

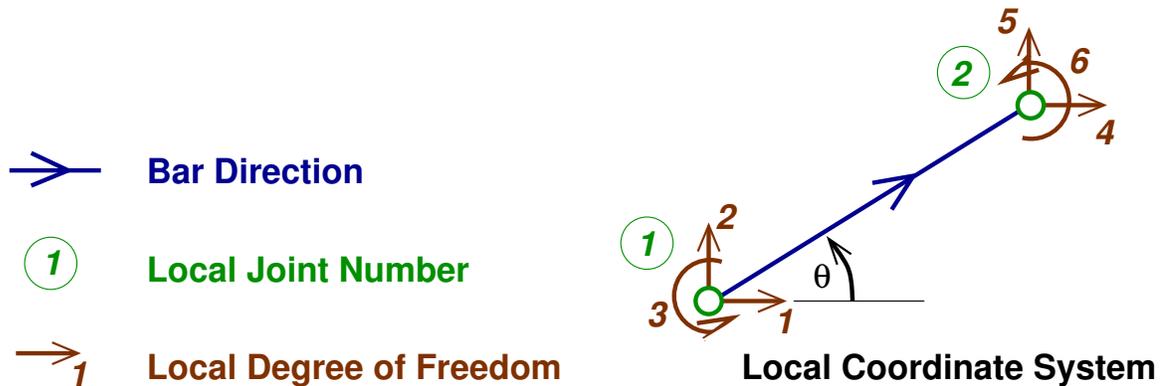
The Matrix Stiffness Method for 2D Frames

CEE 421L. Matrix Structural Analysis

Department of Civil and Environmental Engineering
Duke University Henri P. Gavin

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1. Number the displacement coordinates and reaction coordinates in your frame. In a planar frame, every node has three coordinates: one in the global X-direction, one in the global Y-direction, and one rotation about the global Z-axis (counter-clockwise). Using the method in this handout, every node gets three coordinates.
2. Number each frame element.
3. Node Locations
Write the (x, y) locations of each node using units consistent with E and I . In other words, if E and I are given in kN/cm^2 and cm^4 , write the (x, y) node location in terms of centimeters.
4. Define each element.
Draw each element of your frame individually and draw the local element coordinates in the global directions. For example if element number n is a diagonal frame element, and the global directions are X: horizontal and Y: vertical, draw element number n like this:



where 1,2,3,4,5,6 are the LOCAL coordinates of the frame element in the GLOBAL directions. The local coordinates are always numbered 1,2,3,4 with 1 and 4 pointing in the global X direction (to the right), with 2 and 5 pointing in the global Y direction (up), and with 3 and 6 rotating about the global Z-axis (counter-clockwise). All of these six coordinates will line up with the global structural coordinates that you identified in step 1., above.

5. Element Stiffness Matrices in Global Coordinates, \mathbf{K} .

For each element, find its (6×6) element stiffness matrix, by evaluating the equations on pages 4 - 7 of the handout. You should understand where these equations come from, why this matrix is symmetric, why the diagonal terms are all positive, and what the off-diagonal terms mean.

6. Structural Stiffness Matrix, \mathbf{K}_s .

The structural stiffness matrix is a square, symmetric, matrix with dimension equal to the number of coordinates. In this step we will fill up the structural stiffness matrix using terms from the element stiffness matrices in global directions (from step 5.) This procedure is called matrix assembly.

Recall from step 4. how the LOCAL element coordinates (1,2,3,4,5,6) line up with the GLOBAL structural coordinates. For example, local coordinates (1,2,3,4,5,6) might line up with structural coordinates (13,14,15,7,8,9) of the frame. In that case:

$K(1,1)$ is added to $K_s(13,13)$,

$K(1,2)$ is added to $K_s(13,14)$,

...

$K(2,6)$ is added to $K_s(14,9)$,

...

$K(5,6)$ is added to $K_s(8,9)$,

$K(6,6)$ is added to $K_s(9,9)$,

Add each element into the structural stiffness matrix in this way to build up \mathbf{K}_s .

7. External Loads, $\mathbf{p} = \mathbf{p}_p + \mathbf{p}_e$.

Create the load vector $\mathbf{p} = \mathbf{p}_p + \mathbf{p}_e$, by adding the vector of equivalent forces and moments (negative fixed end forces) for each internal load (collocated with displacement coordinates), \mathbf{p}_e , to the point load vector, \mathbf{p}_p .

8. Deflections, \mathbf{d} .

Find the deflections by solving the linear matrix equation $\mathbf{Kd} = \mathbf{p}$ for \mathbf{d} . You can do this easily in matlab: $\mathbf{d}(\mathbf{q}) = \mathbf{Ks}(\mathbf{q},\mathbf{q}) \setminus \mathbf{p}(\mathbf{q})$, where the vector \mathbf{q} lists the set of displacement coordinates (coordinates without reactions).

9. Internal frame element forces, \mathbf{q} .

Again, recall how the global structural coordinates correspond to with each element's coordinates (1,2,3,4,5,6). For example, in element number "N" from step 6., the local element deflections $v_1, v_2, v_3, v_4, v_5, v_6$ line up with the global deflections $d_{13}, d_{14}, d_{15}, d_7, d_8, d_9$. The internal bar forces can then be computed from:

$$\mathbf{q} = \mathbf{q}_f + \mathbf{T} \mathbf{K} \mathbf{v}$$

Notation

- \mathbf{u} = Element deflection vector in the Local coordinate system
 \mathbf{q} = Element force vector in the Local coordinate system
 \mathbf{k} = Element stiffness matrix in the Local coordinate system
 ... $\mathbf{q} = \mathbf{k} \mathbf{u}$
 \mathbf{T} = Coordinate Transformation Matrix
 ... $\mathbf{T}^{-1} = \mathbf{T}^T$
 \mathbf{v} = Element deflection vector in the Global coordinate system
 ... $\mathbf{u} = \mathbf{T} \mathbf{v}$
 \mathbf{f} = Element force vector in the Global coordinate system
 ... $\mathbf{q} = \mathbf{T} \mathbf{f}$
 \mathbf{K} = Element stiffness matrix in the Global coordinate system
 ... $\mathbf{K} = \mathbf{T}^T \mathbf{k} \mathbf{T}$
 \mathbf{d} = Structural deflection vector in the Global coordinate system
 \mathbf{p} = Structural load vector in the Global coordinate system
 \mathbf{K}_s = Structural stiffness matrix in the Global coordinate system
 ... $\mathbf{p} = \mathbf{K}_s \mathbf{d}$
 \mathbf{q}_f = the vector of fixed end forces from internal applied loads
 in the local element coordinate system.
 \mathbf{f}_f = the vector of fixed end forces from internal applied loads
 in the global element coordinate system
 ... $\mathbf{f}_f = \mathbf{T}^T \mathbf{q}_f$

	Local	Global
Element Deflection	\mathbf{u}	\mathbf{v}
Element Force	\mathbf{q}	\mathbf{f}
Element Stiffness	\mathbf{k}	\mathbf{K}
Structural Deflection	-	\mathbf{d}
Structural Loads	-	\mathbf{p}
Structural Stiffness	-	\mathbf{K}_s