ECO 463

Simultaneous Games - Supplemental Problems

1. Rock, Paper, Scissors. Each of two players simultaneously announces either *Rock* or *Paper* or *Scissors*. *Paper* beats *Rock*, *Rock* beats *Scissors* and *Scissors* beats *Paper*. The player who names the winning object receives \$1 from her opponent; if both players make the same choice, then no payment is made. Each player's preferences are represented by the expected amount of money she receives:

Player 1 \ Player 2	Rock	Paper	Scissors
Rock	0, 0	-1, 1*	1*, -1
Paper	1*, -1	0, 0	-1, 1*
Scissors	-1, 1*	1*, -1	0, 0

   (a) Find all of its mixed strategy equilibria giving both the equilibrium strategies and the payoffs.

   Solution:
   There are no equilibria in pure strategies and none in which one player's strategy is pure and the other's is strictly mixed. For both players to mix with row choosing rock with probability p_R, paper with probability $p_P$and scissors with probability $p_S \equiv 1 - p_R - p_P$ and column similarly using q_R, q_P and $q_S \equiv 1 - q_R - q_P$ we need

$$\begin{aligned} -q_P + (1 - q_R - q_P) &= c \\ q_R - (1 - q_R - q_P) &= c \\ -q_R + q_P &= c \end{aligned}$$

   for row to be willing to mix. These equations have the unique solution $c = 0$ and $q_R = q_P = 1/3$. The analogous requirement for column has the unique solution $p_R = p_P = 1/3$. Thus the unique equilibrium is for each player to play each action with probability $1/3$ and get an expected payoff of \$0.

(b) Find all the mixed strategy equilibria of a modified game in which player 1 is prohibited from announcing *Scissors*.

Solution:

If player 1 is precluded from playing *Scissors*, then player 2's choice of *Rock* is strictly dominated by *Paper*. The resulting game is:

Player 1 \ Player 2	Paper	Scissors
Rock	-1, 1*	1*, -1
Paper	0*, 0	-1, 1*

There are no equilibria in pure strategies and none in which one player's strategy is pure and the other's is strictly mixed. For both players to mix with row choosing *Rock* with probability p, *Paper* with probability $1-p$ and column similarly playing *Paper* with probability q and *Scissors* with probability $1-q$ we need

$$\begin{aligned} -q+1-q &= c \\ -(1-q) &= c \end{aligned}$$

for row to be willing to mix. These equations have the unique solution $q = 2/3$ and $c = -1/3$. Thus Player 2 plays *Paper* with probability 2/3, *Scissors* with probability 1/3 and Player 1 gets an expected payoff of $-\$1/3$. Similarly player 1 chooses *Rock* with probability 1/3, *Paper* with probability 2/3 and player 2 gets an expected payoff of \$1/3.

2. Waiting in line. Two hundred people are willing to wait in line to see a movie at a theater whose capacity is one hundred. Denote person i's valuation of the movie in excess of the price of admission, expressed in terms of the amount of time she is willing to wait, by v_i. That is, person i's payoff if she waits to t_i units of time is $v_i - t_i$. Each person attaches no value to a second ticket, and cannot buy tickets for other people. Assume $v_1 > v_2 > \cdots > v_{200}$. Each person chooses an arrival time. If several people arrive at the same time, then their order in line is determined by their index with lower numbered people going first. If a person arrives to find 100 or more people in line, her payoff is zero. Model the situation as a variant of a discriminatory multi-unit auction, in which each person submits a bid for only one unit, and find its Nash equilibrium.[1]

   (a) What would a supply and demand analysis suggest?

[1] Arrival times for people at movies do not in general seem to conform with a Nash equilibrium. What feature missing from the model could explain the pattern of arrivals?

(b) Will at least 100 people wait in line?

(c) Will the highest value people wait in line? I.e., will anyone with an index greater than 100 see the movie?

(d) Will people choose to wait different amounts of time?

(e) Will anyone that sees the movie wait more than v_{100}?

(f) Will anyone that sees the movie wait less than v_{101}?

Solution: $v_{100} \geq b_1 = b_2 = \cdots = b_{100} = b_j \geq v_{101}$ for at least one $j > 100$.