in negotiations. In contrast to earlier incomplete information models of sanctions, sanctions may not be observed in equilibrium when the sender provides temporary reliefs from sanctions. The expected delay may be longer in this latter case. These findings have direct implications in international bargaining.

The model and the analytical tools that I introduce in this paper can be applied to alternative scenarios. For example, some scholars argue that sanctions may generate a "rally around the flag" effect (Pape 1997), or domestic public opinion may favor keeping the set of actions about which the sanctions have been instituted (McGillivray and Stam 2004), both of which can be formulated as a negative $\phi$. In this case, "rally around the flag" would diminish over time and sanctions would become more effective. On another note, Jonge Oudraat (2000) argues that the literature on sanctions has mostly studied sanctions in isolation, that is, independent of other possible coercive tools, for example, the actual use of force. It is quite possible that economic sanctions can work more effectively when they are a part of a comprehensive strategy. That includes the use of force as an outside option in case sanctions do not bring about the expected behavioral change in the target state leadership. I leave these topics for future research.
References


A Equilibrium Analysis

In equilibrium, each player chooses an optimal action at every subgame. Since sanctions are costly for the sender, once an agreement is reached, it is optimal for him to stop sanctioning forever. I will assume that a player accepts an offer when he is indifferent between accepting and rejecting it given the continuation equilibrium. If player $j$ is better off by accepting player $i$’s offer, then player $i$ can increase his/her payoff by slightly decreasing player $j$’s share, which player $j$ will continue to accept. Therefore player $i$’s optimal offer makes player $j$ indifferent between accepting and rejecting $i$’s offer. Consider an equilibrium at which offers are accepted in every subgame. Given $1$’s strategy for sanctions, $y_{it}$, the conditions for the equilibrium offers that are accepted are given by

$$\frac{1 + x_{St}}{1 - \delta_T} = 1 - [(1 - \phi)y_{St} + \phi [y_{St} - y_{T,t-1}^+]] + \delta_T \frac{1 + x_{T,t+1}}{1 - \delta_T}$$  \hspace{1cm} (1)$$

$$\frac{v - x_{T,t+1}}{1 - \delta_S} = -\psi y_{T,t+1} + \frac{\delta_S}{1 - \delta_S} (v - x_{S,t+2})$$  \hspace{1cm} (2)$$

In the first equation, the left hand side is the target’s payoff if she accepts $x_{St}$. $\frac{1 + x_{St}}{1 - \delta_T}$ is her lifetime utility from consuming $v_T = 1$ and $x_{St}$ forever. Since there is no more sanctioning thereafter, the target does not incur any cost from sanctions from period $t$ on. The right hand side is the target’s payoff if she rejects $x_{St}$. In that case, she consumes $1 - [(1 - \phi)y_{St} + \phi [y_{St} - y_{T,t-1}^+]]$ under sanctions in period $t$, and then her offer of $x_{T,t+1}$ will be accepted in period $t + 1$, which she will consume thereafter along with $v_T = 1$ with no more sanctions.

In the second equation, the left hand side is the sender’s payoff if he accepts the target’s offer of $x_{T,t+1}$ in period $t + 1$ and the right hand side is his payoff if he rejects it. In that case, the sender’s offer of $x_{S,t+2}$ will be accepted in period $t + 2$, and he will pay a one-period cost of $\psi y_{T,t+1}$ for sanctioning the target.

To simplify the expressions, multiply both sides of (1) by $(1 - \delta_T)$, cancel out 1s on both side, and multiply both sides of (2) by $(1 - \delta_S)$ and obtain the following, which I will use in the analysis.

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\[ x_{St} = -(1 - \delta_T) \left[ (1 - \phi)y_{St} + \phi \left[ y_{St} - y_{T,t-1} \right] \right] + \delta_T x_{T,t+1} \tag{3} \]
\[ v - x_{T,t+1} = -(1 - \delta_S)\psi y_{T,t+1} + \delta_S(v - x_{S,t+2}) \tag{4} \]

**Proof of Proposition 1:**

First suppose that \( y_{St} = y_{T,t+1} = 0 \) for all \( t \). Then the game reduces to the standard Rubinstein bargaining game, and (3) and (4) reduce to
\[ x_{St} = \delta_T x_{T,t+1} \]
\[ v - x_{T,t+1} = \delta_S(v - x_{S,t+2}) \]
which yield a unique stationary equilibrium with \( x_{St} = x^*_S \) and \( x_{T,t+1} = x^*_T \).

Now suppose that player \( i \)'s offer of \( x^*_i \) is rejected in period \( t \) and the sender chooses a sanction level of \( y \). From the next period on, the players play the standard Rubinstein game with no more sanctions and player \( j \)'s offer of \( x^*_j \) is accepted, \( j \neq i \). So the sender’s continuation payoff is
\[ -\psi y + \frac{\delta_S}{1 - \delta_S} (v - x^*_j) \]
which is maximized at \( y = 0 \). That is, for every \( t \), \( y_{St} = 0 \) is best response to \( y_{T,t+1} = y_{S,t+2} = 0 \) for all \( t' \geq t \), and \( y_{T,t+1} = 0 \) is best response to \( y_{S,t+2} = y_{T,t+3} = 0 \) for all \( t' \geq t \). This completes the proof.

**Proof of Proposition 2:**

Suppose that \( y_{St} = y_{T,t+1} = y > 0 \) for all \( t \). Then (3) and (4) reduce to
\[ x_{St} = -(1 - \delta_T)(1 - \phi)y + \delta_T x_{T,t+1} \]
\[ v - x_{T,t+1} = -(1 - \delta_S)\psi y + \delta_S(v - x_{S,t+2}) \]
for all \( t > 1 \). Substituting \( x_{St} = x_{S,t+2} = \bar{x}_S(y) \) and \( x_{T,t+1} = \bar{x}_T(y) \), obtain
\[ \bar{x}_S(y) = x^*_S - \frac{1}{1 - \delta_S \delta_T} \left[ (1 - \delta_T)(1 - \phi) - \delta_T(1 - \delta_S)\psi \right] y \]
and
\[ \bar{x}_T(y) = x^*_T - \frac{1}{1 - \delta_S \delta_T} \left[ \delta_S(1 - \delta_T)(1 - \phi) - (1 - \delta_S)\psi \right] y \]
Therefore, it is optimal for the target to offer \( \bar{x}_T(y) \), accept any offer \( x \geq \bar{x}_S(y) \), and reject all other offers.
Next I will show that it is not optimal for the sender to deviate from the stated strategy. It is verified easily that \( x_S^* - \bar{x}_S(y) \geq x_T^* - \bar{x}_T(y) \) and \( x_T^* - \bar{x}_T(y) \geq 0 \) if and only if
\[
\psi \leq \psi^* = \frac{\delta_S(1 - \delta_T)(1 - \phi)}{1 - \delta_S}
\]
Given that the target will accept any offer \( x \geq \bar{x}_S(y) \), it is not optimal for the sender to offer \( x > \bar{x}_S(y) \). Suppose that the sender offers \( x < \bar{x}_S(y) \). Then the target rejects \( x \) and makes the next offer. If the sender does not deviate by choosing a different sanction level after this rejection, the target offers \( \bar{x}_T(y) \) the next period, which the sender accepts. So the sender’s payoff from such deviation is
\[
-\psi y + \frac{\delta_S}{1 - \delta_S}(v - \bar{x}_T(y)) < \frac{1}{1 - \delta_S}(v - \bar{x}_S(y))
\]
where the last inequality follows from \( \bar{x}_S(y) < \bar{x}_T(y) \) and the last term is the sender’s payoff from offering \( \bar{x}_S(y) \), which the target accepts. So such deviation is not optimal for the sender. If the sender also chooses a different level of sanctions, then the players play the SPE with no sanctions from next period on, which gives the sender a payoff of \( \frac{v - x_T^*}{1 - \delta_T} \) in the next period. So the sender’s continuation payoff with a sanction \( y' \neq y \) is
\[
-\psi y' + \frac{\delta_S}{1 - \delta_S}(v - x_T^*) \leq \frac{\delta_S}{1 - \delta_S}(v - x_T^*) < \frac{1}{1 - \delta_S}(v - \bar{x}_T(y)) < \frac{1}{1 - \delta_S}(v - \bar{x}_S(y))
\]
where the inequalities follow from \( y > 0, \bar{x}_S(y) < \bar{x}_T(y) < x_T^* \), and the last term is the sender’s payoff from offering \( \bar{x}_S(y) \), which the target accepts. So such deviation is not optimal for the sender either. Therefore, it is optimal for the sender to offer \( \bar{x}_S(y) \).

Finally, I will show that it is optimal for the sender to impose a sanction of \( y \) after a rejection. Suppose that the target rejects the sender’s offer. If the sender follows his equilibrium strategy, his continuation payoff is given by
\[
-\psi y + \frac{\delta_S}{1 - \delta_S}(v - \bar{x}_T(y))
\]
If he deviates and imposes another level of sanction, the players play the SPE with no sanctions from next period on, in which the target’s offer of $x_T^*$ is accepted the next period. So the best deviation for the sender is to deviate to $y_{St} = 0$. Then his continuation payoff is given by

$$\frac{\delta_S}{1 - \delta_S}(v - x_T^*)$$

So for $y_{St} = y$ to be optimal, it must be that

$$-\psi y + \frac{\delta_S}{1 - \delta_S}(v - \bar{x}_T(y)) \geq \frac{\delta_S}{1 - \delta_S}(v - x_T^*)$$

equivalently

$$(1 - \delta_S)\psi y \leq \delta_S(x_T^* - \bar{x}_T(y))$$

that is,

$$\psi \leq \tilde{\psi} = \frac{\delta_S(1 - \delta_T)(1 - \phi)}{(1 - \delta_S)(1 - \delta_S\delta_T + \delta_S)} < \psi^*.$$

Similarly, suppose that the sender rejects the target’s offer. If the sender follows his equilibrium strategy, his continuation payoff is given by

$$-\psi y + \frac{\delta_S}{1 - \delta_S}(v - \bar{x}_S(y))$$

If he deviates and imposes another level of sanction, the players play the SPE with no sanctions from next period on, in which the sender’s offer of $x_S^*$ is accepted the next period. So the best deviation for the sender is to deviate to $y_{T,t+1} = 0$. Then his continuation payoff is given by

$$\frac{\delta_S}{1 - \delta_S}(v - x_S^*)$$

So for $y_{T,t+1} = y$ to be optimal, it must be that

$$-\psi y + \frac{\delta_S}{1 - \delta_S}(v - \bar{x}_S(y)) \geq \frac{\delta_S}{1 - \delta_S}(v - x_S^*) \Leftrightarrow$$

$$(1 - \delta_S)\psi y \leq \delta_S(x_S^* - \bar{x}_S(y))$$

Since $x_T^* - \bar{x}_T(y) \leq x_S^* - \bar{x}_S(y)$, the last inequality is implied by $\psi \leq \tilde{\psi}$. This completes the proof.
Proof of Proposition 3:
Suppose that $y_{St} = y_S$ and $y_{T,t+1} = y_T$ for all $t$. Then $y_{T,t-1} = y_T$ and $y_{S,t+2} = y_S$ so that (3) and (4) reduce to

$$x_{St} = -(1 - \delta_T) [(1 - \phi)y_T + \phi[y_S - y_T]_+] + \delta_T x_{T,t+1}$$

$$v - x_{T,t+1} = -(1 - \delta_S) \psi y_T + \delta_S (v - x_{S,t+2})$$

for all $t > 1$. Notice that a decrease in $y_T$ decreases $x_{T,t+1}$ and $x_{St}$, which is better for the sender since these are the target’s shares and player pays a smaller cost for sanctions. Then set $y_T = 0$ and $y_S = y$. Substituting $x_{St} = x_{S,t+2} = \bar{x}_S(y)$ and $x_{T,t+1} = \bar{x}_T(y)$ in the equation system above, obtain

$$\bar{x}_S(y) = x^*_S - \frac{(1 - \delta_T)y}{1 - \delta_S \delta_T} \text{ and}$$

$$\bar{x}_T(y) = x^*_T - \frac{\delta_S (1 - \delta_T)y}{1 - \delta_S \delta_T}$$

Therefore, it is optimal for the target to offer $\bar{x}_T(y)$, accept any offer $x \geq \bar{x}_S(y)$, and reject all other offers.

Next I will show that it is not optimal for the sender to deviate from the stated strategy. It is verified easily that $x^*_S - \bar{x}_S(y) \geq x^*_T - \bar{x}_T(y) \geq 0$. Given that the target will accept any offer $x \geq \bar{x}_S(y)$, it is not optimal for the sender to offer $x > \bar{x}_S(y)$. Suppose that the sender offers $x < \bar{x}_S(y)$. Then the target rejects $x$ and makes the next offer. If the sender does not deviate by choosing a different sanction level after this rejection, the target offers $\bar{x}_T(y)$ the next period, which the sender accepts. So the sender’s payoff from such deviation is

$$- \psi y + \frac{\delta_S}{1 - \delta_S} (v - \bar{x}_T(y))$$

$$\leq \frac{\delta_S}{1 - \delta_S} (v - \bar{x}_T(y)) < \frac{1}{1 - \delta_S} (v - \bar{x}_S(y))$$

where the last inequality follows from $\bar{x}_S(y) < \bar{x}_T(y)$ and the last term is the sender’s payoff from offering $\bar{x}_S(y)$, which the target accepts. So such deviation is not optimal for the sender. If the sender also chooses a different level of sanctions, then the players play the SPE with no sanctions from the
next period on, which gives the sender a payoff of \( \frac{v - x_T^*}{1 - \delta_T} \) in the next period. So the sender’s continuation payoff with a sanction \( y' \neq y \) is

\[
- \psi y' + \frac{\delta_S}{1 - \delta_S} (v - x_T^*) < \frac{\delta_S}{1 - \delta_S} (v - x_T^*) < \frac{1}{1 - \delta_S} (v - \bar{x}_T(y)) < \frac{1}{1 - \delta_S} (v - \bar{x}_T(y))
\]

where the inequalities follow from \( y > 0, \bar{x}_S(y) < \bar{x}_T(y) < x_T^* \), and the last term is the sender’s payoff from offering \( \bar{x}_S(y) \), which the target accepts. So such deviation is not optimal for the sender either. Therefore, it is optimal for the sender to offer \( \bar{x}_S(y) \).

Finally, I will show that it is optimal for the sender to impose a sanction of \( y \). Suppose that the target rejects the sender’s offer. If the sender follows his equilibrium strategy, his continuation payoff is given by

\[
- \psi y + \frac{\delta_S}{1 - \delta_S} (v - \bar{x}_T(y))
\]

If he deviates and imposes another level of sanction, the players play the SPE with no sanctions from the next period on, in which the target’s offer of \( x_T^* \) is accepted the next period. So the best deviation for the sender is to deviate to \( y_{St} = 0 \). Then his continuation payoff is given by

\[
\frac{\delta_S}{1 - \delta_S} (v - x_T^*)
\]

So for \( y_{St} = y \) to be optimal, it must be that

\[
- \psi y + \frac{\delta_S}{1 - \delta_S} (v - \bar{x}_T(y)) \geq \frac{\delta_S}{1 - \delta_S} (v - x_T^*)
\]

equivalently

\[
(1 - \delta_S) \psi y \leq \delta_S (x_T^* - \bar{x}_T(y))
\]

that is,

\[
\psi \leq \tilde{\psi} = \frac{\delta_S (1 - \delta_T)}{(1 - \delta_S)(1 - \delta_S \delta_T)}.
\]

Note that \( \tilde{\psi} < \tilde{\psi} \).
Similarly, suppose that the sender rejects the target’s offer. If the sender follows his equilibrium strategy, his continuation payoff is given by
\[ \frac{\delta_S}{1 - \delta_S}(v - \tilde{x}_S(y)) \]
If he deviates and imposes another level of sanction, then the players play the SPE with no sanctions from the next period on, in which the sender’s offer of \( x_S^* \) is accepted the next period. So the best deviation for the sender is to deviate to \( y_{T, t+1} = 0 \). Then his continuation payoff is given by
\[ \frac{\delta_S}{1 - \delta_S}(v - x_S^*) < \frac{\delta_S}{1 - \delta_S}(v - \tilde{x}_S(y)) \]
where the last inequality follows from \( \tilde{x}_S(y) < x_S^* \). So \( y_{T, t+1} = 0 \) is optimal for the sender. This completes the proof.

B Incomplete Information - Sanction Effectiveness

B.1 Complete Information

The SPE with no sanctions is characterized by offers \( x_S^* \) and \( x_T^* \). The offers in an SPE with constant sanctions of \( y \leq v_T \) are given by \( \tilde{x}_S(y) \) and \( \tilde{x}_T(y) \) and the offers in an SPE that alternates between sanctions of \( y \) and no sanctions are given by \( \tilde{x}_S(y) \) and \( \tilde{x}_T(y) \). The offer functions \( \tilde{x}_S(y) \), \( \tilde{x}_T(y) \), \( \tilde{x}_S(y) \) and \( \tilde{x}_T(y) \) are all decreasing in \( y \). When \( \delta_S = \delta_T = \delta \),
\[ x_S^* = \frac{\delta}{1 + \delta} v \quad \text{and} \quad x_T^* = \frac{1}{1 + \delta} v, \]
\[ \tilde{x}_S(y) = x_S^* - \frac{1}{1 + \delta} [(1 - \phi) - \delta \psi] y \quad \text{and} \]
\[ \tilde{x}_T(y) = x_T^* - \frac{1}{1 + \delta} [\delta (1 - \phi) - \psi] y, \]
\[ \tilde{x}_S(y) = x_S^* - \frac{y}{1 + \delta} \quad \text{and} \]
\[ \tilde{x}_T(y) = x_T^* - \frac{\delta y}{1 + \delta}. \]
B.2 Equilibrium under constant sanctions of $y$

Suppose that there is a separating equilibrium in which type $\tau$ makes an offer of $x_T^\tau$, such that $x_T^h \neq x_T^l$.

After observing $x_T^\tau$, the continuation game becomes a complete information game with $v_T = v_{\tau}$. Suppose that the players play an SPE with constant sanctions of $y_{\tau} \leq v_{\tau}$ afterwards. In this case, if the sender rejects $x_T^\tau$, the sender imposes a sanction of $y_{\tau}$ and his offer $x_S(y_{\tau})$ is accepted in the next period. Then for the sender to accept $x_T^\tau$, it must satisfy

$$v - x_T^\tau = -(1 - \delta)\psi y_{\tau} + \delta(v - x_S(y_{\tau}))$$

so that $x_T^\tau = x_T(y_{\tau})$. Suppose that the sender rejects $x_T^\tau$ with probability $\alpha^\tau$. Since the sender is indifferent between accepting and rejecting $x_T(y_{\tau})$, any $\alpha^\tau \in [0, 1]$ is optimal for the sender. The sender’s expected payoff from such a separating equilibrium is calculated as

$$\bar{u}_S(y_h, y_l) = \theta \left[(1 - \alpha^h)(v - x_T(y_h)) + \alpha^h \delta(v - x_S(y_h))\right] + (1 - \theta) \left[(1 - \alpha^l)(v - x_T(y_l)) + \alpha^l \delta(v - x_S(y_l))\right]$$

$$= \theta(v - x_T(y_h)) + (1 - \theta)(v - x_T(y_l))$$

where the last equality follows from that the sender is is indifferent between accepting and rejecting an offer in a separating equilibrium. Since $x_T(y_{\tau})$ is decreasing in $y_{\tau}$, $\bar{u}_S(y_h, y_l)$ is increasing in both $y_h$ and $y_l$. To find the best separating equilibrium for the sender, I will pick $y_l$ and $y_h$ as large as possible below. Since $v_l < v_h$, I will solve for a separating equilibrium with $y_l \leq y_h$.

Assuming that the game starts with the target having been sanctioned to the full extent, for type $h$ not to imitate type $l$, it must be the case that

$$(1 - \alpha^h)x_T^h + \alpha^h \left[ -(1 - \delta)(1 - \phi)y_h + \delta x_S(y_h) \right] \geq (1 - \alpha^l)x_T^l + \alpha^l \left[ -(1 - \delta)(1 - \phi)y_l + \delta x_S(y_l) \right]$$

The left hand side is type $h$’s expected payoff from offering $x_T^h$ and the right hand side is her payoff from offering $x_T^l$. Similarly, for type $l$ not to imitate type $h$, it must be the case that

$$(1 - \alpha^l)x_T^l + \alpha^l \left[ -(1 - \delta)(1 - \phi)y_l + \delta x_S(y_l) \right] \geq (1 - \alpha^h)x_T^h + \alpha^h \left[ -(1 - \delta)(1 - \phi)y_h + \delta x_S(y_h) \right]$$

These two inequalities imply

$$(1 - \alpha^h)x_T^h + \alpha^h \left[ -(1 - \delta)(1 - \phi)y_h + \delta x_S(y_h) \right] = (1 - \alpha^l)x_T^l + \alpha^l \left[ -(1 - \delta)(1 - \phi)y_l + \delta x_S(y_l) \right]$$
Since $\tilde{x}^T_T = \tilde{x}_T(y_T) > -(1 - \delta)(1 - \phi)y_T + \delta \tilde{x}_S(y_T)$, the left and right hand sides of this equation are decreasing functions of $\alpha^h$ and $\alpha^l$, respectively. Also $y_T \leq y_h$ implies $\tilde{x}_T(y_h) \leq \tilde{x}_T(y_T)$ and $\tilde{x}_S(y_h) \leq \tilde{x}_S(y_T)$ so that it must be the case that $\alpha^h \leq \alpha^l$. Since delay is costly, set $\alpha^h = 0$. Then

\[ \tilde{x}^h_T = (1 - \alpha^l)\tilde{x}^l_T + (\alpha^l [-(1 - \delta)(1 - \phi)y_T + \delta \tilde{x}_S(y_T)] \leftrightarrow \]

\[ \alpha^l = \frac{\tilde{x}^l_T - \tilde{x}^h_T}{\tilde{x}^l_T - \delta \tilde{x}_S(y_T) + (1 - \delta)(1 - \phi)y_T} \]

Substitute $\tilde{x}^T_T = \tilde{x}_T(y_T)$ to obtain

\[ \alpha^l = \frac{(\delta(1 - \phi) - \psi)(y_h - y_l)}{(1 - \delta^2)(v + (1 - \phi + \psi)y_l)} \]

$\alpha^l > 0$ since $\psi \leq \tilde{\psi} = \frac{\delta(1 - \phi)}{1 + \delta(1 - \delta)} < \delta(1 - \phi)$. For any $y_T \leq y_h$ such that $\alpha^l \leq 1$, the above strategy and belief system forms a separating equilibrium.

To find the best such separating equilibrium, maximize $y_h$ and $y_l$ subject to $y_T \leq y_T$ and $\alpha^l \leq 1$. Then $\tilde{y}_l = v_T$,

\[ \tilde{y}_h = \min\{v_h, v_T + \frac{(1 - \delta^2)(v + (1 - \phi + \psi)v_T)}{\delta(1 - \phi) - \psi}\} \text{ and } \tilde{\alpha} = \frac{(\delta(1 - \phi) - \psi)(\tilde{y}_h - v_T)}{(1 - \delta^2)(v + (1 - \phi + \psi)v_T)} \]

provide the best equilibrium for sender. Let $\tilde{u}$ denotes the sender’s expected payoff in this equilibrium.

### B.3 Equilibrium under alternating sanctions

The equilibrium analysis is very similar to the previous one. Suppose that there is a separating equilibrium in which type $\tau$ makes an offer of $\tilde{x}^T_T$, such that $\tilde{x}^h_T \neq \tilde{x}^l_T$.

After observing $\tilde{x}^T_T$, the continuation game becomes a complete information game with $v_T = v_T$. Suppose that the players play an SPE with an alternating scheme of sanctions afterwards, in which the sender sanctions the target at $y_T \leq v_T$ when the target rejects his offer and he does not sanction her when he rejects her offer. In this case, if the sender rejects $\tilde{x}^T_T$, the sender does not imposes sanctions and his offer $\tilde{x}_S(y_T)$ is accepted in the next period. Then for the sender to accept $\tilde{x}^T_T$, it must satisfy

\[ v - \tilde{x}^T_T = \delta(v - \tilde{x}_S(y_T)) \]
so that \( \tilde{x}_T^r = \tilde{x}_T(y_r) \). Suppose that the sender rejects \( \tilde{x}_T^r \) with probability \( \alpha^r \). Since the sender is indifferent between accepting and rejecting \( \tilde{x}_T(y_r) \), any \( \alpha^r \in [0, 1] \) is optimal for the sender. The sender’s expected payoff from such a separating equilibrium is calculated as

\[
\tilde{u}_S(y_h, y_l) = \theta \left[ (1 - \alpha^h)(v - \tilde{x}_T(y_h)) + \alpha^h \delta(v - \tilde{x}_S(y_h)) \right] \\
+ (1 - \theta) \left[ (1 - \alpha^l)(v - \tilde{x}_T(y_l)) + \alpha^l \delta(v - \tilde{x}_S(y_l)) \right] \\
= \theta(v - \tilde{x}_T(y_h)) + (1 - \theta)(v - \tilde{x}_T(y_l))
\]

where the last equality follows from that the sender is indifferent between accepting and rejecting an offer in a separating equilibrium. Since \( \tilde{x}_T(y_r) \) is decreasing in \( y_r \), \( \tilde{u}_S(y_h, y_l) \) is increasing in both \( y_h \) and \( y_l \). To find the best separating equilibrium for the sender, I will pick \( y_l \) and \( y_h \) as large as possible below. Since \( v_l < v_h \), I will solve for a separating equilibrium with \( y_l \leq y_h \).

For type \( h \) not to imitate type \( l \), it must be the case that

\[
(1 - \alpha^h)\tilde{x}_T^h + \alpha^h \delta \tilde{x}_S(y_h) \geq (1 - \alpha^l)\tilde{x}_T^l + \alpha^l \delta \tilde{x}_S(y_l)
\]

The left hand side is type \( h \)’s expected payoff from offering \( \tilde{x}_T^h \) and the right hand side is her payoff from offering \( \tilde{x}_T^l \). Similarly, for type \( l \) not to imitate type \( h \), it must be the case that

\[
(1 - \alpha^l)\tilde{x}_T^l + \alpha^l \delta \tilde{x}_S(y_l) \geq (1 - \alpha^h)\tilde{x}_T^h + \alpha^h \delta \tilde{x}_S(y_h)
\]

These two inequalities imply

\[
(1 - \alpha^h)\tilde{x}_T^h + \alpha^h \delta \tilde{x}_S(y_h) = (1 - \alpha^l)\tilde{x}_T^l + \alpha^l \delta \tilde{x}_S(y_l)
\]

As in the previous case, it must be the case that \( \alpha^h \leq \alpha^l \). Since delay is costly, set \( \alpha^h = 0 \). Then

\[
\tilde{x}_T^h = (1 - \alpha^l)\tilde{x}_T^l + \alpha^l \delta \tilde{x}_S(y_l) \\
\alpha^l = \frac{\tilde{x}_T^l - \tilde{x}_T^h}{\tilde{x}_T^l - \delta \tilde{x}_S(y_l)}
\]

Substitute \( \tilde{x}_T^r = \tilde{x}_T(y_r) \) to obtain

\[
\alpha^l = \frac{\delta(y_h - y_l)}{(1 - \delta^2)v} \geq 0
\]

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For any \( y_l \leq y_h \) such that \( \alpha^l \leq 1 \), the above strategy and belief system forms a separating equilibrium.

The sender’s expected payoff from such a separating equilibrium is calculated as

\[
\tilde{u}_S(y_h, y_l) = \theta(v - \tilde{x}_T(y_h)) + (1 - \theta) \left[ (1 - \alpha^l)(v - \tilde{x}_T(y_l)) + \alpha^l \delta (v - \tilde{x}_S(y_l)) \right] \\
= \theta(v - \tilde{x}_T(y_h)) + (1 - \theta)(v - \tilde{x}_T(y_l))
\]

Since \( \tilde{x}_T(y_r) \) is decreasing in \( y_r \), \( \tilde{u}_S(y_h, y_l) \) is increasing in both \( y_h \) and \( y_l \).

To find the best such separating equilibrium, maximize \( y_h \) and \( y_l \) subject to \( y_l \leq v_r \) and \( \alpha^l \leq 1 \). Then \( \tilde{y}_l = v_l \),

\[
\tilde{y}_h = \min \{ v_h, v_l + \frac{(1 - \delta^2)v}{\delta} \} \leq \bar{y}_h \text{ and } \bar{\alpha} = \frac{\delta (\bar{y}_h - v_l)}{(1 - \delta^2)v}
\]

provides the best equilibrium for sender. Let \( \bar{u} \) denotes the sender’s expected payoff in this equilibrium.

### B.4 Comparison of the sender’s payoffs

**Case 1:** \( v_l + \frac{(1-\delta^2)v}{\delta} < v_l + \frac{(1-\delta^2)(v+(1-\psi)v_l)}{\delta(1-\phi)\psi} \leq v_h \)

\[
\tilde{y}_h = v_l + \frac{(1-\delta^2)(v+(1-\psi)v_l)}{\delta(1-\phi)\psi} \text{ and } \bar{y}_h = v_l + \frac{(1-\delta^2)v}{\delta} \text{ in this case. Then } \tilde{x}_T(\tilde{y}_h) < \tilde{x}_T(\bar{y}_h) \text{ is equivalent to } \psi > \frac{(1-\delta^2+\delta)\phi-\delta}{\delta^2}, \text{ which holds if } \delta \text{ is large enough.}
\]

**Case 2:** \( v_l + \frac{(1-\delta^2)v}{\delta} < v_h < v_l + \frac{(1-\delta^2)(v+(1-\phi)v_l)}{\delta(1-\phi)\psi} \)

\( \tilde{y}_h \) drops to \( v_h \) and \( \bar{y}_h \) continues to be equal to \( v_l + \frac{(1-\delta^2)v}{\delta} \) in this case. Since \( \tilde{x}_T(\bar{y}_h) \) increases as \( \bar{y}_h \) drops, \( \tilde{x}_T(\tilde{y}_h) < \tilde{x}_T(\bar{y}_h) \) continues to hold if \( \delta \) is large enough.

**Case 3:** \( v_h \leq v_l + \frac{(1-\delta^2)v}{\delta} < v_l + \frac{(1-\delta^2)(v+(1-\phi)v_l)}{\delta(1-\phi)\psi} \)

\( \bar{y}_h = \tilde{y}_h = v_h \) in this case and \( \tilde{x}_T(v_h) < \tilde{x}_T(v_h) \).

If \( \delta \) is large enough, then \( \tilde{x}_T(\tilde{y}_h) < \tilde{x}_T(\bar{y}_h) \) so that \( \bar{u} > \tilde{u} \).
C Incomplete Information - Issue Salience

C.1 Complete Information Game

Assume that the target derives a payoff of \( \gamma x \) from a share of \( x \) and \( \delta_T = \delta_S = \delta \). Under this assumption, (3) becomes

\[
\gamma x_{St} = -(1 - \delta) [ (1 - \phi) y_{St} + \phi [ y_{St} - y_{T,T-1} ]_+ ] + \delta \gamma x_{T,t+1}
\]

and (4) remains the same

\[
v - x_{T,t+1} = -(1 - \delta) \psi y_{T,t+1} + \delta(v - x_{S,t+2})
\]

The analysis is similar to the analysis in Section A, so I will only characterize the equilibrium offers. To find the SPE without sanctions, set \( y_{St} = y_{T,t-1} = y_{T,t+1} = y \). Then \( x_{St} = x_{S,t+2} = x^*_S \) and \( x_{T,t+1} = x^*_T \) solve the equation system above.

To find an SPE with constant sanctioning, set \( y_{St} = y_{T,t-1} = y_{T,t+1} = y \), \( x_{St} = x_{S,t+2} = \bar{x}_S(y) \) and \( x_{T,t+1} = \bar{x}_T(y) \), which yields

\[
\bar{x}_S(y; \gamma) = x^*_S - \frac{1}{1 + \delta} \left[ \frac{1 - \phi}{\gamma} - \delta \psi \right] y \quad \text{and} \quad \bar{x}_T(y; \gamma) = x^*_T - \frac{1}{1 + \delta} \left[ \frac{\delta(1 - \phi)}{\gamma} - \psi \right] y,
\]

To find an SPE with alternate sanctioning, set \( y_{T,t-1} = y_{T,t+1} = 0 \), \( y_{St} = y \), \( x_{St} = x_{S,t+2} = \bar{x}_S(y) \) and \( x_{T,t+1} = \bar{x}_T(y) \), which yields

\[
\bar{x}_S(y; \gamma) = x^*_S - \frac{y}{\gamma(1 + \delta)} \quad \text{and} \quad \bar{x}_T(y; \gamma) = x^*_T - \frac{\delta y}{\gamma(1 + \delta)}.
\]

The SPE without sanctions is independent of \( \gamma \), offers in SPE with constant or alternate sanctioning are increasing functions of \( \gamma \).

C.2 Equilibrium under constant sanctions of \( y \)

Suppose that there is a separating equilibrium in which type \( \tau \) makes an offer of \( \bar{x}^\tau \), such that \( \bar{x}^h_T \neq \bar{x}^l_T \).

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After observing $\bar{x}_T$, the continuation game becomes a complete information game with $v_T = v_r$. Suppose that the players play an SPE with constant sanctions of $y_r \leq v_r$ afterwards. In this case, if the sender rejects $\bar{x}_T$, the sender imposes a sanction of $y_r$ and his offer $\bar{x}_S(y_r; \gamma_r)$ is accepted in the next period. Then for the sender to accept $\bar{x}_T$, it must satisfy

$$v - \bar{x}_T = -(1 - \delta)\psi y_r + \delta(v - \bar{x}_S(y_r; \gamma_r))$$

so that $\bar{x}_T = \bar{x}_T(y_r, \gamma)$. Suppose that the sender rejects $\bar{x}_T$ with probability $\alpha^r$. Since the sender is indifferent between accepting and rejecting $\bar{x}_T(y_r; \gamma_r)$, any $\alpha^r \in [0, 1]$ is optimal for the sender. The sender’s expected payoff from such a separating equilibrium is calculated as

$$\bar{u}_S(y_h, y_l) = \theta \left[ (1 - \alpha^h)(v - \bar{x}_T(y_h; \gamma_h)) + \alpha^h \delta(v - \bar{x}_S(y_h; \gamma_h)) \right] + (1 - \theta) \left[ (1 - \alpha^l)(v - \bar{x}_T(y_l; \gamma_l)) + \alpha^l \delta(v - \bar{x}_S(y_l; \gamma_l)) \right]$$

where the last equality follows from that the sender is is indifferent between accepting and rejecting an offer in a separating equilibrium. Since $\bar{x}_T(y_r; \gamma_r)$ is decreasing in $y_r$, $\bar{u}_S(y_h, y_l)$ is increasing in both $y_h$ and $y_l$. Since $v_T$ is known to be equal to 1, to find the best separating equilibrium for the sender, I will solve for a separating equilibrium with $y_l = y_h = 1$. Different types of the target will be separated by risk of delay and sanctions in a separating equilibrium. Since $\bar{x}_T(1; \gamma)$ is an increasing function of $\gamma$, type $l$ will make a small offer of $\bar{x}_T(1; \gamma_l)$, which the sender accept with probability 1, and type $h$ offer $\bar{x}_T(1; \gamma_h)$, which the sender will accept with probability $1 - \bar{\alpha}$.

Assuming that the game starts with the target having been sanctioned to the full extent, for type $h$ not to imitate type $l$, it must be the case that

$$(1 - \bar{\alpha})\gamma_h \bar{x}_T(1; \gamma_h) + \bar{\alpha} [- (1 - \delta)(1 - \phi) + \delta \gamma_h \bar{x}_S(1; \gamma_h)] \geq \gamma_h \bar{x}_T(1; \gamma_l) \quad (5)$$

The left hand side is type $h$’s expected payoff from offering $\bar{x}_T^h$ and the right hand side is her payoff from offering $\bar{x}_T^l$. Similarly, for type $l$ not to imitate type $h$, it must be the case that

$$\gamma_l \bar{x}_T(1; \gamma_l) \geq (1 - \bar{\alpha})\gamma_l \bar{x}_T(1; \gamma_h) + \bar{\alpha} [- (1 - \delta)(1 - \phi) + \delta \gamma_l \bar{x}_S(1; \gamma_h)] \quad (6)$$

The target is worse off when her offer is rejected, i.e.

$$\gamma \bar{x}_T(1; \gamma) > -(1 - \delta)(1 - \phi) + \delta \gamma \bar{x}_S(1; \gamma)$$
so an decrease in \( \bar{\alpha} \) increases the target’s expected payoff, so that 5 becomes more relaxed and 6 becomes more constrained. Therefore 5 is relaxed and 6 is binding at the SPE with the smallest \( \bar{\alpha} \) so that

\[
\gamma_l \bar{x}_T(1; \gamma_l) = (1 - \bar{\alpha}) \gamma_l \bar{x}_T(1; \gamma_h) + \bar{\alpha} [- (1 - \delta)(1 - \phi) + \delta \gamma_l \bar{x}_S(1; \gamma_h)] \iff \\
\bar{\alpha} = \frac{\gamma_l [\bar{x}_T(1; \gamma_h) - \bar{x}_T(1; \gamma_l)]}{\gamma_l [\bar{x}_T(1; \gamma_h) - \delta \bar{x}_T(1; \gamma_l)] + (1 - \delta)(1 - \phi)} \in (0, 1)
\]

equivalently

\[
\bar{\alpha} = \frac{\delta(1 - \phi)(\gamma_h - \gamma_l)}{\gamma_l \gamma_h (1 - \delta)(v + \psi) + \delta(1 - \phi)(\delta \gamma_h - \gamma_l) + (1 - \delta^2) \gamma_h (1 - \phi)}
\]

Let \( \bar{u} \) be the sender’s expected payoff from this separating equilibrium:

\[
\bar{u} = (1 - \theta)(v - \bar{x}_T(1; \gamma_l)) + \theta [(1 - \bar{\alpha})(v - \bar{x}_T(1; \gamma_h)) + \bar{\alpha} \delta(v - \bar{x}_S(1; \gamma_h))]
\]

\[
= (1 - \theta)(v - \bar{x}_T(1; \gamma_l)) + \theta(v - \bar{x}_T(1; \gamma_h))
\]

### C.3 Equilibrium under alternating sanctions

The equilibrium analysis is very similar to the previous one. Suppose that there is a separating equilibrium in which type \( \tau \) makes an offer of \( \bar{x}_T^\tau \), such that \( \bar{x}_T^h \neq \bar{x}_T^l \).

After observing \( \bar{x}_T^\tau \), the continuation game becomes a complete information game with \( v_T = v_\tau \). Suppose that the players play an SPE with an alternating scheme of sanctions afterwards, in which the sender sanctions the target at \( y_\tau \leq v_\tau \) when the target rejects his offer and he does not sanction her when he rejects her offer. In this case, if the sender rejects \( \bar{x}_T^\tau \), the sender does not imposes sanctions and his offer \( \bar{x}_S(y_\tau; \gamma_\tau) \) is accepted in the next period. Then for the sender to accept \( \bar{x}_T^\tau \), it must satisfy

\[
v - \bar{x}_T^\tau = \delta(v - \bar{x}_S(y_\tau; \gamma_\tau))
\]

so that \( \bar{x}_T^\tau = \bar{x}_T(y_\tau; \gamma_\tau) \). Suppose that the sender rejects \( \bar{x}_T^\tau \) with probability \( \alpha^\tau \). Since the sender is indifferent between accepting and rejecting \( \bar{x}_T(y_\tau; \gamma_\tau) \), any \( \alpha^\tau \in [0, 1] \) is optimal for the sender. The sender’s expected payoff from such a separating equilibrium is calculated as

\[
\bar{u}_S(y_h, y_l) = \theta \left[ (1 - \alpha^h)(v - \bar{x}_T(y_h; \gamma_h)) + \alpha^h \delta(v - \bar{x}_S(y_h; \gamma_h)) \right] + (1 - \theta) \left[ (1 - \alpha^l)(v - \bar{x}_T(y_l; \gamma_l)) + \alpha^l \delta(v - \bar{x}_S(y_l; \gamma_l)) \right] = \theta(v - \bar{x}_T(y_h; \gamma_h)) + (1 - \theta)(v - \bar{x}_T(y_l; \gamma_l))
\]

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where the last equality follows from that the sender is indifferent between accepting and rejecting an offer in a separating equilibrium. Since $\bar{x}_T(y_T; \gamma_T)$ is decreasing in $y_T$, $\bar{u}_S(y_h, y_l)$ is increasing in both $y_h$ and $y_l$. Since $v_T$ is known to be equal to 1, to find the best separating equilibrium for the sender, I will solve for a separating equilibrium with $y_l = y_h = 1$. Different types of the target will be separated by risk of delay and sanctions in a separating equilibrium. Since $\bar{x}_T(1; \gamma)$ is an increasing function of $\gamma$, type $l$ will make a small offer of $\bar{x}_T(1; \gamma_l)$, which the sender accept with probability 1, and type $h$ offer $\bar{x}_T(1; \gamma_h)$, which the sender will accept with probability $1 - \bar{\alpha}$.

Assuming that the game starts with the target having been sanctioned to the full extent, for type $h$ not to imitate type $l$, it must be the case that

$$\gamma_l \bar{x}_T(1; \gamma_l) \geq (1 - \bar{\alpha}) \gamma_l \bar{x}_T(1; \gamma_h) + \bar{\alpha} \delta \gamma_l \bar{x}_S(1; \gamma_h) \geq \gamma_h \bar{x}_T(1; \gamma_l)$$ \hspace{1cm} (7)

The left hand side is type $h$’s expected payoff from offering $\bar{x}_T^l$ and the right hand side is her payoff from offering $\bar{x}_T^l$. Similarly, for type $l$ not to imitate type $h$, it must be the case that

$$\gamma_l \bar{x}_T(1; \gamma_l) \geq (1 - \bar{\alpha}) \gamma_l \bar{x}_T(1; \gamma_h) + \bar{\alpha} \delta \gamma_l \bar{x}_S(1; \gamma_h)$$ \hspace{1cm} (8)

The target is worse off when her offer is rejected, i.e.

$$\gamma l \bar{x}_T(1; \gamma) > \delta \gamma l \bar{x}_S(1; \gamma)$$

so an decrease in $\bar{\alpha}$ increases the target’s expected payoff, so that 7 becomes more relaxed and 8 becomes more constrained. Therefore 7 is relaxed and 8 is binding at the SPE with the smallest $\bar{\alpha}$ so that

$$\gamma_l \bar{x}_T(1; \gamma_l) = (1 - \bar{\alpha}) \gamma_l \bar{x}_T(1; \gamma_h) + \bar{\alpha} \delta \gamma_l \bar{x}_S(1; \gamma_h) \Leftrightarrow$$

$$\bar{\alpha} = \frac{\bar{x}_T(1; \gamma_h) - \bar{x}_T(1; \gamma_l)}{\bar{x}_T(1; \gamma_h) - \delta \bar{x}_T(1; \gamma_l)} \in (\bar{\alpha}, 1)$$

That is, expected delay is longer with a separating perfect Bayesian equilibrium with alternate sanctioning, however sanctions are on observed on the equilibrium path in this type of equilibrium. Let $\bar{u}$ be the sender’s expected payoff from this separating equilibrium:

$$\bar{u} = (1 - \theta)(v - \bar{x}_T(1; \gamma_l)) + \theta [(1 - \bar{\alpha})(v - \bar{x}_T(1; \gamma_h)) + \bar{\alpha} \delta (v - \bar{x}_S(1; \gamma_h))]$$

$$= (1 - \theta)(v - \bar{x}_T(1; \gamma_l)) + \theta (v - \bar{x}_T(1; \gamma_h))$$

$$> \bar{u}$$

where the last inequality follows from $\bar{x}_T(1; \gamma_l) > \bar{x}_T(1; \gamma_l)$ and $\bar{x}_T(1; \gamma_h) > \bar{x}_T(1; \gamma_l)$.
D Incomplete Information - Adaptability

D.1 Complete Information Game

The complete information equilibrium analysis is given in Section B.1. I will introduce \( \phi \) as an argument to the offer functions computed in that section. That is, let the offers in an SPE with constant sanctions of \( y \) be given by \( x_S(y; \phi) \) and \( x_T(y; \phi) \) and the offers in an SPE that alternates between sanctions of \( y \) and no sanctions be given by \( \tilde{x}_S(y; \phi) \) and \( \tilde{x}_T(y; \phi) \), where

\[
x_S^* = \frac{\delta}{1 + \delta} v \quad \text{and} \quad x_T^* = \frac{1}{1 + \delta} v,
\]

\[
\tilde{x}_S(y; \phi) = x_S^* - \frac{1}{1 + \delta} [(1 - \phi) - \delta \psi] y \quad \text{and} \quad x_T(y; \phi) = x_T^* - \frac{1}{1 + \delta} [\delta (1 - \phi) - \psi] y,
\]

\[
\tilde{x}_S(y; \phi) = x_S^* - \frac{y}{1 + \delta} \quad \text{and} \quad \tilde{x}_T(y; \phi) = x_T^* - \frac{\delta y}{1 + \delta},
\]

when \( \delta_S = \delta_T = \delta \). The offer functions \( \tilde{x}_S(y; \phi), \tilde{x}_T(y; \phi), \tilde{x}_S(y; \phi) \) and \( \tilde{x}_T(y; \phi) \) are all decreasing in \( y \).

D.2 Equilibrium under constant sanctions of \( y \)

Suppose that there is a separating equilibrium in which type \( \tau \) makes an offer of \( x_T^\tau \), such that \( x_T^h \neq x_T^l \).

After observing \( x_T^\tau \), the continuation game becomes a complete information game with \( v_T = v_{\tau} \). Suppose that the players play an SPE with constant sanctions of \( y_{\tau} \leq v_{\tau} \) afterwards. In this case, if the sender rejects \( x_T^\tau \), the sender imposes a sanction of \( y_{\tau} \) and his offer \( \tilde{x}_S(y_{\tau}; \phi_{\tau}) \) is accepted in the next period. Then for the sender to accept \( x_T^\tau \), it must satisfy

\[
v - \tilde{x}_T^\tau = -(1 - \delta) \psi y_{\tau} + \delta(v - \tilde{x}_S(y_{\tau}; \phi_{\tau}))\]

so that \( x_T^\tau = \tilde{x}_T(y_{\tau}, \phi) \). Suppose that the sender rejects \( x_T^\tau \) with probability \( \alpha^* \). Since the sender is indifferent between accepting and rejecting \( x_T(y_{\tau}; \phi_{\tau}) \),
any $\alpha^* \in [0, 1]$ is optimal for the sender. The sender’s expected payoff from such a separating equilibrium is calculated as

$$
\bar{u}_S(y_h, y_l) = \theta \left[ (1 - \alpha^h)(v - \bar{x}_T(y_h; \phi_h)) + \alpha^h \delta(v - \bar{x}_S(y_h; \phi_h)) \right] + (1 - \theta) \left[ (1 - \alpha^l)(v - \bar{x}_T(y_l; \phi_l)) + \alpha^l \delta(v - \bar{x}_S(y_l; \phi_l)) \right]
$$

$$
= \theta(v - \bar{x}_T(y_h; \phi_h)) + (1 - \theta)(v - \bar{x}_T(y_l; \phi_l))
$$

where the last equality follows from that the sender is indifferent between accepting and rejecting an offer in a separating equilibrium. Since $\bar{x}_T(y; \phi)$ is decreasing in $y$, $\bar{u}_S(y_h, y_l)$ is increasing in both $y_h$ and $y_l$. Since $v_T$ is known to be equal to 1, to find the best separating equilibrium for the sender, I will solve for a separating equilibrium with $y_l = y_h = 1$. Different types of the target will be separated by risk of delay and sanctions in a separating equilibrium. Since $\bar{x}_T(1; \phi)$ is an increasing function of $\phi$, type $l$ will make a small offer of $\bar{x}_T(1; \phi_l)$, which the sender accept with probability 1, and type $h$ offer $\bar{x}_T(1; \phi_h)$, which the sender will accept with probability $1 - \bar{\alpha}$.

Assuming that the game starts with the target having been sanctioned to the full extent, for type $h$ not to imitate type $l$, it must be the case that

$$
(1 - \bar{\alpha})\bar{x}_T(1; \phi_h) + \bar{\alpha} [-(1 - \delta)(1 - \phi_h) + \delta \bar{x}_S(1; \phi_h)] \geq \bar{x}_T(1; \phi_l) 
$$

(9)

The left hand side is type $h$’s expected payoff from offering $\bar{x}_T^l$ and the right hand side is her payoff from offering $\bar{x}_T^l$. Similarly, for type $l$ not to imitate type $h$, it must be the case that

$$
\bar{x}_T(1; \phi_l) \geq (1 - \bar{\alpha})\bar{x}_T(1; \phi_h) + \bar{\alpha} [-(1 - \delta)(1 - \phi_l) + \delta \bar{x}_S(1; \phi_h)] 
$$

(10)

The target is worse off when her offer is rejected, i.e.

$$
\bar{x}_T(1; \phi) > -(1 - \delta)(1 - \phi) + \delta \bar{x}_S(1; \phi)
$$

so an decrease in $\bar{\alpha}$ increases the target’s expected payoff, so that 9 becomes more relaxed and 10 becomes more constrained. Therefore 9 is relaxed and 10 is binding at the SPE with the smallest $\bar{\alpha}$ so that

$$
\bar{x}_T(1; \phi_l) = (1 - \bar{\alpha})\bar{x}_T(1; \phi_h) + \bar{\alpha} [-(1 - \delta)(1 - \phi_l) + \delta \bar{x}_S(1; \phi_h)] \iff \\
\bar{\alpha} = \frac{\bar{x}_T(1; \phi_h) - \bar{x}_T(1; \phi_l)}{[\bar{x}_T(1; \phi_h) - \delta \bar{x}_T(1; \phi_l)] + (1 - \delta)(1 - \phi_l)} \in (0, 1)
$$
equivalently
\[\tilde{\alpha} = \frac{\delta(\phi_h - \gamma_l)}{(1 - \delta)(v + \psi + 1) + (\delta \phi_h - \phi_l)}\]

Let \(\tilde{u}\) be the sender’s expected payoff from this separating equilibrium:
\[
\tilde{u} = (1 - \theta)(v - \bar{x}_T(1; \phi_l)) + \theta [(1 - \tilde{\alpha})(v - \bar{x}_T(1; \phi_h)) + \tilde{\alpha} \delta(v - \bar{x}_S(1; \phi_h))]
= (1 - \theta)(v - \bar{x}_T(1; \phi_l)) + \theta(v - \bar{x}_T(1; \phi_h))
\]

### D.3 Equilibrium under alternating sanctions

In this case, the equilibrium analysis is straightforward. Since \(\bar{x}_S(y_r; \phi_r)\) and \(\bar{x}_T(y_r; \phi_r)\) are independent of \(\phi\), there is no need for separation. In sender’s most favorable equilibrium, the target offers \(\bar{x}_T(1; \phi_l) = \bar{x}_T(1; \phi_h)\), which the sender accepts. In particular, they play the SPE with alternate sanctions with \(y = 1\) (Proposition 3). That is, there is no delay in this equilibrium. Let \(\tilde{u}\) be the sender’s expected payoff from this separating equilibrium:
\[
\tilde{u} = (1 - \theta)(v - \bar{x}_T(1; \phi_l)) + \theta(v - \bar{x}_T(1; \phi_h)) > \bar{u}
\]
where the last inequality follows from \(\bar{x}_T(1; \phi_l) > \bar{x}_T(1; \phi_h)\) and \(\bar{x}_T(1; \phi_h) > \bar{x}_T(1; \phi_l)\).

### E Random Offers

Assume that the sender is selected with probability \(\pi_S\) and the target is selected with probability \(\pi_T = 1 - \pi_S\) to make an offer. Assume that \(\delta_S = \delta_T = \delta\).

Consider an equilibrium in which the sender alternates between \(y_0\) and \(y_1\), where \(y_1 \geq y_0\). \(y_0 = y_1 = 0\) corresponds to an SPE with no sanctions, \(y_0 = y_1 > 0\) corresponds to an SPE with constant sanctions, and \(y_0 < y_1\) corresponds to an SPE with alternating levels of sanctions.

Suppose that the sender alternates between \(y_0\) and \(y_1\). Let \(s \in \{0, 1\}\) denote the state of the game. \(s = 0\) if the target was sanctioned at the low level of \(y_0\) the previous period and \(s = 1\) if the target was sanctioned at the high level of \(y_1\) the previous period. Let \(x_{is}\) be player \(i\)’s offer if the state of the game is \(s\) and \(i\) is selected to make an offer, which happens with
probability \( \pi_i \). Then \( x_i \) are determined as follows: First consider \( s = 0 \). If the sender is selected to make an offer, then \( x_{S0} \) satisfies

\[
\frac{1 + x_{S0}}{1 - \delta} = 1 - (y_1 - \phi y_0) + \frac{\delta}{1 - \delta} \left[ 1 + \pi_s x_{S1} + \pi_T x_{T1} \right]
\]

The left hand side the target lifetime payoff if she accepts \( x_{S0} \). In this case, there will be no more sanctions and she will consume \( x_{S0} \) and \( v_T = 1 \) thereafter. The right hand side is her lifetime payoff if she rejects \( x_{S0} \). Since \( s = 0 \), the sender imposes a sanction of \( y_1 \) after the target rejects \( x_{S0} \), which imposes on her a cost of \((1 - \phi)y_1 + \phi [y_1 - y_{t-1}]_+ = (1 - \phi)y_1 + \phi [y_1 - y_0]_+ = y_1 - \phi y_0 \). The state of the game switches to \( s = 1 \) the next period and the offer of the randomly selected player is accepted, which provides an expected share of \( \pi_S x_{S1} + \pi_T x_{T1} \) there after. Rewrite this equality as

\[
x_{S0} = -(1 - \delta)(y_1 - \phi y_0) + \delta \left[ \pi_S x_{S1} + \pi_T x_{T1} \right]
\]

If the target is selected to make an offer, then \( x_{T0} \) satisfies

\[
\frac{v - x_{T0}}{1 - \delta} = -\psi y_1 + \frac{\delta}{1 - \delta} \left[ v - (\pi_S x_{S1} + \pi_T x_{T1}) \right] \iff \quad \text{where the left hand side of the first equality is the sender's lifetime utility if he accepts the target's offer of } x_{T0}, \text{ and the right hand side is his lifetime utility if he rejects it. In this case, he imposes a sanction of } \psi y_1, \text{ which costs him } \psi y_1, \text{ the state of the game switches to } s = 1 \text{ the next period and the offer of the randomly selected player is accepted, which provides an expected share of } v - (\pi_S x_{S1} + \pi_T x_{T1}) \text{ there after.}
\]

Consider \( s = 1 \). If the sender is selected to make an offer, then \( x_{S1} \) satisfies

\[
\frac{1 + x_{S1}}{1 - \delta} = 1 - (1 - \phi)y_0 + \frac{\delta}{1 - \delta} \left[ 1 + \pi_S x_{S0} + \pi_T x_{T0} \right]
\]

The left hand side the target lifetime payoff if she accepts \( x_{S1} \). The right hand side is her lifetime payoff if she rejects \( x_{S1} \). Since \( s = 1 \), the sender imposes a sanction of \( y_0 \) after the target rejects \( x_{S1} \), which imposes on her a cost of \((1 - \phi)y_0 + \phi [y_0 - y_{t-1}]_+ = (1 - \phi)y_0 + \phi [y_0 - y_1]_+ = (1 - \phi)y_0 \). The state of the game switches to \( s = 0 \) the next period and the offer of the randomly selected player is accepted, which provides an expected share of \( \pi_S x_{S0} + \pi_T x_{T0} \) there after. Rewrite this equality as

\[
x_{S1} = -(1 - \delta)(1 - \phi)y_0 + \delta \left[ \pi_S x_{S0} + \pi_T x_{T0} \right]
\]
If the target is selected to make an offer, then $x_{T1}$ satisfies

$$
\frac{v - x_{T1}}{1 - \delta} = -\psi y_0 + \frac{\delta}{1 - \delta} \left[ v - (\pi_S x_{S0} + \pi_T x_{T0}) \right] \Leftrightarrow
$$

$$
x_{T1} = (1 - \delta)(v + \psi y_0) + \delta \left[ \pi_S x_{S0} + \pi_T x_{T0} \right]
$$

Let

$$A_{S0} = -(1 - \delta)(y_1 - \phi y_0),$$

$$A_{T0} = (1 - \delta)(v + \psi y_0),$$

$$A_{S1} = -(1 - \delta)(1 - \phi)y_0$$

$$A_{T1} = (1 - \delta)(v + \psi y_0)$$

and rewrite the equation system that solves for $x_{is}$ as follows:

$$x_{S0} = A_{S0} + \delta \left[ \pi_S x_{S1} + \pi_T x_{T1} \right]$$

$$x_{T0} = A_{T0} + \delta \left[ \pi_S x_{S1} + \pi_T x_{T1} \right]$$

$$x_{S1} = A_{S1} + \delta \left[ \pi_S x_{S0} + \pi_T x_{T0} \right]$$

$$x_{T1} = A_{T1} + \delta \left[ \pi_S x_{S0} + \pi_T x_{T0} \right]$$

Let

$$B_0 = \pi_S x_{S0} + \pi_T x_{T0}$$ and

$$B_1 = \pi_S x_{S1} + \pi_T x_{T1}$$

be the target’s expected share when the state of the game is 0 and 1, respectively. Solving for the equation system yields

$$x_{S0} = B_0 - \pi_T (A_{T0} - A_{S0})$$

$$x_{T0} = B_0 + \pi_S (A_{T0} - A_{S0})$$

$$x_{S1} = B_1 - \pi_T (A_{T1} - A_{S1})$$

$$x_{T1} = B_1 + \pi_S (A_{T1} - A_{S1})$$

and

$$B_0 = \frac{(\pi_S A_{S0} + \pi_T A_{T0}) + \delta (\pi_S A_{S1} + \pi_T A_{T1})}{1 - \delta^2}$$ and

$$B_1 = \frac{(\pi_S A_{S1} + \pi_T A_{T1}) + \delta (\pi_S A_{S0} + \pi_T A_{T0})}{1 - \delta^2}$$

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Then

\[
\frac{\partial B_0}{\partial y_1} = \frac{-\pi_S + \pi_T \psi}{1 + \delta} = \frac{-\pi + (1 - \pi)\psi}{1 + \delta} \quad \text{and} \quad \frac{\partial B_1}{\partial y_1} = \delta \frac{\partial B_0}{\partial y_1}
\]

so that \(\frac{\partial B_0}{\partial y_1} < 0\) and \(\frac{\partial B_1}{\partial y_1} < 0\) if and only if \(\psi < \frac{\pi}{1 - \pi}\). In this case, an increase in \(y_1\) decreases \(B_0\) and \(B_1\), benefiting the sender.

\[
\frac{\partial B_0}{\partial y_0} = \frac{\pi_S \phi + \delta (\pi_T \psi - \pi_S (1 - \phi))}{1 + \delta} \quad \text{and} \quad \frac{\partial B_1}{\partial y_0} = \frac{\delta \pi_1 \phi + (\pi_T \psi - \pi_S (1 - \phi))}{1 + \delta}
\]

and

\[
\frac{\partial B_1}{\partial y_0} > 0 \implies \frac{\partial B_0}{\partial y_0} > 0
\]

\(\frac{\partial B_1}{\partial y_0} > 0\) is equivalent to

\[
\psi > \tilde{\psi} = \frac{\pi (1 - (1 + \delta) \phi)}{1 - \pi}
\]

That is, if \(\psi > \tilde{\psi}\), then decreasing \(y_0\) decreases both \(B_0\) and \(B_1\), benefiting the sender.

**No Sanctions Equilibrium**

Set \(y_0 = y_1 = 0\) in the solution above and obtain

\[
A_{S0} = A_{S1} = 0, \quad A_{T0} = A_{T1} = (1 - \delta) v
\]

so that

\[
B_0 = B_1 = \pi_T v = (1 - \pi) v \equiv B^r
\]

and

\[
x_{S0} = x_{S1} = \pi_T \delta v = (1 - \pi) \delta v \equiv x^r_S
\]

\[
x_{T0} = x_{T1} = \pi_T v + \pi_S (1 - \delta) v = (1 - \pi \delta) v \equiv x^r_T
\]

The following summarizes the SPE with no sanctions:
Proposition 9 The following is an SPE with no sanctions. The sender offers \( x_{St} = x^r_S \), accepts any offer \( x_{Tt} \leq x^r_T \), rejects all other offers, and does not sanction the target after a rejection. The target offers \( x_{Tt} = x^r_T \), accepts any offer \( x_{St} \geq x^r_S \), and rejects all other offers.

This SPE with no sanctions will be used to enforce sanctions in equilibria with sanctions below.

Constant Sanctions Equilibrium

Set \( y_0 = y_1 = y \) in the solution above and obtain

\[
A_{S0} = A_{S1} = -(1 - \delta)(1 - \phi)y, \\
A_{T0} = A_{T1} = (1 - \delta)(v + \psi y)
\]

so that

\[
B_0 = B_1 = (1 - \pi)(v + \psi y) - \pi(1 - \phi)y = B^r - [\pi(1 - \phi) - (1 - \pi)\psi]y \equiv \bar{B}
\]

and

\[
\bar{x}^r_S(y) = x_{S0} = x_{S1} = (1 - \pi)\delta v + [(1 - \pi)\delta \psi - (1 - (1 - \pi)\delta)(1 - \phi)]y
\]

\[
= x^r_S - [(1 - (1 - \pi)\delta)(1 - \phi) - (1 - \pi)\delta \psi]y,
\]

\[
\bar{x}^r_T(y) = x_{T0} = x_{T1} = (1 - \pi\delta)v + [(1 - \pi\delta)\psi - \pi\delta(1 - \phi)]y
\]

\[
= x^r_T - [\pi\delta(1 - \phi) - (1 - \pi\delta)\psi]y
\]

Suppose that the players revert to the no sanctions equilibrium if \( S \) ever fails to sanction \( T \) at the level \( y \). Then \( S \) would not deviate if

\[-\psi y + \frac{\delta}{1 - \delta} (v - \bar{B}) \geq \frac{\delta}{1 - \delta} (v - B^r)\]

equivalently,

\[\psi \leq \bar{\psi}^r = \frac{\delta \pi(1 - \phi)}{1 - \delta \pi}\]

The following proposition characterizes a class of SPE with constant sanctions:

Proposition 10 Suppose that \( \psi \leq \bar{\psi}^r \). Then for any \( y \in [0, 1] \), the following is an SPE with constant sanctions of \( y \). The sender offers \( \bar{x}^r_S(y) \), accepts any offer \( x_{Tt} \leq \bar{x}^r_T(y) \), rejects all other offers, and imposes a sanction of
on the target after a rejection. The target offers $\bar{x}^r_T(y)$, accepts any offer $x_{St} \geq \bar{x}^r_S(y)$, and rejects all other offers. If the sender deviates by choosing another sanction level, then the players play the SPE with no sanctions of Proposition 9 in the continuation game.

### Alternating Sanctions Equilibrium

Set $y_0 = 0$, $y_1 = y$ in the solution above and obtain

$$A_{S0} = -(1-\delta)y,$$
$$A_{T0} = (1-\delta)(v + \psi y)$$
$$A_{S1} = 0$$
$$A_{T1} = (1-\delta)v$$

so that

$$B_0 = (1-\pi)v - \frac{\delta}{1+\delta} [\pi - (1-\pi)\psi]y \equiv \tilde{B}_0$$
$$B_1 = (1-\pi)v - \frac{1}{1+\delta} [\pi - (1-\pi)\psi]y \equiv \tilde{B}_1$$

and

$$x_{S0}(y) = B_0 - (1-\pi)(1-\delta)(v + (1+\psi)y)$$
$$x_{T0}(y) = B_0 + \pi(1-\delta)(v + (1+\psi)y)$$
$$x_{S1}(y) = B_1 - (1-\pi)(1-\delta)v$$
$$x_{T1}(y) = B_1 + \pi(1-\delta)v$$

Suppose that the players revert to the no sanctions equilibrium if $S$ ever fails to sanction $T$ at the level $y$. Then $S$ would not deviate if

$$-\psi y + \frac{\delta}{1-\delta}(v - \tilde{B}_1) \geq \frac{\delta}{1-\delta}(v - B^r)$$

equivalently,

$$\psi \leq \tilde{\psi}^r = \frac{\pi}{(1-\pi) + (1-\delta^2)}$$

The following proposition characterizes a class of SPE with alternating sanctions:
Proposition 11 Suppose that $\psi \leq \bar{\psi}$. Then for any $y \in [0, 1]$, the following is an SPE with an alternating scheme of sanctions. If the state of the game is $s$, the sender offers $x_{Ss}(y)$, accepts any offer $x_{Tt} \leq x_{Tt}(y)$, rejects all other offers, the target offers $x_{Tt}(y)$, accepts any offer $x_{St} \geq x_{St}(y)$, and rejects all other offers. After a rejection of an offer, the sender imposes a sanction of $y$ if $s = 0$. If the sender deviates by choosing another sanction level, then the players play the SPE with no sanctions of Proposition 9 in the continuation game.

Also, $\bar{B}_1 < \bar{B}$ always holds and $\bar{B}_1 < \bar{B}$ holds if $\delta > \tilde{\delta} = \frac{\pi(1-\phi)+(1-\pi)\psi}{2(\pi+(1-\pi)\psi)}$, where $\tilde{\delta} < \frac{1}{2}$. This completes the equilibrium analysis.