

BASIC GAMES

All of the following game models assume an interaction between *two actors* ("players") who have *two distinct choices* ("strategies") each, between which they choose simultaneously (without knowing the other player's choice when choosing). Consequently, there are four possible outcomes, and the actual outcome depends on the choices made by *both* players (no one player's choice forecloses the choice of the other). It is assumed that each player derives a different utility from each outcome and therefore has a clear, ordered preference among the four outcomes. These *ordinal preferences* are indicated in the payoff tables (matrixes) by the numbers one through four with $4 > 3 > 2 > 1$ (" $>$ " indicates better than). The *players* are called "Row" (player's choice of strategies is depicted by the two rows) and "Column" (choice depicted by the two columns). The *strategies* for each player are called "X" and "Y," though the actual choice represented by "X" need not be identical for both players.* For any outcome in *notation* XX, YY, XY or YX, the first letter indicates the strategy chosen by the player in question, the other letter his "opponent's" choice. For each game, one *2x2 table* of the payoff structure is shown, although several tables can be obtained from each game (from its preference ordering) by (1) interchanging rows, (2) interchanging columns, and (3) interchanging both rows and columns.† A player has a *dominant strategy* if choosing this one over the other strategy yields a higher payoff to her, irrespective of which strategy the other player chooses. (Under the standard rationality assumptions of Game Theory, a player who has a dominant strategy will choose that strategy.) An *equilibrium* [outcome] is an outcome from which neither player can *unilaterally* depart without a loss of utility (decrease in payoff obtained).‡ A *Pareto-optimal* outcome "has the property that there is no other outcome of the game in which neither player gets a smaller payoff."¹ If an equilibrium is *not Pareto-optimal*, it is here called a *deficient equilibrium*.

Stag Hunt

The two strategies are to cooperate in the hunt of the stag (X) or to chase a passing rabbit (Y), thus letting the stag escape [cf. J.-J. Rousseau for the original story ...].

Preference ordering:

Row = Column: $XX > YX > YY > XY$

- no dominant strategy

- 2 equilibria: XX (Pareto-optimal) and YY (deficient)

	X	Y
X	4	3
Y	1	2

Chicken

The two strategies are to swerve (X) or to hold (Y), risking a crash.

Preference ordering:

Row = Column: $YX > XX > XY > YY$

- no dominant strategy

- 2 deficient equilibria: YX and XY

- the Pareto-optimal (XX) outcome is unstable

	X	Y
X	3	4
Y	2	1

[OVER]

* A cooperative outcome, for instance, may require functionally equivalent but not identical action (meeting a friend from California half-way requires me to go West and her to go East). The same logic applies to the "Y" strategy.

† An additional possibility is to exchange players, but this is only an option if preferences of Row = Column.

‡ Note that allowing a player to change his strategy (while the other player maintains his) effectively relaxes the assumption of choosing simultaneously.

¹ Rapoport, Anatol and M. Guyer. "A Taxonomy of 2x2 Games." In *General Systems* vol.11 (1966): 205.

Prisoner's Dilemma

The two strategies are to keep quiet (X, "cooperate") or to implicate the other prisoner (Y, "defect").

Preference ordering:

Row = Column: $YX > XX > YY > XY$

- Y is the dominant strategy for both players
- 1 deficient equilibrium: YY
- the Pareto-optimal outcome (XX) is unstable

	X	Y
X	3 3	1 4
Y	4 1	2 2

Deadlock

The two strategies are to cooperate (X) or to defect (Y).

Preference ordering:

Row = Column: $YY > XY, YX > XX^*$

- Y is the dominant strategy for both players
- 1 Pareto-optimal equilibrium: YY

	X	Y
X	1 1	2 3
Y	3 2	4 4

Harmony

The two strategies are to cooperate (X) or to defect (Y).

Preference ordering:

Row = Column: $XX > YX, XY > YY^\dagger$

- X is the dominant strategy for both players
- 1 Pareto-optimal equilibrium: XX

	X	Y
X	4 4	3 2
Y	2 3	1 1

Note that the preference orderings of "Deadlock" and "Harmony" are identical if you exchange X and Y; in any 2x2 table depicting "Harmony" payoffs, switching the labels of both rows *and* columns (top/left strategy is now mutual defection), yields a matrix of "Deadlock" preferences.

Coordination

The story behind this game scenario is that two people want to meet, but that there are two possible locations, indicated by the strategies (going to location X or to location Y). There are two basic versions of the game. In version I, it is assumed that neither player has a preference for either location (strategy/policy). In version II, also often called the "Battle of the Sexes," it is assumed that preference for location exist and differ, but that the preference for meeting at the same location (cooperating on the same policy) is stronger than the preference for the location.[‡]

Coordination I

Preference ordering:

Row = Column: $XX = YY > XY = YX$

- no dominant strategy
- 2 equilibria: XX, YY

	X	Y
X	1 1	0 0
Y	0 0	1 1

Coordination II

Preference ordering:

Row: $X > Y; XX > YY > XY > YX$

Column: $Y > X; YY > XX > YX > XY$

- no dominant strategy
- 2 equilibria: XX and YY

	X	Y
X	4 3	2 2
Y	1 1	3 4

* It is conventional to make the additional assumption that the outcome of mutual defection (YY) is most preferred (4,4). However, the basic logic of a single-play game (dominant strategy for both leads to stable YY) can also work if YY is only the second-most preferred outcome for one or both of the players, as long as $YY > XY$ and $YX > XX$.

† Here it is customary to make the additional assumption that the cooperative outcome yields the 4,4 payoff. However, ... (see footnote for "Deadlock").

‡ Keep in mind that the payoffs 1 through 4 are on *ordinal* not interval scales. Therefore, no meaningful arithmetic operations can be performed on the numbers: 4 and 3 cannot be "added up," and the sum cannot be assumed to be equal for both players, i.e. playing the game twice, once meeting at X, once meeting at Y cannot be assumed to yield automatically an equal utility to both players.