Optimal Allocation of Exclusivity Contracts

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Motivation

Exclusivity contracts are valuable.

- Sponsorships
  - e.g., sponsors in sports events and teams.
- Advertisement
  - e.g., internet ads.
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  - e.g., sponsors in sports events and teams.

- **Advertisement**
  - e.g., internet ads.

- **Distribution agreements**
  - e.g., fast moving consumer goods, brand name products.

- **Franchising**
  - e.g., car dealerships.
Manufacturer and Its Retailers

Acura dealers in North Carolina
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Manufacturer and Its Retailers

Lexus dealers in North Carolina
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Overview of the Talk

Optimal allocation and pricing in the presence of exclusivity.

- Revenue-maximizing posted price procedures:
  - Inflated prices;
  - Improved revenues.

- Suboptimality of posted prices (simultaneous and dynamic).

- Theoretical barriers for:
  - Finding revenue-maximizing (optimal) procedures: mechanism design;
  - Practical implementation of optimal procedures: computational complexity.

- A hybrid auction-pricing procedure:
  - Easy to implement;
  - Revenue-dominates posted prices;
  - Optimal in a perfect competition setting.
Relation to the Literature

- **Supply chain contracts**
  - Cachon (2003), Chen (2007), etc.

- **Externalities in supply chain**
  - Bernstein and Federgruen (2004), Netessine and Zhang (2005), Adida and DeMiguel (2011), etc.

- **Allocation and pricing on networks with positive and negative externalities**
  - Positive externality: Haphpanah et al. (2013), Candogan et al. (2012);

- **Allocation and pricing with interdependent valuations (externalities)**
  - Jehiel, Moldovanu, and Stacchetti (1996), Jehiel and Moldovanu (2001), etc.

- **Multi-dimensional mechanism design**
  - Jehiel, Moldovanu, and Stacchetti (1999), Manelli and Vincent (2007), etc.
A monopolistic seller has unlimited supply of identical items (e.g., contracts) allocated among $N$ unit-demand buyers.

Buyer $i$’s type: a vector $\mathbf{v}_i = (w_i, v_i)$

- $w_i$: buyer $i$’s (exclusivity) valuation for the item if none of her competitors gets an item.
- $v_i$: buyer $i$’s (non-exclusivity) valuation for the item if there is a competitor who also obtains the item.
- $w_i \geq v_i \geq 0$; $w_i - v_i$ is the exclusivity premium.

Information structure

- $\mathbf{v}_i$ is private information; drawn independently (across buyers) from a joint CDF $F_i$ with support $\Omega = [\underline{w}, \overline{w}] \times [\underline{v}, \overline{v}]$ (PDF $f_i$).
Example: Consider two buyers with independent valuations $v_i \sim U[0, 1]$. 
**Example:** Consider two buyers with independent valuations $v_i \sim U[0, 1]$.

- If no exclusivity valuations
  - $P^0 = 0.5$, the probability that a buyer accepts is 0.5, and the expected revenue $R^0$ is $2 \times 0.25 = 0.5$. 
Example: Consider two buyers with independent valuations $v_i \sim U [0, 1]$.

- $w_i = v_i + \varepsilon_i$, $\varepsilon_i \sim U [0, 1]$ and $v_i \perp \varepsilon_i$
  - $P^* = 0.75$, and the expected revenue $R^*$ is 0.75.
Consider ex ante identical buyers, i.e., \((w_i, v_i)\) is independently and identically distributed across buyers.

(i) The optimal price with exclusivity is greater than that without exclusivity, \(P^* \geq P^0\).

(ii) The optimal expected revenue with exclusivity is greater than that without exclusivity, \(R^* \geq R^0\).

However, posted prices are NOT optimal among all allocation and pricing procedures.
Definition of Mechanism

- WLOG, consider direct mechanisms that allocate items based on buyers’ reports, i.e., \( \vec{v} = (\hat{v}_i, \hat{v}_{-i}) \in \Omega^n \).
- A direct (deterministic) mechanism specifies, for each \( \vec{v} \in \Omega^n \)
  - Allocation: \( p_i : \Omega^n \rightarrow \{0, 1\} \) determines whether buyer \( i \) gets the item.
  - Payments: \( m_i : \Omega^n \rightarrow \mathbb{R} \) is the payment from buyer \( i \) to the seller.
- If buyer \( i \) does not participate, she does not get any item.
- Buyer \( i \)'s ex post utility when she reports her type as \( \hat{v}_i \), while her true type is \( v_i \), and when other buyers report \( v_{-i} \), is

\[
U_i (\hat{v}_i, v_i, v_{-i}) = w_i p_i (\hat{v}_i, v_{-i}) \prod_{j \neq i} (1 - p_j (\hat{v}_i, v_{-i})) \\
+ v_i p_i (\hat{v}_i, v_{-i}) \left( 1 - \prod_{j \neq i} (1 - p_j (\hat{v}_i, v_{-i})) \right) \\
- m_i (\hat{v}_i, v_{-i}).
\]
Revenue Maximization Problem

The seller’s Revenue Maximization Problem (General-RMP) is

\[
\max \left\{ p_i, m_i \right\}_{i=1}^n \sum_{i=1}^n \int m_i (v_i, v_{-i}) \, dF(v) 
\]

subject to

(EPIC) \quad U_i (v_i, v_i, v_{-i}) \geq U_i (\hat{v}_i, v_i, v_{-i}) \quad \text{for all } i \text{ and all } v_i, \hat{v}_i, v_{-i},

(EPIR) \quad U_i (v_i, v_i, v_{-i}) \geq 0 \quad \text{for all } i \text{ and all } v_i, v_{-i},

(Feasibility) \quad \sum_{i=1}^n p_i (v) \leq n \text{ and } 0 \leq p_i (v) \leq 1 \quad \text{for all } i.

- For general two-dimensional \( v \), there exist no belief-free mechanisms (Manelli and Vincent, 2007).
Local Exclusivity Setting

The scope of exclusivity may be limited to a geographic area, to a market segment, or to a reference group of perceived peers/competitors.
Acura dealers in North Carolina
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- Buyers’ competition network: a publicly known network \((N, E)\)
  - \(E\): the 0-1 adjacency matrix, \(e_{ij} = 1\) if and only if buyer \(i\) considers buyer \(j, j \neq i\), to be competitors.
  - \(S(i) \subseteq N \setminus \{i\}\) denote the set of buyer \(i\)’s competitors, i.e., \(S(i) = \{j \in N : e_{ij} = 1\}\).
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- **Local Linear Exclusivity (LLE)**
  - Buyer \(i\)’s exclusivity is
    \[ w_i = v_i + \sum_{j \in S(i)} \alpha_{ij} v_j. \]
  - Non-negative matrix \(A = [\alpha_{ij}]\) is publicly known.

**Bounded Local Linear Exclusivity (BLLE)**

For every \(j\)
\[ \sum_{f \in S(i) : j \in f} \alpha_{ij} \leq 1. \]
If buyer \(j\) does not get the item, its competitors cannot realize more than 100% of the value \(j\) would have realized if allocated the item.
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    \]
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Optimal Mechanism with Local Exclusivity

- Exclusivity premium, $\gamma_i \triangleq \sum_{j \in S(i)} \alpha_{ij} v_j$.
- Define virtual valuation, $\psi_i \triangleq v_i - \frac{1 - F_i^\gamma(v_i)}{f_i^\gamma(v_i)}$.

**Proposition**

For any set of realizations $\{v_i\}_{i \in N}$, the seller’s revenue maximization problem can be stated as a point wise maximization problem

$$\max \sum_{i=1}^{n} \left( \psi_i + \gamma_i \prod_{j \in S(i)} (1 - p_j(v)) \right) p_i(v)$$

subject to

(Feasibility) $\sum_{i=1}^{n} p_i(v) \leq n$ and $0 \leq p_i(v) \leq 1$ for all $i$,

(Monotonicity) $p_i(v_i, v_{-i})$ is increasing in $v_i$. 
### Optimal Mechanism with Local Exclusivity

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<th>BLLE</th>
<th>LLE</th>
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<tr>
<td><strong>Public Information</strong></td>
<td>Non-Exclusive only (straightforward)</td>
<td>Exclusive or Non-Exclusive (could be hard)</td>
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- The maximum independent set problem is a special case of the optimal mechanism design problem (a canonical NP-hard problem).
  - A reasonable procedure implementing optimal allocation and pricing is unlikely to be found.
  - A full characterization of settings which guarantee existence of a reasonable implementation is unlikely to be established.
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- Theoretical barriers for:
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  - Practical implementation of optimal procedures: computational complexity.

- A hybrid auction-pricing procedure:
  - Easy to implement;
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Hybrid Auction-Pricing Procedure

Parameters

Phase I (auction): an ascending auction for exclusivity with reserve $r$ and upper threshold $R$ (Exclusivity Valuations Ascending Auction).

Phase II (pricing): posted price $P$.

Three parameters: $[r, R, P]$

- $r$: the reserve price in the ascending auction for exclusivity.
- $R$: the maximum price in the ascending auction for exclusivity.
- $P$: the posted price for Phase II.
The seller separates exclusive and non-exclusive allocations and attempts to sell exclusivity first.

- The seller sets the reserve price $r$.
- The seller runs the ascending auction for exclusivity.
- The auction ends
  1. by allocating exclusively at a price smaller than $R$,
  2. by the seller’s cancelling exclusivity auction when the auction price reaches $R$. In this case, the seller offers items to both buyers at the predetermined fixed price $P$. 

Hybrid auction-pricing procedure revenue-dominates posted prices.
It is optimal in the perfect competition setting, i.e., \((N, E)\) is a complete graph.

**Proposition**

*In a perfect competition setting with LLE, there exist parameters \(r, R,\) and \(P\) such that*

i) If buyers bid/respond truthfully, the outcome of the hybrid auction-pricing procedure matches that of the seller’s optimal mechanism.

ii) Bidding truthfully in the hybrid auction-pricing procedure is a perfect Bayesian equilibrium.
Hybrid Auction-Pricing Procedure

Implications

- Sell exclusivity first.
- Use posted prices for non-exclusive allocation.
- Adjustments to non-exclusive sales procedures are insufficient.
- How close is the hybrid auction-pricing procedure to the optimal mechanism in general?
We study the optimal allocation and pricing of exclusivity contracts.

- Provide revenue-maximizing posted price procedures:
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- Extensions:
  - Dynamic pricing in the presence of exclusivity;
  - Computationally examining how well simultaneous and dynamic pricing perform relative to the optimal mechanism;
  - Implementable revenue-maximizing procedures on positive externalities.
Example: Supply Chain Contracts (1/2)

- Stage 1
  - A monopolistic supplier is selling $K$ wholesale-price contracts to $N$ retailers.
  - If getting the contract, the retailer can purchase from the supplier and sell in the market; otherwise, gets zero.
  - Stochastic market demand $D$
    - $a_i^\forall D$: demand when retailer $i$ gets the contract non-exclusively.
    - $a_i^w D$: demand when retailer $i$ gets the contract exclusively.
    - $\mathbf{a} = (a_i^w, a_i^\forall)$ is private information, $0 \leq a_i^\forall \leq a_i^w \leq 1$, and $\sum_{i=1}^{N} a_i^\forall \leq 1$. 

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  - $a = (a_i^w, a_i^v)$ is private information, $0 \leq a_i^v \leq a_i^w \leq 1$, and $\sum_i a_i^v \leq 1$.

Stage 2

- The supplier’s per unit production cost $c_s$, and the supplier charges the retailer $c_s$ per unit purchased.
- The retailer decides how much to purchase from the supplier.
- No goodwill penalty on either the supplier or retailers; No salvage value; No marginal cost per unit incurred on the retailer.
- Demand realized.
• $\Pi_c$: total profit of coordinated supply chain.
• Retailer $i$’s profit:
  • Non-exclusivity: $v_i = a_i^v \Pi_c$.
  • Exclusivity: $w_i = a_i^w \Pi_c$.
• Supplier’s profit: 0 (note that the supplier gets profits from selling the contract).
• $v_i = (w_i, v_i)$ is retailer $i$’s (private) type.
• Extended to the general framework with coordinated buyback contracts that give zero second-stage profit to the supplier.
General Framework: Supply Chain Contracts (1/3)

- Goodwill penalty for the supplier and retailers: $g_s, g_r$.
- Retailer’s marginal cost per unit: $c_r$.
- Salvage value: $v$.
- Expected total demand: $\mu = E_D[D]$; Demand distribution $G$; Proportion: $a_i = a_i^w$ or $a_i^v$ or 0.
- Retailer $i$’s order quantity: $q_i$; Vector of order quantity: $q = (q_1, q_2, \cdots, q_N)$.
- Retailer $i$’s expected sales: $S_i(q_i) = E_D[\min \{q_i, a_iD\}]$.
- Retailer $i$’s lost sales: $L_i(q_i) = E_D[\max \{0, a_iD - q_i\}]$.
- Retailer $i$’s leftover inventory: $l_i(q_i) = E_D[\max \{0, q_i - a_iD\}]$. 
Consider a standard buyback contract \((w, b)\).

Retailer \(i\)’s optimal profit:

\[
\pi_i^* (q_i^* (w, b)) = a_i (\Pi_r - g_r \mu),
\]

where

\[
\Pi_r (w, b) = (p + g_r - c_r - w) G^{-1} \left( \frac{p + g_r - c_r - w}{p - v + g_r - b} \right) \\
- (p - v + g_r - b) \int_0^{G^{-1} \left( \frac{p + g_r - c_r - w}{p - v + g_r - b} \right)} G (D) dD.
\]

The supplier’s profit under retailer’s optimal order quantity:

\[
\pi_s (q^* (w, b)) = \sum_{i=1}^{N} a_i (\Pi_s (w, b) - g_s \mu),
\]

where

\[
\Pi_s (w, b) = (w + g_s - c_s) G^{-1} \left( \frac{p + g_r - c_r - w}{p - v + g_r - b} \right) \\
- (b + g_s) \int_0^{G^{-1} \left( \frac{p + g_r - c_r - w}{p - v + g_r - b} \right)} G (D) dD.
\]
\[ \Pi_c = \max_q \pi_r(q) + \sum_{i=1}^{N} \pi_i(q_i) + \sum_{i=1}^{N} a_i (g_r + g_s) \mu. \]

Buyback contract that achieve \( \Pi_c \) (for some \( \lambda \geq 0 \))

\[
p + g_r - c_r - w^c = \lambda (p + g_s + g_r - c_s - c_r)
\]

and

\[
p - v + g_r - b^c = \lambda (p - v + g_s + g_r).
\]

\[ \Pi_r (w^c, b^c) = \lambda \Pi_c \text{ and } \Pi_s (w^*, b^*) = (1 - \lambda) \Pi_c. \]

Supplier and retailers’ profits:

\[
\pi_i^c(q^*(w^c, b^c)) = a_i (\lambda \Pi_c - g_r \mu);
\]

\[
\pi_s^c(q^*(w^c, b^c)) = \sum_{i=1}^{N} a_i ((1 - \lambda) \Pi_c - g_s \mu).
\]

In the complete information setting (\( a^w_i \) and \( a^y_i \) are publicly known)

- The supplier extracts all the surplus of retailers in the first stage.
- \((w^c, b^c)\) is also optimal for the supplier in the two-stage game.
How to Sell Contracts?

The seller should post a single inflated price.

Lemma

A single posted price mechanism is revenue-optimal among two-price menus.
Additive Exclusivity Valuation

- \( w_i = v_i + \theta_i^0 \), where \( \theta_i^0 \) is publicly known additive premium.
- Virtual valuation: \( \psi_i \triangleq v_i - \frac{1 - F_i^v(v_i)}{f_i^v(v_i)} \).
- The feasible set of allocation: \( \mathcal{P}_{add} = \{ p: \sum_{i=1}^{n} p_i(v) \leq K, \quad 0 \leq p_i(v) \leq 1, \text{ and } p_i(v) \text{ is increasing in } v_i \text{ for all } i \} \).

Proposition

The optimal mechanism, i.e., functions \( (p^*(v), m^*(v)) \), satisfies

\[
p^*(v) = \arg \max_{p \in \mathcal{P}_{add}} \left\{ \sum_{i=1}^{n} \left( \psi_i + \theta_i^0 \prod_{j \in S(i)} (1 - p_j(v)) \right) p_i(v) \right\}
\]

and \( m_i^*(v) = v_i p_i^*(v) + \theta_i^0 p_i^*(v) \prod_{j \in S(i)} (1 - p_j^*(v)) - \int_{v_i}^{v} p_i^*(t, v_{-i}) \, dt \).
Multiplicative Exclusivity Valuation

- \( w_i = \theta_i^1 v_i \), where \( \theta_i^1 \) is publicly known multiplier.
- The feasible set of allocation: \( \mathcal{P}_{mul} = \{ p : \sum_{i=1}^n p_i(v) \leq K, 0 \leq p_i(v) \leq 1, \text{ and } p_i(v) \left(1 + (\theta_i^1 - 1) \prod_{j \in S(i)} (1 - p_j(v))\right) \text{ is increasing in } v_i \text{ for all } i \} \).

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and

\[
m_i^*(v) = v_i p_i^*(v) \left(1 + (\theta_i^1 - 1) \prod_{j \in S(i)} (1 - p_j^*(v))\right)
\]

\[- \int_{v_i}^{V_i} p_i^*(t, v_i) \left(1 + (\theta_i^1 - 1) \prod_{j \in S(i)} (1 - p_j^*(t, v_i))\right) dt.\]
By standard technique, we write the ex post payment as

\[ m_i(v_i, v_{-i}) = v_i p_i(v_i, v_{-i}) \]

\[ + \left( \sum_{j \in S(i)} \alpha_{ij} v_j \right) p_i(v_i, v_{-i}) \prod_{j \in S(i)} (1 - p_j(v_i, v_{-i})) \]

\[ - \int_{v_i}^{v_i} p_i(t, v_{-i}) \, dt. \]
Parallelogram Valuations

(0,0) (1,1) (1,2)
Finite Types, Grid Index=4

Parallelogram Valuations

(0,0) (1,1) (1,2)
Parallelogram Valuations

(0,0) → (1,1) → (1,2)
Types are \((w_i, v_i)\) with \(w_i = v_i + \varepsilon_i\). Discretize the type space \(\Omega\):
\[
\Omega(M) = \{(i/(M - 1) + j/(M - 1), i/(M - 1)) : i, j = 0, 1, \ldots, M - 1\}.
\]

- \(M\): the number of possible outcomes of \(v_i\) or \(\varepsilon_i\).
- Discrete uniform distributions on both \(v_i\) and \(\varepsilon_i\).

| Grid Index | \(M\) | \(|\Omega|\) | Seller’s Expected Revenue |
|------------|-------|-------------|--------------------------|
| 2          | 4     | 1.188       |
| 3          | 9     | 1.056       |
| 4          | 16    | 0.995       |
| 5          | 25    | 0.961       |
| 6          | 36    | 0.938       |
| 7          | 49    | 0.924       |
| 8          | 64    | 0.915       |
| 9          | 81    | 0.907       |
| 10         | 100   | 0.901       |
Concluding Remarks

We study the optimal allocation and pricing of exclusivity contracts.

- Provide revenue-maximizing posted price procedures:
  - Inflated prices; improved revenues;
  - Suboptimality of posted prices (simultaneous and dynamic).

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