Quantitative Susceptibility Mapping and Susceptibility Tensor Imaging

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Magnetization and Susceptibility

\[ B_0 = \mu_0 H_0 \]

\[ m = \chi H_0 \]

\[ m \Rightarrow \delta B \]

\[ B = B_0 + \delta B \]

\[ B_0 \] is perturbed by local magnetization induced by susceptibility
Superposition of Field Perturbation

- Paramagnetic: $\chi > 0$
- Diamagnetic: $\chi < 0$

Voxel Effect of Field Inhomogeneity

- $\delta f = \frac{\gamma}{2\pi} \delta B$
- $R'_2 = 2\pi \cdot \text{FWHM} / 2 = \gamma \Delta B / 2$

Full Width at Half Maximum
- FWHM (Hz) or $\Delta B$ (Tesla)
Multi-Echo Gradient Echo Images

Magnitude: T2* Decay

\[ S(t) = S_0 e^{-t/T_2} \cdot e^{-t/T_2'} = S_0 e^{-t/T_2^*} \]

\[ \frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} \]

OR \[ R_2^* = R_2 + R_2' \]
R2* Mapping and Contrast

\[
\begin{align*}
T2^* & \quad \infty & \quad 25\,\text{ms} \\
R2^* & \quad 0 & \quad 40\,\text{Hz}
\end{align*}
\]

- White matter
- Blood vessels
- Globus pallidus
- Red nucleus
- Substantia nigra

What Is in the Phase?

**Sources of phase**
- Phase wraps
- Receiver coil
- Objects outside the FOV
- Objects inside the FOV

**background**
- **tissue**
**Sphere mean value property**

\[ \theta = \theta_{\text{tissue}} + \theta_b \]

\[ \nabla^2 \theta_b = 0 \]

Harmonic function

\[ \text{Mean phase over a sphere} \]

\[ S \otimes \theta = \theta_b \]

Phase at the center

\[ \theta - S \otimes \theta = \theta_{\text{tissue}} - S \otimes \theta_{\text{tissue}} = (\delta - S) \otimes \theta_{\text{tissue}} \]

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**Summary of Phase Processing**

Raw Phase ➔ Unwrapped Phase ➔ Filtered Phase

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QSM and STI
Chunlei Liu, Duke University

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**Mapping “Apparent” Magnetic Susceptibility**

\[ \theta(r) = 2\pi \cdot \delta f(r) \cdot TE \]

\[ FT\{\delta f(r)\} = \left(\frac{1}{3} - \frac{k^2}{k^2}\right) \chi(k) \frac{\gamma}{2\pi} B_0 \]

**Difficulty: “Magic Angle” Effect**

\[ FT\{\delta f(r)\} = \left(\frac{1}{3} - \frac{k^2}{k^2}\right) \chi(k) \frac{\gamma}{2\pi} B_0 \]
Quantitative Susceptibility Mapping (QSM)

- Raw Phase
- Unwrapped Phase
- Tissue Phase
- Susceptibility (ppm)

Phase of one voxel is affected by all surrounding voxels
Susceptibility is an intrinsic property of a voxel

Sources of Susceptibility in Tissue

**Endogenous**

(ppm cm³ mol⁻¹; cgs)

<table>
<thead>
<tr>
<th></th>
<th>H₂O</th>
<th>O₂</th>
<th>Fe</th>
<th>Ca</th>
<th>Ca²⁺</th>
<th>Na</th>
<th>Na⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptibility</td>
<td>-12.96</td>
<td>+3,449</td>
<td>Ferro</td>
<td>+40</td>
<td>&lt; 0</td>
<td>+16</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

**Exogenous**

(ppm cm³ mol⁻¹; cgs)

<table>
<thead>
<tr>
<th></th>
<th>Gd</th>
<th>Gd-DTPA</th>
<th>FeO</th>
<th>Mn</th>
<th>MnCl₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptibility</td>
<td>+185,000</td>
<td>+27,930</td>
<td>+7,200</td>
<td>+511</td>
<td>+14,350</td>
</tr>
</tbody>
</table>
**Myelin Dominates Gray/ White Contrast**

CTRL

FA

Susceptibility

Shiverer (no myelin)

FA

Susceptibility


**Magnetic Susceptibility is Anisotropic**

Susceptibility at Orientation #

1 2 3 4

B₀

Magnetic susceptibility is orientation dependent

C. Liu, MRM 2010; 63: 1471-1477
Relate Susceptibility Tensor to MR Signal

Magnetic flux density seen by a spin

\[ B = \mu_0 (1 - \sigma + A\chi)(H_0 + h) \]

Susceptibility inclusion

Susceptibility tensor 3x3

Applied field vector

Demagnetizing field vector

Find the Demagnetizing Field

\[ B_0 = \mu_0 (1 + \chi)(H_0 + h) \]

\[ \nabla \cdot B_0 = 0 \]

\[ H \cdot (\nabla \cdot \chi) + \nabla \cdot h = 0 \]

\[ h = -FT^{-1} \left\{ k \frac{k^T FT \{\chi\} H}{k^2} \right\} \]
In the laboratory frame of reference

\[
\Delta B = \mu_0 FT^{-1} \left\{ A FT \{ \chi \} H - k \frac{k^T FT \{ \chi \} H}{k^2} \right\}
\]

\[
\theta = \mu_0 FT^{-1} \left\{ A \hat{H}^T FT \{ \chi \} \hat{H} - \hat{H} \bullet k \frac{k^T FT \{ \chi \} \hat{H}}{k^2} \right\} \gamma H t
\]

Susceptibility tensor is symmetric
6 unknowns
Color-Coded Principal Susceptibility

Magnetic Susceptibility Anisotropy in the Human Brain


Quantifying Iron Deposition

Iron Content (μg Fe/gww)

Reference (AAS)
Reference (ICP)
Reference (Colorimetry)

Susceptibility (0.01 ppm)

RN
Pu
SN
GP
SS
GP, CI, Pu, SCC, SS


Susceptibility Tensor Imaging and Tractography

Anterior Commissure
Hipocampal Commissure

DTI
STI


STI in vivo
Summary

Artifacts → Contrast

Oxygen: BOLD fMRI
Iron: Cortical gray matter and nucleus
Myelin: White matter