Modeling Cylinder Flow Vortex Shedding with Enforced Motion Using a Harmonic Balance Approach

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In recent years, new aeromechanical problems have been encountered in turbomachinery. In particular, non-synchronous vibrations (NSV) in blades have been observed by engine companies and occur as a result of flow instabilities. As a first step towards better understanding the NSV in turbine engine configurations, the two-dimensional shedding flow about a circular cylinder is investigated in this study. The governing nonlinear, unsteady Navier-Stokes equations are solved using a novel harmonic balance method. This method requires one to two orders of magnitude less computational time than conventional time-marching computational fluid dynamic (CFD) techniques. In this paper, results are presented for a stationary cylinder in crossflow and a cylinder with enforced motion. A unique phase error method is used to determine the shedding frequency for the stationary cylinder case. The lock-in effect for the prescribed motion case is observed, and preliminary results show that cylinder motion does not significantly affect the unsteady lift for cylinder oscillation amplitudes up to approximately ten percent of the cylinder's diameter.

Nomenclature

\begin{align*}
h & = \text{Cylinder Plunge Coordinate} \\
D & = \text{Cylinder Diameter} \\
Re & = \text{Reynolds Number} \\
St & = \text{Strouhal Number} \\
C_L & = \text{Unsteady Lift Coefficient}
\end{align*}

Introduction

Non-synchronous vibrations (NSV) of turbine engine blades are the result of the interaction of an aerodynamic instability, such as vortex shedding, with blade vibrations. It has been observed by most engine companies and can ultimately lead to blade fracture. The overall goal of NSV research efforts is to develop a fast and systematic method for engine companies to prevent and eliminate NSV. As an initial step towards better understanding the NSV in turbine engine configurations, it is advantageous to investigate the well-known two-dimensional shedding flow about a circular cylinder case that exhibits similar NSV features. Currently, three different cylinder test cases are being investigated: a stationary cylinder, a cylinder with forced oscillation, and a cylinder on an elastic support. The focus of this paper is only on the first two cases. The low Reynolds flow over a cylinder serves as a useful test case because a large amount of experimental and numerical data is available for comparison (see for example Blevins, Williamson, Sarpkaya, Anagnostopoulos, Tanida, Norberg, and McMullen). If a cylinder is placed in a low Reynolds number flow (47 \( < \Re < 180 \)), vortices are shed alternately and two-dimensionally from the top and bottom of the cylinder. For Reynolds numbers greater than 180, the flow aft of the cylinder becomes three-dimensional due to spanwise instabilities. As a result, this preliminary study only considers flow in the Reynolds number range of 47 \( < \Re < 180 \). The purpose of the study is to use a newly developed frequency domain harmonic balance technique for modeling nonlinear periodic unsteady flows, namely cylinder vortex shedding with enforced motion. This method is much more efficient than conventional time-marching algorithms for periodic unsteady flows.

Theoretical Development

This study utilizes the novel harmonic balance (HB) approach developed by Hall et al. (see also Thomas et al.) to study the flow over a single isolated cylinder. In this case, the flow is governed by the Navier-Stokes equations. In the past, most researchers have employed a time-domain approach or a time-linearized frequency domain approach to model the fluid flow in turbomachinery. Yet, each method has its limitations. In particular, the time-domain approach takes considerable computational time, and the time-linearized frequency domain approach cannot model dynamic nonlinearities. However, the HB technique does not have these shortcomings. The HB method

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requires considerably less computational time than traditional time-marching CFD methods, and it can model nonlinear effects brought about by large flow disturbances.

The harmonic balance solution method requires the user to input the frequency of the flow instability. However, the purpose of the study is to find the NSV frequency as one of the unknowns of the solution. One simple method is to choose a wide range of frequencies and compute the solution residual of the harmonic balance equation for each case. However, previous studies by McMullen, et al.\(^7\) indicate the solution residual drops off suddenly and only at the precise NSV frequency. As a result, this method of searching for the NSV frequency can easily miss the correct frequency, and it requires numerous runs of the harmonic balance code.

Therefore, a novel frequency search technique has been developed by Hall et al.\(^13\) For a given frequency, the final HB solution residual will have a constant phase change per iteration. When an incorrect frequency is input, the harmonic balance method will predict that the phase change per iteration of any unsteady first harmonic quantity, such as unsteady lift, stays nearly constant. Once this phase angle is calculated for a couple different frequencies, it becomes readily apparent that the phase is nearly linearly related to the assumed frequency. The exact frequency can be calculated by determining the frequency for which the phase is zero. This can be accomplished by simple interpolation with as few as two HB solutions.

### Results

The frequency search method is demonstrated for the stationary cylinder in cross flow. A relationship between Reynolds number and Strouhal number is determined and compared with existing computational (McMullen\(^7\)) and experimental data (Williamson\(^14\)). When the cylinder is forced to oscillate at a given amplitude and frequency, there is a range of forcing frequencies over which the shedding frequency will "lock-in" to the driving frequency. This synchronization region was measured by Koopman at low Reynolds numbers and is also determined using the HB technique and compared with Koopman’s\(^3\) experimental data. Furthermore, the unsteady lift is calculated for both the stationary cylinder and the cylinder with enforced motion.

### Stationary Cylinder in Cross Flow

As an initial illustration of the merits of the harmonic balance method, the first goal is to reproduce the experimentally determined results for the stationary cylinder case. The problem is modeled using the HB method and a 129x65 mesh for various Reynolds numbers.\(^13\) Using the two frequency search methods described previously, Fig. 1 and Fig. 2 show the HB solution residuals and phase change in unsteady lift per iteration for a number of assumed Strouhal numbers at a Reynolds number of 170 when using two harmonics in the HB method. As can be seen from the plots, the predicted Strouhal number is approximately 0.1865.

The frequency search procedure was repeated for a range of Reynolds numbers in the laminar flow regime, and a corresponding Strouhal number was determined for each Reynolds number. The resulting Strouhal numbers were then compared with experimental data collected by Williamson\(^14\) and McMullen’s\(^7\) numerical results, and all are shown in Fig. 3. The HB method shows excellent agreement with the experimentally determined values.

### Stationary Cylinder Lift

In addition to determining the frequency of the fluid dynamic instability, the amplitude of the lift on the cylinder is calculated. At the shedding frequency for a range of Reynolds numbers, the amplitude of the first harmonic lift acting on the shedding cylinder is determined. Fig. 4 shows the unsteady lift amplitude versus Reynolds number. By extrapolating to the Reynolds number of zero oscillating lift, the onset of the vor-
Cylinder shedding can be determined. This occurs at about a Reynolds number of 47, which is approximately the same as the value determined from other nonlinear dynamic computational techniques.\textsuperscript{15}

**Cylinder with Prescribed Motion**

Next, the cylinder is forced to oscillate at a specified amplitude and frequency. Over a range of frequencies, the flow shedding frequency "locks-in" to the frequency of the vibrating cylinder. Outside of this region, the cylinder flow sheds at a frequency near that determined by the flow Strouhal number (see Fig. 3). Koopman experimentally determined the lock-in region for Reynolds numbers of 100 and 200. In a first attempt to replicate the data, the code was run at a Reynolds number of 150 with a fixed amplitude for many different Strouhal numbers, and the solution residual was noted for each case. The HB solution residual within the lock-in region converges to machine zero. Outside of this region, physically, two distinct frequencies are present, however the current HB method can only handle one frequency, so the HB solution will not converge. Fig. 5 shows this behavior for two different Strouhal numbers, one inside and one outside of the lock-in region.

This procedure was repeated for various amplitudes, and an estimate of the left and right bounds on the lock-in region is determined based on the HB solution behavior. The results of the study show that mesh size has only a small effect, however adding more harmonics improves the results greatly. Further computation is required to establish a complete lock-in boundary for the HB results. A comparison between Koopman’s data and the results obtained so far using the HB method is shown in Fig. 6.
Fig. 7 Magnitude of the Imaginary Part of the Unsteady Lift Coefficient versus Strouhal Number \((Re = 80, \ h/D = 0.14, \) and a 129x65 Mesh with \(R_{\text{outer boundary}} = 20\)).

Unsteady Cylinder Lift for Prescribed Motion

The unsteady lift is also calculated for the prescribed motion case, and as a verification tool, the results are compared with the experimental data of Tanida, et al.\(^5\) for a Reynolds number of 80 and a non-dimensional amplitude of \(h/D = 0.14\). A plot of the imaginary component of the lift coefficient versus Strouhal number is shown in Fig. 7. The HB method shows remarkable agreement with the experimental data. In the plot, LB and RB refer to the left and right bounds of the lock-in region. Outside of the lock-in region, two frequencies are present, so the current HB method cannot be used to find the unsteady lift coefficient. A sensitivity study has been conducted, and there does not appear to be a substantial benefit to keeping more than two harmonics in the HB solution for the parameter range studied here.

Finally, Fig. 8 shows a plot of the amplitude of the unsteady lift coefficient as a function of the frequency within the lock-in region divided by the shedding frequency with no enforced motion. The results demonstrate that when compared with the stationary lift values, the amplitude of lift for oscillation amplitudes up to about \(h/D = 0.10\) is nearly the same. Therefore, the lift coefficient appears to be independent of the prescribed amplitude when the cylinder is driven at an amplitude of approximately ten percent or less of the cylinder’s diameter. Further study of the effects of higher vibratory amplitudes is currently being conducted. This has important implications for the study of non-synchronous vibrations of turbomachinery blades because it may not be necessary to couple the NSV aerodynamic solution with blade motion, which is a much easier computation. The plot also shows that the size of the lock-in region decreases as the Reynolds number is increased. Furthermore, the phase shift within the lock-in region is calculated as well as the equivalent linear aerodynamic damping from the fluid forces acting on the structure. As in a linear vibration problem, these results indicated an abrupt 180 degree phase shift between shedding force and cylinder motion as the cylinder vibration frequency passes through the natural shedding frequency.

Conclusions

To validate a novel harmonic balance approach as an efficient method to solve for NSV frequency and amplitude, an initial study is conducted for the flow over a stationary cylinder and a cylinder with prescribed motion. Good agreement between experiments and the HB method are achieved for both the stationary cylinder and enforced motion cases. A surprising result of the study is that cylinder motion did not significantly affect the amplitude of lift for prescribed amplitudes up to approximately ten percent of the cylinder diameter. Further computation is required to determine at exactly what amplitude the lift becomes greater than that for the stationary case. This is significant because it implies that it may not be necessary to couple the NSV aerodynamic solution with blade motion for small amplitudes and thus would require much less computation than a fully, coupled aeroelastic solution.

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References


