In this course, we will apply numerical methods to solve constrained optimization problems. Mathematically speaking, in a constrained optimization problem we are asked to find values for \( n \) parameters or design variables, \( \mathbf{x} = [x_1, x_2, x_3, \ldots, x_n]^T \), that minimizes a specified objective function \( J = f(\mathbf{x}) \) such that a set of inequalities are satisfied.

In many design problems, the objective is to simply minimize the total cost (or price) of the product you are designing. In other design problems, the objective could be to maximize an aspect of the performance, to maximize profits, etc. The suitability of the optimized design depends on more than the cost (or performance, profits, etc). Most systems must also meet a range of other criteria (e.g., safe enough, reliable enough, strong enough). These criteria are expressed as inequality constraints, and depend upon the set of design variables. As long as the inequalities are met the criteria are satisfied. The inequalities constrain the best value of the objective. In a structural design, for example, some inequality constraints might be that: (i) the stress remains less than a yield stress value, (ii) the compression force in a column remains less than the buckling load of the column, and (iii) the deformation of the system remains within certain limits. Note that any inequality \( a \leq b \) can be written \( a - b \leq 0 \) or \( a/b - 1 \leq 0 \). In general, we will write a set of \( m \) inequality constraints as \( g_1(\mathbf{x}) \leq 0 \), or \( g_1(\mathbf{x}) \leq 0 \), \( g_2(\mathbf{x}) \leq 0 \), \( \cdots \), \( g_m(\mathbf{x}) \leq 0 \).

With this notation, a constrained optimization problem may be written:

\[
\text{minimize} \quad J = f(x_1, x_2, \ldots, x_n) \quad \text{such that} \quad \begin{align*}
g_1(x_1, x_2, \ldots, x_n) & \leq 0 \\
g_2(x_1, x_2, \ldots, x_n) & \leq 0 \\
& \vdots \\
g_m(x_1, x_2, \ldots, x_n) & \leq 0
\end{align*}
\]  

In certain rare cases, we can write actual equations for \( J = f(\mathbf{x}) \) and \( \mathbf{g}(\mathbf{x}) \leq 0 \), and use calculus to find an equation for the constrained optimum. In the vast majority of practical problems, however, the equations are simply much too complicated for this approach. In such cases computer-aided analysis can automate the evaluation of the cost and feasibility of a particular trial design. Further, computer-aided optimization allows designers to rapidly determine feasible and possibly “optimal” designs.

Because the formulation of optimum design problems is standardized for application to automated computational optimization methods, an important part of the design process is to cast the design problem into the context that is suitable for numerical optimization.

The formulation of design optimization problems is the focus of this assignment.
1. (25 points) Design Conceptualization —

Conceptualize the design of a mechanical device (i.e., without using electronics of any form) to be used from a small boat by a research worker who wishes to collect samples of air above large lakes at particular altitudes, up to a maximum of 500 ft above lake level. After release, the device must not be attached to the boat and must ascend to within 10 ft of an easily adjustable, pre-determined height. It must return to the water surface with a 0.5 liter sample of air from that height and then float on the surface until picked up.¹ Invent mechanisms that would trigger the sampling of the air at a specified height and which would allow the device to float on the surface after sampling. Identify the design variables that would characterize this device. Specify the performance of this device in terms of cost or some other characteristic. What might be some other criteria by which one may evaluate this device?

Prepare your solution in the form of a figure with labels and one or two paragraphs describing your invention and identifying the design variables and the performance measure.

2. (60 points) Optimization-Oriented Design Analysis and Formulation —

These problems are found in:

(a) (15 points) Problem 2.4
(b) (15 points) Problem 2.12
(c) (15 points) Problem 2.17
(d) (15 points) Problem 2.22

¹Adapted from G.Pahl and E.Beitz, *Engineering Design, a systematic approach*, Springer 1993