PRACTICAL GUIDELINES FOR THE NATURAL EXCITATION TECHNIQUE (NExT) AND THE EIGENSYSTEM REALIZATION ALGORITHM (ERA) FOR MODAL IDENTIFICATION USING AMBIENT VIBRATION

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Modal identification methodologies are used to determine the dynamic characteristics of systems. In structural engineering, these dynamic parameters play an important role in the understanding of the dynamic behavior of bridges, building, and other structures. Modal identification has been used for the verification of structural designs,[1–4] the study of the reliability of a structure to extreme dynamic loads such as earthquakes or hurricanes,[5] and it has been the focus of a large number of structural health monitoring (SHM)[6–11] and model updating techniques.[12–15] These methodologies can be classified in input–output and output-only methods. Input–output methodologies require measuring both, the force exciting the structure and the reaction of the structure to this excitation (i.e., acceleration, strain, displacement, etc.) In contrast, output-only methodologies do not require measuring the forces acting on the structure. In this case, the excitation is assumed stochastic with some specific characteristics.

Controlled excitation is difficult to achieve in most civil structures because of their size and because the influence of external loads such as wind and traffic is difficult to isolate. In addition, structure owners are usually reluctant to close structures for testing because of the lost of productivity due to the structure's down time. The George Washington Bridge over the Hudson River in New York city, for example, is crossed by an estimated 300,000 vehicles each day. The closing of this structure would create major traffic problems in Manhattan. Therefore, output-only modal identification using ambient excitation such as traffic, or wind is preferred and frequently the only viable option for modal identification of civil infrastructure.

Several methodologies are available to identify modal characteristics based on ambient vibration. Among the most common methods are the stochastic subspace identification (SSI) and the eigensystem realization algorithm (ERA) when applied with the natural excitation technique (NExT). These methodologies are based on the assumptions that the structure behaves within a linear range, the structure is time invariant, and the forces applied to the structure are uncorrelated with the response of the system. SSI can be applied directly to ambient vibration records[16] and have fewer parameters to consider when compared with other methods.[17] The ERA was originally deployed for impulse excitation.[18,19] NExT can be used to obtain records with the same characteristics as free response data from ambient vibration, enabling the use of the ERA with ambient vibration.

This paper focuses on the use of the NExT and the ERA for output-only modal identification of civil infrastructure. This paper is divided into two parts. First, a brief description of the methodologies is presented. This description is followed by a discussion of the effect of each of the parameters involved in these techniques. The goal of this paper is to provide guidance on the parameter selection for NExT–ERA. A complete discussion of the methodologies and comparison with other techniques can be found elsewhere.[18–22] The data of the second experimental phase of the American Society of Civil Engineers (ASCE)-SHM benchmark problem is used as an example through this section. These tests were performed in a four-story scaled building at the University of British Columbia in 2002,[20] and the data is available for download at http://mase.wustl.edu/wusceel/asce.shm/.

IDENTIFICATION PROCESS

Modal identification using the NExT and ERA has three main steps. First, the ambient excitation data is processed with NExT to obtain a signal with the same characteristics of a free vibration data. Parameters such as the number of points for fast Fourier transforms (FFTs) and sampling frequency of the data are important for this step. Then, the ERA is used to obtain a numerical model of the system in steady-state form based on this free vibration signal. Here, the number of expected modes of vibration, and the number of rows and columns of the Hankel matrix are parameters of the algorithm that can affect the identification process. Finally, natural frequencies, mode shapes, and damping ratios are calculated from the identified numerical model. Other factors related to the data acquisition and cleansing of the data, such as aliasing and quantization problems, will have a direct effect on the identification process. These factors are not discussed in this paper.

Natural Excitation Technique

Natural excitation technique was first used for the modal testing of a wind turbine under wind excitation[21,22] and it

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has been used in a large number of studies. Consider a multi-degree-of-freedom structure described by the differential equation of motion

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{y}(t)$ is the vector of displacements at time t, $\mathbf{F}(t)$ is the vector of forces at each time interval, **M**, **C**, and **K** are the mass, damping, and stiffness matrices respectively, and $(\dot{\ })$ indicates the derivative with respect to time. Assuming that the excitation is a white noise input, it can be shown that the cross-correlation function between the vector of acceleration with a reference acceleration signal $R_{\ddot{\mathbf{y}},\ddot{y}_i}(\tau)$ is

$$\mathbf{M}\ddot{R}_{\ddot{\mathbf{y}},\ddot{y}_i}(\tau) + \mathbf{C}\dot{R}_{\ddot{\mathbf{y}},\ddot{y}_i}(\tau) + \mathbf{K}R_{\ddot{\mathbf{y}},\ddot{y}_i}(\tau) = 0 \tag{2}$$

This equation has the same form as the homogeneous differential equation of motion and implies that the cross-correlation function $R_{\ddot{\mathbf{y}},\ddot{y}_i}(\tau)$ has the same characteristics as a free vibration signal from the structure. The cross-correlation function can be estimated using two different methods: (1) a direct procedure and (2) via FFTs.[23] The direct procedure uses time domain data while the estimation via FFTs requires calculating spectral density functions. Considering discrete records of acceleration, the cross-correlation function between two channels of acceleration i and j is estimated using the direct approach using the equation:

$$\ddot{R}_{\ddot{y}_i,\ddot{y}_j}(k\Delta t) = \frac{1}{N-k}\sum_{l=1}^{L-k}\ddot{y}_i(l)\ddot{y}_i(l+k)< \tag{3}$$

where Δt and L are the time step and the total number of points of the acceleration record respectively. The second method to estimate the correlation function requires the estimation of the cross-spectral density function and taking its inverse Fourier transform.[23] The power spectral density estimation can easily be performed by Welch's method[23] and it is available in mathematical programs such as Matlab. In this process, the time record is divided in data blocks, and the power spectral density is calculated by averaging the squared magnitudes of the spectral FFT of each data block. The analyst should determine the number of points for the FFT, number of data points overlapping between each block, and the record length or number of blocks for the calculations. In addition, the analyst needs to decide what reference channel (or channels) should be used to calculate the cross-correlation function, and the type of tapering function (i.e., Hanning or Hamming) used for each data block. It is important to mention that the number of points of the resultant cross-correlation functions is equal to the number of points used for the FFT, and its sampling frequency is equal to the sampling frequency of the data record.

Eigensystem Realization Algorithm

Considering the state-space representation for a discrete system

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \tag{4}$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \tag{5}$$

where $\mathbf{x}(k)$ is the vector of states, **u** is the vector of system inputs (i.e., forces applied to the structure) and **y** is the vector of system outputs (i.e., acceleration measurements used modal identification) at the k-th step, and **A**, **B**, **C**, and **D** are the discrete-time state-space matrices.[24]

The eigensystem realization algorithm uses the principles of minimum realization to obtain a state-space representation of the structure. A realization is the estimation of the system matrices **A**, **B**, and **C** from the response of the structure. There are an infinite number of matrices **A**, **B**, **C**, and **D**, each of different dimensions, that can be used to describe the input/output relationship of the structure. However, we are interested in the case with the smallest number of states. This realization is called a minimum realization. The eigenvalues of **A** are complex conjugates and are called the poles of the system. Each complex conjugate pair corresponds to one mode of vibration and can be used to determine the natural frequency and the damping ratio of the structure. The imaginary part represents the damped natural frequency $(\pm\omega_n\sqrt{1-\xi_n^2})$ and the real part corresponds to $-\xi_n \cdot \omega_n$, where ω_n stands for the natural frequency and ξ_n for the damping ratio of the n-th mode of vibration.

When the ERA is used in conjunction with NExT, matrices **B** and **D** cannot be calculated because the input to the system is not known. Nevertheless, the modal parameters of the system can be calculated from the matrices **A** and **C**. The first step in applying ERA is forming the Hankel matrix[18,19]

$$\mathbf{H}(k) = \begin{bmatrix} \mathbf{y}(k+1) & \mathbf{y}(k+2) & \cdots & \mathbf{y}(k+m) \\ \mathbf{y}(k+2) & \mathbf{y}(k+3) & \cdots & \mathbf{y}(k+m+1) \\ & & \ddots & \\ \mathbf{y}(k+n) & \mathbf{y}(k+n+1) & \cdots & \mathbf{y}(k+m+n) \end{bmatrix} \tag{6}$$

where the number of rows and columns of the Hankel matrix is $n \times N$ and M respectively, and N is the number of sensors available for identification. The second step is to perform the singular value decomposition of $\mathbf{H}(0)$

$$\mathbf{H}(0) = \mathbf{R}\mathbf{\Sigma}S^T \tag{7}$$

where $\mathbf{H}(0)$ is the Hankel matrix at $k = 0$, **R** and **S** are m by m and n by n orthonormal matrices respectively, and $\mathbf{\Sigma}$ is a m by n matrix with nonnegative numbers in the diagonal. Under ideal conditions, the matrix $\mathbf{\Sigma}$ will be

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_g & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{8}$$

where $\mathbf{\Sigma}_g$ is a g by g matrix and g is the system order or number of poles. In reality, the diagonal terms of the matrix $\mathbf{\Sigma}$ are nonzero produced by noise in the data acquisition process and numerical truncation. A minimum realization is obtained by eliminating the smaller singular values, resulting in a minimum order system that represents the structure. It can be shown that the system matrices **A** and **C** can be calculated as:

$$\mathbf{A} = \mathbf{\Sigma}^{-1/2}\mathbf{R}^T\mathbf{H}(1)S\mathbf{\Sigma}^{-1/2} \tag{9}$$
$$\mathbf{C} = \mathbf{E}^T\mathbf{R}\mathbf{\Sigma}^{1/2} \tag{10}$$

where the small singular values have been eliminated from the matrix $\mathbf{\Sigma}$ and $\mathbf{E} = [\,\mathbf{I} \quad \mathbf{0}\,]$ with **I** being an m by m identity matrix and **0** a matrix of appropriated dimensions. For the

Fig. 1: Benchmark structure

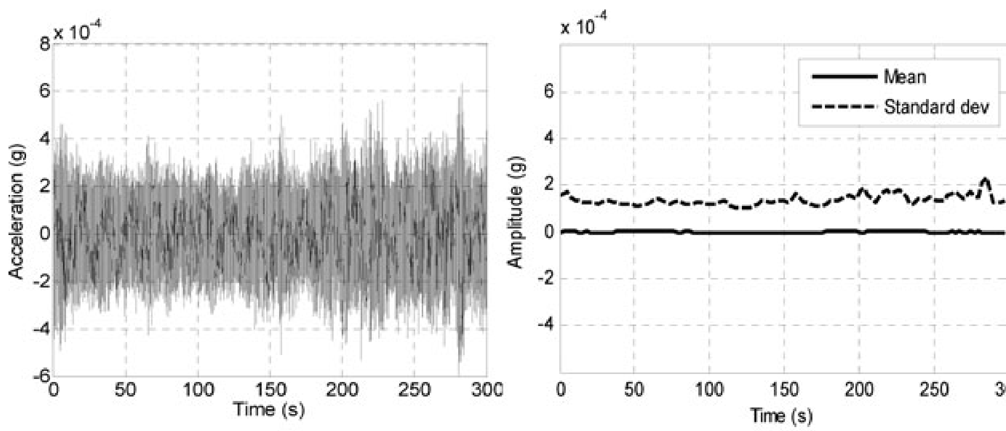
successful implementation of the ERA, the analyst needs to decide the number of rows and columns of the Hankel matrix **H**, and the expected number of natural frequencies (or poles). In addition, the analyst should be able to judge the results of the identification based on a particular set of metrics. The following section provides some practical guidelines to perform the identification and evaluate the estimated modal parameters.

PARAMETER SETUP: PRACTICAL GUIDELINES

The successful implementation of NExT and ERA for modal identification requires the analyst to setup over five different parameters. The number of points of the FFT, and the reference channel used to calculate the cross-correlation function are the two main parameters for NExT. The ERA has three parameters to be determined: the number of rows and columns of the Hankel matrix (m and n), and the number of poles to identify. In addition, the length and sampling frequency of the time domain records affect the modal identification procedure. Usually, this is an iterative procedure where the analyst performs the identification with a set of parameters, analyzes the output of this process and changes the parameters depending on the results obtained. Once the parameters are set for a specific structure, subsequent analysis for the same structure can be performed with little or no changes. This section describes best practices to perform modal identification using NExT–ERA, including guidelines to select these parameters appropriately. The records from the experimental phase of the ASCE-SHM benchmark problem are used to illustrate the change in parameters.

Test Descriptions

Acceleration records from the experimental phase of the International Association of Structural Control (IASC)-ASCE structural health monitoring benchmark problem are used to demonstrate the application of NExT and ERA. Figure 1 shows a picture of the two-bay by two-bay four-story steel structure used for this study. Ambient, hammer, and shaker excitations were used during the experiment. Sixteen data channels were collected during each test, 15 corresponding to acceleration records and one corresponding to the excitation force when the shaker or hammer was used. Each story had three accelerometers, one in the center beam measuring acceleration in the east–west direction and two in the east and west sides of the building measuring in the north–south direction. The structure was modified in nine different configurations by changing braces and loosening bolts. This paper considers the acceleration records recorded with the fully braced system under ambient excitation only. Figure 2a shows a representative acceleration record corresponding to the braced configuration of the structure with ambient vibration. Figure 2b shows the first two moments of the signal (mean and standard deviation) calculated in



(a) Acceleration record (b) Mean and standard deviation

Fig. 2: Representative acceleration record

windows of 1000 points with 500 points overlapping. The mean and standard deviation are relatively constant, giving an indication of the stationarity of the process. The experimental records are readily available for download from the ASCE structural health monitoring committee page. More information about the experimental benchmark problem can be found in Reference 25 or in the committee webpage.

Record Selection

A successful modal identification starts with understanding the available data records. The sampling frequency, noise-to-signal ratio, and stationarity of the records are key factors. Here, we assume that the data records have a relatively low noise-to-signal ratio and focus on the stationarity and the sampling frequency of the records. Natural excitation technique and ERA can successfully be used for modal identification using records that are not entirely stationary, but having stationary records such as the one shown in Fig. 2 produces the best results. Records are weakly stationary if the mean and the correlation functions do not change with time, that is, when the mean at time $\mu(t)$ is constant, and the correlation function $R_{xx}(t, t+\tau) = R_{xx}(\tau)$ for any time t. A record is strongly stationary when all possible moments and joint moments of the signal do not change with time.[26] If part of the record is stationary and part of a record is non-stationary, such as those obtained in bridges having periods of continuous traffic versus times of sporadic traffic, the non-stationary part can be discarded. Before starting the modal identification procedure, it is useful to create a few cross-spectral density functions and determine if clean peaks are shown within the spectrum. These peaks show the existence of possible natural frequencies. If peaks are not found either use a higher sampling rate to capture the dynamics of the system, or check the sensor location or noise-to-signal ratio. These spectral plots can also be used to identify the range of frequencies to be identified.

Sampling Frequency, Number of Points for FFT, Record Length, and Reference Channel

An important step in the modal identification using NExT and ERA is to calculate a high-quality cross-correlation function. The three parameters that affect the performance of NExT when calculating the cross-correlation function via FFT are the record length, number of points for the FFT, overlapping, and sampling frequency of the data record. Here the number of points for the FFT corresponds to the window length or data block. These parameters affect the length and frequency content of the cross-correlation function. The record length and the amount of overlapping between windows affect the number of averages for the spectral density function calculations and it should be considered in conjunction with the number of points to be used in the FFT. Figure 3 shows two cross-power spectral density plots. Both plots are calculated based on the same acceleration record and using 1024 points for the FFT. The sampling frequency of the acceleration record is 200 Hz, making one data block equal to 5.12 s. The solid line in Fig. 3 was calculated with 60 s of acceleration, while dashed uses 30 s. As Expected, the cross-spectral density calculated with the longer record is smoother than the cross-spectral density calculated with the shorter record. Longer records are preferred for noisy data, while relatively short records can be used for relatively clean data. The record length should be selected based on the expected frequency of the structure. Lower frequency structures such as cable-stayed bridges require longer records to capture the same number of cycles than higher frequency structures.

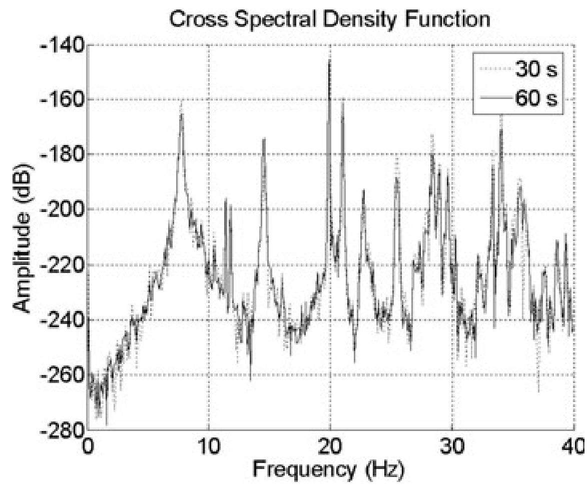


Fig. 3: Cross-spectral density function with two record lengths

The sampling frequency determines the frequency range where modal parameters are identified. Having a record with a high sampling frequency can produce a lack of performance in the identification of some low frequency modes, especially when they are closely spaced. Often the data should be resampled to focus the methodology in a specific frequency range. The sample frequency should be at least twice the higher frequency of interest to comply with the Nyquist criterion but it is recommended to select a sampling frequency not much higher than that. Anti-aliasing filters should be used when resampling the data. Otherwise the modal identification procedure could incorrectly identify high frequency modes at a lower frequency due to aliasing on the signals. However, it is important to consider that anti-aliasing filters could appear as poles on the signals due to filter's dynamic characteristics. Considering the cross-spectral density shown in Fig. 3, one can argue that most of the data of interest is below 40 Hz, and therefore we can resample the data to 80 Hz.

The sampling frequency and the number of points for the FFT affect the length of the cross-correlation record as the sampling frequency of the record is equal to the sampling frequency of the cross-correlation, and the number of points in the cross-correlation function is equal to the number of points of the FFT. Selecting high sampling frequencies and keeping the number of points of the FFT constant can produce either too short or long cross-correlation records. Figure 4 shows the free vibration record calculated with the original record (Fig. 2) sampled at 200 Hz and with the record resampled at 80 Hz. Both cross-correlation functions were calculated with 1024 points in the FFT. The first cross-correlation is just over 2.5 s while the second one is over 6 s. The free decay is more evident in the cross correlation calculated with 80 Hz because it goes further in time even though it has the same number of

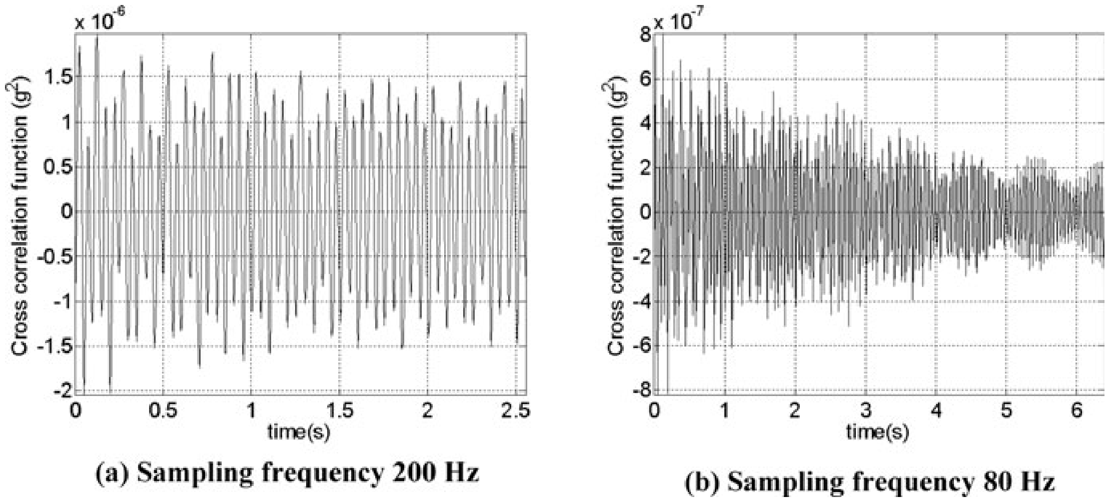


Fig. 4: Cross-correlation functions

points. Even though short cross-correlations not displaying a clear visible free decay can be successfully used for identification, having long records of the correlation allows the analyst to judge the quality of the correlation estimate visually.

Natural excitation technique requires a reference channel to calculate the cross-correlation function. One option is to choose a reference channel with high amplitude, low noise-to-signal ratio, and far from a node of vibration of the modes of interest. Using signals collected from sensors close to a node of vibration of a particular mode can create challenges in the identification this mode. An alternative to select a single reference channel is to perform the modal identification several times with different reference channels each time. This allows the creation of a diagram similar to stabilization diagrams,[27] allowing a more thorough modal identification. In addition, the ERA can support data using several reference channels. In this case the Hankel matrix entries become matrices instead of vectors. This method is explained in detail in the following section.

Rows and Columns of the Hankel Matrix

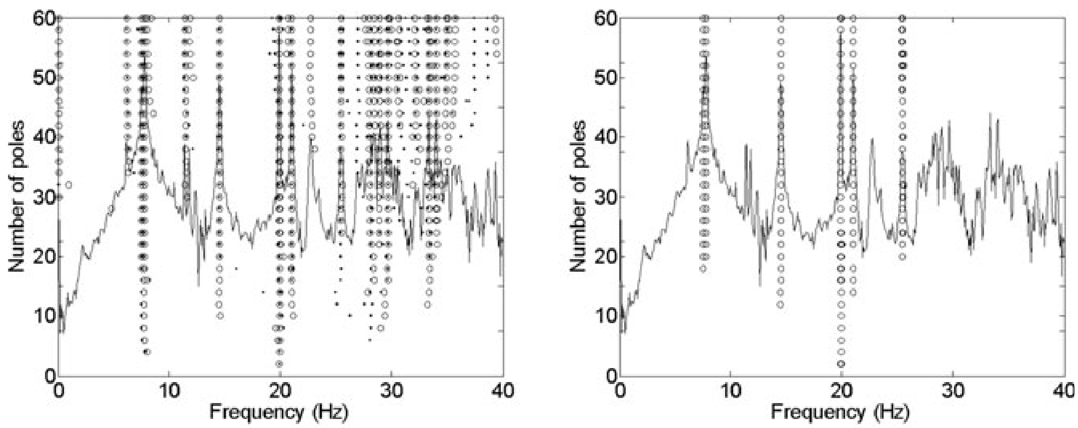
The number of rows and columns of the Hankel matrix are two of the main parameters of the ERA. The number of rows and columns is chosen based on the number of expected natural frequencies on the system. If the analyst is familiar with the system dynamics or has a numerical model of the structure, a first approximation to the number of natural frequencies can be guessed based on experience or a preliminary numerical model. If the dynamics of the structure is not known, a first approximation to the number of modes can be obtained by counting the peaks of the cross-spectral density function.

A rule of thumb for the number of columns of the Hankel matrix is to use four times the number of expected modes (twice the number of expected poles). It is important to mention that forcing a low-rank Hankel matrix could lead to missing some physical modes of vibration but this is usually a good starting point. The number of rows is a set based on the number of points available in the cross-spectral density function. The goal is to use as much data from the spectral density function as possible without including noisy signals found at the end of the cross-correlation function. Considering, for example, the cross-correlation function shown in Fig. 4b, the first 5 s of data, where a clear decay is present, are used to form the Hankel matrix. A few cycles of vibration should be used to ensure that the modal identification is performed successfully. Therefore, the cross-correlation record should be longer for structures with low natural frequencies.

Model Order and Stabilization Diagrams

Specifying the correct number of poles, or model order, is probably the most important step in the modal identification procedure. If the model order is too high, fictitious modes of vibration can be included in the identified results. If the model order is too small, some of the modal parameters might not be identified. Stabilization diagrams are an effective tool to determine the correct number of poles for the ERA.[28–30] The idea behind stabilization diagrams is to repeat the identification process with a different number of poles each time. Stable poles should remain constant for all or most of the iterations. Figure 5a shows an example of a stabilization diagram.

Stabilization diagrams are also used to explore the use of different reference channels for the calculation of the cross-spectral density functions. Different modes can be identified when different reference channels are selected for NExT, and therefore the final modal identification could be a mix of modes identified with different reference channels. Figure 5a shows the stabilization diagrams for two reference channels. The dots indicate the identified poles using the acceleration record of the 4th floor in the east–west direction as a reference channel for NExT and the circles indicate the poles identified when the 4th floor, north–south acceleration record was used. In this particular structure, a possible mode at 23 Hz is only observed when using the north–south acceleration record as a reference channel.



(a) Stabilization diagram (b) Filtered stabilization diagram

Fig. 5: Stabilization diagrams

MAC and Damping Filter

An additional step can be used to evaluate the identified parameters from the stabilization diagram. The damping ratio and modal assurance criterion (MAC) values can be used to automatically identify stable modes in the stabilization diagram.[27] Modes with damping ratios lower or higher than certain threshold limit can be automatically discarded. These modes commonly correspond to numerical modes. The damping estimation in these techniques has a larger variation when compared with the estimation of the natural frequencies, and mode shapes and the selected threshold limits should reflect this variation. In addition, the correlation between the identified mode shapes at a specific frequency can be compared using the MAC. The MAC value is metric used to determine the similarity (linearity) of one-mode shape when compared with another reference mode. Modal assurance criterion values range from zero to one, zero corresponding to no consistent mode shapes, and one corresponding to consistent shapes.[31] The MAC value is calculated using the equation

$$MAC_{a,b} = \frac{\left\{\sum_{j=1}^{n} \phi_{a,j}\phi_{b,j}\right\}^2}{\left\{\sum_{j=1}^{n} (\phi_{a,j})^2\right\}\left\{\sum_{j=1}^{n} (\phi_{b,j})^2\right\}} \qquad (11)$$

where $\phi_{a,j}$ is the j-th coordinate of the mode shape a. The MAC is used here to determine stable mode shapes on the stabilization diagram. As the real mode shape is not known, MAC is used to compare the identified modes in each iteration of the stabilization diagram at a particular frequency. Any mode shape is selected as a reference mode and compared with all other modes at the same frequency. The result of this operation is a vector of MAC indicating which modes are similar to the reference shape. A MAC matrix can be calculated selecting each mode identified as a reference mode. Here each column corresponds to different reference mode shapes. The diagonal of the matrix should be equal to one since it is the MAC calculated using one-mode shape. Columns with a high mean value indicate that the selected reference shape for that particular column has been identified several times in the stabilization diagram. Once a stable reference shape is selected unstable modes can be discarded. As a rule of thumb, MAC values higher than 0.95 indicate a good correlation between two mode shapes. Mode shapes with lower MAC values can be considered fictitious modes and discarded from the identification. Figure 5b shows the same stabilization diagram presented in Fig. 5a but including only the modes with MAC values higher than 0.95 and damping ratios lower than 5%. The steps to obtain this graph are the following: (1) discard all modes with more than 5% damping, (2) select the first identified natural frequency, (3) search for modes with frequencies within 0.005 Hz of the selected frequency, (4) calculate the MAC between these modes, (5) if the number of modes with MAC values greater then 0.95 is more than 20, accept the selected modes as a true mode of vibration, (6) identify the following unknown mode of vibration and repeat the process starting at (3). Six natural frequencies could be clearly identified after this process corresponding to the first three bending mode in the north–south direction (7.49 and 19.9 Hz and 25.45 Hz, respectively), the first two bending modes in the east–west direction (7.77 and 19.90 Hz, respectively) and a rotational mode (14.47 Hz). Note that the frequencies of the first two modes of the structure are closely spaced but the methodology was successful in identifying them.

CONCLUSIONS

This paper discusses the steps and parameters to perform an effective modal identification using the NExT and the ERA. The steps for a successful modal identification can be summarized in: (1) data record selection, (2) calculation of cross-correlation functions, (3) identification of system matrices, and (4) calculation of modal parameters. The record length, number of points for the FFT and the record sampling frequency directly affect the calculations of the cross-correlation function. The number of rows and columns for the Hankel matrix as well as the number of expected modes are important parameters for the identification of the system matrices using the ERA. The use of stabilization diagrams, MAC values, and damping ratios are used to differentiate numerical modes from real modes. Acceleration records from the IASC-ASCE experimental structural health monitoring benchmark problem were used to illustrate the use of the methodology. Six natural frequencies, including two closely spaced modes were successfully detected.

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