



Chemo-Mechanical Consolidation of Clays: Analytical Solutions for a Linearized One-Dimensional Problem

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Abstract. Consolidation (and swelling) of clayey soils caused by change in chemistry of pore fluid is addressed. Such phenomena are caused by changes in the concentration of various species in the solution and result primarily from a stress-independent deformation of individual clusters, and from a mechanical weakening or strengthening of the clay solid matrix in the presence of stress. Second, transport of chemicals that involves concentration gradients induces additional driving forces of osmotic consolidation due to semipermeable membrane nature of clay. In this paper an extension of Terzaghi's model of the mechanical consolidation to incorporate chemical loading of soil is proposed. A linearized model is used to solve analytically two one-dimensional problems of consolidation of a homogeneous layer simulating a landfill liner with drained or undrained boundaries. The numerical results show a strong dependence of distribution of pore pressure on the chemical load and chemically induced settlements of soil to be comparable to the mechanical ones.

Key words: chemo-mechanical consolidation, swelling pore-fluid chemistry.

1. Introduction

This paper addresses consolidation and swelling induced in soils by a nonmechanical cause: change in pore fluid chemistry. The change of chemical content of pore fluid constitutes chemical loading which may affect hydraulic and mechanical behavior of rocks and soils in a number of engineering as well as natural environmental problems. Engineering importance of chemical swelling has been traditionally associated with the soil heave due to change in pore water salinity and with electro-osmotic technologies. More recent technologies, where chemical consolidation plays a major role are electro-kinetic remediation and settlement of liners of landfills or impoundments affected by chemicals. In particular, it was found that an alteration of physico-chemical interaction between solid and pore fluid caused by a change in chemical content of pore fluid may induce deformation of soils, see Bolt (1956), Greenberg *et al.* (1973), Sridharan and Venkatappa Rao (1973), Mitchell (1993), Barbour and Fredlund (1989), Fernandez and Quigley (1991). Possible microscopic and macroscopic mechanisms leading to the observed changes in hydraulic conductivity and in mechanical properties of clays are discussed by Hueckel *et al.* (1997) and Hueckel (1997).

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In this paper we take a macroscopic point of view on chemically related consolidation. In such an approach consolidation or swelling may be correlated to a number of macroscopic chemical variables, in first place concentrations of chemicals and their gradients. These variables contribute to chemical strain through several different mechanisms. First, macroscopic deformation of soil may develop due to shrinking or swelling of individual clusters, and in the presence of stress an additional deformation develops due to weakening or strengthening of solid as a result of the interaction of the inter-platelet water with pore solution. Following Fernandez and Quigley (1991) we shall adopt the term ‘chemical consolidation’ for such deformation, as opposed to the term ‘osmotic consolidation’ used e.g. by Barbour and Fredlund (1989).

The second group of mechanisms of chemical strain is linked to concentration gradients. Most notable is that resulting from osmosis in clay. Osmotic process occurs when dense packed clay behaves as imperfect (semi-permeable) membrane (Young and Low, 1965). The osmotic effect stands for the dependence of the relative pore fluid velocity on gradient of osmotic pressure or gradient of concentration of chemicals in the fluid (see e.g. Greenberg *et al.* 1973). A contribution to consolidation associated with the chemical gradient induced flow is called chemico-osmotic consolidation, see Mitchell (1993), or osmotically induced consolidation, see Barbour and Fredlund (1989). The former term will be used here.

Another chemical strain component (ultrafiltration consolidation) may arise due to a non-advective flux of chemicals transported through porous media caused by pore pressure gradient, Katchalsky and Curran (1965). For low permeability clays the ultrafiltration may significantly influence the transport, and indirectly consolidation.

Additional phenomenon, which may contribute to strain in clayey material is the change in density of water being desorbed from or re-adsorbed to the surface of minerals due to changes in chemistry of pore liquid. The amount of desorbed water due to contamination with some organics may be significant (see Fernandez and Quigley, 1988) while the difference of density between adsorbed water and free water may hypothetically amount to 30% , see Skipper *et al.* (1991). Thus, the process of adsorption or desorption of water may affect pronouncedly the balance of mass of pore liquid and thus consolidation and transport.

Modeling of consolidation coupled with transport of chemicals has been initiated by Mitchell *et al.* (1973), and Greenberg *et al.* (1973). Extending the classical Terzaghi’s theory (1923, 1943) of consolidation they included osmotic and ultrafiltration effects. The model was further developed by Barbour and Fredlund (1989), who added the dependence of porosity on increment of osmotic pressure of pore liquid–chemical consolidation. Sherwood (1994) used Biot’s (1941) theory for deformable chemically insensitive shales saturated with fluid to model effects of transport of chemicals through semi-permeable membrane with an uncoupled pore pressure and rock deformation. Models for ion-induced consolidation of biological materials developed by, e.g., Eisenberg and Grodzinsky (1987), and Myers *et al.* (1995) include concentration dependent elasticity moduli as well as ‘chemical stress’ as a function of concentration. Deformational effects of chemicals on clays, and in particular irreversible deforma-

tion have been addressed by Hueckel (1995, 1997) within chemo-plasticity model for clays coupled with concentration sensitive permeability. Boundary-value problems in chemical consolidations are usually limited to one-dimensional problems, solved mainly numerically (Mitchell *et al.* 1973; Barbour and Fredlund, 1989).

The purpose of this paper is to develop analytical solutions for a phenomenological linear model of consolidation and swelling of clayey materials subjected to chemical loading. The basic lines of the original Terzaghi (1923, 1943) approach (definitions of mechanical and geometrical quantities, and assumptions related to the mechanical behavior of the soil) are followed. Transport is considered as occurring through molecular diffusion for a single chemical only. For the nonmechanical interaction of the chemical with the soil minerals the assumption of local chemical equilibrium is used. The mechanical behavior is restricted to small strain range. Two sets of boundary conditions with only one or both permeable boundaries, corresponding to classical problems of mechanical consolidation are considered.

2. Phenomenological Background and Constitutive Assumptions

We shall consider a water saturated soil which contains a significant fraction of smectitic clay. Following Terzaghi's (1923, 1943) theory of one-dimensional mechanical consolidation, we assume that soil forms a uniform laterally infinite layer. As for the material we assume that the pores of soil are completely filled with liquid, the liquid and the solid components are incompressible, the coefficient of permeability does not depend on the soil's deformation, the rheological nature of the consolidation is exclusively due to the low permeability of the soil, strains are small, the lateral deformation of the soil is excluded, and the applied stress is the same for any horizontal section of the soil. In addition to the usual mechanical load, clay is subjected to chemical load consisting in permeation by a single contaminant at concentration c .

From the micro-structural point of view a basic unit of smectite is a cluster formed by a stack of parallel platelets held together by inter-platelet adsorbed water. This water contains ions which balance the electrical potential of the charged surface of the clay mineral. As a result, the adsorbed water behaves like the solid in the sense that it does not move with respect to solid mineral when a gradient of pore pressure is applied. However, when chemicals, in particular salts or organics are added to the pore water, their physico-chemical interaction with clay particles may lead to an exchange of ions residing on the surface with ions from the liquid, or sorption of the chemicals on the surface of minerals, and change of osmotic pressure in inter-platelet water with respect to free water (Bolt, 1956). This may result in change in the thickness of inter-platelet water. Consequently, interparticle forces change and the clusters shrink or swell, making possible particle displacement and rotation. On the macroscopic level, this leads to deformation and changes both hydraulic and mechanical properties of soil. Possible mechanisms of such changes are discussed by Hueckel *et al.* (1997).

In what follows, soil is considered as two-constituent material. In addition, it is assumed that adsorbed water is a part of liquid. The latter assumption has in the context of consolidation theory a number of advantages and some disadvantages over an alternative attributing the adsorbed water to solids (Hueckel, 1992b). The assumption of liquid incompressibility understood as the capacity of volume change due to pressure change (Bear, 1972) refers to free liquid and adsorbed water taken separately. This does not preclude that a chemically driven transition from the adsorbed state to free water may involve a remarkable change of mass or volume of the water involved, as discussed above. This latter effect will be considered expressing the density of adsorbed water under transition as a unique function of concentration of the chemical in the bulk solution (postulate of local chemical equilibrium) independently on the advancement of consolidation. As far as the volume changes of solids, in addition to the mechanical incompressibility, it will be assumed that volume changes in solids due to chemical changes are zero.

Given the two driving forces of the consolidation the superposition of the material response to the mechanical and chemical loadings is assumed. Whenever possible, the original Terzaghi notation is maintained. For a constant composition of the pore liquid, the decrease Δn in the total porosity from the initial n_0 is proportional to the increase in the effective stress from an initial value $\bar{\sigma}_0$ to a final value $\bar{\sigma}$,

$$\Delta n = n_0 - n = m_{vc}(\bar{\sigma} - \bar{\sigma}_0), \quad (1)$$

where m_{vc} is the coefficient of volume decrease due to the mechanical loading. A reduction of the effective stress (unloading) from an initial value $\bar{\sigma}$ to a final value $\bar{\sigma}'$ is supposed to cause an increase in the porosity

$$\Delta n = n' - n = m_{vs}(\bar{\sigma}' - \bar{\sigma}), \quad (2)$$

where m_{vs} is called the coefficient of volume expansion due to mechanical unloading. In general, the values of coefficients m_{vc} , and m_{vs} may have different values allowing for expression of an irreversibility of mechanical response of the soil. Following Terzaghi's approach, porosity in the above relationships is defined as the ratio of total pore volume in a representative material element to the initial volume of the element. Given the assumed incompressibility of solid minerals, and the lack of chemical or physico-chemical processes decreasing the volume of solid matter, the change of such defined porosity is an equivalent of a measure of volumetric deformation of soil. Furthermore, following to Terzaghi's hypothesis, the effective stress, $\bar{\sigma}$, is assumed to be equal to the difference between the total stress in the soil sample, σ , and the pressure in the pore fluid, u

$$\bar{\sigma} = \sigma - u. \quad (3)$$

The effective stress concept in chemically active clay environment has been discussed focusing on the role of adsorbed water in the interparticle transmission of forces (Bolt, 1956, Lambe, 1960, Mitchell, 1962, Sridharan and Vankatappa Rao, 1973,

and Morgenstern and Balasubramoniam, 1980). The experiments of Sridharan and Venkatappa Rao (1973), and Morgenstern and Balasubramoniam (1980), were re-examined by Hueckel (1992a) in the light of parallel and series systems of stress transmission in solid and in load-bearing adsorbed water. It was concluded that for relatively dense clays with the dominance of face-to-face contacts between platelets, the series system may be considered as more appropriate. In such a system effective stress was found equal to specific interparticle repulsion forces. Therefore, Terzaghi's effective stress principle is still valid for these clays. However, additional strain resulting from the chemically induced change in interparticle repulsive forces needs to be added to the mechanically induced strain (Hueckel, 1992a).

The second driving force of the consolidation is the change in the chemistry of the pore liquid represented by an increase or decrease of the concentration of the contaminating chemical. It is assumed that for a given mechanical loading, the chemically induced changes of the total porosity are immediate and reversible. The latter assumption is only an approximation valid for monotonically increasing concentration. In reality, upon removal of contaminant, at constant, sufficiently high effective stress, a permanent strain has been observed (see, e.g., Fernandez and Quigley, 1991, and Hueckel *et al.* 1997) considered as chemo-plastic effect, see Hueckel (1995, 1997). In what follows only monotonic changes in concentration will be considered. A corresponding change in porosity, at constant effective stress is assumed proportional to a monotonic change in the mass concentration of the chemical, from an initial value c_0 to a value c

$$\Delta n = n_0 - n = m_c(c - c_0), \quad (4)$$

where concentration c is defined as ratio of mass of the chemical present in liquid to the total volume of liquid (adsorbed and free) in a representative element of porous body, m_c denotes the coefficient of volume change due to the variation of concentration of the chemical, and by convention it is positive when the increase of the concentration is accompanied by the decrease of porosity. In what follows coefficient m_c is considered to be constant. Thus, by superposition, the total change of the porosity induced by the mechanical and/or chemical loading may be expressed as follows

$$\Delta n = m_v(\bar{\sigma} - \bar{\sigma}_0) + m_c(c - c_0), \quad (5)$$

where m_v is equal to m_{vc} or m_{vs} , depending on the sign of changes of the effective stress. In what follows the coefficients m_v and m_c are assumed to be independent of the deformation of soil and concentration of the chemical in liquid. Note, that this formulation does not require mass transfer between adsorbed and free water to be included. It needs to be stressed that in order to induce a homogeneous chemical consolidation of clay layer as a continuum the acting chemical should be miscible in water, Sridharan, and Venkatappa Rao (1973), Fernandez and Quigley (1985, 1988). Water non-miscible contaminants tend to penetrate clay through discrete, concentrated macro-cracks and affect much less the clay matrix.

Equation (5) represents the central constitutive assumption of the proposed model. An analogous equation has been used by Barbour and Fredlund (1989), with the notable difference that the chemical part of the volume change was assumed to be proportional to osmotic pressure change, rather than concentration. Finally, it must be emphasized that the crucial assumption of the one-dimensional theory of zero lateral strains applies separately to their mechanical and chemical components. In reality, it may be possible that the two nonzero strain components compensate one for another (as is the case in elasto-plastic deformation in oedometer), giving rise to a complicated strain and stress pattern.

The fundamental quantity which is of particular interest in any problem of consolidation is the settlement of the soil. Due to the incompressibility of solid minerals and their chemical insensitivity discussed above, settlement of the soil is determined by the change in its total porosity. The change, ds , of the thickness of a horizontal layer of soil with the original small thickness dz is equal to $ds = \Delta n dz$. Thus, using relationship (5), the integral

$$S(t) = \int_0^L [m_v(\bar{\sigma} - \bar{\sigma}_0) + m_c(c - c_0)] dz \quad (6)$$

defines the total settlement or swelling of any finite layer of soil with the initial thickness L .

The porosity changes in Equation (5) are constrained by solute mass balance, which on the other hand is linked to seepage law. The transport of chemicals through low permeability clayey materials occurs primarily due to advective and diffusive mechanisms. Additional chemo-hydraulic couplings are reported to play a significant role, see, e.g., Greenberg *et al.* (1973), Carnahan and Jacobsen (1990), and Yeung and Mitchell (1993). The linear equation adopted here for fluid flow through porous material includes the dependence on the gradient of pore pressure and on the gradient of concentration (chemico-osmotic effect), and may be represented as follows:

$$v = -\frac{k}{\gamma_l} \frac{\partial u}{\partial z} + k_c \frac{\partial c}{\partial z}, \quad (7)$$

where v is discharge velocity of fluid (difference between average pore velocity of liquid and velocity of solid, referred to the total volume of the medium), k is coefficient of permeability (or hydraulic conductivity), k_c is coefficient of osmotic permeability and γ_l is unit weight of liquid. The linear equation for the nonadvective component of flux of the chemical, J , (the generalized first Fick's law) depends on gradient of concentration of the chemical, and on gradient of pore pressure (ultrafiltration or reverse osmosis effect) and reads

$$J = -D \frac{\partial c}{\partial z} + D_u \frac{\partial u}{\partial z}, \quad (8)$$

where D is the effective coefficient of diffusion, and D_u is coefficient of ultrafiltration. The latter parameter is according to the nonequilibrium thermodynamics a product

of actual concentration of the chemical and of another coefficient representing the ultrafiltration effect, see Katchalsky and Curran (1965).

Note, that pore pressure u is the one in free water only. However, since no explicit use of the equilibrium equation is made, there is no need to introduce any average stress for the liquid phase.

3. Governing Equations

3.1. EQUATION OF CHEMO-MECHANICAL CONSOLIDATION

Mass conservation of the liquid and that of the solid is considered additionally assuming that the variation of mass density of the pore liquid due to change of pressure is insignificant for the process considered (see Appendix A), and that the increment of the total porosity is balanced by the volume of liquid which is squeezed out of the soil element (discharge velocity), diminished by the change in volume of liquid during desorption of water from the surface of minerals. Thus, in the linear, one-dimensional case the mass balance of liquid reads

$$\frac{\partial n}{\partial t} + \frac{\partial v}{\partial z} = -n_0 \xi \frac{\partial c}{\partial t}, \quad (9)$$

where the assumption of chemical equilibrium for the process of sorption or desorption of water is adopted. Parameter ξ represents a relative change of total density of pore liquid due to a change of the amount of adsorbed water induced by unit increment of the concentration of the chemical. Note also that in the mass balance for the whole liquid we do not need to include the mass transfer between adsorbed and free water. Also, as related to the volume of total representative element discharge, velocity v needs not to be considered separately for free and adsorbed water.

By the definition of the effective stress, Equation (3) and the assumption that the total stress in the soil σ is constant, the increment of pore pressure, u , is uniquely related to the increment of the effective stress, $\bar{\sigma}$,

$$\frac{\partial u}{\partial t} = -\frac{\partial \bar{\sigma}}{\partial t}. \quad (10)$$

A change of effective stress and of concentration of the chemical per unit time induce a change of the total porosity n which, by Equation (5), reads

$$\frac{\partial n}{\partial t} = -m_v \frac{\partial \bar{\sigma}}{\partial t} - m_c \frac{\partial c}{\partial t}. \quad (11)$$

Combining the above equation with the equation of mass balance (9) and using the assumption that the total stress is constant, (10), one obtains a relation between the time derivatives of pore pressure and concentration, and the gradient of discharge velocity

$$\frac{\partial v}{\partial z} + m_v \frac{\partial u}{\partial t} - (m_c - n_0 \xi) \frac{\partial c}{\partial t} = 0. \quad (12)$$

Using the equation for discharge velocity (7), Equation (12) may be reduced to the relation for pore pressure and concentration

$$m_v \frac{\partial u}{\partial t} - \frac{k}{\gamma_1} \frac{\partial^2 u}{\partial z^2} - (m_c - n_0 \xi) \frac{\partial c}{\partial t} + k_c \frac{\partial^2 c}{\partial z^2} = 0 . \quad (13)$$

The above equation links the change in pore pressure in a one-dimensional soil medium subjected to a constant mechanical loading and an evolution of concentration of the chemical in pore liquid. Since, in general, the mass concentration, c , is an independent quantity, an additional equation which rules the transport of the chemical is needed.

3.2. TRANSPORT EQUATION

The conservation of mass of a chemical transported through a fully saturated soil requires that the change of partial density (the product of concentration and porosity) be balanced by the mass accumulation due to advective and nonadvective flux of the chemical (see Appendix B). For the linear and one-dimensional case, the balance thus involves four variables

$$n_0 \frac{\partial c}{\partial t} + c_0 \frac{\partial n}{\partial t} + c_0 \frac{\partial v}{\partial z} + \frac{\partial J}{\partial z} = 0, \quad (14)$$

where J is the nonadvective flux of the chemical defined by relationship (8). Combining Equations (10) and (11) with Equation (14) to eliminate the rate of porosity, one obtains that

$$(n_0 - c_0 m_c) \frac{\partial c}{\partial t} + c_0 m_v \frac{\partial u}{\partial t} + c_0 \frac{\partial v}{\partial z} + \frac{\partial J}{\partial z} = 0. \quad (15)$$

Then, using postulates (7) and (8) the equation which governs the transport of the chemical can be reduced to a form involving only two variables

$$(n_0 - c_0 m_c) \frac{\partial c}{\partial t} - (D - c_0 k_c) \frac{\partial^2 c}{\partial z^2} + c_0 m_v \frac{\partial u}{\partial t} - \left(c_0 \frac{k}{\gamma_1} - D_u \right) \frac{\partial^2 u}{\partial z^2} = 0. \quad (16)$$

Equation (16) takes into account the coupled nature of the transport in the material which undergoes chemico-osmotic consolidation. Note that because of the linearization of the problem, convective terms have been neglected in Equations (9) and (14). From the physical point of view, the convective terms are not expected to be of significance in clays due to the extremely low fluid velocity. In a particular case, the above equation takes on the form of an ordinary diffusion equation for concentration since the advective contaminants transport is disregarded, and the coupling with the process of consolidation through its sensitivity to the change of porosity, as well as effect of ultrafiltration are negligible. It should be noted that Equation (16) specializes to the case of chemical consolidation for $k_c = 0$, and to osmotic consolidation when $m_c = 0$.

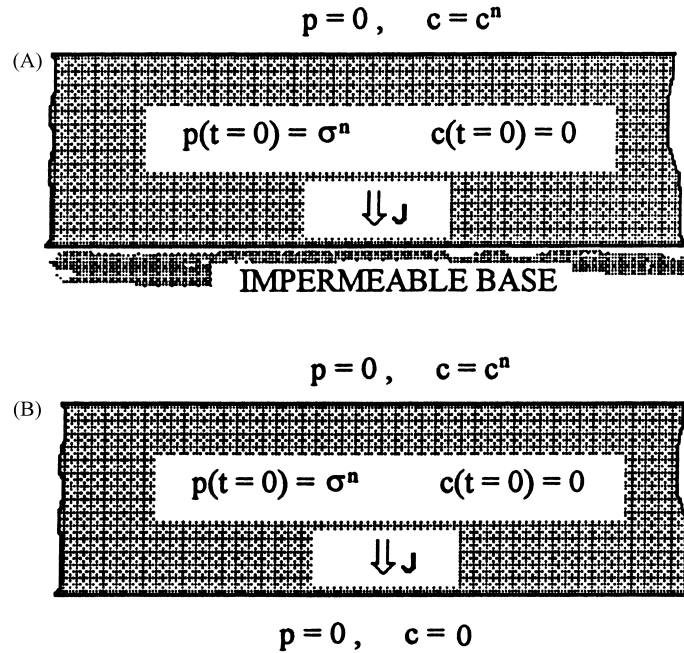


Figure 1. Impermeable (A) and permeable (B) base soil layer.

4. Solutions for the Consolidation Caused by Consecutive Mechanical and Chemical Loadings

Differential Equations (12) and (16) constitute a linear system of coupled equations for chemo-mechanical consolidation for pore pressure and concentration as dependent variables. In this section, solutions of this system will be developed with a particular set of initial and boundary conditions that correspond to the consolidation of a soil layer subject first to mechanical loading and after reaching a steady state, loaded chemically at the upper boundary (see Figure 1). Such a formulation, while leading to a special type of initial/boundary conditions, corresponds to a technically important problem of the consolidation of a clay liner underneath a sanitary landfill. The sets of boundary conditions for pore pressure are identical with those in the classical Terzaghi formulation. As will be seen from the governing equation set, (12) and (16), further developed in Section 4.2, during the chemical phase it is a coupled system in variables c and u . The liquid at the upper boundary may freely flow out from the material through a drain (porous stone). The pressure of the liquid at this boundary is equal to zero. The mechanical loading and mass concentration of the chemical (when it is applied) at the upper boundary are kept constant. Two different conditions at the lower boundary are considered. In the first case (Case A in Figure 1) the lower boundary is impermeable (undrained condition) and thus the fluxes of liquid and of the chemical are equal to zero. In the second case (Case B) the boundary is permeable (drained condition) and washed by pure water, i.e. pressure

and concentration are set as equal to zero. For the reference purpose solutions for the two boundary-value problems when the transport equation is not coupled with consolidation are considered in Appendix C.

4.1. MECHANICAL CONSOLIDATION

A constant total normal traction p' is applied at the upper boundary of the water saturated soil layer as in the classical consolidation problem (Terzaghi, 1943), governed by the equation for pore pressure

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial z^2} = 0, \quad (17)$$

where $\alpha = (k/m_v \gamma_1)$ is called the coefficient of mechanical consolidation. The upper boundary pore pressure initial-boundary conditions are

$$u(z, 0) = p' \quad \text{and} \quad u(L, t) = 0. \quad (18)$$

At the lower boundary, the undrained and drained conditions (Cases A and B, respectively) imply the following homogeneous boundary conditions for pressure:

Case A:

$$\frac{\partial}{\partial z} u(0, t) = 0, \quad (19)$$

Case B:

$$u(0, t) = 0. \quad (20)$$

The solutions to the above problems obtained by the method of separation of variables are classical and may be found in numerous sources (see, e.g., Mei, 1995).

4.2. CHEMICAL CONSOLIDATION

It is assumed that the consolidation due to mechanical loading has reached a steady state, defined by the new height of the layer $L^* = L - m_v p' L$. At the upper boundary, a solution with a constant concentration of the chemical is now applied. The mechanical loading at the boundary p' is not changed and fluid pressure is equal to zero. The chemo-consolidation problem is defined by a system of two differential equations, (13) and (16). Homogeneous initial conditions are imposed for pore pressure and mass concentration of the chemical

$$u(z, 0) = c(z, 0) = 0. \quad (21)$$

For simplicity, the counting of time for the chemical consolidation is set again at zero. Since the pressure is equal to zero and concentration of the chemical is maintained constant at the upper boundary

$$u(L^*, t) = 0, \quad \text{and} \quad c(L^*, t) = c_L. \quad (22)$$

The two sets of boundary conditions for a lower boundary corresponding to impermeable or permeable and washed boundaries are

Case A:

$$\frac{\partial}{\partial z}u(0, t) = 0, \quad \frac{\partial}{\partial z}c(0, t) = 0, \quad (23)$$

Case B:

$$u(0, t) = 0, \quad c(0, t) = 0. \quad (24)$$

The solutions to the above coupled problems may be simplified by isolating the time derivatives of concentration and pressure ($\partial c/\partial t$) and ($\partial u/\partial t$), by substitution of Equation (13) into (16), and vice versa. Then, the governing equations for rates of pore pressure and of concentration can be written separately

$$\frac{\partial u}{\partial t} = +E \frac{\partial^2 u}{\partial z^2} + F \frac{\partial^2 c}{\partial z^2}, \quad (25)$$

$$\frac{\partial c}{\partial t} = +G \frac{\partial^2 u}{\partial z^2} + H \frac{\partial^2 c}{\partial z^2}, \quad (26)$$

where

$$E = \alpha - \frac{D_u(m_c - n_0\xi)}{n_0m_v(1 - c_0\xi)}, \quad F = \frac{D(m_c - n_0\xi)}{n_0m_v(1 - c_0\xi)} - \frac{k_c}{m_v}, \quad (27)$$

$$G = \frac{-D_u}{n_0(1 - c_0\xi)}, \quad H = \frac{D}{n_0(1 - c_0\xi)}. \quad (28)$$

In order to solve Equations (25) and (26) we apply the method developed for modeling diffusion of heat and moisture into solids (see Crank, 1956) and for the problem of multicomponent diffusion (see Toor, 1964). Multiplying Equations (25) and (26) by undetermined constants q_1 and q_2 , respectively, and adding the products the couple (25) and (26) is reduced to a single equation as

$$\frac{\partial}{\partial t}(q_1u + q_2c) - (Eq_1 + Gq_2)\frac{\partial^2}{\partial z^2}u - (Fq_1 + Hq_2)\frac{\partial^2}{\partial z^2}c = 0. \quad (29)$$

This equation can be expressed as a diffusion equation for a new variable $\phi = q_1u + q_2c$ if

$$Eq_1 + Gq_2 = Qq_1, \quad Fq_1 + Hq_2 = Qq_2, \quad (30)$$

where Q is another undetermined constant. The above linear algebraic equations for q_1 and q_2 have a solution only if the corresponding determinant is equal to zero, i.e.

$$GF - (E - Q)(H - Q) = 0. \quad (31)$$

Since in general the above equation has two different roots Q_1 and Q_2 ,

$$Q_{1,2} = \frac{1}{2}[H + E \pm \sqrt{(H - E)^2 + 4GF}], \quad (32)$$

two pairs of constants q_1 and q_2 , which correspond to roots Q_1 and Q_2 may exist. Roots Q_1 and Q_2 need to be positive to yield physically meaningful solutions for the ordinary diffusion equations (Crank, 1956). Denoting these parameters as q_{11} , q_{21} , and q_{12} , q_{22} , respectively, and assuming without loss of generality that q_{11} and q_{22} are equal to one, Equations (30) allow us to determine the constants q_{12} and q_{21} . Substituting Equations (30) into Equations (29) gives the pair of uncoupled diffusion equations for ϕ_i as independent variables

$$\frac{\partial}{\partial t}\phi_i - Q_i \frac{\partial^2}{\partial z^2}\phi_i = 0, \quad i = 1, 2, \quad (33)$$

where

$$\phi_1 = u + q_{21}c \quad \text{and} \quad \phi_2 = q_{12}u + c. \quad (34)$$

The two sets of the initial and boundary conditions for variables ϕ_i can be determined using definitions (34) and conditions (21)–(24) as follows:

$$\phi_1(z, 0) = p', \quad \phi_2(z, 0) = q_{12}p', \quad (35)$$

$$\phi_1(L^*, t) = q_{21}c_L, \quad \phi_2(L^*, t) = c_L, \quad (36)$$

Case A:

$$\frac{\partial}{\partial z}\phi_i(0, t) = 0, \quad i = 1, 2, \quad (37)$$

Case B:

$$\phi_i(0, t) = 0, \quad i = 1, 2. \quad (38)$$

Using the method of separation of variables (see, e.g., Mei (1995)), one may find the following solutions to the above boundary-value problems:

Case A:

$$\phi_i(z, t) = \phi_{i0} + (\phi_{ib} - \phi_{i0}) \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{1/2 + n} \cos(\lambda_n z) \exp[-(\lambda_n)^2 Q_i t], \quad (39)$$

where

$$\begin{aligned} \lambda_n &= (n + \frac{1}{2}) \frac{\pi}{L^*}, \\ \phi_{1b} &= p', \quad \phi_{2b} = q_{12}p', \quad \phi_{10} = q_{21}c_L, \quad \phi_{20} = c_L, \end{aligned} \quad (40)$$

Case B:

$$\begin{aligned} \phi_i(z, t) = & \phi_{i0} \frac{z}{L^*} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{1}{n} \times \\ & \times [(\phi_{i0} - \phi_{ib})(-1)^n + \phi_{ib}] \sin(\lambda_n z) \exp[-(\lambda_n)^2 Q_i t], \end{aligned} \quad (41)$$

where $\lambda_n = n\pi/L^*$ and coefficients ϕ_{ib} and ϕ_{i0} are the same as for Case A. Knowing ϕ_1 and ϕ_2 , the distributions of pressure and concentration can be found from Equations (34):

$$u(z, t) = \frac{\phi_1 - q_{21}\phi_2}{1 - q_{11}q_{21}}, \quad c(z, t) = \phi_2 - q_{12} \frac{\phi_1 - q_{21}\phi_2}{1 - q_{12}q_{21}}. \quad (42)$$

Finally, the settlement of the upper boundary, $S(t)$, defined through Equation (6) takes on the form

$$S(t) = m_v p' L - \frac{m_v(\bar{\phi}_1 - k_{21}\bar{\phi}_2)}{1 - k_{12}k_{21}} + m_c \bar{\phi}_2 - m_c k_{12} \frac{\bar{\phi}_1 - k_{21}\bar{\phi}_2}{1 - k_{12}k_{21}}, \quad (43)$$

where $\bar{\phi}_i = \int_0^{L^*} \phi_i dz$ are defined as

Case A:

$$\bar{\phi}_i(t) = \phi_{i0} L^* + \frac{2L^*}{\pi^2} (\phi_{ib} - \phi_{i0}) \sum_{n=0}^{\infty} \frac{1}{(1/2 + n)^2} \exp[-(\lambda_n)^2 Q_i t], \quad (44)$$

Case B:

$$\begin{aligned} \bar{\phi}_i(t) = & \frac{1}{2} \phi_{i0} L^* - \frac{2L^*}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{n^2} \times \\ & \times [\phi_{i0} - \phi_{ib})(-1)^n + \phi_{ib}] [(-1)^n - 1] \exp[-(\lambda_n)^2 Q_i t], \end{aligned} \quad (45)$$

and the parameters λ_n , ϕ_{ib} , and ϕ_{i0} are the same as for the respective solutions for ϕ_i .

4.3. NUMERICAL RESULTS AND DISCUSSION

In order to develop a simple numerical example, the following input data will be assumed. A layer of the clayey material 1 m thick is first loaded mechanically at the top by the vertical stress which corresponds to the weight of a 10 m column of water, prompting purely mechanical consolidation. When the steady state is reached, the layer is loaded chemically by water solution of NaCl at concentration 0.23 kg/l and chemically induced consolidation occurs. The values of material parameters are taken from the paper by Barbour and Fredlund (1989) (see Table I). For such a data set, the effects of ultrafiltration and change of volume of pore liquid due to desorption of water are neglected ($D_u = 0$ and $\xi = 0$).

Table I. Material parameters for the considered problem

Material property	Value
γ	$1.0 \times 10^4 \text{ kg/m}^2\text{s}^2$
m_v	$5.0 \times 10^{-7} \text{ m s}^2/\text{kg}$
m_c	$0.105 \times 10^{-3} \text{ m}^3/\text{kg}$
k	$1.0 \times 10^{-10} \text{ m/s}$
D	$2.5 \times 10^{-10} \text{ m}^2/\text{s}$
k_c	$2.14 \times 10^{-13} \text{ m}^5/\text{kg s}$

The distribution of pore pressure during mechanical consolidation for a few time instants are shown in Figures 2(a) and 2(b). Although the material has a relatively low hydraulic conductivity (10^{-8} cm/s), only two years in Case A, and one year in Case B are needed until the pore pressure excess is dissipated. As one could expect, since the drained lower boundary accelerates the flow out of the fluid from the soil, the dissipation of pore pressure is faster in this case than in the undrained case. For the chemical phase, the distribution of pore pressure and of concentration due to chemical and chemico-osmotic consolidation are given in Figures 3, 4, and 5. About 200 years and 100 years are needed for Cases A and B, respectively, until the concentration and pore pressure reach steady states. Again the drainage of the boundaries significantly influences the evolution of distributions of concentration and pressure. In order to examine the contributions to the pressure build-up during chemico-osmotic consolidation, the variations of pore pressure due to chemical and osmotic mechanisms are considered separately (parameters m_c or k_c are assumed to be zero, respectively) as shown in Figures 5 and 6. It is seen that for the data assumed in the example the chemical mechanism causes development of a small positive pore pressure, while the osmotic mechanism induces a large negative pressure.

The evolution curves for settlement of the layer as a function of time, corresponding to the mechanical and chemico-osmotic consolidation are compared in Figures 7(a) and 7(b). The final value of the settlement due to the applied mechanical loading (10 m of water) is visibly larger than the settlement due to the chemico-osmotic consolidation. Although, this is related to the specific boundary conditions chosen in this example, they represent a load system typical for a clay barrier. Clearly, for Case A the chemico-osmotic component may increase the total consolidation to over 50%. The time after which the settlement of the layer under the mechanical loading reaches a steady state is comparable with that for chemical loading when both chemical and osmotic mechanisms of consolidation are taken into account. The settlement due to chemical and chemico-osmotic mechanisms exhibit a remarkable difference in the ranges of time when the mechanisms contribute most to the total consolidation induced by the chemical loading (this was also noticed by Barbour and Fredlund (1989)). Chemical consolidation in the problem is clearly ‘slower’ than its osmotic counterpart. In particular, it should be noticed that for steady state

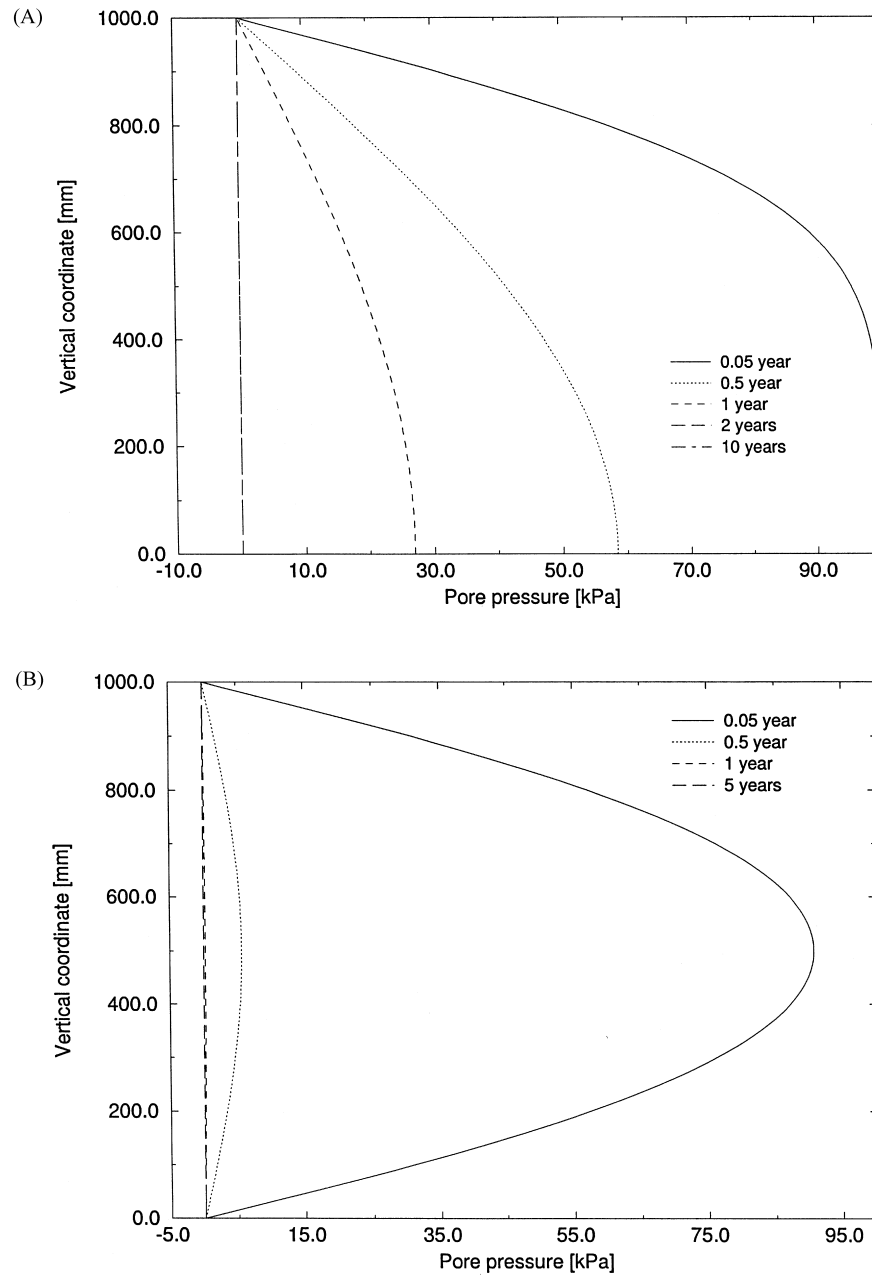


Figure 2. Pore pressure distribution for mechanical consolidation of soil layer with impermeable (A) and permeable (B) base.

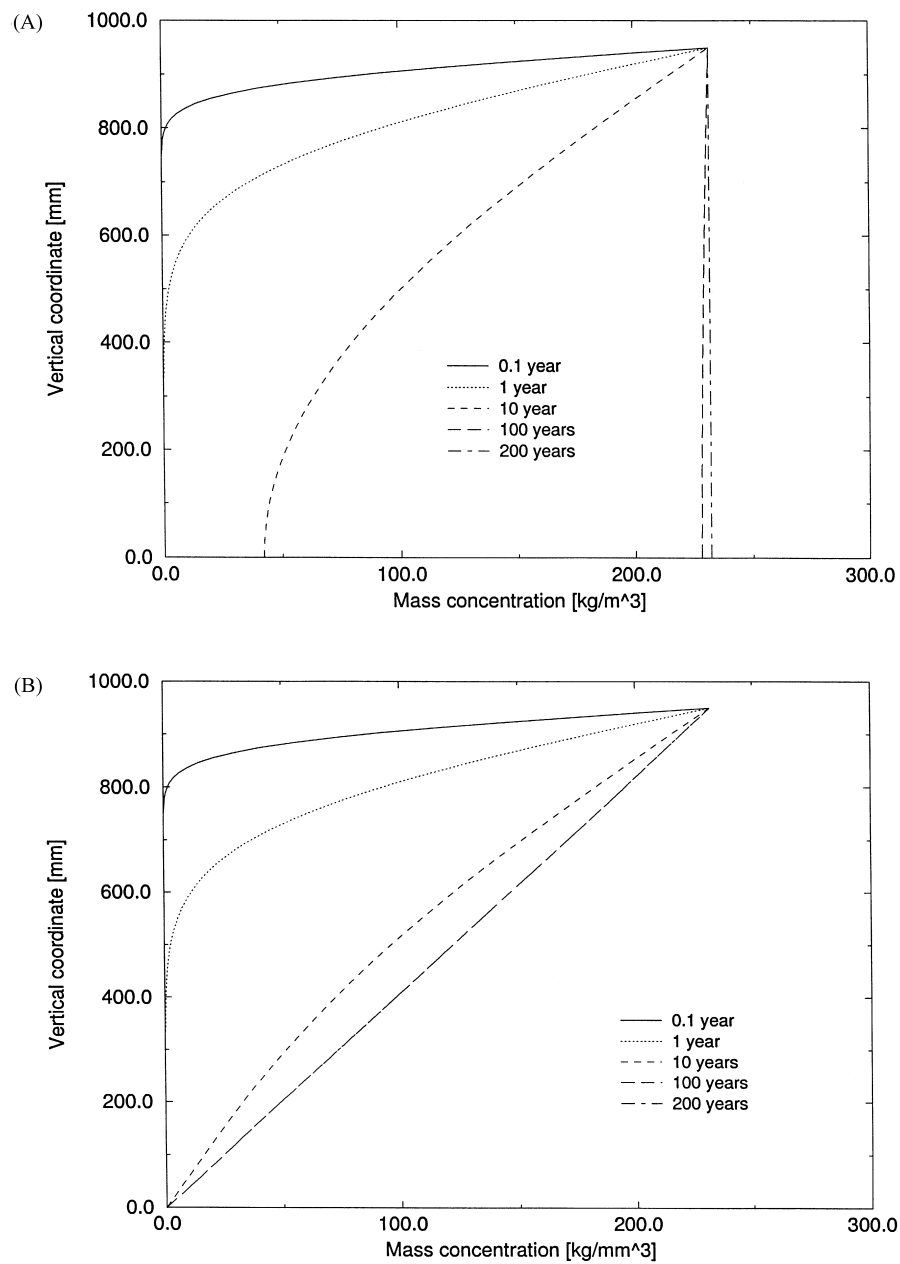


Figure 3. Distribution of concentration during chemico-osmotic consolidation of soil layer with impermeable (A) and permeable (B) base.

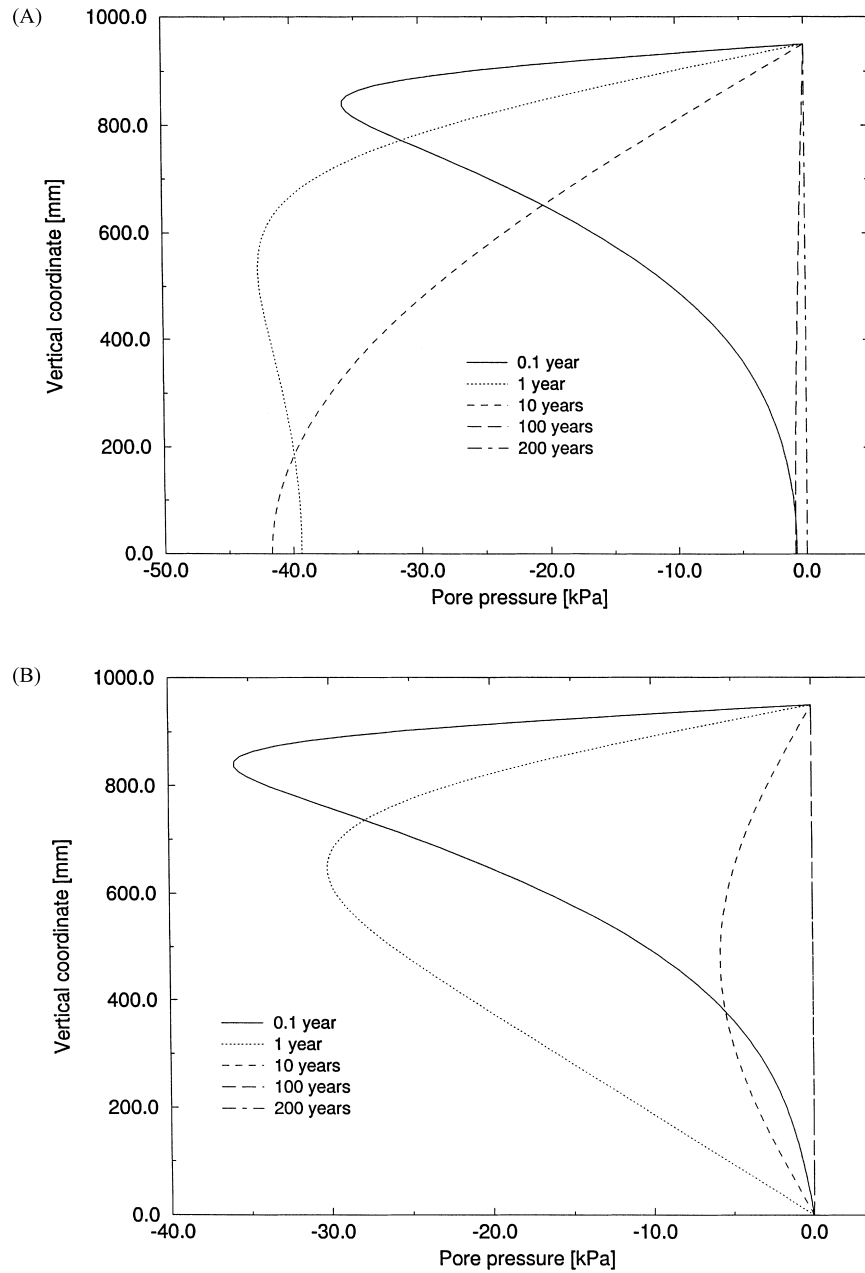


Figure 4. Pore pressure distribution for chemo-osmotic consolidation of soil layer with impermeable (A) and permeable (B) base.

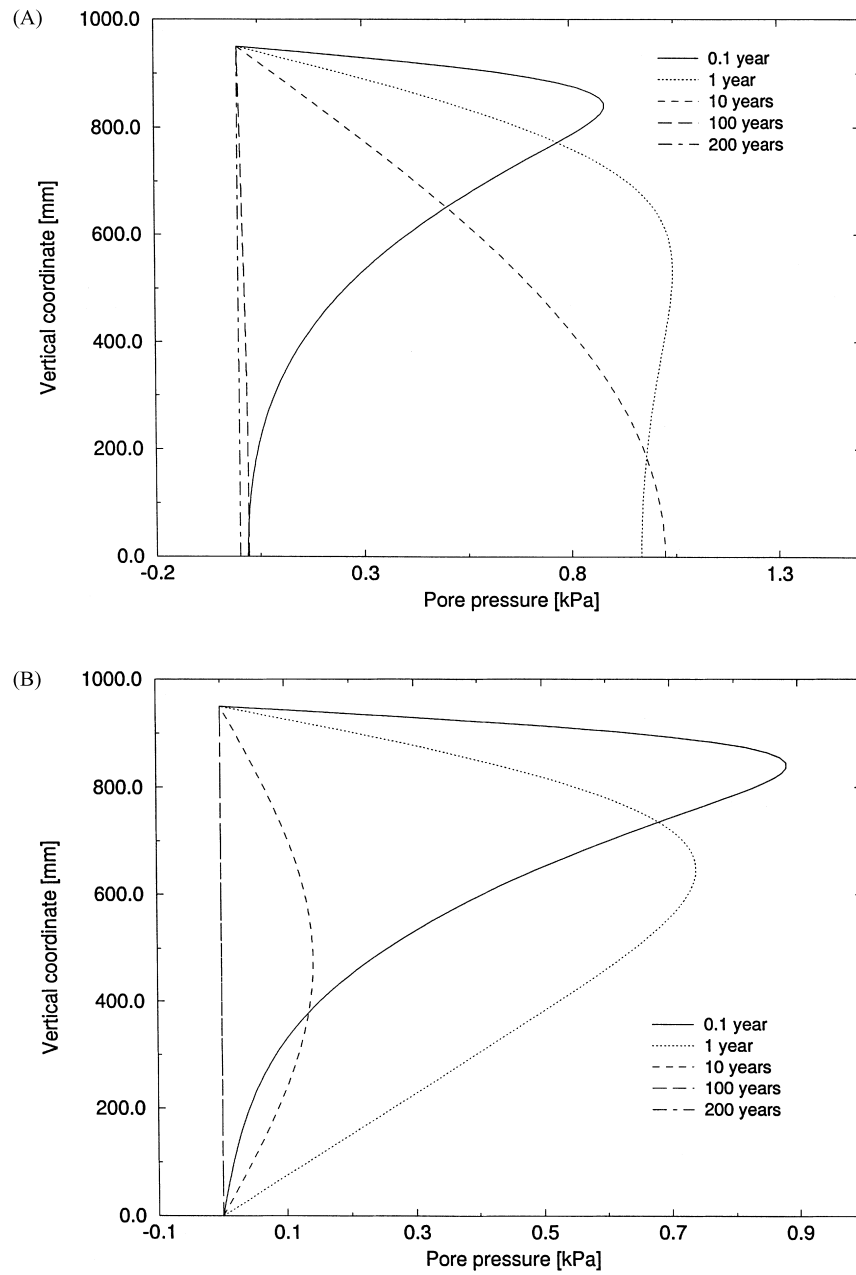


Figure 5. Pore pressure distribution for chemical consolidation of soil layer with impermeable (A) and permeable (B) base.

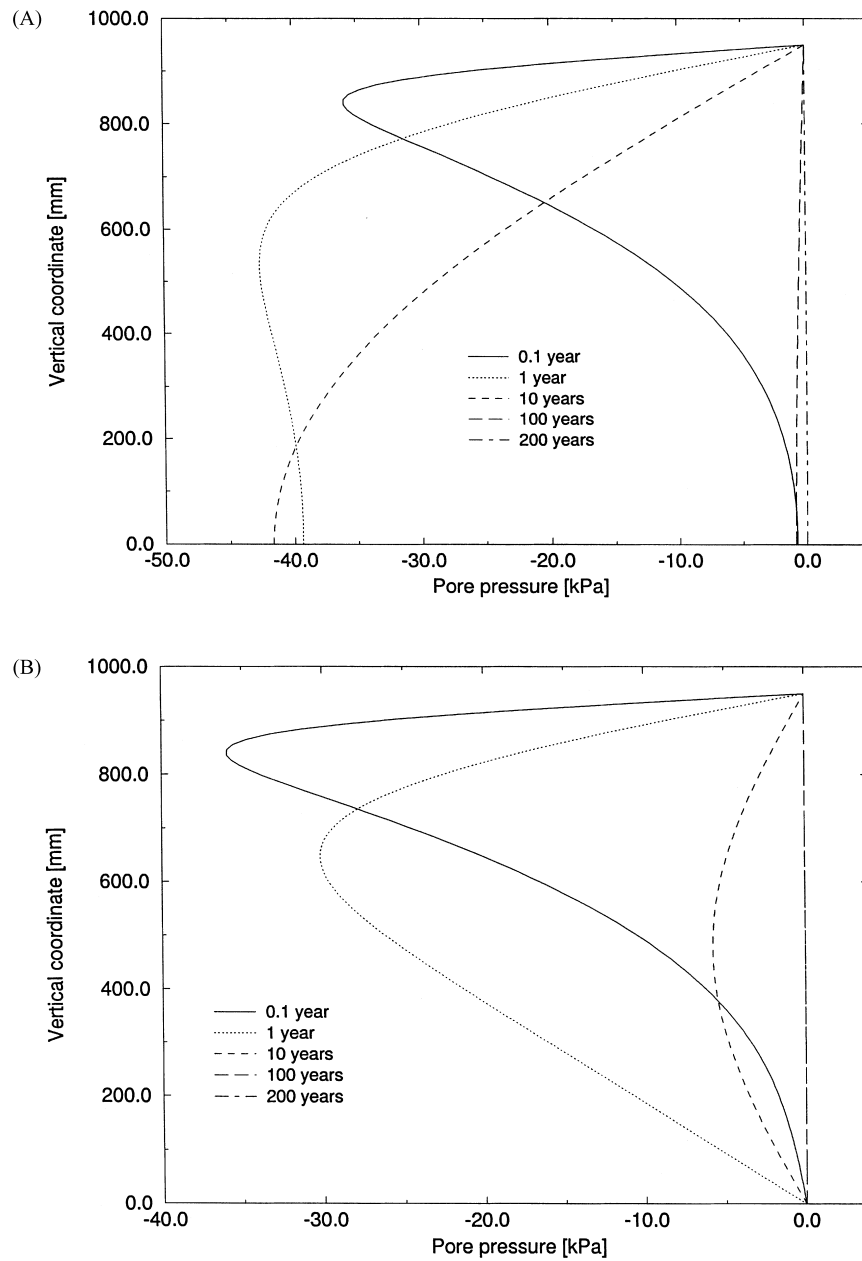


Figure 6. Pore pressure distribution for osmotic consolidation of soil layer with impermeable (A) and permeable (B) base.

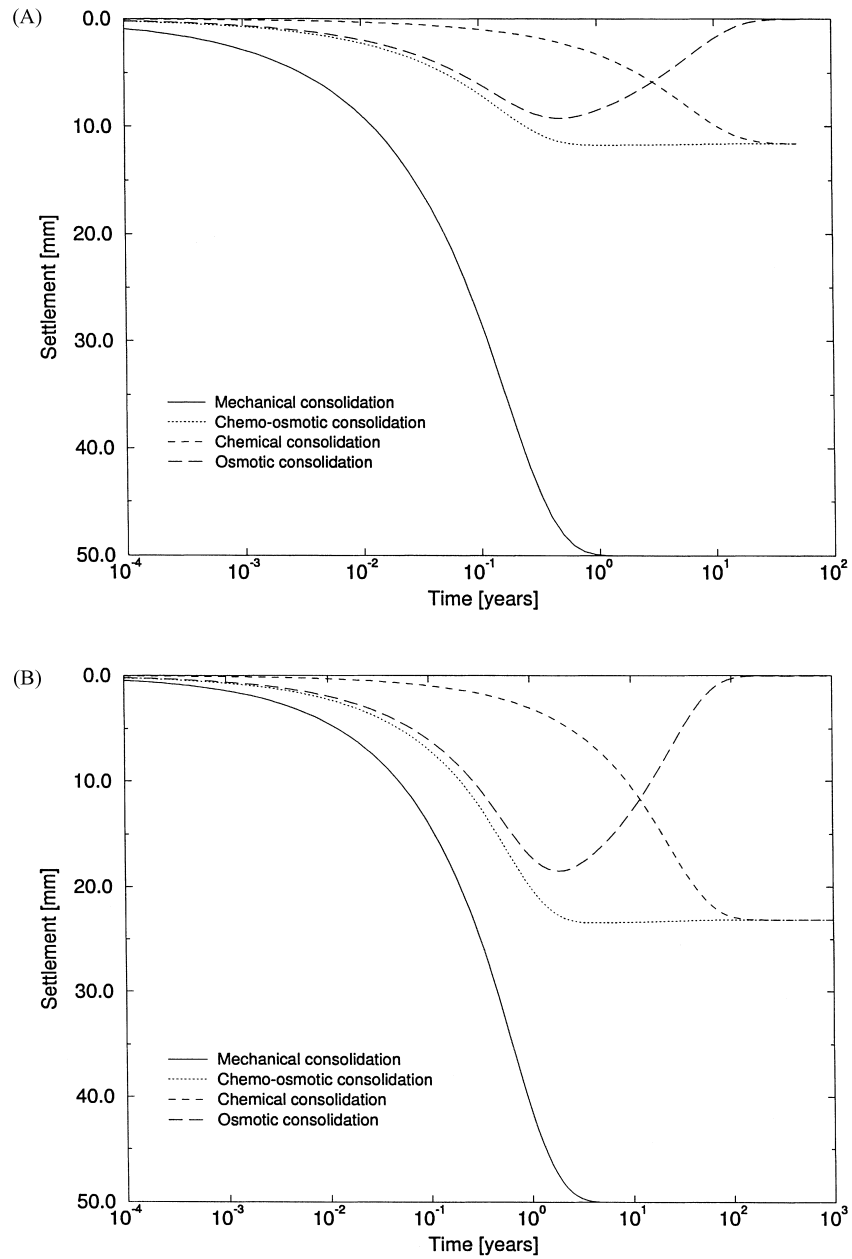


Figure 7. Evolution of settlement for impermeable (A) and permeable (B) base soil layer.

when the concentration gradient is constant, the settlement due to chemico-osmotic consolidation for both undrained and drained lower boundary decreases to zero. Such behavior of chemically consolidated material has not been predicted by the numerical model developed by Barbour and Fredlund (1989).

5. Conclusions

The model of chemically induced strain proposed includes the effect of contamination on deformation of clay. The model neglects advection of chemicals due to the externally applied pressure difference during the chemical load phase, and disregards changes in hydraulic conductivity of the liner material caused by contaminants. The analytical solutions were obtained for one-dimensional consolidation which allow us to evaluate the mechanical, chemical, and chemico-osmotic contributions to the consolidation due to contamination, as well as the role of consolidation in diffusion-dominated transport through a saturated porous layer. The solutions can be directly applied also to swelling problems.

The results presented demonstrate the significance of the modeling of coupled chemico-mechanical phenomena to predict the consolidation of chemically sensitive materials. The example shows that for a realistic set of conditions, about 200 years from the beginning of contamination are needed for the undrained case, and 100 years for the drained bottom, for the concentration and pore pressure reach steady states. The contributions of the chemical and chemico-osmotic mechanisms to the pressure buildup during chemico-osmotic consolidation may be visibly of a different order. The chemical mechanism in the considered case causes development of a small positive pore pressure, while a large negative pressure is induced by the chemico-osmotic mechanism.

The total settlement may be substantially increased by the chemico-osmotic consolidation (up to 50% in the considered case). The osmotic component as induced by the negative pressure increase component slows down the global chemical component of the settlements. Interestingly, at the steady state of the process, the osmotic component of consolidation settlement returns to zero. Although the proposed solution refers to a particular loading case (a single chemical is applied when the mechanical consolidation reaches the steady state) any other setup of the external driving forces of consolidation are allowed due to the linearity of the model. For a better approximation of the real conditions, further studies are necessary of the coupled problems including sorption and reaction of contaminants with solid and a full coupling of transport and deformation of the solid matrix. On the other hand, the linearity of the problem and the goal of obtaining an analytical solution excluded many physically important features of the phenomenon discussed, namely effects of sorption and reaction of the contaminants with soil minerals, as well as the dependence of intrinsic permeability on concentration. The inclusion of these effects requires numerical solutions.

Appendix A: Continuity Equation for Fluid Flow for Concentration Affected Porosity

According to the principle of mass conservation the macroscopic equation of continuity of mass of fluid saturating porous material derived by spatial averaging reads (see, e.g., Hassanizadeh (1986))

$$\frac{\partial}{\partial t}(\rho \bar{n}) + \nabla \cdot (\rho \bar{n} \mathbf{v}^f) = 0, \quad (\text{A1})$$

where \mathbf{v}^f is the velocity of fluid, ρ is intrinsic density of pore fluid, and \bar{n} stands for the total porosity of the material defined as the ratio of the actual total pore volume in a representative region of averaging and volume of the region. It is assumed here that the pore volume V^p includes both free liquid (liquid which may flow) and liquid adsorbed on the surface of soil minerals. Introducing the discharge velocity of fluid $\mathbf{v} = \bar{n}(\mathbf{v}^f - \mathbf{v}^s)$, where \mathbf{v}^s is the velocity of solid material, and neglecting the convective components of the time derivative of density and porosity, the last equation may be rearranged as

$$\frac{\partial \bar{n}}{\partial t} + \bar{n} \nabla \cdot (\mathbf{v}^s) + \nabla \cdot \mathbf{v} = -\frac{\bar{n}}{\rho} \frac{\partial \rho}{\partial t}. \quad (\text{A2})$$

From the continuity equation of mass for incompressible solid phase it results that

$$\frac{\partial \bar{n}}{\partial t} = (1 - \bar{n}) \nabla \cdot \mathbf{v}^s, \quad (\text{A3})$$

while the relation between variation of porosity \bar{n} and the porosity used in Terzaghi's approach n may be written as (see Kaczmarek and Hueckel, 1998)

$$\frac{\partial \bar{n}}{\partial t} = (1 - \bar{n}) \frac{\partial n}{\partial t}. \quad (\text{A4})$$

The continuity Equation (2) takes on the form

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{v} = -n_0 \xi \frac{\partial c}{\partial t}, \quad (\text{A5})$$

where porosity \bar{n} in Equation (A5) is replaced by n_0 because initially both porosities are equal, and due to linearization the initial values are considered only. Due to the earlier hypothesis, the rate of density of the fluid mass resulting from the change in the amount of adsorbed water was assumed to be proportional to the rate of concentration of the chemical, i.e. $(\partial \rho / \rho \partial t) = \xi (\partial c / \partial t)$.

Appendix B: Continuity Equation for Chemical Transport for Concentration Affected Porosity

The conservation of mass of a chemical transported in fluid saturating porous material is (Hassanizadeh, 1986)

$$\frac{\partial}{\partial t}(c\bar{n}) + \nabla \cdot (c\bar{n}\mathbf{v}^c) = 0, \quad (\text{B6})$$

where c is the average concentration and \mathbf{v}^c denotes the average velocity of the chemical. Introducing the decomposition of the total flux of the chemical into its components due to transport with the solid and fluid advection, and nonadvective transport

$$c\bar{n}\mathbf{v}^c = c\bar{n}\mathbf{v}^s + c\mathbf{v} + \mathbf{J}, \quad (\text{B7})$$

where \mathbf{v}^s , \mathbf{v} , and \mathbf{J} are velocity of solid, discharge velocity, and nonadvective flux of the chemical, and neglecting the convective components of the time derivative of concentration and porosity, we get

$$\bar{n} \frac{\partial c}{\partial t} + c \frac{\partial \bar{n}}{\partial t} + c\bar{n}\nabla \cdot \mathbf{v}^s + c\nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{J} = 0. \quad (\text{B8})$$

Inserting relations (A3) and (A4) into the above equation, and replacing porosity \bar{n} by porosity n in the first term of the equation (small changes of porosity are taken into account), we obtain that the continuity equation of the chemical reads

$$n \frac{\partial c}{\partial t} + c \frac{\partial n}{\partial t} + c\nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{J} = 0. \quad (\text{B9})$$

Appendix C: Chemical Consolidation – an Uncoupled Formulation

For the purpose of reference an uncoupled problem will be now considered. The first problem describes the consolidation induced by changes in water pressure caused by diffusion alone. To this end we assume that the influence of the porosity changes due to flow of liquid on the transport of chemical may be disregarded. Moreover, we neglect the effect of change of mass of desorbed water and ultrafiltration coupling. Then, Equations (13) and (16) may be written as

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial z^2} = \frac{m_c}{m_v} \frac{\partial c}{\partial t} - \frac{k_c}{m_v} \frac{\partial^2 c}{\partial z^2}, \quad (\text{C10})$$

$$\frac{\partial c}{\partial t} - \beta \frac{\partial^2 c}{\partial z^2} = 0, \quad (\text{C11})$$

where $\beta = (D/n_0)$. Since the second equation does not depend on the fluid pressure, it may be solved independently of the first equation. This solution will determine the right-hand side of the differential equation for the pore pressure. In what follows,

the solutions to the above problem for the two earlier introduced cases (impermeable and permeable lower boundaries) are the considered. The solution to the diffusion equation for the concentration of the chemical (C11) with the initial and boundary conditions (21), (22), (23), and (24) are obtained by a method of separation of variables (see Mei, 1995)

Case A:

$$c(z, t) = c_L \left[1 - \frac{2}{L^*} \sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_n} \cos(\lambda_n z) \exp[-(\lambda_n^*)^2 \beta t] \right],$$

where

$$\lambda_n^* = (n + \frac{1}{2}) \frac{\pi}{L^*}, \quad (\text{C12})$$

Case B:

$$c(z, t) = \frac{c_L}{L^*} \left[1 + 2 \sum_{n=0}^{\infty} \frac{1}{\lambda_n} \sin(\lambda_n z) \exp[-(\lambda_n^*)^2 \beta t] \right],$$

where

$$\lambda_n^* = \frac{n\pi}{L^*}. \quad (\text{C13})$$

The solution to the inhomogeneous equation for the pore pressure (C10) with homogeneous initial and boundary conditions (21), (22), (23) and (24), is found in the form of the superposition of eigenfunctions (Mei, 1995)

Case A:

$$u(z, t) = \sum_{n=0}^{\infty} T_n(t) \cos(\lambda_n^* z), \quad (\text{C14})$$

Case B:

$$u(z, t) = \sum_{n=0}^{\infty} T_n(t) \sin(\lambda_n^* z), \quad (\text{C15})$$

where the expansion coefficients $T_n(t)$ must be found by substitution of solutions (C14) and (C15) into Equation (C10). The right-hand side of the latter equation is defined using the solution to the transport equation (C11). As a result, by the orthogonality of eigenfunctions and making use the initial condition (21), one obtains that the coefficients $T_n(t)$ are defined as

Case A:

$$T_n(t) = \frac{2c_L(m_c\beta - k_c)(-1)^n}{m_v L^* \lambda_n^* (\alpha - \beta)} \times \exp[-(\lambda_n^*)^2 \alpha t] \{1 - \exp[-(\lambda_n^*)^2 (\alpha - \beta)t]\}, \quad (\text{C16})$$

Case B:

$$T_n(t) = \frac{2c_L(m_c\beta - k_c)}{m_v \lambda_n (\alpha - \beta)} \exp[-(\lambda_n^*)^2 \alpha t] \{\exp[-(\lambda_n^*)^2 (\alpha - \beta)t] - 1\}, \quad (\text{C17})$$

The settlement of the soil layer reads

Case A:

$$S(t) = -\frac{2c_L(m_c\beta - k_c)}{L^*(\alpha - \beta)} \sum_{n=0}^{\infty} \frac{1}{\lambda_n^2} \exp[-(\lambda_n^*)^2\alpha t] \{\exp[-(\lambda_n^*)^2(\alpha - \beta)t] - 1\} + m_c c_L L^* - \frac{2m_c c_L}{L^*} \sum_{n=0}^{\infty} \frac{1}{\lambda_n^2} \exp(-\lambda_n^2\beta t), \quad (\text{C18})$$

Case B:

$$S(t) = -\frac{2c_L(m_c\beta - k_c)}{L^*(\alpha - \beta)} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\lambda_n^2} \exp[-(\lambda_n^*)^2\alpha t] \times \{\exp[-(\lambda_n^*)^2(\alpha - \beta)t] - 1\} + m_c c_L L^* - \frac{2m_c c_L}{L^*} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \exp(-\lambda_n^2\beta t), \quad (\text{C19})$$

where λ_n in the above equations are identical to those in Equations (C12) and (C13), respectively.

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