Mathematics for Political and Social Research

(i.e., Extended Math Camp)

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http://people.duke.edu/~das76/MooSieBook.html

https://www.youtube.com/channel/UCrA2SLUKnV6yjdgIfDwFeGg

Course Description

This online, free course is designed to introduce and/or review the mathematical tools underlying most work in quantitative social science, particularly political science. This includes both statistical and formal modeling techniques. Topics to be covered include: (i) basic building blocks, (ii) calculus, (iii) probability, (iv) linear algebra, and (v) multivariate calculus and optimization. The course is intended to prepare students for later methods classes, provide students with a basic understanding of a variety of mathematical skills, and explore the importance of these skills in the context of social science research. Upon successful completion of the course students will be able to:

- apply basic mathematical operations,
- understand when and why certain mathematical operations are required by social scientists,
- recognize and hopefully avoid the various pitfalls of faulty application of mathematics, and
- attain basic mathematical literacy with respect to research published in the discipline.

Course Format

The course material has been broken up into individual video lectures and again into modules that closely mirror the course's textbook. All videos are posted on YouTube and free to access; a link to the channel is provided at the top of the page. You may want to subscribe in case additional content is added later. More resources can be found on the webpage linked to on the previous line.

The topics of each module in each lecture are provided below, along with the corresponding reading from the textbook. It is recommended that you do the listed reading before watching the lecture, just as you would do the reading before coming to class. Comfort with math, and later expertise, is all about repetition.

The course is meant to be modular, and you can skip around to a certain degree. For example, you can easily understand most of the three lectures on linear algebra right after Lecture 2. Further, the topics do vary a bit in difficulty. Lectures 12, 14, 15, and 16 cover more advanced topics than are often taught in first year graduate courses and can therefore often be put off for later. Finally, one of the advantages of this format is the ability to pause; if at any time you begin feeling overwhelmed or frustrated, simply pause and come back to it later. And do leave time to work through the course; trying to watch all 30+ hours of video in a week is definitely not recommended!

As there is no way to learn mathematics without actually practicing mathematics, the course offers multiple ways to practice. The textbook has examples you can work through in each chapter and exercises at the end of each chapter. Selected answers to these exercises, with some worked problems, can be found on the website listed at the top of this document. There are additional problem sets, one for each lecture, also posted there, all in one document. Solutions to these problems there as well in a different document. Finally, at the end of the each lecture is a problem session module in which I work through some example problems relevant to that lecture. Though you are of course free to do as much or as little of these problems as you would like, the more you do, the better you will understand the material and be able to use it in your studies. As importantly, the more you do, the less not understanding the material will prove a detriment to your studies.

Readings and topics follow. I hope you enjoy the course!

Readings

The textbook the course is:

Moore, Will H. and David A. Siegel. 2013. *A Mathematics Course for Political and Social Research.* Princeton, NJ: Princeton University Press.

Other potentially helpful sources include:

• Simon, Carl and Lawrence Blume. (SB) 1994. Mathematics for Economists. New York:

Norton.

• Hagel, Timothy. 1995. Basic Math for Social Scientists: Concepts. Volume 108.

• Ross, Sheldon M. Introduction to Probability Models. San Diego: Academic Press.

• ——. 1996. Basic Math for Social Scientists: Problems and Solutions. Volume 109.

• Gill, Jeff. 2006. Essential Mathematics for Political and Social Research. Cambridge Press.

• Fox, John. 2009. A Mathematical Primer for Social Statistics.

• Chiang, Alpha and Kevin Wainwright. 2005. Fundamental Methods of Mathematical Economics. Boston: McGraw Hill.

• Namboodiri, Krishnan. 1984. Matrix Algebra: An Introduction. Sage Publications.

• Thompson, Silvanus P. and Martin Gardner. Calculus Made Easy.

• http://Math.com

• http://mathworld.wolfram.com/

Video Topics (with Links and Readings)

Introduction: http://youtu.be/NQyQTctveDQ

Part I: Building Blocks

Lecture 1 Reading: Chapters 1 and 2 of Moore and Siegel

Lecture 1, Module 1 (Numbers): http://youtu.be/VrblmS0VOMo

Lecture 1, Module 2 (Sets and Basic Set Notation): http://youtu.be/uflzHj51FNA

Lecture 1, Module 3 (Constants, Variables, and Level of Measurement): http://youtu.be/lqUvsyYlfEc

Lecture 1, Module 4 (Operators and Some Rules for Operators): http://youtu.be/vNCz4cQUDSU

Lecture 1, Module 5 (More on Sets, Set Operators): http://youtu.be/xb-9JKxXLn8

Lecture 1, Module 6 (Properties of Reals and Other Sets and Spaces): http://youtu.be/efYNpBzegzc

Lecture 1, Module 7 (Algebra Review 1, Expressions): http://youtu.be/3oxB9Hy7UNE

Lecture 1, Module 8 (Algebra Review 2, Solving Equalities): http://youtu.be/QMFxkCvqHFg

Lecture 1, Module 9 (Algebra Review 3, Simplifying Expressions): http://youtu.be/fKwTx9mY5bE

Lecture 1, Module 10 (Introduction to Proofs): http://youtu.be/9IrZDt5-wuk

Lecture 1, Module 10b (More on Necessity and Sufficiency): http://youtu.be/2imCMooxOqc

Lecture 1, Problem Session: http://youtu.be/wBdrMi0SOzQ

Lecture 2 Reading: Chapters 3 and 4 of Moore and Siegel

Lecture 2, Module 1 (Functions 1—Definitions): http://youtu.be/SrkXssCc6Jo

Lecture 2, Module 2 (Functions 2—Some Properties): http://youtu.be/9V_-_3rcvf0

Lecture 2, Module 3 (Functions 3—Examples of Functions): http://youtu.be/z3rvMOG9_sY

Lecture 2, Module 4 (Functions 4—Additional Properties): http://youtu.be/0ptOg6d-F2g

Lecture 2, Module 5 (Relations 1—Use, Definitions, Examples): http://youtu.be/V7QVhXGpDNk

Lecture 2, Module 6 (Relations 2—Characteristics and Utility Representation): http://youtu.be/DYUYOYWcLfg

Lecture 2, Module 7 (Sequences and Series): http://youtu.be/hts7LMsG290

Lecture 2, Module 8 (Limits of Sequences and Series): http://youtu.be/2ohar8xQBQ0

Lecture 2, Module 9 (More on Sets—Open, Closed, Compact, Convex): http://youtu.be/uzB83sQSrDk

Lecture 2, Module 10 (Limits of Functions and Continuity): http://youtu.be/kFnC6rRNbrs

Lecture 2, Problem Session: http://youtu.be/mAXNQmxON_E

Part II: Calculus in One Dimension

Lecture 3 Reading: Chapter 5 of Moore and Siegel

Lecture 3, Module 1 (Why Calculus?): http://youtu.be/iRP_Q6SSBrA

Lecture 3, Module 2 (Instantaneous and Discrete Change): http://youtu.be/YVA6WG3Yg4Y

Lecture 3, Module 3 (Notation): http://youtu.be/y220JJnAr6g

Lecture 3, Module 4 (Limits and Derivatives and the Definition of a Derivative): http://youtu.be/Rmz47ipBRz4

Lecture 3, Module 5 (Examples of Derivatives Computed from the Definition): http://youtu.be/uX5le2V2wXA

Lecture 3, Module 6 (A Brief Introduction to Partial Derivatives and Multivariate Functions): http://youtu.be/s0K9DrCi158

Lecture 3, Problem Session: http://youtu.be/g64f--KV0d0

Lecture 4 Reading: Chapter 6 of Moore and Siegel

Lecture 4, Module 1 (Computing Derivatives): http://youtu.be/s2ytDQGwvuI

Lecture 4, Module 2 (The Derivative is a Linear Operator): http://youtu.be/CDEISbolTfo

Lecture 4, Module 3 (Polynomials and Powers): http://youtu.be/OE6MGxIuVHc

Lecture 4, Module 4 (Chain Rule): http://youtu.be/N559AWzpvf4

Lecture 4, Module 5 (Derivatives of Exponentials): http://youtu.be/ZBQTFEYyOhM

Lecture 4, Module 6 (Derivatives of Logarithms): http://youtu.be/U_VIWrwh8dk

Lecture 4, Module 7 (Product (and Quotient) Rule): http://youtu.be/LX_st-fHH6A

Lecture 4, Module 8 (Putting It All Together): http://youtu.be/KbqppU1zPCI

Lecture 4, Problem Session: http://youtu.be/4By5oI7kQ10

Lecture 5 Reading: Chapter 7 of Moore and Siegel

Lecture 5, Module 1 (What are Integrals? and How are They Used?): http://youtu.be/GkBahLMD3ck

Lecture 5, Module 2 (Definite Integral as a Limit of a Sum): http://youtu.be/0y1o-7AKqS8

Lecture 5, Module 3 (Indefinite Integral and the Antiderivative): http://youtu.be/73bbUMB7rco

Lecture 5, Module 4 (Fundamental Theorem of Calculus): http://youtu.be/jZzfN3MP1lA

Lecture 5, Module 5 (Integrals of Functions): http://youtu.be/XapcygYvRoo

Lecture 5, Module 6 (The Integral is a Linear Operator): http://youtu.be/OMLU6QDAdJQ

Lecture 5, Module 7 (Integration by Substitution): http://youtu.be/QHNdlXt__mc

Lecture 5, Module 8 (Integration by Parts): http://youtu.be/l4ydnFXbNt8

Lecture 5, Module 9 (Putting It All Together): http://youtu.be/nYL0sRWYvsw

Lecture 5, Problem Session: http://youtu.be/9YRdobtuCbM

Lecture 6 Reading: Chapter 8 of Moore and Siegel

Lecture 6, Module 1 (Extrema in One Dimension): http://youtu.be/K9lXm1F4cvk

Lecture 6, Module 2 (Higher-order Derivatives): http://youtu.be/T8Ief_ck8qg

Lecture 6, Module 3 (Concavity and Convexity): http://youtu.be/zSUTLw6r9js

Lecture 6, Module 4 (Critical and Inflection Points): http://youtu.be/Y5Sz2YME5m8

Lecture 6, Module 5 (First Derivative Test): http://youtu.be/Gatr-Ui-Sxs

Lecture 6, Module 6 (Second Derivative Test): http://youtu.be/4bwJwykVm08

Lecture 6, Module 7 (Summary of Method of Finding Extrema): http://youtu.be/p1zMh1zNV_0

Lecture 6, Module 8 (Examples: Maximizing Utility and Linear Models and OLS Regression): http://youtu.be/S4o3zREA5hc

Lecture 6, Problem Session: http://youtu.be/vsxdk3VWsa0

Part III: Probability

Lecture 7 Reading: Chapter 9 of Moore and Siegel

Lecture 7, Module 1 (Introduction to Probability): http://youtu.be/1HclKOWgIOU

Lecture 7, Module 2 (Classical Probability): http://youtu.be/_Fop6_GGws0

Lecture 7, Module 3 (Notation, Rules and Properties): http://youtu.be/rNLyLOQoVus

Lecture 7, Module 4 (Computing Probabilities): http://youtu.be/782cvyO_UnE

Lecture 7, Module 5 (Bayes' Rule): http://youtu.be/GkBB6qEuLDU

Lecture 7, Module 6 (Odds and Relative Risk Ratios): http://youtu.be/TotsUW2xnlk

Lecture 7, Problem Session: http://youtu.be/9mHULv6Hv5M

Lecture 8 Reading: Chapter 10 of Moore and Siegel

Lecture 8, Module 1 (Random Variables and Distributions): http://youtu.be/CQUnbYG0LBA

Lecture 8, Module 2 (Empirical Distributions): http://youtu.be/Hiv6H3gEgD0

Lecture 8, Module 3 (The Probability Mass Function): http://youtu.be/_Vd5C2ZYbxw

Lecture 8, Module 4 (The Cumulative Distribution Function): http://youtu.be/tLTC8FryUgc

Lecture 8, Module 5 (Bernoulli and Binomial Distributions): http://youtu.be/DSg-4_nv5yQ

Lecture 8, Module 6 (Event Count Distributions): http://youtu.be/h6N0zn1w5go

Lecture 8, Module 7 (Expectations and Moments of Discrete Random Variables): http://youtu.be/LoDpAFYlD9g

Lecture 8, Module 8 (Expected Utility): http://youtu.be/lmCyCzsfuto

Lecture 8, Problem Session: http://youtu.be/63zpNMc0Oao

Lecture 9 Reading: Chapter 11 of Moore and Siegel

Lecture 9, Module 1 (Introduction to Continuous Probability Distributions): http://youtu.be/wB-FioDqiD8

Lecture 9, Module 2 (PDF, CDF, and Parameters): http://youtu.be/sivT85zz3PU

Lecture 9, Module 3 (Joint Distributions): http://youtu.be/n09-IJXilsc

Lecture 9, Module 4 (Expectations and the Uniform Distribution): http://youtu.be/igvBmZfVLnY

Lecture 9, Module 5 (Moments of Continuous Distributions): http://youtu.be/puvKTJYAVmA

Lecture 9, Module 6 (Stochastic Dominance): http://youtu.be/6iE_5y4r2FI

Lecture 9, Module 7 (Normal (Gaussian) Distribution): http://youtu.be/ln1t_KLRqkM

Lecture 9, Module 8 (Logistic Distribution): http://youtu.be/0F_D-aFfXEw

Lecture 9, Module 9 (Duration and other Frequently Used Distributions): http://youtu.be/IzGkgo45too

Lecture 9, Problem Session: http://youtu.be/_djqlhmD51o

Part IV: Linear Algebra

Lecture 10 Reading: Chapter 12 of Moore and Siegel

Lecture 10, Module 1 (Of Scalars and Vectors): http://youtu.be/jQ8-asNn9EU

Lecture 10, Module 2 (Vector Algebra): http://youtu.be/-1860RpSlnk

Lecture 10 Module 3 (Vector Multiplication): http://youtu.be/xYRl5PXSSxg

Lecture 10, Module 4 (Matrices): http://youtu.be/eW5XoZhQAE8

Lecture 10, Module 5 (Matrix Algebra): http://youtu.be/ncGdSW50dUc

Lecture 10, Module 6 (Matrix Operators): http://youtu.be/HbLxS8aL1n0

Lecture 10, Module 7 (Matrix Inverse): http://youtu.be/aaxpunlIP4A

Lecture 10, Module 8 (Properties of Vectors and Matrices): http://youtu.be/9ScMH2BEe10

Lecture 10, Module 9 (Matrices and OLS): http://youtu.be/1mUlWW9wXNk

Lecture 10, Problem Session: http://youtu.be/xvaUD0dajU4

Lecture 11 Reading: Chapter 13 of Moore and Siegel

Lecture 11, Module 1 (Linear Algebra and Vector Spaces): http://youtu.be/vOX2iROHL6U

Lecture 11, Module 2 (Linear Independence and Spanning Vectors): http://youtu.be/ntulTl6XQSQ

Lecture 11, Module 3 (Solving Systems of Equations): http://youtu.be/ek7TaYeRZMk

Lecture 11, Module 4 (Substitution): http://youtu.be/hk4JmJkB-oA

Lecture 11, Module 5 (Elimination): http://youtu.be/Sez4R7du73U

Lecture 11, Module 6 (Matrix Inversion): http://youtu.be/Md_R7EmGADY

Lecture 11, Module 7 (Cramer's Rule): http://youtu.be/kiykoATSlAs

Lecture 11, Module 8 (Applications in Statistics and Game Theory): http://youtu.be/Wt0V3umilHQ

Lecture 11, Problem Session: http://youtu.be/Ar50peCqMIY

Lecture 12 Reading: Chapter 14 of Moore and Siegel

Lecture 12, Module 1 (Eigenvalues): http://youtu.be/MmOwpcUF2CE

Lecture 12, Module 2 (Computing Eigenvalues): http://youtu.be/s-9rNuuhIOg

Lecture 12, Module 3 (Eigenvectors): http://youtu.be/pnyHMrxQJNA

Lecture 12, Module 4 (Matrix Decomposition and Eigenvector Centrality): http://youtu.be/-dLBjPDmhfo

Lecture 12, Module 5 (Markov Chains and Stochastic Processes): http://youtu.be/FKJpIRZdmaU

Lecture 12, Module 6 (Ergodicity and Limiting Distributions): http://youtu.be/4z4jMa6pves

Lecture 12, Module 7 (Computing Steady States): http://youtu.be/_h3Po7xqIiQ

Lecture 12, Problem Session: http://youtu.be/rnwzkPqh7AA

Part V: Multivariate Calculus

Lecture 13 Reading: Chapter 15 of Moore and Siegel through 15.2.2, 15.2.5

Lecture 13, Module 1 (Functions of Several Variables): http://youtu.be/8iTJbWY2J5w

Lecture 13, Module 2 (Partial Derivatives): http://youtu.be/W9vPDK_Zkxk

Lecture 13, Module 3 (Multidimensional Integrals): http://youtu.be/X99j8C2Iy1k

Lecture 13, Problem Session: http://youtu.be/U8XgBODp3vA

Lecture 14 Reading: Chapter 15 of Moore and Siegel from 15.2.2 through end, other than 15.2.5

Lecture 14, Module 1 (Gradients): http://youtu.be/kSMxUEYElwk

Lecture 14, Module 2 (Total Derivatives): http://youtu.be/EahI4dG4a3s

Lecture 14, Module 3 (The Jacobian): http://youtu.be/EzBkAd9tuFQ

Lecture 14, Module 4 (The Hessian): http://youtu.be/msb5ffgZEQk

Lecture 14, Module 5 (Concavity and Convexity): http://youtu.be/0VpB4kJ2FsE

Lecture 14, Problem Session: http://youtu.be/1q0h06CrPeM

Lecture 15 Reading: Chapter 16 of Moore and Siegel

Lecture 15, Module 1 (Unconstrained Optimization I: Method): http://youtu.be/YCT5IbFP_Tw

Lecture 15, Module 2 (Unconstrained Optimization II: Examples): http://youtu.be/yiOSKnh2QiE

Lecture 15, Module 3 (Equality Constraints I: Method): http://youtu.be/oDtwQh79Nh4

Lecture 15, Module 4 (Equality Constraints II: Examples): http://youtu.be/RvC8er7sY0M

Lecture 15, Module 5 (Inequality Constraints I: Method): http://youtu.be/hTsNcDGkpGg

Lecture 15, Module 6 (Inequality Constraints II: Examples): http://youtu.be/fgsM2vQnvNQ

Lecture 15, Problem Session: http://youtu.be/-I0efcSSVH0

Lecture 16 Reading: Chapter 17 of Moore and Siegel

Lecture 16, Module 1 (Optima and Comparative Statics): http://youtu.be/KN9tdiv2X30

Lecture 16, Module 2 (Implicit Differentiation in One Dimension): http://youtu.be/z-ID3lpOu8M

Lecture 16, Module 3 (Implicit Differentiation in Multiple Dimensions): http://youtu.be/O6w8f9RVRu8

Lecture 16, Problem Session: http://youtu.be/6TbzEXHw9ns