Single versus Multiple Sourcing: Drivers of Supplier Diversification

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Motivation: Reasons for Supplier Diversification

- **Managing supply risks.**
  - Diversification due to supply risks (e.g., catastrophic events; random capacity, uncertain yield, etc).

- **Managing cost efficiency.**
  - Diseconomies of scale (tendency to source from multiple suppliers).
  - Economies of scale (tendency to source from a single supplier).

- **Managing marginal revenue.**

  Etc.
An Example

- Manufacturer with linear per unit revenue $r$.
- Supplier $i = 1, \ldots, n$
  - per unit procurement cost $c_i$.
  - probability of producing/delivering any unit $p_i$
    (independent, proportional yield).
- Manufacturer’s expected profit as a function of quantity ordered

$$\Pi_i(q) = (r - c_i) p_i q.$$
An Example

\[ \Pi(q) \]

\[ (r-c_1)p_1q \]

\[ (r-c_2)p_2q \]
An Example

\[ \Pi(q) \]

\[ (r-c_1)p_1q \]

\[ (r-c_2)p_2q \]
An Example

\[ \Pi(q) \]

\[ (r-c_1(q))p_1(q)q \]

\[ (r-c_2(q))p_2(q)q \]
An Example

\[ \Pi(q) \]

\[ (r-c_1(q))p_1(q)q \]

\[ (r-c_2(q))p_2(q)q \]
An Example

\[ \Pi(q) \]

\[ (r-c_1(q))p_1(q)q \]

\[ (r-c_2(q))p_2(q)q \]
Overview of the Results

We study the impact of supply risk, revenue, and cost considerations on manufacturer’s supplier selection decision. The manufacturer seeks to maximize expected profit.

- Notion of Concave/Linear Risk: concavity/linearity of the expected delivered quantity.
  - Risk is linear if and only if it has a form of proportional yield.
  - Shape of risk function is critical for risk-driven diversification.
  - Ranking of suppliers is ambiguous for concave risk.

- A way to resolve trade-off between economies of scale and concave risk.
  - Risk-adjusted cost for a supplier: expected unit cost at the optimal sole-sourcing order.
  - Risk-adjusted costs provide managerial guidance on optimal supplier selection strategy.
Supply risk management and supplier diversification

- All-or-nothing (Bernoulli) yield uncertainty: Anupindi and Akella (1993, Model I), Swaminathan and Shanthikumar (1999), Tomlin and Wang (2005), Tomlin (2006), etc.
- EOQ inventory setting and proportional yield: Henig and Gerchak (1990), Parlar and Wang (1993), Fadıloğlu et al. (2008), Tajbakhsh et al. (2010), etc.

Sourcing strategies with asymmetric information

- Yang et al. (2009), Chaturvedi and Martínez-de-Albéniz (2011), Gümüş et al. (2012), Özer and Raz (2011), etc.
The manufacturer’s optimization problem is

$$\max_{q \in S} E_W \left[ V \left( \sum_{i=1}^{n} Y_i (q_i) \right) - \sum_{i=1}^{n} c_i \left( Y_i (q_i) \right)^{1+\gamma_i} \right],$$

where $W = (W_1, W_2, ..., W_n)$. 
Model: Supply Risk

- A monopolistic manufacturer sells at an exogenous rate and procures a component from a subset of $n$ potential suppliers
  - $q_i$: order quantity from supplier $i$.
- Supplier’s risk function (as in Dada et al., 2007)
  - $W_i$: random variable, independent across suppliers.
  - $K_i(q_i, w_i)$: production function, concave in $q_i$.
- Supplier $i$’s delivered quantity
  $$Y_i(q_i) = \min \{q_i, K_i(q_i, W_i)\}.$$
- Supplier $i$’s expected delivered quantity
  $$Z_i(q_i) = E_{W_i}[Y_i(q_i)].$$
Consider a "pay-for-output" model (e.g., Parlar and Wang, 1993).

Manufacturer faces linear procurement cost
  - Traditional constant per-unit wholesale contract.

Manufacturer faces economies or diseconomies of scale in procurement
  - For example, through the use of a quantity discount.

Manufacturer’s payment to supplier $i$

$$C_i (q_i) = c_i (Y_i (q_i))^{1+\gamma_i}$$

- $c_i$: constant base cost.
- $\gamma_i$: measure of cost efficiency, $\gamma_i = 0$, linear production cost; $\gamma_i \in (-1, 0)$, economies of scale; $\gamma_i > 0$, diseconomies of scale.
Model: Revenue Function

\[ V \left( \sum_{i=1}^{n} Y_i(q_i) \right), \text{ increasing in } \sum_{i=1}^{n} Y_i(q_i). \]

- Newsvendor revenue

\[ V \left( \sum_{i=1}^{n} Y_i(q_i) \right) = E_D \left[ r \min \{ D, \sum_{i=1}^{n} Y_i(q_i) \} \right] \]

- \( D \): random demand. \( r \): revenue per unit.

- Fixed-order linear revenue function (e.g., Chaturvedi and Martínez-de-Albéniz, 2011)

\[ V \left( \sum_{i=1}^{n} Y_i(q_i) \right) = r \sum_{i=1}^{n} Y_i(q_i) \text{ with } q \in S \]

- \( S = \{ q : \sum_{i=1}^{n} q_i \leq Q \text{ and } q_i \geq 0 \text{ for all } i \}. \)

- Service level constraint (e.g., Federgruen and Yang, 2008)

\[ V \left( \sum_{i=1}^{n} Y_i(q_i) \right) = \Pr (l_0 + \sum_{i=1}^{n} Y_i(q_i) \geq D) - (1 - \alpha) \]

- \( l_0 \): inventory of the final product. \( \alpha \): maximum shortfall probability.

- Risk attitudes (e.g., Eeckhoudt et al., 1995).
Model: Supplier Selection

\[
\max_{q \in S} E_W \left[ r \left( \sum_{i=1}^{n} Y_i (q_i) \right) - \sum_{i=1}^{n} c_i \left( Y_i (q_i) \right)^{1+\gamma_i} \right]
\]

Feasible Set of q
Definition: Risk is said to be *linear [concave]* if and only if

\[ Z_i(q_i) = E_{W_i} [Y_i(q_i)] = E_{W_i} [\min \{ q_i, K_i(q_i, W_i) \}] \]

is linear [concave].

- Let \( Z_i(q_i) = p_i q_i \) with \( p_i \geq 0 \) when risk is linear.

**Lemma**

*The expected delivered amount \( Z_i(q_i) \) is linear in \( q_i \) if and only if the production function \( K_i(q_i, W_i) \) has the form of proportional yield.*

- Bernoulli risk is a special case of linear risk.
Manufacturer’s optimization problem is

$$\max_{q \in S} \sum_{i=1}^{n} (r - c_i) p_i q_i.$$ 

Proposition

Suppose suppliers have linear per unit costs $c_i < r$ and $Z_i(q_i) = p_i q_i$ with $p_i \geq 0$. The manufacturer orders only from supplier $i^*$ where

$$i^* = \arg\max_i (r - c_i) p_i.$$ 

- Sole sourcing is always an optimal strategy.
The manufacturer’s optimization problem is

\[ \max_{q \in S} \sum_{i=1}^{n} (r - c_i) Z_i(q_i). \]
1. Hedging Supply Risk

Concave Risk, Identically Linear Cost
The manufacturer’s optimization problem is

$$\max_{q \in S} \sum_{i=1}^{n} (r - c_i) Z_i (q_i).$$

- Ranking of suppliers depends on the order quantity.
1. Hedging Supply Risk
Concave Risk, Identically Linear Cost

\[ Z(q) \]

\[ Z_1(q) \]

\[ Z_2(q) \]

\[ q \]
1. Hedging Supply Risk

Concave Risk, Identically Linear Cost
1. Hedging Supply Risk

Concave Risk, Linear Cost

The manufacturer’s optimization problem is

$$\max_{q \in S} \sum_{i=1}^{n} (r - c_i) Z_i(q_i).$$

- Ranking of suppliers depends on the order quantity.

**Proposition**

Suppose $Z_i$ are strictly concave. Let $Q_i$ be the optimal order quantity if sole sourcing from supplier $i$. Diversification is an optimal strategy if and only if

$$(r - c_i)Z_i'(Q_i) < \max_{j \neq i} (r - c_j)Z_j'(0), \text{ for all } i.$$

- Sole sourcing is not optimal with identical suppliers.
- Diversification might be optimal even when one supplier dominates in terms of cost and risk.
1. Hedging Supply Risk
Concave Risk, Linear Cost

\[ \Pi(q) \]

\[ (r-c_1)Z_1(q) \]

\[ (r-c_2)Z_2(q) \]
2. Inducing Cost Efficiency
Economies of Scale, Linear Risk

- Linear risk: $Z_i(q) = p_i q$.

Manufacturer's optimization problem is

$$\max_{q \in S} r p_i q_i - EW \sum_{i=1}^{n} c_i \left( Y_i(q_i) \right)^{1+\gamma_i}.$$

- Economies of scale ($\gamma_i < 0$)

**Proposition**

*Suppose $\gamma_i < 0$. Then, it is optimal for the manufacturer to source from a single supplier.*
Cost Efficiency and Supply Risk

- Economies of Scale: sole sourcing.
- Concave Supply Risk: diversification.
- Economies of Scale + Concave Supply Risk
  - Source from a single supplier? multiple suppliers?
  - Ranking of suppliers?

Manufacturer's optimization problem is

\[
\max_{q \in S} \ E_W \left[ \ r \left( \sum_{i=1}^{n} Y_i(q_i) \right) - \sum_{i=1}^{n} c_i \left( Y_i(q_i) \right)^{1+\gamma_i} \right].
\]

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Manufacturer’s optimization problem is
\[
\max_{q \in S} \sum_{i=1}^{n} \left( rZ_i(q_i) - c_i E_{W_i} \left( \min \{ q_i, W_i \} \right)^{1+\gamma_i} \right).
\]

Feasible set of order quantities \( q \) is
\[
S = \left\{ q : \sum_{i=1}^{n} q_i \leq Q \text{ and } q_i \geq 0 \text{ for all } i \right\}.
\]
Manufacturer’s optimization problem in terms of $z_i$ is

$$\max_{z \in S_z} \sum_{i=1}^{n} \left( rz_i - c_i E_{W_i} \left[ \left( \min \left\{ Z_i^{-1} (z_i), W_i \right\} \right)^{1+\gamma_i} \right] \right).$$

Feasible set of expected delivered quantities $z$ is

$$S_z = \left\{ z : \sum_{i=1}^{n} Z_i^{-1} (z_i) \leq Q \text{ and } z_i \geq 0 \text{ for all } i \right\}.$$
Cost Efficiency and Supply Risk
Transformation of the Optimization Problem

- Manufacturer’s optimization problem in terms of \( z_i \) is

\[
\max_{z \in S_z} \sum_{i=1}^{n} (rz_i - c_i T_i(z_i)) .
\]

- Feasible set of expected delivered quantities \( z \) is

\[
S_z = \left\{ z : \sum_{i=1}^{n} Z_i^{-1}(z_i) \leq Q \text{ and } z_i \geq 0 \text{ for all } i \right\}.
\]

- \( T_i(z_i) = E_{W_i} \left[ \left( \min \left\{ Z_i^{-1}(z_i), W_i \right\} \right)^{1+\gamma_i} \right] \)
Cost Efficiency and Supply Risk
Transformation of the Optimization Problem

- Manufacturer’s optimization problem in terms of $z_i$ is

$$\max_{z \in S_z} \sum_{i=1}^{n} (r z_i - c_i T_i(z_i)) .$$

- Feasible set of expected delivered quantities $z$ is

$$S_z = \left\{ z : \sum_{i=1}^{n} Z_i^{-1}(z_i) \leq Q \text{ and } z_i \geq 0 \text{ for all } i \right\} .$$

- $T_i(z_i) = E_{W_i} \left[ \left( \min \left\{ Z_i^{-1}(z_i), W_i \right\} \right)^{1+\gamma_i} \right]$

**Lemma**

$S_z$ is convex. With $-1 < \gamma_i < 0$, $T_i(z_i)$ is strictly increasing and strictly concave in $z_i$. 
Risk-adjusted expected per-unit cost

\[ \tilde{c}_i \overset{\text{def}}{=} c_i T_i (Z_i (Q)) / Z_i (Q). \]

- \( \tilde{c}_i \) is the cost associated with ordering from a single supplier.
- \( \tilde{c}_i \) is critical to determine the optimal sourcing strategy.

**Proposition**

There exist thresholds \( k_i \) and \( \bar{k}_i \) for supplier \( i \), with \( k_i \leq \bar{k}_i \leq r \) and that do not depend on \( c_i \), such that:

(i) It is optimal to order from a single supplier \( i \) if and only if \( \tilde{c}_i \leq k_i \).

(ii) An optimal ordering strategy does not involve any orders from supplier \( i \) if and only if \( \tilde{c}_i > \bar{k}_i \).
Economies of scale $\gamma_i \in (-1, 0)$ and concave risk.
Cost Efficiency and Supply Risk
Effect of Cost with Two Suppliers

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Concluding Remarks

We study the impact of supply risk, revenue, and cost considerations on manufacturer’s supplier selection decision. The manufacturer seeks to maximize expected profit.

- **Notion of Concave/Linear Risk:** concavity/linearity of the expected delivered quantity, $Z_i(q)$.
  - Risk is linear if and only if it has a form of proportional yield.
  - Shape of risk function is critical for risk-driven diversification.
  - Ranking of suppliers is ambiguous for concave risk.

- **Resolving trade-off between economies of scale and concave risk.**
  - Risk-adjusted cost for a supplier, $\tilde{c}_i$: expected unit cost at the optimal sole-sourcing order.
  - Risk-adjusted costs provide managerial guidance on optimal supplier selection strategy.

- **Future directions:**
  - Economies of scale + concave revenue.
  - Multi-period setting with inventory decisions and learning about supplier risk.
The risk-adjusted expected economies-of-scale parameter $\tilde{\gamma}_i^*$. 

Lemma

For $\pi_i^* \in (0, 1)$, there exists $\tilde{\gamma}_i^* \in (-1, 0)$ such that

$$
\frac{T_i(Z_i(\pi_i^*Q))}{T_i(Z_i(Q))} = (\tilde{\pi}_i^*)^{1+\tilde{\gamma}_i^*}.
$$
Cost Efficiency and Supply Risk

Types of Sourcing

- **Three types of sourcing:**
  - **No Sourcing:** \( q_i = 0 \) for all \( i \).
  - **Sole Sourcing:** \( q_i = Q \) for some supplier \( i \).
  - **Multiple Sourcing:** there exist at least two suppliers \( i, j \) such that \( q_i > 0 \) and \( q_j > 0 \).

- Let \( \pi^*_i \) be the proportion of the total order \( Q \) that is ordered from supplier \( i \) in the optimal strategy: \( \pi^*_i Q = q^*_i \).

- The optimization problem can be restated as

\[
\max_{\chi^S, \chi^M} \left[ \sum_{i=1}^{n} \chi^S_i \left[ r Z_i(Q) - c_i T_i(Z_i(Q)) \right] + \chi^M \sum_{i=1}^{n} \left[ r Z_i(\pi^*_i Q) - c_i T_i(Z_i(\pi^*_i Q)) \right] \right]
\]

subject to

\[
\sum_{i=1}^{n} \chi^S_i + \chi^M \leq 1, \ \chi^S_i, \chi^M \in \{0, 1\}.
\]
Cost Efficiency and Supply Risk

Effect of Revenue

Proposition

(i) No sourcing is optimal if and only if \( r \leq \min_i \tilde{c}_i \).

(ii) There exists \( R \geq 0 \) such that an optimal strategy type is the same as type of \( \theta^* \) if \( r \geq R \).
Proposition

Consider a setting with a manufacturer and two suppliers facing economies of scale in production. There exist constants \(d_1, d_2\), a linear increasing function \(L^0\), and increasing functions \(L^1(x)\) and \(L^2(x)\) that define the manufacturer’s optimal strategy type as presented in the following figure.
We study the impact of supply risk, revenue, and cost considerations on manufacturer’s supplier selection decision. The manufacturer seeks to maximize expected profit.

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