A parametric statistical generalization of uniform-hazard earthquake ground motions

Bryce W. Dickinson, A.M.ASCE 1 and Henri P. Gavin, M.ASCE 2

ABSTRACT

Sets of ground motion records used for seismic hazard analyses typically have intensity measures corresponding to a particular hazard level for a site (perhaps conditioned on a particular intensity value and hazard). In many cases the number of available ground motions that match required spectral ordinates and other criteria (such as duration, fault rupture characteristics, and epicentral distance) may not be sufficient for high-resolution seismic hazard analysis. In such cases it is advantageous to generate additional ground motions using a parameterized statistical model calibrated to records of the smaller data set. This study presents a statistical parametric analysis of ground motion data sets that are classified according a seismic hazard level and a geographic region, and which have been used extensively for structural response and seismic hazard analyses. Parameters represent near-fault effects such as pulse velocity and pulse period, far field effects such as velocity amplitude and power spectral attributes, and envelope characteristics. A systematic fitting of parameterized pulse functions to the individual ground motion records, of parameterized envelopes to individual instantaneous ground motion amplitudes, and of parameterized power spectral density functions to averaged power spectra result in probability distributions for ground motion parameters representative of particular seismic hazard levels for specific geographical regions. This methodology presents a means to characterize the variability in a set of ground motions records in terms of physically meaningful parameters.

INTRODUCTION

Earthquake engineers and engineering seismologists have appreciated the importance of uncertainty in earthquake ground motions for over sixty years (Housner 1947). Earthquake ground motion data sets, even when scaled to a common intensity measure, exhibit significant variability in spectral and temporal characteristics (Housner 1947), (Cornell 1968), (Conte and Peng 1997). Despite the growing availability of ground motion records, ground motion characteristics having an intensity with a certain return period at a site can be known only approximately due to natural variability in earthquake source mechanisms, seismic wave propagation and local site effects. Consequently, expectations about the potential ground motion characteristics at a site are suitably interpreted on a probabilistic basis.

Procedures to generalize and simulate strong ground motions may be generally divided into three main classifications: computational geophysical models, empirical attenuation models, and empirical stochastic models. Computational geophysics employs finite difference or finite element methods to numerically solve elastodynamic wave equations in order to simulate wave propagation through a modeled media (Graves 1998). Attenuation models attempt to relate peak ground motions and spectral response values to parameters describing the earthquake source, the path of the wave propagation, and the local and regional geophysical conditions through empirical relationships (Power et al. 2008). Stochastic models are employed to describe and reproduce the stochastic characteristics of a set of observed ground motions without necessarily considering the geophysical properties of the event (Shinozuka and Deodatis 1988). Researchers have further developed methods to combine attenuation and stochastic process models (Boore 2005b).

Probabilistic Seismic Hazard Analysis (PSHA) combines local seismic hazard models, attenuation models, and site effects to determine the spectral accelerations and other ground motion characteristics of a given return period at a site (Petersen et al. 2008). Performance-based design of structures with nonlinear behavior, on the other hand, requires realistic ground motions representative of the hazard designated by the PSHA.

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(McGuire 1995). Considering the dispersion in intensity measures, even when conditioned on an intensity measure with a specific hazard level, a large number of records may be necessary for a high-resolution performance-based design.

To this end we present a general approach toward the statistical characterization of ground motions from a sample of representative ground motion records. Parameter values that describe the stochastic content, the long-period velocity pulse, and the time-modulating envelope function are determined individually for each record. For this study we make use of sets of ground motion records that have been widely appropriated and which remain in use by many researchers. This methodology may be applied as easily to other sets of ground motions.

BACKGROUND, LITERATURE REVIEW, AND MOTIVATION

Methods for simulating earthquake ground motions typically describe strong ground motions as nonstationary stochastic processes. A number of methods have been proposed for generalizing and simulating the nonstationary behavior of earthquake ground motions (Grigoriu 1995), (Conte and Peng 1997), (Michaelov et al. 1999), (Shinozuka et al. 1999), (Iyengar 2001), (Mukherjee and Gupta 2002), (Wen and Gu 2004), (Gu and Wen 2007), (Gu and Wen 2007). The more common of these stochastic process descriptors involve time-dependent spectral parameters and a time modulating envelope function. The well-known Kanai-Tajimi ground motion spectrum (Kanai 1957), (Tajimi 1960) models ground motion as the response of a damped linear oscillator excited by broadband acceleration. The power spectral density function for in this model contains two physically meaningful parameters: ground frequency and damping. To achieve nonstationarity, such a stationary stochastic process may be multiplied by a deterministic envelope function (Jennings et al. 1968), (Iyengar and Iyengar 1969), (Liu 1970), (Lin and Yong 1987). Shinozuka and Deodatis comprehensively reviewed stochastic ground motion simulation methodologies including the filtering of Gaussian white noise, the filtering of Poisson processes, the spectral representation of stochastic processes, and simulation based on stochastic wave theory (Shinozuka and Deodatis 1988). A companion review paper (Kozin 1988) discusses autoregressive moving average (ARMA) models to parametrically match target ground motions. Many of these models efficiently simulate the evolving amplitude and frequency content of a recorded ground motion and contain parameters that depend on the records chosen for model calibration. The Point Source Stochastic Simulation Model is a commonly-used numerical procedure for simulating ground motion records (Boore 2003), (Boore 2005b). This method makes use of a deterministic envelope function applied to a filtered white noise process, in which the power-spectral density and duration of the record are related empirically to the earthquake fault rupture, path effects, and site conditions. This method has been shown to be effective in simulating the stochastic content of ground motions (Boore 2003) and has bridged a gap between geophysical attenuation relationships and stochastic process models.

Rather than attempting to model the range of ground motions that a given structure may experience over its lifetime, critical excitation methods have been introduced (Takewaki 2005) which assume there is a single worst case input excitation that will cause the most significant response in a given structure. This critical excitation is based on the properties of the elastic or inelastic structural system using the input energy and the energy input rate as the measures of criticality. With this method, a building may be designed to withstand the worst case input, which is certainly important for critical structures. This method is not amenable to estimating exceedance rates or other probabilistic measures.

Computational geophysical models enable simulation of ground motions with characteristics that may be under-represented in catalogs of recorded ground motions and associated empirical models. Such characteristics include forward directivity effects, residual ground displacement, basin effects, and fault-zone channeling. These methods involve the coupling of source models with wave propagation models in order to simulate ground motion activity throughout a geophysical region (Graves 1998) (Bielak et al. 2005) (Olsen et al. 2008).

Increased awareness of the destructive capability of near-fault velocity pulses has motivated several
studies examining the response of structures and their contents subjected to this type of forcing (Bertero et al. 1978), (Rao and Jangid 2001), (Zhang and Iwan 2002a), (Zhang and Iwan 2002b), (Mavroeidis and Papageorgiou 2003), (Mavroeidis et al. 2004), (Tian et al. 2007). The energy wave released upon the slip of a fault can exhibit a number of characteristics including forward directivity, permanent translation, and radiation effects (Mavroeidis and Papageorgiou 2003). When the fault rupture velocity is near the shear wave velocity, seismic energy accumulates at the wave front as the rupture propagates; ground velocity records at sites that are close to the rupture and located ahead of the slip exhibit a distinct high-amplitude pulse in the fault-normal component. Such pulses are typically observed near the beginning of the record (Somerville 2003), (Tothong and Baker 2007). Mavroeidis and Papageorgiou have modeled near-fault earthquake ground motion records containing long-period velocity pulses (Mavroeidis and Papageorgiou 2003). This model is parameterized by the period, the amplitude, the number of cycles, and the phase of the pulse, (all of which have a clear physical interpretation) and the ultimate generation of synthetic near-fault ground motions is accomplished through the superposition of a coherent, long-period velocity pulse with incoherent seismic radiation. A number of subsequent studies have examined the mechanisms and modeling of near-fault ground motions in order to study the resulting structural response (Somerville 2003), (Bray and Rodriguez-Marek 2004), (Tian et al. 2007). A quantitative measure of pulse-like characteristics determined that 91 of the 3551 records in the PEER-NGA database contains a pulse (Baker 2007). The destructive potential of ground motions containing a strong velocity pulse indicates a need to include a pulse simulation model in stochastic process models for the simulation of demanding ground motions.

Stochastic methods for the simulation of ground motion records must therefore contain several key characteristics. The process model must be capable of simulating the nonstationarity of strong ground motions. It must be capable of the inclusion of both long-period velocity pulses as well as incoherent radiation effects. Ideally, the model will be simple with a small number of physically meaningful parameters. Current methods in simulating ground motions that include a near-fault velocity pulse and stochastic content have not yet considered the probability distributions of the model parameters or the relation of these probability distributions to particular seismic hazard levels or geographical regions.

The present study aims to quantify the statistical variation of ground motion parameters fit to samples of ground motion records and to generate additional ground motions containing these same statistical characteristics with a model that includes the superposition of a long-period velocity pulse and a nonstationary stochastic acceleration record. Several investigators have assembled samples of ground motion records representative of a given hazard level as designated by a PSHA (Iervolino and Cornell 2005), (McGuire 1995), (Somerville et al. 1997), (Tothong and Baker 2007). In this study models are calibrated to the ground motion data sets developed for the SAC steel project (Somerville et al. 1997). The five data sets explored in this study were developed to represent the hazard probabilities of exceedance of 2% in 50 years and 10% in 50 years in Los Angeles and Seattle. A set containing 40 records with near fault effects is also explored. These data sets have been regarded as statistical samples for the potential ground motion for a given hazard level and specific geographical region criteria. As a result, the SAC ground motions have been used widely for seismic analysis and performance-based design (Chang et al. 2002), (Lee and Foutch 2002), (Wen and Song 2003), (Yun et al. 2002), (Morgan and Mahin 2008).

Statistical analysis of the model parameters for each of the near-fault, Los Angeles, and Seattle data sets allow for waveform parameters to be drawn from distributions with means, variances, and parameter correlations calibrated to samples of historic ground motion records. In a companion study (Gavin and Dickinson 2010), correlations between input model parameters and spectral responses are plotted as “correlation spectra.” These spectra are used to reduce the number of input parameters to only those which associate with variability in structural response.

**GROUND MOTION PARAMETERIZATION**

In this study synthetic earthquake ground motions are generated through the superposition of a stochastic
ground velocity record with a single, coherent, long-period velocity pulse. The stochastic ground motion velocity record is generated by first simulating an enveloped and unscaled stochastic ground acceleration record, $a_u(t)$,

$$a_u(t_i) = \hat{e}(t_i; \tau_0, \tau_1, \tau_2, \tau_3) \sum_{k=1}^{N+1} \left[ \hat{S}(f_k; f_g, \zeta_g) \right]^{1/2} \cos(2\pi f_k t_i + \theta_k),$$

(1)

where $\hat{S}(f_k; f_g, \zeta_g)$ is the power spectrum of the stochastic content of the acceleration record parameterized by ground frequency, $f_g$, and a ground damping, $\zeta_g$,

$$\hat{S}(f_k; f_g, \zeta_g) = \frac{(2\zeta_g f_k/f_g)^2}{(1 - (f_k/f_g)^2)^2 + (2\zeta_g f_k/f_g)^2},$$

(2)

shown in Figure 1, and $\hat{e}(t; \tau_0, \tau_1, \tau_2, \tau_3)$ is an envelope function. For digital simulation, discrete frequencies $f_k$ are uniformly spaced from $f_{lo}$ to $f_{hi}$; $f_k = f_{lo} + (k - 1)(f_{hi} - f_{lo})/N$. The phase angle at each frequency, $\theta_k$, is a uniformly distributed random variable between 0 and $2\pi$ (Shinozuka and Jan 1972; Shinozuka and Deodatis 1991). In this study, $N = 1024$ and the frequency limits, $f_{lo}$ and $f_{hi}$, depend on the type of ground motion to be simulated. The envelope function used here separates the ground motion into four time periods (Jennings et al. 1968): a time of no ground acceleration, $\tau_0$, the time the ground motion takes to reach its peak acceleration, $\tau_1$, the time the acceleration is at the plateau peak region, $\tau_2$, and an exponential decay time constant, $\tau_3$,

$$\hat{e}(t_i; \tau_0, \tau_1, \tau_2, \tau_3) = \begin{cases} ((t_i - \tau_0)/\tau_1)^2 & \text{for } \tau_0 \leq t \leq \tau_0 + \tau_1 \\ 1 & \text{for } \tau_0 + \tau_1 \leq t \leq \tau_0 + \tau_1 + \tau_2 \\ \exp[-(t_i - \tau_0 - \tau_1 - \tau_2)/\tau_3] & \text{for } \tau_0 + \tau_1 + \tau_2 \leq t \end{cases},$$

(3)

as shown in Figure 2.

An unscaled stochastic ground velocity record $v_u(t)$ is calculated from $a_u(t)$ by integrating frequency components between $f_{lo}$ and $f_{hi}$ in the frequency-domain (Boore 2005a), (Dickinson 2008). The scaled stochastic ground velocity record is $v_s(t) = v_u(t)(V_s/\max|v_u(t)|)$, where $V_s$ is a parameter denoting the peak of the stochastic ground velocity record. The artificial ground velocity record is then found by combining the scaled stochastic ground velocity with a velocity pulse,

$$\hat{v}(t_i) = v_s(t_i) + v_p(t_i).$$

(4)
FIG. 2. Envelope function parameters.

The pulse model has five parameters: the peak pulse velocity, $V_p$, the period of the pulse, $T_p$, the number of cycles in the pulse, $N_c$, the location of the pulse, $T_{pk}$, and the phase of the pulse, $\phi$,

$$v_p(t; V_p, T_p, N_c, T_{pk}, \phi) = V_p \exp \left[ -\frac{\pi^2}{4} \left( \frac{t - T_{pk}}{N_c T_p} \right)^2 \right] \cos \left( 2\pi \frac{t - T_{pk}}{T_p} - \phi \right),$$  \hspace{1cm} (5)

as shown in Figure 3. This function is essentially a single Gabor wavelet in which the scale and shift parameters are continuous and have a clear physical interpretation. The artificial ground acceleration record $a_g(t)$ is found by differentiating $v_g(t)$ from frequencies of 0 to $f_{hi}$ in the frequency-domain.

In summary, twelve individual parameter values are used in this study to completely characterize a given ground motion record: pulse amplitude, $V_p$, pulse period, $T_p$, number of pulse cycles, $N_c$, pulse arrival time, $T_{pk}$, pulse phase, $\phi$, peak velocity of the stochastic content, $V_s$, central ground frequency, $f_g$, ground damping, $\zeta_g$, time of no ground acceleration, $\tau_0$, time to peak acceleration, $\tau_1$, duration of plateau region, $\tau_2$, and acceleration decay time constant, $\tau_3$. 

FIG. 3. Pulse parameters for $\phi = 0$. 

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PARAMETER ESTIMATION

This section describes the estimation of the twelve ground motion parameters for five of the ground motion data sets developed for the SAC steel project. Estimates of parameter values and their confidence intervals were computed using the Levenberg-Marquardt method in three related nonlinear least squares problems.

The pulse parameters were estimated first. To extract long-period data from the ground motion records, each acceleration record was bandpass filtered between 0.1 Hz and 1.5 Hz and integrated once to give the velocity record and integrated again to give the displacement record for the long-period content of the ground motion. These long-period acceleration, velocity, and displacement records were visually examined to determine the presence of a coherent pulse. Pulses are typically visible in the velocity record and are even more apparent in the displacement record. Numerical values for the peak pulse velocity, \( V_p \), the period of the pulse, \( T_p \), the number of cycles in the pulse, \( N_c \), the location of the pulse, \( T_{pk} \), and the phase of the pulse, \( \phi \), were identified such that the pulse equation (5) roughly matched the long-period and high-amplitude content of the velocity record. These values served as the initial guess for a nonlinear least squares fit of the pulse equation to the filtered velocity record,

\[
\chi^2(V_p, T_p, N_c, T_{pk}, \phi) = \frac{1}{2} \sum_{i=1}^{m} \left[ v(t_i) - v_p(t_i; V_p, T_p, N_c, T_{pk}, \phi) \right]^2.
\]

Parameter constraints were enforced during the fitting process to ensure that parameter estimates remained within physically-motivated bounds, as shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( V_p )</th>
<th>( T_p )</th>
<th>( N_c )</th>
<th>( T_{pk} )</th>
<th>( \phi )</th>
<th>( f_g )</th>
<th>( \zeta_g )</th>
<th>( \tau_0 )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( \tau_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>(-\pi)</td>
<td>0.5</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum</td>
<td>500</td>
<td>9.5</td>
<td>5.0</td>
<td>20</td>
<td>(\pi)</td>
<td>10</td>
<td>3.0</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Units</td>
<td>cm/s</td>
<td>s</td>
<td>s</td>
<td>Hz</td>
<td>rad</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
<td>s</td>
</tr>
</tbody>
</table>

Nonlinear least squares methods sometimes converge to local minima and occasionally different initial guesses for the pulse parameters resulted in different parameter estimates. In such cases, decisions were made as to which estimate was most reasonable. Individual pulse records were designated as “easy,” “medium,” or “difficult” to fit depending on the ease of finding a pulse, the quality of the fit, the speed of convergence, and the difficulty of the choices made in the process.

An “easy” to fit record exhibits a single distinct velocity pulse: a clear peak or set of peaks in the record that is longer in period and higher amplitude than any other surrounding peak. The pulse is often the main oscillatory component in the low-frequency bandpass filtered data set, though some records contain additional higher frequency content. An “easy” pulse to fit typically consists of approximately one cycle, different initial guesses converge to the same parameter estimates within a few iterations, and the modeled pulse displacements are close to the actual displacement record and are occasionally detectable in the acceleration record as well. Examples of records with “easy” to fit pulses to are shown in Figure 4.

Records which contained both a long-period, low amplitude, pulse and a short-period, high amplitude pulse were fit with “medium” difficulty. Sometimes an extremely high amplitude peak (approximately 150% or higher than other peaks in the record) would be selected as the pulse over some slightly longer-period but significantly lower amplitude peaks. Conversely, sometimes a very long-period pulse would be selected over other peaks with much shorter periods and not significantly larger amplitudes. If additional reasoning is needed in picking a peak for the pulse fit, the pulse closer to the beginning of the record was chosen. Usually once a choice is made as to which peak constituted the desired pulse, the resulting fit would converge to a desirable result within a few iterations. Another reason a fit might be labeled “medium” in difficulty would
be due to the need to constrain the parameter estimates to sensible values. For example, the number of pulse cycles, $N_c$, was constrained to a minimum value of 0.5. The nonlinear least squares procedure for a few of the records attempted to drive the number of cycles below this value, however, in order to compensate for this, the pulse amplitude value, $V_p$, would grow arbitrarily large, leading to nonsensical results. Changes in parameter values throughout the convergence process were carefully examined for consistency before choosing a final result. Examples of records with “medium” pulse-fitting difficulty from each of the ground motion suites are shown in Figure 5.

A record labeled as “difficult” to fit was typically one for which the least squares method would have trouble converging or would converge to different results based on different initial guesses with no clear distinction between pulse options. In some cases, particularly in the Seattle records, it was so difficult to isolate a distinct pulse that it was decided that no discernible pulse was present in the record. In such cases, the pulse velocity would be assigned a value of zero. Several other records contained what appeared to be multiple cycle pulses where convergence was difficult due either to the variability of results from different initial guesses or simply the inability of the converged result to capture the pulse behavior. In most cases, either convergence (or at least general stability in parameters) was achieved that would capture most of any pulse-like characteristic of the record based on choices that aimed to fit long-period and larger amplitude trends towards the beginning of the record. The greatest problem with the difficult pulse fits was that the pulse parameters were outliers; the method would frequently sacrifice capturing the high amplitude

FIG. 4. Easy pulse fits for each data set, — = fit, - - = data. (a) near fault, nf11; (b) LA 10%/50yr, la03; (c) LA 2%/50yr, la30; (d) SE 10%/50yr, se14; (e) SE 2%/50yr, se23

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oscillations for longer period trends. Examples of records with “difficult” to fit pulse to are shown in Figure 6. The numbers of “easy,” “medium,” difficult,” and “no pulse” records for each of the ground motion suites are shown in Table 2.

To find \( V_s \), the peak of the stochastic part of the velocity record, the acceleration time history was bandpass filtered between 0.1 and 20 Hz and integrated once. The velocity pulse fit was then subtracted from the overall velocity record to leave only the stochastic part of the velocity record. The peak of the stochastic velocity record, \( V_s \), was then determined and recorded.

To fit the envelope the instantaneous acceleration amplitude, \( e(t) \), was computed from the original
acceleration record, $a(t_i)$, and its Hilbert transform, $\tilde{a}(t_i)$,

$$
e(t_i) = \sqrt{a(t_i)^2 + \tilde{a}(t_i)^2}.
$$

The envelope function, equation (3), was fit to the instantaneous acceleration amplitudes normalized to a peak value of 1. The envelope fit function was divided by $\sqrt{2}$ so that the approximation would pass through the normalized data. In fitting the envelope function, the residuals were weighted by the square root of the data such that the higher amplitude values of the normalized acceleration would be more significant,

$$
\chi^2(\tau_0, \tau_1, \tau_2, \tau_3) = \frac{1}{2} \sum_{i=1}^{m} \left[ \frac{e(t_i)/\max(e(t_i)) - \tilde{e}(t_i; \tau_0, \tau_1, \tau_2, \tau_3)/\sqrt{2}}{1/\sqrt{e(t_i)}} \right]^2.
$$

It was found that this weighting method resulted in envelope fits that reached their maximum value very close to the time of the peak ground acceleration (within the first half of the ground motion record), which was considered preferable to envelopes that had maxima toward the end of the record. Subtracting the pulse acceleration prior to fitting the envelope did not noticeably affect envelope parameters. Initial guesses for the four envelope parameters were chosen by visual inspection. Despite a variety of initial guesses with long values of $\tau_2$, the least squares method consistently removed the plateau region indicating that this parameter could be excluded from the envelope function. Examples of envelope functions from each of the ground motion suites are shown in Figure 7.
The power spectral density of the difference between each acceleration record and its pulse was estimated using Welch’s averaged periodogram method and normalized to a maximum of 1. The normalized spectra were then averaged over all records for each of the five data sets to obtain an averaged PSD for each of the five sets. Each of the five average PSD’s were then fit with equation (2). In fitting power spectra the residuals were weighted by the square root of $S(f_i)$ such that the higher amplitude values of the power spectrum were considered more significant.

$$\chi^2(f_g, \zeta_g) = \frac{1}{2} \sum_{i=1}^{m} \left[ \frac{S(f_i) - \hat{S}(f_i; f_g, \zeta_g)}{1/\sqrt{S(f_i)}} \right]^2$$

(9)

This method was applied consistently to each of the five average PSD functions and the resulting parameter values for the ground damping, $\zeta_g$, and ground frequency, $f_g$, were recorded along with their associated standard errors. The averaged PSD’s were easily fit by the Levenberg-Marquardt method; parameter estimates converged within a few iterations. The power spectral density fits for each of the ground motion suites are shown in Figure 8.

PARAMETER DISTRIBUTIONS

The systematic fitting of waveform expressions to the SAC ground motion data sets provide a statistical sample of the characteristics of each suite. The near fault set provides 40 sample measurements of ten
FIG. 8. Power spectra fits for each data set, — = fit, - - = data. (a) near fault; (b) LA 10%/50yr; (c) LA 2%/50yr; (d) SE 10%/50yr; (e) SE 2%/50yr

of the model parameters and the others provide 20 sample measurements of ten of the model parameters. The remaining two parameters, ground frequency, \( f_g \), and ground damping, \( \zeta_g \), are obtained from spectra averaged over each set, thereby giving one value per set. The means and variances of these statistical samples are provided in Table 3. In this table the mean values for ground frequency, \( f_g \), and ground damping, \( \zeta_g \), were obtained via the nonlinear least squares fitting of the averaged power spectral density functions for each data set. The reported standard deviations for these two parameters are simply the standard parameter errors as determined by the fitting procedure. Although the parameter describing the initial period of no ground motion, \( \tau_0 \), was required for fitting the envelopes, it simply represents an arbitrary amount of measurement time before the strong ground motion begins. As such, it has no effect in computing structural responses. Therefore, the distributions of values for \( \tau_0 \) were not explored further. Since this time period is removed from models of simulated ground motions, the arrival in time of the pulse, \( T_{pk} \), was also adjusted in order to account for this change, by subtracting \( \tau_0 \) from \( T_{pk} \).

The parameter estimates show that the 2% in 50 years Los Angeles data set has the largest velocity amplitudes, the near fault data set has the second largest amplitude parameters. The mean velocity amplitudes for the Los Angeles 2% in 50 years records are a little more than twice the mean velocity amplitudes for the Los Angeles 10% in 50 years records. The mean velocity amplitudes for the Seattle 2% in 50 years records are about three times larger than the mean velocity amplitudes for the Seattle 10% in 50 years records. For the near-fault data set the pulse amplitudes are 60 percent larger than the amplitude of the stochastic
TABLE 3. Fit statistics and offsets for five data sets

<table>
<thead>
<tr>
<th>suite</th>
<th>parameter</th>
<th>( V_p )</th>
<th>( T_p )</th>
<th>( N_c )</th>
<th>( T_{pk} )</th>
<th>( \phi )</th>
<th>( V_s )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( \tau_3 )</th>
<th>( f_g )</th>
<th>( \zeta_g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>la10in50</td>
<td>mean</td>
<td>45.28</td>
<td>2.05</td>
<td>0.80</td>
<td>4.97</td>
<td>2.71</td>
<td>34.83</td>
<td>4.77</td>
<td>0.14</td>
<td>4.48</td>
<td>0.78</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>std. dev.</td>
<td>22.81</td>
<td>1.25</td>
<td>0.82</td>
<td>3.32</td>
<td>1.42</td>
<td>16.77</td>
<td>2.69</td>
<td>0.14</td>
<td>3.10</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>se10in50</td>
<td>mean</td>
<td>11.60</td>
<td>1.95</td>
<td>0.84</td>
<td>6.11</td>
<td>3.26</td>
<td>22.22</td>
<td>12.25</td>
<td>0.14</td>
<td>10.40</td>
<td>1.33</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>std. dev.</td>
<td>11.97</td>
<td>0.65</td>
<td>0.30</td>
<td>2.47</td>
<td>1.50</td>
<td>11.91</td>
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<td>0.10</td>
<td>8.28</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>offset</td>
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<td>0.5</td>
<td>0.8</td>
<td>-\pi</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Mean pulse periods ranged from 2.58 s to 2.85 s across the five data sets. The c.o.v. for the pulse period is about 60% for all data sets. Based on these waveforms, pulse period statistics are relatively insensitive to the hazard level or geographical location.

The central frequency of the stochastic content of Seattle ground motions (\( \approx 1.20 \) Hz) is higher than that of Los Angeles ground motions (\( \approx 0.75 \) Hz), and the duration of Seattle ground motions is about three times those of Los Angeles. In all five data sets the bandwidth (\( \zeta f_g \)) of the stochastic content is about 1.0/s.

Cumulative distributions for each of the nine ground motion parameters for the five data sets are shown in Figures 9 through 13. The associated standard errors for every parameter estimate are displayed as horizontal error bars. The standard error bars shown are truncated by the parameter value bounds provided in Table 1. Several parameters have physically-motivated minimum values as shown in the last row of Table 3. These offsets are subtracted from each of the sample values prior to the estimation of the mean and variance of each distribution. These offset values must then be added to the randomly generated parameter value prior to the ground motion simulation. For example, the number of cycles in a pulse, \( N_c \), for records in the “la10in50” set, can be considered as 0.5 plus a lognormal random variable with a mean of 0.8 and a standard deviation of 0.8. This procedure serves the purpose of generating a better fit of the cumulative distribution function as well as ensuring that physically meaningful parameter minimums are enforced during simulations. Empirical cumulative distributions of the parameter estimates match lognormal and Gamma distribution functions. The difference between the distributions is indiscernible in their upper tails.

SUMMARY AND CONCLUSIONS

The generation of synthetic ground motions in this study is accomplished via the superposition of a coherent velocity pulse with a stochastic acceleration record corresponding to a specific power spectral density function and multiplied by a time modulating envelope function. Estimation of twelve ground motion waveform parameters was accomplished via a systematic nonlinear least squares analysis for 120 of the SAC ground motion records. Five parameters describe the pulse: pulse amplitude, \( V_p \), pulse period, \( T_p \), number of cycles, \( N_c \), pulse arrival time, \( T_{pk} \), and pulse phase, \( \phi \). One parameter describes the peak of the stochastic content of the velocity, \( V_s \). Four parameters describe the envelope: the time before ground motion begins, \( \tau_0 \), the period of increasing amplitude, \( \tau_1 \), a plateau region of strong ground motion, \( \tau_2 \), and a period of slow amplitude decay, \( \tau_3 \). And two parameters describe the power spectral density function: the ground frequency, \( f_g \), and the ground damping, \( \zeta_g \). The ground frequency and ground damping parameters
FIG. 9. Parameter estimates, standard errors, and cumulative distributions of ground motion parameter estimates for the SAC “nrfault” set. ⊙=parameter estimates; ⊢⊣=standard parameter errors; •=Gamma distribution; —=lognormal distribution. (a) $V_p$; (b) $T_p$; (c) $N_c$; (d) $T_{pk}$; (e) $\phi$; (f) $V_s$; (g) $\tau_1$; (h) $\tau_2$; (i) $\tau_3$

were fit for the average power spectral density for each data set rather than for individual records because power spectra computed from short-duration transient waveforms have a high variance. Sample means, variances, and distribution functions were calculated for each of the other ten parameters for each data set and parameter distributions are represented well by lognormal and Gamma distribution functions.

The pulse period statistics ($T_p \approx 2.7 \pm 60\%$), the bandwidth statistics ($\zeta_{fg} \approx 1.0/s \pm 10\%$) and the coefficient of variation of the peak stochastic velocity (50%) are similar across the five suites of ground motion analyzed in this study. The central frequency of the stochastic ground motion $f_{g}$ is about 1.20 Hz in the Seattle records and about 0.75 Hz for the Los Angeles records. The duration of the Seattle records is about three times longer than the Los Angeles records and velocity pulses are relatively inconsequential in the Seattle records, which are generally far-field records from subduction-zone ruptures.

This analysis provides a quantitative measure of the variability among ground motions through a parameterized ground motion waveform model. A companion study (Gavin and Dickinson 2010) uses this information to investigate correlations between parameters, to identify parameters having a significant impact on structural response, and to develop response-spectrum-compatible models for bi-axial synthetic ground motions representative of the original data sets. Provided that ground motion parameters can be identified or scaled for a sufficiently large number of hazard levels, this simulation procedure enables high resolution fragility analyses.

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FIG. 10. Parameter estimates, standard errors, and cumulative distributions of ground motion parameter estimates for the SAC “la10in50” set. ○=parameter estimates; ±±=standard parameter errors; •=Gamma distribution; —=lognormal distribution. (a) \( V_p \); (b) \( T_p \); (c) \( N_c \); (d) \( T_{pk} \); (e) \( \phi \); (f) \( V_s \); (g) \( \tau_1 \); (h) \( \tau_2 \); (i) \( \tau_3 \)
FIG. 11. Parameter estimates, standard errors, and cumulative distributions of ground motion parameter estimates for the SAC “la2in50” set. ○=parameter estimates; ⊗⊣=standard parameter errors; ●=Gamma distribution; —=lognormal distribution. (a) $V_p$; (b) $T_p$; (c) $N_c$; (d) $T_{pk}$; (e) $\phi$; (f) $V_r$; (g) $\tau_1$; (h) $\tau_2$; (i) $\tau_3$


Gu, P. and Wen, Y. (2007). “A record-based method for the generation of tridirectional uniform hazard-


FIG. 13. Parameter estimates, standard errors, and cumulative distributions of ground motion parameter estimates for the SAC “se2in50” set. ⊙=parameter estimates; ⊥=standard parameter errors; •=Gamma distribution; —=lognormal distribution. (a) $V_p$; (b) $T_p$; (c) $N_c$; (d) $T_{pk}$; (e) $\phi$; (f) $V_r$; (g) $\tau_1$; (h) $\tau_2$; (i) $\tau_3$


