Signatures of the Mediterranean outflow from a North Atlantic climatology

2. Diagnostic velocity fields

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Abstract. Part 1 of this study is a descriptive analysis of the spreading of Mediterranean Water based on high-resolution maps of salinity and density in the eastern basin. In this second part of our study, velocity fields for two representative isopycnal surfaces of the Mediterranean outflow ($\sigma_0 = 29.70$ and $\sigma_0 = 29.90$) are estimated from a diagnostic model that combines climatological hydrographic data from the National Oceanic Data Center with long-term direct measurements of water exchange through the Strait of Gibraltar. The model is constrained by geostrophic dynamics, conservation of mass, no-flux conditions at the continental shelf, and specified flow through the Strait of Gibraltar. Our principal data source is a recently assembled database of the North Atlantic that consists of climatological mean property fields averaged on isopycnal surfaces. The mean fields are based on more than 80 years (1909–1990) of data and have a nominal horizontal resolution of 0.5°. To provide boundary conditions at the Strait of Gibraltar, we use the results of a model developed from data collected during the Gibraltar Experiment in 1985. The estimated velocity fields show Mediterranean Water exiting the Strait of Gibraltar, following the southern Iberian coast, and then entering the Tagus Basin, where it turns anticyclonically to create a reservoir of this water mass. The flow continues northward along the eastern boundary, penetrating into the Rockall Channel. Finally, the model flow fields do not show a significant westward advection of Mediterranean waters into the subtropical gyre.

1. Introduction

The hydrographic properties of the warm and salty Mediterranean Water that spreads into the open North Atlantic have been extensively studied during the past century. Earlier efforts to establish the circulation and governing dynamics of the Mediterranean outflow have generally been limited to the use of local, synoptic hydrographic data [Baringer, 1993; Baringer and Price, 1997; Ochoa and Bray, 1991; Zenk, 1975; Zenk and Armi, 1990]. While these studies, and others discussed in detail in part 1 of this paper, have done much to decipher the local dynamics and pathway of Mediterranean Water, direct evidence for the long-term fate of these outflow waters is still lacking. Two unsettled questions concerning the fate of the Mediterranean Water are whether or not there is a westward advection of this water into the subtropical Atlantic and whether or not this water mass is advected northward to latitudes near Rockall Channel [Iorga and Lozier, this issue; Reid, 1994]. The interest in these pathways is particularly focused on the degree to which Mediterranean Water influences the source waters at the deep water formation sites in the Norwegian-Greenland Seas. To achieve an understanding of this influence, an assessment of the climatological pathways in the eastern North Atlantic is necessary. As a first step toward this goal, Iorga and Lozier [this issue] analyzed the salinity and density fields in the eastern North Atlantic using a recent, high-resolution climatological database of the North Atlantic [Lozier et al., 1995]. They provided a descriptive analysis of the Mediterranean Water pathway derived from its salinity signature. The reader is referred to Iorga and Lozier [this issue] for background material on past synoptic studies of the Mediterranean outflow waters. In this paper a quantitative analysis of the climatological Mediterranean pathway is presented. Specifically, stream function and velocity fields have been computed in the eastern North Atlantic from a diagnostic model that is based on geostrophic dynamics and the conservation of mass and constrained by no-flux boundary conditions and a flow specification at the Strait of Gibraltar.

There have been several past efforts at the estimation of the long-term flow field in the eastern North Atlantic with diagnostic models. A decade ago, Hogg [1987] used an inverse model in conjunction with Levitus climatological data to estimate the Montgomery stream function on two isopycnal surfaces ($\sigma_1 = 31.8$ and $\sigma_1 = 32.3$) in the eastern North Atlantic. He found that at the depth of the Mediterranean outflow the flow is down the temperature tongue (westward) with the advective thermal flux balanced by a lateral diffusive thermal flux. However, the eastern extent of Hogg's model domain was at 23°30'W; thus his model domain did not encompass the source region for the Mediterranean waters. Additionally, the two isopycnal surfaces that Hogg used for his computation are...
centered at the upper and lower edge of the climatological salinity tongue, rather than at the core, which is located between \( \sigma_1 = 32.00 \) and \( \sigma_1 = 32.20 \). Using a simple numerical model based on the salinity conservation equation and a given circulation scheme, Richardson and Mooney [1975] showed that for Peclet numbers corresponding to the North Atlantic (3–30) the tongue-like penetration of salty Mediterranean Water into the subtropical gyre could be explained by diffusive processes. They found that the shape of the tongue (its shift to the south and west and the fact that it becomes narrower away from the Strait of Gibraltar), as well as its westward extent, can be attributed to advective processes. Other studies have modeled the flow field in the North Atlantic but without a particular focus on the Mediterranean outflow and its fate as it flows from the Strait of Gibraltar. For example, Mazé et al. [1997] determined the vertical distribution of zonal transports near the eastern boundary of the North Atlantic. Even though their focus was not on the Mediterranean Water, Mazé et al. [1997] argue that a direct entry of these waters into the ocean interior can only occur at Iberian latitudes through westward Meddy propagation. This conclusion is in agreement with inverse model output from Paillot and Mercier [1996], which shows Mediterranean Water (at 1000 m) turning northward out of the Gulf of Cadiz with no evidence of a direct westward advection of Mediterranean Water across the North Atlantic basin. Paillot and Mercier’s model also gives a southern branch of Mediterranean Water along the African coast, which they argue is not realistic. Supporting Hogg’s finding of a westward flow are Bogden et al.’s [1993] model results, where the time-averaged geostrophic velocity field in the North Atlantic was estimated from observations of density, wind stress, and bottom topography, with a prescribed Ekman pumping at the surface and no normal flow condition at the bottom. Bogden et al. [1993] found a westward flow of \( O \) (1 cm s\(^{-1}\)) below the thermocline in a narrow range of latitudes between 25\( ^\circ \) and 32\( ^\circ \), well to the south of the Mediterranean tongue axis.

Our model differs from past diagnostic models of Mediterranean outflow in that it takes into account the eastern boundaries, it includes the Strait of Gibraltar outflow, and it uses a recent high-resolution climatological database [Lozier et al., 1995]. Additionally, our model solution is approached in steps in order to examine the effect of each model equation and constraint on the final flow field. Our approach is purposely simplistic in that we seek to obtain a climatological flow field that to first order, is geostrophic and conserves mass.

The system of equations in our model leads to an overdetermined system that is solved with a least squares fit. Climatological properties, such as salinity, temperature, and pressure, and derived properties, such as dynamic height and specific volume anomaly, are used as model input along with the specification of the outflow at the Strait of Gibraltar. Model output consists of high-resolution stream function fields on selected isopycnal surfaces for the region offshore of the European and African coasts. In the following sections the climatological database and the Strait of Gibraltar flow specification are discussed (sections 2.1 and 2.2, respectively). In section 2.3 the governing equations are formulated, and an overview of the solution method is presented in section 2.4. Three intermediate solutions and the final model solution are discussed in section 3. Section 4 presents solution errors and also contains a model evaluation by analyzing the degree to which the derived flow fields are in geostrophic balance and the degree to which they conserve mass. In section 5 the divergence of the salinity flux field is examined in an attempt to understand the distribution of salinity in the North Atlantic basin. Finally, a summary is given in section 6.

2. Database and Methods

2.1. Hydrographic Database

The climatological mean hydrographic properties of the North Atlantic estimated by Lozier et al. [1995] are used for the calculations in this study. These means were constructed from 87 years (1904–1990) of hydrographic station data archived at the National Oceanic Data Center (NODC). For the purpose of this study we have selected data within the domain encompassing the eastern North Atlantic basin between 25\( ^\circ \) and 60\( ^\circ \)N and 0\( ^\circ \) and 40\( ^\circ \)W. The reader is referred to part 1 of this study [Iorga and Lozier, this issue] for a discussion of the spatial and temporal distribution of the 26,533 stations covering this domain. In the preparation of the climatological means, hydrographic properties from historical station data were projected onto an isopycnal surface and then spatially averaged and smoothed on that surface, with the smoothing scale set by the local data density. For our study we projected properties onto two isopycnal surfaces, \( \sigma_{0.5} = 29.70 \) (at ~800 m) and \( \sigma_{0.5} = 29.90 \) (at ~1000 m), chosen as representative of the upper and lower cores of the Mediterranean outflow, respectively. Absolute flow fields are estimated for these two surfaces. Table 1 lists the potential density values corresponding to these isopycnal surfaces (and also to another one, \( \sigma_{0.5} = 29.50 \), used in this study) but referred to other reference pressures (0 and 1000 dbar). In the eastern North Atlantic a nominal resolution of 0.5\( ^\circ \) was achieved for the mean property fields on each of these isopycnals. Further details on the quality control and processing of the data are given by Lozier et al. [1995], while a detailed description of the salinity fields and depths associated with these isopycnals is given by Iorga and Lozier [this issue].

<table>
<thead>
<tr>
<th>( \sigma_{0.5} )</th>
<th>( \sigma_0 )</th>
<th>( \sigma_1 )</th>
<th>( P_{\text{Gulf of Cadiz}}/P_{\text{NA}} )</th>
<th>( T_{\text{Gulf of Cadiz}}/T_{\text{NA}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.50</td>
<td>27.27</td>
<td>31.70</td>
<td>535/650</td>
<td>11.4/10.4</td>
</tr>
<tr>
<td>29.70</td>
<td>27.47</td>
<td>31.90</td>
<td>670/835</td>
<td>11.1/9.2</td>
</tr>
<tr>
<td>29.90</td>
<td>27.67</td>
<td>32.10</td>
<td>830/1060</td>
<td>10.9/8.0</td>
</tr>
</tbody>
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The approximate mean temperature and pressure of each isopycnal in the Gulf of Cadiz and in the open North Atlantic (NA) are given.

2.2. Strait of Gibraltar Flow Specification

Along the Strait of Gibraltar there are two prominent sills: the Camarinal Sill at ~5\(^\circ\)45’W, with a depth of 286 m, and the Spartel Sill, at ~6\(^\circ\)6W, slightly deeper at 316 m. The model used for this study is constrained west of Spartel Sill, at 6\(^\circ\)15’W, between 35\(^\circ\)45’ and 36\(^\circ\)15’N, by a specified flow field that simulates the two-layer exchange through the Strait of Gibraltar. The flow is defined so that it conserves an outflow transport of 0.72 Sv and an inflow transport of 0.68 Sv, in accordance with the results of the model developed by Bryden et al. [1994], which is based on the data collected during the Gibraltar Experiment in 1985. The vertical velocity distribution of the flow at this site (Plate 1) is derived from the velocity profile described by Johnson et al. [1994] in such a way that it distributes the total transport uniformly over the width of the
Strait of Gibraltar, which for our model is 0.5° of latitude. On the basis of this vertical velocity profile the zonal velocities are $u_0 = -0.06$ m s$^{-1}$ (westward) for $\sigma_{0.5} = 29.70$ and $u_0 = -0.14$ m s$^{-1}$ (westward) for $\sigma_{0.5} = 29.90$. These velocities serve as model constraints.

2.3. Formulation of the Equations

Potential density coordinates ($\sigma$ coordinates), which are curvilinear and orthogonal, are used in our model formulation. With such a coordinate system the three-dimensional (3-D) gradient of a scalar $A$ and the 3-D divergence of the velocity vector $u = (u, v, w)$, are given as:

$$\nabla A = i \frac{\partial A}{\partial x} + j \frac{\partial A}{\partial y} + k \sigma \frac{\partial A}{\partial \sigma}$$

$$\nabla \cdot u = \sigma \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial \sigma} \right] = \nabla_h \cdot u + \sigma \frac{\partial w}{\partial \sigma},$$

where $\sigma$ is the short-hand notation for $\partial \sigma/\partial z$, $u_h = (u, v)$ and $\nabla_h$ and $\nabla_v$ are the 2-D lateral gradient and divergence operators defined in $\sigma$ coordinates as

$$\nabla_h = \sigma \left[ i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} \right],$$

$$\nabla_v = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}.$$

2.3.1. Dynamic equation. On the basis of the assumption that the ocean is in hydrostatic equilibrium and in geostrophic balance, conservation of momentum in Cartesian coordinates yields

$$\frac{\partial P}{\partial z} = -\rho \mathcal{Q}$$

$$f u = \frac{1}{\rho} \frac{\partial P}{\partial x}$$

$$f = -\frac{1}{\rho} \frac{\partial P}{\partial y},$$

where $\rho$ is the density, $\mathcal{Q}$ is the acceleration due to gravity, $f$ is the Coriolis parameter, and $P$ is the pressure. To describe the flow field expressed in (3)-(5), a modification of the Montgomery stream function termed pressure anomaly stream function, $\Psi$ [Zhang and Hogg, 1992], is defined for the laterally nondivergent quantity $u_h$;

$$\hat{k} \times u_h = -\nabla_v \Psi,$$

where $\Psi$ is defined as

$$\Psi = P' \delta - \int_{P_0}^{P} dP,$$

where $\delta$ is the specific volume anomaly and $P'$ is given by $P - P_0$, where $P$ is the mean pressure on the isopycnal surface. Pressure anomaly $P'$ is used instead of pressure $P$ in the definition of the pressure anomaly stream function in order to minimize the error term ($P' \nabla \cdot u_h$) introduced by the definition of a stream function on an isopycnal [Hogg, 1987; Zhang and Hogg, 1992]. An evaluation of this error term for our study surfaces shows it to be 2 orders of magnitude smaller than the gradient of the pressure anomaly stream function, except for the eastern Gulf of Cadiz (east of 8°W), where it is 1 order of magnitude smaller than the gradient of the stream function. The explicit representation is given by

$$\Psi(x, y) - \Psi_{k+1}(x, y) = \int_{P_0}^{p_{k+1}} \delta dP + \frac{P_{k+1}}{\delta} \delta_{k+1} = \int_{P_0}^{p_k} \delta dP + P_k \delta_{k+1} - P_k \delta_{k+1}.$$
2.3.2. Mass conservation equation. Because seawater is approximately incompressible, conservation of mass can be expressed through the continuity equation (in \( \sigma \) coordinates) as

\[
\nabla \cdot \mathbf{u}_h + \sigma \frac{\partial w}{\partial \sigma} = 0,
\]

(9)

where \( \mathbf{u}_h = (u, v) \) represents the 2-D horizontal velocity for an incompressible fluid on an isopycnal and \( w \) is the cross-isopycnal velocity. In general, the second term in (9) is several orders of magnitude smaller than the first term; therefore we neglect it in our model for purposes of simplification. In (9), \( \mathbf{u} \) is replaced by the stream function (equation (6)), thus yielding the formulation of mass conservation used in our model:

\[
\nabla \cdot \left[ \frac{1}{f} \mathbf{k} \times \nabla \psi \right] = 0
\]

(10)

2.3.3. No-flux boundary conditions and the Strait of Gibraltar flow. The no-flux boundary condition that constrains the system is simply a statement that there should be no
difference in the stream function between adjacent horizontal boundary points, namely:

$$\Psi_k(x_1, y_1) - \Psi_k(x_2, y_2) = 0,$$  \hspace{1cm} (11)

where \((x_1, y_1)\) and \((x_2, y_2)\) are adjacent boundary points. Boundary points are taken as the first onshore point of intersection between the grid and the land. Such a condition is implemented for all land boundaries, while open ocean boundaries are not constrained. The one exception to the latter constraint is that the two points defining the Strait of Gibraltar are constrained using a finite difference form of (6):

$$\Psi_k(x_1, y_1) - \Psi_k(x_2, y_2) = -f \cdot \Delta y \cdot U_0,$$ \hspace{1cm} (12)

where \(\Delta y\) is the grid increment and \(U_0\) is the zonal velocity at the Strait of Gibraltar, as described in section 2.2.

2.4. Solution Method

The dynamic equation (8) the continuity equation (10) the no-normal flow conditions (11), and the Strait of Gibraltar flow specification (12) constitute the model equations. The latter two are used as equality constraints for the model since we choose to have these conditions met exactly. A staggered grid is used for the centered, finite difference form of the governing equations. Water properties \((P, T, S, \text{and derived properties})\) are placed at the corners of the 0.5° square, while the unknown stream functions are evaluated in the middle of each square defined by the four adjoining property points. The set of equations are then cast into matrix form:

$$A\mathbf{p} = \mathbf{d} + \mathbf{e},$$ \hspace{1cm} (13)

where the vector \(\mathbf{p}\) contains the unknowns and the matrix \(A\) holds the terms multiplying these unknowns. The data vector \(\mathbf{d}\) contains the inhomogeneous terms from the dynamic equation and from the boundary condition at the Strait of Gibraltar, while \(\mathbf{e}\) is the residual vector that is to be reduced to a minimum in a least squares sense. The resulting system is fully overdetermined. An optimal solution for 5600 unknowns is obtained by minimizing the residuals for over 16,850 equations and equality constraints.

Since different equations have different magnitudes for the individual terms and residuals, equation scaling is necessary for the system of equations represented by (13). Scaling assures that each equation will have equal consideration in the model solution and that the scaled residual field will have the same variance for all equations. For an overdetermined system, only row scaling is effective [Zhang, 1991]. Theoretically, if equation errors are correlated, the scaling factor should be set by error variance. Practically, without any prior knowledge of correlation, it is usually assumed that the equation errors are not correlated. In this case the scaling factors are chosen such that the residuals for all equations have the same noise level [Zhang, 1991].

The equations are additionally weighted to allow for model solutions that give priority to one or more equations or constraints. With the weighting of the dynamic and mass conservation equations at unity the boundary conditions were assigned a weighting factor of 10² to satisfy the no-flux constraints at an accuracy of 10⁻⁶. The specified velocity at the Strait of Gibraltar required a weighting factor of 10⁴ to assure the same noise level. Additionally, preliminary model runs showed the necessity of downweighting by a factor of 10 the dynamic equation east of 7°30'W because it is here that the geostrophic assumption is the weakest [Ochoa and Bray, 1991].

A solver for a linearly quadratic system of equations [Paige and Saunders, 1982] was used to minimize the residuals in (13) while preserving the equality constraints set by the boundary conditions. This solver is particularly suited for large, sparse matrices, such as matrix \(A\) in (13). Model output consists of the stream function at each grid point for \(\sigma_{0.5} = 29.70\) and for \(\sigma_{0.5} = 29.90\), corresponding to the upper and lower core of the Mediterranean outflow, respectively.

3. Diagnostic Model Results

Our solution method described in section 2 is approached in four consecutive steps in order to understand how each equation/constraint affects the estimated flow field. In this section we present three intermediate solutions to the flow field before our final flow estimate. The first step is simply a geostrophic calculation, with only the dynamic equation and a single level of no motion used to compute the stream function/velocity fields for each selected isopycnal surface. The system of equations is even-determined for this case. For our second step the conservation of mass is added and no-flux conditions are imposed as constraints at the boundaries that define the continental shelf. The passageway at the Strait of Gibraltar is left unconstrained. With the addition of mass conservation and the boundary conditions the system of equations becomes overdetermined. The third step is similar to the previous one, with the addition that the outflow velocity at the Strait of Gibraltar is specified, as described in section 2.2 and 2.3.3. For each of these three intermediate steps, two solutions are computed. One uses a shallow level of no (slow) motion, while the other uses a deep level of no motion. For the three intermediate steps, results from only one surface \((\sigma_{0.5} = 29.90)\) are shown for the sake of brevity. Differences between these surfaces for these intermediate steps are unimportant to this study. The final solution, which uses all constraints and the weighted shallow/deep level of no motions, is presented for both surfaces. A detailed description of these four steps is given in the following sections.

3.1. Purely Geostrophic Solution

The stream function for the isopycnal surface \(\sigma_{0.5} = 29.90\), computed under the assumption of a shallow level of no (slow) motion \((\sigma_{0.5} = 29.50)\), is shown in Figure 1a. The appropriateness of this reference level can be assessed by the model's ability to reproduce known features of the ocean circulation within this spatial domain. For example, the North Atlantic Current, which extends to the depth of the Mediterranean outflow [Lozier et al., 1995], is known to flow east-northeastward after branching from the Gulf Stream [e.g., Sy, 1988; Krauss, 1986], yet this model configuration shows a southwestward flow in the northwestern corner of our model domain. This reversal clearly is the result of an inappropriate assumption (for this particular area) of \(\sigma_{0.5} = 29.50\) as the level of no (slow) motion. In reality the North Atlantic Current is expected to be stronger on the isopycnal surface \(\sigma_{0.5} = 29.50\) than on this deeper isopycnal. However, we do expect the geostrophic flow to be in general agreement with past observations where a shallow level of no motion is appropriate. For example, a poleward boundary current along the western African coast, as noted by Barton [1989], is evident in Figure 1a. With such a reference level choice it is also observed that the
Mediterranean outflow takes a wide turn around Cape St. Vincent and then traces an anticyclonic pathway within the Tagus Basin. North of the Estremadura Promontory, this flow evidently converges into a weak poleward current that feeds into the Bay of Biscay. Similarly, a deep level of no (slow) motion $\sigma_2 = 36.97$ is used to compute the geostrophic velocity field on $\sigma_s = 29.90$ (Figure 1b). It is evident that the change from a shallow to a deep level of no (slow) motion induces substantial differences in the computed velocity fields, illustrating the importance of the appropriate selection of a level of no (slow) motion. Notably, the North Atlantic Current now flows east-northeastward as expected with this formulation and as observed [Sy, 1988; Krauss, 1986]. Interestingly, the flow around the Mediterranean tongue remains anticyclonic, and the poleward eastern boundary current off Africa is also evident, though now strengthened. The fact that the $\sigma_{0.5} = 29.70$ and $\sigma_{0.5} = 29.90$ surfaces have poleward flow off the African coast and off the Iberian Peninsula, regardless of whether $\sigma_{0.5} = 29.50$ or $\sigma_2 = 36.97$ is used as a reference level indicates that the velocity field at the two levels of no (slow) motion are similar. To assess the representativeness of this purely geostrophic solution, we have determined the extent to which the flow field is horizontally nondivergent. This estimate is given by a residual, $\gamma(x, y)$, defined as

$$\gamma(x, y) = (\nabla \cdot \mathbf{u}) = \sigma_s \left[ \frac{\partial u}{\sigma_s} + \frac{\partial v}{\sigma_s} \right]$$

(14)

The residual fields for both isopycnal surfaces are shown in Figure 2. Though there is an area, in the middle portion of the domain, where the residuals are virtually zero, the southern and northern portions of the domain are filled with nonzero residuals. To judge the importance of these residuals, we have compared their magnitude to the magnitude of the individual terms on the right-hand side of (14). These terms are of the order of $0.04 \times 10^{-6}$ s$^{-1}$ for $\sigma_{0.5} = 29.50$ and $0.06 \times 10^{-6}$ s$^{-1}$ for $\sigma_2 = 36.97$. As seen in Figure 2, in many areas of the model domain the total divergence is only 10–20% of these values, but in some areas the residuals approach 50% of these values. (The range of values in Figure 2a is from $-0.02$ to $-0.016 \times 10^{-6}$ s$^{-1}$, and the range of values in Figure 2b is from $-0.015$ to $0.025 \times 10^{-6}$ s$^{-1}$.) This horizontal nondivergence serves as the motivation to add mass conservation as a model constraint.

A second test of the model representativeness is a determination of the flow field at the Strait of Gibraltar. Model results from this configuration, where the throughflow at the strait was not defined a priori, yield an inflow of 0.111 m s$^{-1}$ when the shallow reference level is used and an inflow of 0.038 m s$^{-1}$ when the deep reference level is used. These numbers compare to the estimated outflow of $-0.014$ m s$^{-1}$. (Refer to section 2.2 for the discussion of this estimate.) Such discrepancy clearly points to the need to have the flow at the strait constrained. Additionally, it is apparent from the flow fields in Figures 1a and 1b that streamlines intersect the coast, particularly in the southern portion of the domain. This model feature, linked to the issue of mass conservation, will be amended in the model configuration discussed in the next section.

### 3.2. Intermediate Solution I

In this section, mass conservation and no-flux boundary conditions are added to the model. The Strait of Gibraltar remains
unconstrained; no velocity is imposed at this site. As in the previous case, the velocity field for $\sigma_{0.5} = 29.90$ is computed for each isopycnal surface using the shallow (Figure 3a) and deep (Figure 3b) level of no (slow) motion. Overall, the effect of mass conservation and the imposed boundary conditions is to produce fields that are less noisy and with flow that now generally runs parallel to the coastlines. Otherwise, the overall circulation schemes are generally unchanged, with a few ex-

Figure 2. The residuals (multiplied by $10^6$) of the mass conservation statement written for the purely geostrophic solution for the isopycnal surface $\sigma_{0.5} = 29.90$ referenced to (a) $\sigma_{0.5} = 29.50$ and (b) $\sigma_1 = 36.97$. The contour interval is $0.001 \text{ s}^{-1}$. Dashed contours are used for negative values.

Figure 3. Same as for Figure 1, but for intermediate solution I. (a) and (b) The contour interval is $0.2 \text{ m}^2 \text{ s}^{-2}$. 
A continuous poleward current carrying Mediterranean Water along the eastern continental shelf to the Porcupine Bank is now evident in Figure 3a. Such a flow has been previously observed and described by Hill and Mitchelson-Jacob [1993], Reid [1979, 1994], and M. S. McCartney and C. Mauritzen (On the origin of warm water inflow to the Nordic Seas, submitted to Deep-Sea Research, 1999, hereinafter referred to as McCartney and Mauritzen, submitted manuscript, 1999). A cyclonic recirculation is now evident in the Gulf of Cadiz (shown by the dashed contour in Figure 3a) when a shallow level of no (slow) motion is used. This recirculation is in accordance with the salinity signals of the climatological fields [Iorga and Lozier, this issue]. Finally, in Figure 3b the estimated stream function field now shows a strong poleward current off Africa flowing into the Strait of Gibraltar and a weak equatorward current west of the Iberian continental slope, indicating that a deep level of no (slow) motion is inadequate for the eastern region of our domain.

Finally, we note that this model solution, in contrast to the purely geostrophic solution, produces an outflow (−0.033 m s⁻¹) at the Strait of Gibraltar when the shallow reference level is used. When the deep reference level is employed, an inflow, 0.429 m s⁻¹, results. Again, these values are to be contrasted with the observed estimate of −0.14 m s⁻¹. Though the addition of mass conservation and no-flux boundary conditions has created a more realistic flow condition at the strait when the shallow reference level is used, the magnitude of the modeled outflow does not closely match the magnitude estimated from observations. In the next section we explore the changes in the model solution when the flow at the strait is used as a model constraint.

3.3. Intermediate Solution II

Using the same conditions as in the previous section, yet now constraining the specified velocity at the Strait of Gibraltar (section 2.2), a new set of solutions is computed for 0 = 29.90 using a shallow (Figure 4a) and a deep (Figure 4b) level of no motion. The solutions in Figures 4a and 4b exhibit two notable differences from intermediate solution I (Figures 3a and 3b). First, the specification of the outflow at the Strait of Gibraltar, in conjunction with the mass conservation equation, produces a poleward current west of the Iberian Peninsula that is stronger in Figure 4a than in Figure 3a. This can be attributed to the fact that the constrained velocity at the Strait of Gibraltar is greater than the computed velocity from intermediate solution I. With the deeper level of no (slow) motion the addition of a constrained outflow reestablishes a poleward current off the Iberian coast (Figure 4b). Second, the poleward current off Africa is dramatically reduced in magnitude from intermediate solution I (Figure 3b) to intermediate solution II (Figure 4b). These differences are attributed to the fact that the previous model solution, produced by geostrophy in combination with mass conservation, produced an inflow for the Strait of Gibraltar.

3.4. Final Solution

The final solution uses all model constraints and equations, as in intermediate solution II, yet additionally uses levels of no (slow) motion specific to distinct regions of the model domain, as described in section 2.3.1. A description of the final solution will be given in this section with a discussion of the model and solution errors presented in section 4.

The resultant flow field for 0 = 29.70 (Figures 5a and 5b) and 0 = 29.90 (Figures 5c and 5d) are represented by both
stream function and velocity fields. On \( \sigma_{0.5} = 29.70 \), Mediterranean Water flows through Zenk's "gateway" [Zenk and Armi, 1990], located east of the Gorringe Bank, and then diverges as it enters into the Tagus Basin. Part of this flow continues westward along the Gorringe Bank and, near 15°W, turns northwestward across the Atlantic [Zenk and Armi, 1990], joining the North Atlantic Current near 45°N. Another fraction of the Mediterranean Water turns anticyclonically in the Tagus
Basin, creating a reservoir of Mediterranean Water, as described by Daniault et al. [1994]. This portion of the Mediterranean outflow continues northward along the continental slope, as previously described. Another feature of note on this shallow surface is the recirculation in the Gulf of Cadiz. Part of the Mediterranean Water flowing westward along Gorringe Bank turns southward, feeding a local recirculation centered at -10°30'W and 35°N, observed by Daniault et al. [1994] and described by Iorga and Lozier [this issue] from an investigation of the climatological property fields. Another feature previously described [Daniault et al., 1994] is the westward diversion of this poleward current, around the Galicia Bank. Our results reproduce the skirting of the flow as well as its eastward turn into the Bay of Biscay. Our model results show a cyclonic recirculation for the Bay of Biscay. Finally, on this surface the northward penetration of the North Atlantic Current into the Nordic Seas and the eastward current along 50°N are reproduced by our model, in agreement with other model results [Bogden et al., 1993; Martel and Wunsch, 1992].

As on the shallower surface, the isopycnal surface \( \sigma_{0.5} = 29.90 \) (Figures 5c and 5d) also exhibits a poleward current that flows along the continental slope, penetrating northward from the Strait of Gibraltar to Porcupine Bank. In the Gulf of Cadiz and along the Iberian coast the flow is similar to that for the upper surface, displaying a westward deflection around the Gorringe Bank and an anticyclonic turning west of the Iberian Peninsula. There are four notable differences, however. First, the Mediterranean poleward current is stronger for \( \sigma_{0.5} = 29.90 \) than for \( \sigma_{0.5} = 29.70 \). This is expected since our analysis of the climatological salinity field showed the core to be centered on \( \sigma_{0.5} = 29.90 \) off the Iberian Peninsula. Second, the North Atlantic Current is stronger on the upper surface than on the lower surface, as expected with this surface-intensified current. Third, there is a substantial increase in the strength of the westward flow along 35°N from the upper to the lower layer. This flow appears to derive from the poleward eastern boundary current along the African coast and not from the current carrying Mediterranean Water that emanates from the Gulf of Cadiz. This westward flow feeds the southern limb of an anticyclonic circulation in the subtropical basin believed to be, at this depth (-1000 m), a recirculating branch of the North Atlantic Current [Arhan, 1990]. Finally, no recirculation appears for the \( \sigma_{0.5} = 29.90 \) flow, west of the Gulf of Cadiz. On the basis of the climatological analysis [Iorga and Lozier, this issue] this recirculation is believed to extend to this depth. As will be discussed in section 4, we believe the recirculation does not appear on this surface because of the neglect of vertical divergence in the model.

There is general agreement that a poleward current carrying waters of Mediterranean origin flows along the eastern North Atlantic basin, reaching into the Bay of Biscay and also to Porcupine Bank at ~50°N [Reid, 1979, 1994; Hill and Michelson-Jacob, 1993; Daniault et al., 1994; McCartney and Mauritzen, submitted manuscript, 1999]. However, several studies [Hill and Michelson-Jacob, 1993; McCartney and Mauritzen, submitted manuscript, 1999], using tracer and water mass analyses, have raised doubts about the possibility of this water penetrating past 60°N into the Norwegian-Greenland Seas, as suggested by Reid [1979, 1994] from a study of the distribution of salt, silica, and oxygen. These studies reject the hypothesis of a further poleward penetration of the boundary current, in part, on the basis of the fact that the Mediterranean Water exits at intermediate depth (~1000 m), yet the Wyville-Thompson Ridge lies at 500 m. Hill and Michelson-Jacob [1993] and McCartney and Mauritzen (submitted manuscript, 1999) doubt the possibility of a nearly 500 m rise in the isopycnal that carries Mediterranean Water. Limited data at depth prevent an extension of our model domain past 60°N; however, the climatological pressure fields show a northward shoaling of the isopycnals in the eastern basin. As seen in Figure 6, both surfaces, \( \sigma_{0.5} = 29.70 \) (Figure 6a) and \( \sigma_{0.5} = 29.90 \) (Figure 6b), rise steeply north of 55°N. The upper core of the Mediterranean outflow, found at 800 m depth near the continental shelf at 55°N, rises 450 m over a distance of ~500 km. Thus its depth near 60°N, at the Wyville-Thompson Ridge, is ~350 m less than the sill depth. The lower core \( \sigma_{0.5} = 29.90 \) (Figure 6b) keeps an average depth of ~1000–1050 m along the continental slope to 58°N from which it rises to ~400 m at the Wyville-Thompson Ridge and to 200 m north of this ridge. It is noted here that the standard deviation associated with these climatological pressures is of the order of ~60–80 m at the entrance to the Rockall Channel. Finally, we note that as the Mediterranean Water moves away from the Strait of Gibraltar, the depth of its core rises in isopycnal space. Daniault et al. [1994] noted that the maximum salinity in the Cape St. Vincent region is at 1250 m on the isopycnal surface \( \sigma_1 = 32.29 \) (~\( \sigma_{0.5} = 30.04 \)), while downstream, along the Iberian slope the maximum salinity is at 900 m on the isopycnal surface \( \sigma_1 = 32.10 \) (~\( \sigma_{0.5} = 29.85 \)). Iorga and Lozier [this issue] illustrate the same behavior for the climatological Mediterranean outflow. This isopycnal transition increases the likelihood that Mediterranean water will reach the high-latitude convection sites.

4. Model Evaluation

Errors in our solution stem from both an imperfect fit to our set of governing equations and uncertainty in the climatological properties. We will address both of these error sources in this section, referring to the former contribution as model error and the latter as data error. Error that arises because of imperfect physics in our set of governing equations could be assessed through a model/data comparison, yet a quantitative comparison is thwarted by the lack of the observed velocity field over the domain. As presented in section 3, we have relied on a qualitative "match" to observed features over the model domain to assess the adequacy of the chosen model physics. The next step in the evolution of this model is to add salt, heat, and potential vorticity conservation equations as model constraints. Changes in the model solution and an evaluation of model error will indicate the improvement in model physics made by such an addition.

4.1. Model Error

As mentioned earlier, the solution to this overdetermined system is found by minimizing the residuals in (13). As such, the model error is represented by the sum of the square of the residuals. In order to assess the spatial pattern of the error fields we have chosen to map the residuals themselves. The magnitude of the residuals is compared to the magnitude of the terms in the model equations in order to assess the adequacy of the solution.

Our choice to weight heavily the boundary conditions, namely, the no-normal flow boundary condition and the Strait of Gibraltar velocity specification, leads to a flow field where geostrophy and mass conservation are not strictly met. In this section we evaluate the degree to which the estimated flow...
fields are in geostrophic balance and the degree to which they conserve mass. The weighting applied to the boundary conditions (section 2.3) provided a computed westward zonal velocity at the Strait of Gibraltar of $u_0 = -0.1445 \text{ m s}^{-1}$, which is 103% of the specified constraint. Likewise, the model solution yielded $u_0 = -0.0585 \text{ m s}^{-1}$ on $\sigma_0 = 29.70$, which is 97% of the specified constraint. Lower weighting factors led to a disregard of the constraints such that the simulated flow ran into land or inadequate outflow at the Strait of Gibraltar was produced. Larger weighting factors for the constraints yielded nonconvergent solutions or uniform fields that did not reproduce well-known features such as the North Atlantic Current.

In the case of a purely geostrophic flow, the left-hand side of (8), written using $\sigma_0 = 29.70$ as surface $k$ and $\sigma_0 = 29.90$ as surface $k + 1$, should be balanced by the right-hand side of (8). In reality, because the estimated field is not purely geostrophic, the two sides of (8) will differ by a residual field $e(x, y)$ determined as

$$e(x, y) = \Psi_k(x,y) - \Psi_{k+1}(x,y) - \int_{\sigma_k}^{\sigma_{k+1}} \delta P dP + P_{\sigma_k}^\delta - P_{\sigma_{k+1}}^\delta$$

(15)

The magnitude of $e(x, y)$ relative to $\Psi_k$ and $\Psi_{k+1}$ determines the degree to which the computed fields are geostrophic. A comparison of the residual fields (Figure 7) to the model flow fields (Figure 5) reveals that the model flow fields are, to first order, in geostrophic balance. Nonzero residuals appear along the eastern boundary north of 47$^\circ$N and south of 34$^\circ$N, most noticeably, within the eastern Gulf of Cadiz. The Gulf of Cadiz exception is expected since we downweighted the geostrophic constraint in this locale, as discussed earlier. It is supposed that the residuals along the boundaries result from strong boundary constraints. While regions where geostrophy is not strictly met are identifiable, it is important to note that in these locales the residual is still 1 order of magnitude smaller than the computed stream function fields. In other areas the residuals are 2 or more orders of magnitude less than the stream function fields.

The extent to which the flow field is horizontally nondivergent can be estimated by a residual $\gamma(x, y)$ given earlier by (14). The computed residual fields are shown in Figures 8a and 8b for the isopycnal surfaces $\sigma_0 = 29.70$ and $\sigma_0 = 29.90$, respectively. Of primary importance is that the residual field is 1–2 orders of magnitude smaller than the individual terms on the right-hand side of (14). Thus the estimated flow field is found to be laterally nondivergent, to a first approximation. This contrasts to the residual field computed for the purely geostrophic solution where the residuals were found to be a much larger fraction of the individual terms in the mass conservation statement, (14). Interestingly, as the isopycnals substantially rise north of 50$^\circ$N, the residuals do increase in value, suggesting that vertical divergence becomes relatively important in this locale. Such divergence may be associated with the convection of North Atlantic Central Water, generally north of 50$^\circ$N, which produces Subpolar Mode Water (McCARTNEY and MAURITZEN, submitted manuscript, 1999). On the isopycnal sur-
face $\sigma_{0.5} = 29.90$, vertical divergence is also relatively important along the African coast as fresh North/South Atlantic Central Waters penetrate alongshore. The vertical divergence west of the Estremadura Promontory (shown on both surfaces but more evident on $\sigma_{0.5} = 29.90$) is possibly associated with the anticyclonic turn of the Mediterranean Water at this locale. Of final note is the nonnegligible divergence in the Gulf of Cadiz. Neglect of vertical divergence in this area may explain why the modeled flow field does not show a recirculation in this vicinity on the $\sigma_{0.5} = 29.90$ surface.

4.2. Data Error

To establish the solution error associated with the uncertainty in the climatological properties, we used Monte Carlo simulations. Specifically, we generated a series of normally distributed pseudo-random values for each input variable.
(pressure, specific volume anomaly, and dynamic height) given its mean and standard deviation at each grid point [Lozier et al., 1995]. Fifty random values were needed in order for the mean and standard deviation of the random values to converge to the observed mean and standard deviation. Each of the 50 randomly generated fields, each a valid realization of the climatological field, was then used as input into the diagnostic model so that 50 model solutions resulted. Fifty solutions were sufficient to produce a convergent mean field. The standard deviation of these 50 solution fields serves as a measure of the error that is created from uncertainty in the climatological input data. The standard deviations of the pressure anomaly stream function are shown in Figures 9a and 9c for $\sigma_{0.5} = 29.70$ and $\sigma_{0.5} = 29.90$, respectively. Of note is the random nature of the error fields and also the fact that the values are of the order of $10^{-3}$ times the value of the mean stream function fields (Figure 5). A view of the standard deviations for the velocity fields (Figures 9b and 9d) also shows the small magnitude of these
5. An Analysis of the Salinity Flux Divergence

To begin to understand the partitioning between advective and diffusive processes in the distribution of the Mediterranean waters, velocity fields from section 3.4 (the final solution) are superposed over the mean salinity fields for $\sigma_{0.5} = 29.70$ (Plate 2a) and $\sigma_{0.5} = 29.90$ (Plate 2b). From these maps it is evident that the salinity distribution in the eastern North Atlantic basin is connected to advective pathways. However, the salinity signal south of 35°N and east of 20°W is relatively strong despite the absence of an advective pathway originating from the Strait of Gibraltar. This signal indicates the importance of small-scale diffusive processes and/or Meddies and/or vertical advection in these locales. In this section we further examine the diffusive/advective balance of the Mediterranean outflow by analyzing the climatological salinity conservation statement for a steady flow on an isopycnal surface:

$$\nabla \cdot (u_s S) = \nabla \cdot (K_s \nabla S),$$

where $K_s$ is the eddy diffusion coefficient. Using the definition for the 3-D divergence (equation (1)), (16) becomes

$$\nabla \cdot (u_s S) = \nabla \cdot (K_s \nabla S) - \sigma_{0.5} \frac{\partial w}{\partial \sigma} - \sigma_{0.5} \frac{\partial \sigma}{\partial \sigma}. \quad (17)$$

An examination of the salinity flux itself ($u_s S$) yields maps quite similar to the maps in Plates 2a and 2b, so they are not reproduced here. Instead, we choose to evaluate the horizontal divergence of the salinity flux, which is the left-hand side of (17). The degree to which the flux is divergent depends on the strength of either the diffusive processes or the vertical advective processes (or both) in the maintenance of the salinity field, as expressed in (17). Without knowledge of the eddy diffusion coefficients and without knowledge of the vertical velocities, both of which are difficult to measure in the field and to estimate diagnostically, we cannot easily discern the relative contributions of the three terms on the right-hand side of (17).
Figure 8. Residuals (multiplied by $10^6$) for the mass conservation equation using the final solution on (a) $\sigma_{0.5} = 29.70$ and (b) $\sigma_{0.5} = 29.90$. The contour interval is 0.0005 s$^{-1}$ for both Figures 8a and 8b.

Figure 9. (a) The expected error field (multiplied by $10^3$) for the pressure anomaly stream function of the final solution for $\sigma_{0.5} = 29.70$ (b) The expected error field for the velocity field of the final solution for $\sigma_{0.5} = 29.70$. (c) Same as Figure 9b but for $\sigma_{0.5} = 29.90$. (d) Same as Figure 9b but for $\sigma_{0.5} = 29.90$. For Figures 9a and 9c the contour interval is 0.1 m$^2$s$^{-1}$. 
The horizontal divergence of the salinity flux can be decomposed into

$$\nabla \cdot (u_S) = \nabla \cdot u_h + u_h \cdot \nabla \cdot S,$$

(18)

where the first term on the right-hand side represents the salinity flux divergence due to a horizontally divergent flow field and the second term represents salinity flux divergence due to flow across isohalines. We have examined each of these components as part of our analysis.

The overall pattern of the salinity flux divergence fields (Figures 10a and 10b) generally matches the pattern set by the horizontally divergent mass field (Figures 8a and 8b), indicating that the term $\nabla \cdot u_h$ dominates the term $u_h \cdot \nabla \cdot S$ in the estimate of the salinity flux divergence. This dominance is confirmed by a separate evaluation of these terms; the $\nabla \cdot u_h$ field for both isopycnals is shown in Figure 11, and the $u_h \cdot \nabla \cdot S$ field is shown in Figure 12. Overall, the salinity flux divergence created by flow across isohalines is an order of magnitude smaller than the divergence created by a horizontally divergent flow field. Thus the divergence of the salinity flux is principally the result of a laterally divergent flow field.

The modeled flow is principally along isohalines, as suggested by Figure 12 and also as shown in Plate 2. The exceptions to along-isohaline flow occur in different areas for each surface. For the upper surface the flow is strictly along isohalines off the Iberian Peninsula and into the Bay of Biscay. Cross-isohaline flow on this surface occurs in the western portion of the domain, past 20°W and north of ~37°N. On the deeper surface the northwest portion of the domain exhibits along-isohaline flow, while the area where the Mediterranean tongue generally resides is seen to exhibit cross-isohaline flow. Such differences between the surfaces reflect the differences in the strength and spatial distribution of the diffusive fluxes and vertical advective processes between the two surfaces. Interestingly, the area to the southwest of the Gulf of Cadiz, considered to be a preferred pathway of Meddies, shows no sign on either surface of significant cross-isohaline flow. This may suggest that the Meddies are at least partially advected by the background mean flow.

Finally, the errors associated with the salinity flux divergence were computed using the Monte Carlo methods described earlier. The 50 model solutions were used to create 50 estimates of the salinity flux divergence, from which standard deviation fields were calculated. These standard deviation fields (not shown) for the two isopycnal surfaces of interest ($\sigma_{0.5} = 29.70$ and $\sigma_{0.5} = 29.90$) are 2 orders of magnitude smaller than the mean. Thus we conclude that the uncertainty in the climatological mean has little impact on the estimate of the salinity flux divergence.

6. Summary

High-resolution (0.5°) stream function and velocity fields have been computed for two selected isopycnal surfaces at the depth of the Mediterranean outflow. The model used to estimate these fields is based principally on geostrophic dynamics and the conservation of mass. Additionally, no-normal flow boundary conditions and a flow specification at the Strait of Gibraltar are used as model constraints. The computed velocity fields reproduce Mediterranean Water entering the Gulf of Cadiz through the Strait of Gibraltar. From there it flows northward, confined to the southern Iberian shelf. In the western Gulf of Cadiz, part of the Mediterranean Water is cyclonically recirculated on the upper isopycnal surface $\sigma_{0.5} = 29.70$, while the other part turns northward at Cape St. Vincent and enters the Tagus Basin. In the Tagus Basin, Mediterranean Water diverges, with part of it continuing northward along the Iberian continental slope and part deflected westward up to
Figure 10. The along-isopycnal salinity flux divergence (multiplied by $10^6$) on (a) $\sigma_{0.5} = 29.70$ and (b) $\sigma_{0.5} = 29.90$. For both surfaces the contour interval is 0.01 practical salinity units (psu) s$^{-1}$, and dashed lines represent negative values.

Figure 11. The component of the salinity flux divergence caused by the divergence of the horizontal velocity field $\nabla \cdot \mathbf{u}$ for (a) $\sigma_{0.5} = 29.70$ and (b) $\sigma_{0.5} = 29.90$. For both surfaces the contour interval is 0.0025 psu s$^{-1}$, and dashed lines represent negative values.
From here this westward branch turns anticyclonically within the Tagus Basin, creating a reservoir of Mediterranean Water. Downstream, west-northwest of the Estremadura Promontory, it rejoins the along-slope branch. From this locale, Mediterranean Water continues its northward penetration along the continental slope and slightly diverges westward as it flows around Galicia Bank. North of Galicia Bank, the Mediterranean Water mainly turns eastward into the Bay of Biscay, flowing along the northern Iberian slope. Here it is partially recirculated, while the rest of it keeps moving north-northwestward still confined to the upper slope. On both selected surfaces ($\sigma_{0.5} = 29.70$ and $\sigma_{0.5} = 29.90$), at $-48^\circ$N and $10^\circ$W, a convergence between a North Atlantic Current branch moving eastward and Mediterranean Water flowing northwestward is observed in the modeled fields. Such convergence explains the erosion of the salinity signal observed in the climatological field, north of Bay of Biscay, by Iorga and Lozier [this issue]. West of Goban Spur (at $-49^\circ$N and $11^\circ$W), a poleward current carrying Mediterranean Waters splits into two branches. One branch is deflected westward until it converges with the North Atlantic Current, while the slope branch keeps flowing along the continental shelf into the Rockall Channel. Climatological pressure fields indicate that Mediterranean Water lies on isopycnal surfaces that are not blocked by the Wyville-Thompson Ridge. Reid's [1994] finding of a westward current of Mediterranean Water that crosses the North Atlantic is not confirmed by our model. While our model exhibits a westward current at $-35^\circ$N, it derives its waters from the poleward, eastern boundary current off Africa and not from the Mediterranean outflow emanating from the Strait of Gibraltar.

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References
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Figure 12. The component of the salinity flux divergence created by flow across isohalines, $u_n \cdot \nabla \phi$, for (a) $\sigma_{0.5} = 29.70$ and (b) $\sigma_{0.5} = 29.90$. For both surfaces the contour interval is 0.01 psu s$^{-1}$, and dashed lines represent negative values.


