## CORPORATE FINANCE

## LECTURE 4

TAXES AND THE MARGINAL INVESTOR

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In this lecture we consider the effect of government tax policy on valuing certain cash flows. In particular we consider the effect of corporate taxes and personal income taxes on the "cost of capital". We start by considering a world where there is a corporate tax but no personal income tax. Such situation leads to the traditional MM formula which implies that there is a tax advantage to debt.

Let the corporate tax rate be $\tau_{C}$. The value of a levered firm would then be just the value of the unlevered firm plus the present value of the tax shields (this is just the APV formula which we will study later). Thus:

Value of Levered firm = Value of Unlevered firm + PV of tax shields.

To derive this formula, consider a firm whose only asset is a perpetual cash flow of \$1M and whose corporate tax rate is $40 \%$. Suppose that municipal bonds exist and that these are tax free and earn a return of $10 \%$ (i.e. everyone can borrow and lend tax-free, at the personal level, of $10 \%$ ). One can replicate a cash flow of $\$ 1 \mathrm{M}(1-0.4)=\$ 0.6 \mathrm{M}$ every period by investing in $\$ 1 \mathrm{M}(1-0.4) / 0.10=\$ 6 \mathrm{M}$ worth of perpetual bonds.

Equivalently, consider the following table:

## REPLICATING THE UNLEVERED FIRM

|  | $t=0$ | $t=1$ |
| :---: | :---: | :---: |
| Before Tax Cash Flow | $\$ 1 \mathrm{M}$ | $\$ 1 \mathrm{M}$ |
| After Tax Cash Flow | $\$ 0.6 \mathrm{M}$ | $\$ 0.6 \mathrm{M}$ |
| Replicating Investment | $\$ 0.6 \mathrm{M} / 0.1=$ <br> $\$ 6.0 \mathrm{M}$ | $\$ 0.6 \mathrm{M}$ |

The after-tax income to the corporation (and to the individual as there are no personal taxes) in this example is $\$ 0.6 \mathrm{M}$ a year, after payment of the $40 \%$ corporate tax. An investment of $\$ 6 \mathrm{M}$ in tax exempt securities would also yield an after tax income of $\$ 0.6 \mathrm{M}$ a year. Thus the value of the unlevered cash flows of the firm is $\$ 6 \mathrm{M}$. This could have been directly obtained as $\$ 1 \mathrm{M}(1-0.4) / 0.1=\$ 6 \mathrm{M}$.

Now suppose the firm issues $\$ 2 \mathrm{M}$ face value in perpetual debt. The interest rate of the debt must be $10 \%$ (if it is less than $10 \%$ no one will purchase the debt). At $10 \%$, the interest payment per year is $\$ 0.2 \mathrm{M}$. Redoing the table above, we obtain:

## REPLICATING THE LEVERED FIRM

|  | $\mathrm{t}=0$ | $\mathrm{t}=1$ | $\mathrm{t}=2$ and so on |
| :---: | :---: | :---: | :---: |
| Before Tax and Interest Cash <br> Flow |  | \$1M | \$1M |
| Interest Payment |  | \$0.2M | \$0.2M |
| Net Income After Interest Before <br> Taxes |  | \$0.8M | \$0.8M |
| Taxes |  | \$0.32M | \$0.32M |
| After Tax Equity Cash Flow |  | \$0.48M | \$0.48M |
| Replicating equity investment | $\begin{aligned} & \$ 0.48 \mathrm{M} / 0.1= \\ & \$ 4.8 \mathrm{M} \end{aligned}$ | \$0.48M | \$0.48M |
| Replicating debt investment | $\begin{aligned} & \$ 0.2 \mathrm{M} / 0.1= \\ & \$ 2.0 \mathrm{M} \end{aligned}$ | \$0.2M | \$0.2M |

The total value of the firm is thus the value of equity plus the value of debt. Thus the total value of the firm is $\$ 4.8 \mathrm{M}+\$ 2.0 \mathrm{M}=\$ 6.8 \mathrm{M}$. Thus the additional value from undertaking $\$ 2 \mathrm{M}$ in debt is $\$ 0.8 \mathrm{M}$.

An alternative way to calculate this is as follows:

|  | $t=0$ | $t=1$ | $t=2$ and so on |
| :--- | :--- | :--- | :--- |
| Interest Payment |  | $\$ 0.2 \mathrm{M}$ | $\$ 0.2 \mathrm{M}$ |
| Tax Shield From Interest Payment |  | $\$ 0.08 \mathrm{M}$ | $\$ 0.08 \mathrm{M}$ |
| Replicating equity investment | $\$ 0.08 / 0.1=\$ 0.8 \mathrm{M}$ | $\$ 0.08 \mathrm{M}$ | $\$ 0.08 \mathrm{M}$ |

Thus an equity investment of $\$ 0.08 \mathrm{M}$ will replicate the tax shield. Hence we obtain the formula:

## Value of Levered firm = Value of Unlevered firm + PV of tax shields

In this world with perpetual debt (with no personal taxes) we can also obtain the following formula:

$$
\text { PV of tax shields }=\tau_{c} D
$$

where $D$ is the amount of debt that is undertaken and $\tau_{c}$ is the corporate tax rate. Again, in this example, the debt is $\$ 2 \mathrm{M}$ and the tax rate times the debt is $\$ 0.4(\$ 2 \mathrm{M})=\$ 0.8 \mathrm{M}$ which agrees with our answer above. I caution that this additional formula is correct only in the case of perpetual debt and no personal taxes.

Brealey and Myers present a similar example using Merck's balance sheet. Merck is a firm that has no debt in its balance sheet. The top panel in Table 18-3a (see B\&M, page 502) shows Merck's balance sheet (both the book value and the market value) while panel b shows Merck's balance sheets after it decides to borrow $\$ 1$ billion in perpetual debt. The $\$ 1$ billion in perpetual debt with a corporate tax rate of $35 \%$ is worth $\$ 350 \mathrm{M}$ (again we are assuming
that there are no personal taxes). Thus Merck's total value increases by $\$ 350 \mathrm{M}$ to $\$ 134,506 \mathrm{M}$. Its equity value captures the entire increase and increases to $\$ 125,877 \mathrm{M}$.

In the above approach we ignored the personal tax status of investors. But personal taxes do affect the tax advantage to debt relative to equity because they affect the before-tax yields that investors demand on these instruments. If debt at the personal level is taxed at a higher rate than equity at the personal level, investors demand a higher before-tax yield on debt. This higher before-tax yield on debt will reduce the tax advantage of debt. In what follows, we quantify the effect of personal taxes on the tax advantage (or disadvantage) from levering up the firm.

To emphasize the dependence of the tax advantage to leverage, consider an investor who has a tax rate of $\tau_{p}$ on debt income and $\tau_{e}$ on equity income. These two tax rates need not be the same for a variety of reasons. First, capital gains are often taxed at a different rate than dividend and interest income. This was true before the 1986 Tax Reform Act. A lower capital gains tax is again the law under the 1991 and 1997 tax packages. Thus for such individuals, the marginal tax rate on interest income is higher than that on capital gains income. Also, even with the same rate of taxation on capital gains, dividends and interest income, the effective rate on capital gains would still be lower because the capital gains tax is paid only when gains are realized; that is, capital gains taxation can be delayed, so the "value" of capital gains taxation is the present value of the taxes that will be paid in the future. Further, capital gains can be avoided altogether at death (even for one's heirs). Finally, there are investors like corporations who have specific deductions that make the tax rates on dividends and interest income different. Thus, it is reasonable to assume that there are different tax rates for interest income and equity income.

Now, consider a firm that has a cash flow before tax and interest of $C$. Suppose the firm is unlevered. Then the cash flow after taxes is $C\left(1-\tau_{c}\right)$. If the shareholder has access to borrowing and lending at the tax free rate of $r$, then we have the following table:

## REPLICATING THE UNLEVERED FIRM

|  | $\mathrm{t}=0$ | $\mathrm{t}=1$ | $\mathrm{t}=2$ and so on |
| :---: | :---: | :---: | :---: |
| Operating Cash Flow |  | C | C |
| After Corporate Tax Cash Flow |  | $C\left(1-\tau_{c}\right)$ | $C\left(1-\tau_{c}\right)$ |
| After Corporate And Personal Taxes Cash Flow |  | $C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ | $C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ |
| Replicating investment | $C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right) / r$ | $C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ | $C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ |

Thus the "value" of the unlevered firm is

$$
\frac{C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)}{r}
$$

Consider now a levered firm that has debt of market value $D$ and interest $r_{D}\left(r_{D}\right.$ is not independent of the distribution of personal tax rates and $r$, but we ignore that for now). Then we can replicate the debt and equity cash flows of the levered firm as follows:

## REPLICATING THE LEVERED FIRM

|  | $\mathrm{t}=0$ | $\mathrm{t}=1$ | $\mathrm{t}=2$ and so on |
| :---: | :---: | :---: | :---: |
| Operating Cash <br> Flow |  | C | C |
| Interest |  | $r_{D} D$ | $r_{D} D$ |
| Before Tax, <br> After Interest CF |  | $C-r_{D} D$ | $C-r_{D} D$ |
| $\begin{array}{r} \text { After Tax } \\ \text { Corporate Equity CF } \end{array}$ |  | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)$ | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)$ |
| After Corporate and Personal Taxes Cash Flow |  | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ |
| Replicating Investment (Equity) | $\underset{/ r}{\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)}$ | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ | $\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)$ |
| After Personal Tax Debt CF |  | $r_{D} D\left(1-\tau_{p}\right)$ | $r_{D} D\left(1-\tau_{p}\right)$ |
| Replicating <br> Investment (Debt) | $r_{D} D\left(1-\tau_{p}\right) / \mathrm{r}$ | $r_{D} D\left(1-\tau_{p}\right)$ | $r_{D} D\left(1-\tau_{p}\right)$ |

Thus the "value" of the levered firm is:

$$
\begin{array}{r}
\frac{\left(C-r_{D} D\right)\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)}{r}+\frac{r_{D} D\left(1-\tau_{p}\right)}{r} \\
=\frac{C\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)}{r}+\frac{r_{D} D}{r}\left[\left(1-\tau_{p}\right)-\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)\right]
\end{array}
$$

Since the first term represents the "value" of the unlevered firm, the gain to leverage is:

$$
\frac{r_{D} D}{r}\left[\left(1-\tau_{p}\right)-\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)\right]
$$

We can simplify this further if the individual who has personal tax rates of $\tau_{e}$ and $\tau_{p}$ determines the yield (and coupon rate) on debt. For then

$$
r_{D}=\frac{r}{\left(1-\tau_{p}\right)}
$$

and thus the gains to leverage is: (*)

$$
D\left[1-\frac{\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)}{1-\tau_{p}}\right]
$$

Brealey and Myers (page 505) develop a diagrammatic derivation of this formula and you may want to take a look at this.

Suppose the individual that we are considering is taxed at the same rate of interest and dividend income, i.e., $\tau_{p}=\tau_{e}$. Then the gains to leverage would be just $\tau_{c} D$, the same formula that we got when there were no personal taxes. Thus the traditional MM approach is correct when the tax rate on equity income and debt income is identical.

The gains to leverage formula above (Equation * above) depends on the personal tax rates of the investor under consideration. Since different investors have different tax rates on debt and equity income, on whose personal tax rates should we focus? Those of the marginal investor, a fundamentally important notion.

The marginal investor is the investor who determines the market prices of the securities under consideration.

This statement by itself is not a very illuminating one. In what follows we will explain this concept by using different examples.

Let us return to the discussion above where we computed the gains to leverage. Suppose the equilibrium pretax return on equity is $r_{E}$ and that on debt is $r_{D}$. Then investors will specialize in holding debt and equity instruments as follows:

Group I: Prefer Debt $\left(1-\tau_{p}\right) r_{D}>\left(1-\tau_{e}\right) r_{E}$
Group II: Prefer Equity $\left(1-\tau_{p}\right) r_{D}<\left(1-\tau_{e}\right) r_{E}$
Group III: Indifferent $\left(1-\tau_{p}\right) r_{D}=\left(1-\tau_{e}\right) r_{E}$

The first group obtains a higher after-tax yield from debt and hence will buy debt. The second group obtains a higher after-tax yield from equity and hence will buy equity. The third group obtains the same yield from debt and equity and so is indifferent.

If both debt and equity coexist in the economy, there will be investors in all three groups. We claim that investors in Group III are the "marginal investors" since they hold both debt and equity and thus determine the relative prices of debt and equity.

## MILLER'S EQUILIBRIUM

Miller's equilibrium refers to Merton Miller's (1977) argument that individuals in Groups I (and Group III) will demand debt while individuals in group II (and Group III) will demand equity. Corporations will compete to supply debt to investors in Group I until corporations as a group have exhausted all the gains to leverage, i.e., the capital available with investors in Group I is exhausted. Thus, given the marginal investors (Group III), it should be the case that the gains to further leverage must be zero, i.e.,

$$
\left(1-\tau_{p}\right)=\left(1-\tau_{c}\right)\left(1-\tau_{e}\right)
$$

If we substitute the above in the gains to leverage formula above (Equation *), it turns out that the gains to leverage is zero and the value of the firm is independent to the debt-equity ratio!

This in turn implies that

$$
r_{D}=\frac{r_{E}}{1-\tau_{c}}
$$

Essentially, for Miller's equilibrium to occur, we need at least some investors who are tax disadvantaged toward debt income at the personal level and thus demand a higher pretax yield for debt instruments. Hence, even though there is a corporate tax advantage to debt, the
personal tax disadvantage to debt may outweigh the corporate tax advantage to debt (it does for Group II investors who hold equity but not for Group I investors who hold debt). For the "marginal investor" in Group III who holds both debt and equity the personal tax disadvantage to debt just offsets the corporate tax advantage to debt.

Miller's equilibrium thus implies that the pretax yield on debt is higher than the pretax return on equity. Most of us would complain that this is inconsistent with the real world. We need to remember that in Miller's approach we are comparing riskless debt and riskless equity. The yields that we observe in the real world however are those for riskless debt (or less risky debt) and risky equity and thus are not directly informative on whether debt is taxdisadvantaged relative to equity.

## ON THE REASONABLENESS OF MILLER'S EQUILIBRIUM

To understand how realistic Miller's ideas are, we consider the following three examples from Brealey and Myers. The first example looks at a situation consistent with the tax policy before the 1986 Tax Reform Act when the corporate rate was $46 \%$, the top interest and dividend tax rate was $50 \%$ and the capital gains rate was $20 \%$. In addition, this example assumes that one effectively pay a tax rate of $10 \%$ on realized capital gains and could convert dividends into capital gains. The table on page 506 shows the calculations in this case. In this table, we see that debt is tax disadvantaged at the personal level and the net tax advantage to equity is $0.4 \%$.

The next example considers a more recent period, when the corporate rate was $34 \%$ and interest income was taxed at $39.6 \%$. Again, the assumption is made that capital gains are
effectively taxed at half the statutory rate, or $14 \%$ in this case. The first table on page 507 shows the calculations in this case. Here, the net tax advantage to debt is $4.5 \%$.

The last example looks at a situation where investors have to pay tax on dividends (half their equity income) at a rate of $39.6 \%$, and are taxed effectively at $15 \%$ on the capital gains portion of their equity income. The bottom table on page 507 shows these calculations. The net tax advantage to debt is $12.8 \%$.

As such, Miller's conclusions that there is no tax advantage to leverage because of the offsetting personal tax disadvantage to debt is not correct. On the other hand, the traditional MM position overstates the tax advantage to debt. Thus, a tax advantage to debt does exist, though it is less than what it is commonly believed to be. On Wall Street, many analysts blindly assume that the tax advantage of debt is equal to the top statutory tax rate of $35 \%$. We see here that this may be a faulty assumption.

## THE "MARGINAL INVESTOR" AGAIN

The "marginal investor" is the investor who determines the prices of the securities that firms issue. In a majority of cases, it is clear as to who is the marginal investor. If I own the firm, I am the marginal investor. On other occasions, it is more difficult to assess who the marginal investor is.

In the case of municipal bonds, which are tax exempt, the marginal investor is likely to be someone with a high tax bracket (though perhaps not the highest). For example, a couple years ago there were sharp increases in the prices of municipal bonds. In particular, municipal bond yields fell more than Treasury yields. An article in the Wall Street Journal
interpreted these larger price increases in municipal bonds as evidence that the marginal investor had shifted: suddenly, individuals in the highest tax rate were the marginal investors.

If we are considering preferred stock, it is likely that the marginal investor is a corporation that has short term surplus cash (remember that corporations get a deduction for dividend income reducing their effective rate of taxation on dividend income).

