Instability of a Shelfbreak Front

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ABSTRACT

In an attempt to understand whether local instabilities can account for the observed frontal variability in the Middle Atlantic Bight, a linear stability analysis was conducted for a wide range of background density and velocity fields. Three-dimensional perturbations superposed on a continuously stratified shelfbreak front were investigated using the hydrostatic primitive equations. Model results indicate that the shelfbreak frontal jet is unstable over the wide parameter range dictated by the observed velocity and density structure. Model growth rates, on the order of one day, and wavelengths of \( \sim 10-50 \) km compare favorably to observations, suggesting that local baroclinic/barotropic instabilities are a likely source for the strong temporal and spatial variability of the shelfbreak front in the Middle Atlantic Bight.

1. Introduction

Offshore of the eastern United States a sharp temperature and salinity front at the continental shelf break separates the relatively cool and fresh shelf waters from the warm and salty waters of the North Atlantic. In the Middle Atlantic Bight this shelfbreak front is a highly dynamical feature that is often severely contorted by large amplitude meandering (Beardsley et al. 1985). Aligned with this front is an alongshore current that is geostrophic, to first order (Flagg 1977). The strong temporal and spatial variability of this frontal jet is evident from the meandering paths of 10-m drifters that were released near Georges Bank in 1996 (Fig. 1), as part of the Global Oceans Ecosystems Dynamics (GLOBEC) Northwest Atlantic program (R. Beardsley and R. Limbourn 2000, personal communication). These drifters were entrained into the shelfbreak jet from the onshore side of the front and then generally advected downstream to the southwest, collectively demonstrating the continuity of the frontal jet from Georges Bank to Cape Hatteras. Along their path through the Middle Atlantic Bight, drifters were entrained from, and detrained to, both sides of the current, although offshore exchange was predominant (Lozier and Gawarkiewicz 2001).

From Lozier and Gawarkiewicz’s analysis of 10-m and 40-m drifter data over a three-year period (1995–97), it is apparent that the meandering is restricted neither by season nor by locale along the shelf break. Evidence of such ubiquitous meandering and frontal exchange renews speculation that a possible source for the observed variability in the Middle Atlantic Bight is local frontal instability. Such a line of study was first pursued over twenty years ago by Flagg and Beardsley (1978), who used a two-layer front in geostrophic balance to study the stability of the front south of New England. For realistic topography beneath the front, Flagg and Beardsley found that the most unstable modes had characteristic \( e \)-folding times greater than 50 days. Thus, they concluded that some other mechanism must be responsible for the observed wavelike features on the New England shelf/slope. In a later study, Gawarkiewicz (1991) also employed a layered model to simulate the wintertime and summertime fronts south of New England. This study, which differed from the Flagg and Beardsley (1978) study in that it used unbounded bottom topography offshore, produced much shorter characteristic growth scales (on the order of 4–10 days), particularly when a subsurface front (which is characteristic of the summer frontal configuration in the Middle At-
Atlantic Bight) was used as the background density field. While these times are short enough so that an initial perturbation could grow rapidly enough to be of consequence, observations suggest that growth is more rapid, with timescales on the order of 1–2 days (Garvine et al. 1988). Gawarkiewicz’s study (1991) illustrated the dependence of shelfbreak frontal instabilities on the background stratification. Such dependence suggests that layered models, though analytically tractable, possibly limit the range of baroclinic and barotropic instabilities in a frontal region.

In an effort to further explore the full range of frontal instabilities at a shelfbreak front, we employ a spectral method that was developed by Moore and Peltier (1987) to solve for the linear instabilities of a background geostrophic jet that is characterized by continuous vertical and cross-stream stratification. Moore and Peltier used the method to study the stability of two-dimensional atmospheric fronts perturbed by disturbances that are governed by the linearized primitive equations. They found that primitive equation dynamics captured modes that were either absent or distorted by approximated equations (i.e., semigeostrophic and quasigeostrophic momentum equations). The method and the model dy-
The primary goal of this work is to use a linearized primitive equation stability model to ascertain whether front instabilities can account for the observed temporal and spatial variability within the Middle Atlantic Bight. Primitive equation dynamics are particularly important in the region of the Middle Atlantic Bight where large Rossby number flows and outcropping isopycnals render traditional quasigeostrophic stability theory inappropriate. Recent field work in the Middle Atlantic has shown that significant changes in frontal jet strength and structure can occur within a matter of days (Frantantoni et al. 2001). The magnitude of these changes in the horizontal and vertical velocity shear over a few days can be greater than the seasonal change that has been determined from climatological data (Linder and Gawarkiewicz 1998). Thus, a stability study contrasting a climatological winter and climatological summer jet would not illuminate the full range of instabilities present in the Middle Atlantic Bight since it would not cover the full range of background states. Our intent is to assess the stability characteristics for an envelope of probable jet structures. Thus, we have chosen to conduct a thorough parameter study of a shelfbreak jet, with the bounds of the parameter study set by the range of environmental conditions within the Middle Atlantic Bight, as assessed by a recent field program (Gawarkiewicz 1998). Thus, a stability study contrasting a climatological winter and climatological summer jet would not illuminate the full range of instabilities present in the Middle Atlantic Bight since it would not cover the full range of background states. Our intent is to assess the stability characteristics for an envelope of probable jet structures. Thus, we have chosen to conduct a thorough parameter study of a shelfbreak jet, with the bounds of the parameter study set by the range of environmental conditions within the Middle Atlantic Bight, as assessed by a recent field program (Gawarkiewicz 1998), as well as by Linder and Gawarkiewicz’s climatology. The purpose of our parameter study is to understand the dependence of stability characteristics on the vertical and horizontal shear of the background density and velocity fields. For investigations into the dependence of stability characteristics on a bathymetric slope, the reader is referred to past studies (e.g., Orlanski 1969; Flagg and Beardsley 1978; Xue and Mellor 1993; and references therein).

In sum, the objective of this study is to answer three main questions: 1) What background density and velocity structures will yield an unstable jet at a shelfbreak front? 2) What are the temporal and spatial scales of the dominant unstable modes? 3) Do the predicted temporal and spatial scales approximate those found in the Middle Atlantic Bight? Overall, this study aims to improve our understanding of the mechanisms that control the exchange of heat, freshwater, sediment, and other properties between the coastal ocean and the open ocean. In the next section, the model equations and background fields are presented and the model solution, convergence, and verification are discussed. Section 3 contains the results of the parameter study using an idealized shelfbreak jet. Model results using background jets derived from synoptic and climatological data in the Middle Atlantic Bight are presented in section 4, followed by the summary and conclusions in section 5.

2. Methods

Our computation of the linear instabilities of a background geostrophic jet with continuous stratification is based on the method developed by Moore and Peltier (1987) and modified by Xue and Mellor (1993) to include a topographic slope. In this section we will briefly describe the model equations and solution methods. The reader is referred to Moore and Peltier (1987) and Xue and Mellor (1993) for more details.

(a) Model equations and solution method

The model used in this study consists of a steady background current flowing along the slope of idealized bathymetry (Fig. 2a). The model uses a Cartesian coordinate system, where \( x \) is the offshore axis (positive offshore), \( y \) the alongshore axis (positive upstream), and \( z \) the vertical axis (positive upward). Bathymetry is given by \( h(x) \) and the alongshore background flow is given by \( V(x, z) \). [Functional forms for \( h(x) \) and \( V(x, z) \) are given in section 2b.] The background flow is in thermal wind balance with a mean density field, \( \rho(x, z) \), according to \( f \frac{V_y}{V} = B \), where \( f \) is a constant Coriolis parameter and \( B \) is the mean buoyancy defined by \( B = -g \frac{\rho(x, z)}{\rho_0} \), with \( \rho_0 \) the reference density and \( g \) the gravitational acceleration. In order to assess the stability of this basic state, three-dimensional velocity and density perturbations are superposed onto a two-dimensional background velocity and density field. The evolution of the perturbations is governed by the hydrostatic primitive equations, linearized about a geostrophic background state:

\[
\begin{align*}
\frac{\partial u}{\partial t} + \nu_x + f \omega &= -\pi_x \quad (2.1) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + \nu_y + w \frac{\partial v}{\partial z} + f \omega &= -\pi_y \quad (2.2) \\
0 &= -b + \pi_z \quad (2.3) \\
\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + w \frac{\partial \omega}{\partial z} &= 0 \quad (2.4) \\
b + u \frac{\partial B}{\partial x} + V \omega + w B &= 0. \quad (2.5)
\end{align*}
\]
The Cartesian components of the perturbation velocity are \((u, v, w)\), \(\pi\) is perturbation pressure divided by \(\rho_0\), and \(b\) is the perturbation buoyancy. Equations (2.1)–(2.3) are the equations of motion with no frictional or external forces, while Eqs. (2.4) and (2.5), derived from the conservation of mass, represent the continuity equation for an incompressible fluid and the conservation of density, respectively. Boundary conditions stipulate that normal flow across solid boundaries is zero \([u = 0\text{ at } x = 0\text{ and } w = -uh, \text{ at } z = -h(x)]\) and that disturbances vanish at distances far from the coast. Additionally, the rigid-lid approximation \((w = 0\text{ at } z = 0)\) is imposed. The system is assumed to be inviscid with respect to vertical mixing. Recent observations at the shelfbreak front have suggested that vertical diffusion coefficients are quite small \((\text{Re} \ll 1)\) \cite{Rehmann2000}. With such limited diffusion, an estimate for the spindown time is on the order of 50 days, much larger than either the wave periods or growth rates that, as will be shown, are quite small \((\text{Re} \ll 1)\). With such a mapping it is convenient to define transport variables: \((u_\ast, v_\ast, b_\ast) = (uh, vh, bh)\). Additionally, all variables are cast in dimensionless form using the following scaling:

\[
x' = x/L_o, \quad y' = y/L_o, \quad \zeta' = \frac{\zeta}{h}, \quad t' = t_f, \quad u' = \frac{u}{u_o}, \quad v' = \frac{v}{u_o}, \quad \omega' = \frac{\omega}{(L_o/u_o)\omega}, \quad h' = h/H_o, \quad \pi' = \frac{\pi}{(H_o/L_o)\pi}, \quad b' = \left(\frac{H_o}{L_o}\frac{f_o}{L_o}\right)b_o, \quad B' = \frac{B}{H_o N_o^2}, \quad V' = \frac{V}{V_o},
\]

where \(L_o\) and \(H_o\) are the horizontal and vertical length scales, respectively. \(u_o\) is the typical perturbation velocity times \(H_o, N_o^2\) is the Brunt–Väisälä frequency, and \(V_o\) is the maximum of the background velocity, \(V(x, z)\). For all model runs \(L_o = 100\text{ km}, H_o = 200\text{ m},\) and \(f_o = 9.37 \times 10^{-5}\text{ s}^{-1}\) (corresponding to a latitude of 40°N); \(V_o\) and \(N_o\) are varied for the model runs, as explained in section 2b. Solutions for the three-dimensional perturbations are sought in the form

\[
(u', \omega') = \text{Re}[((u^\ast(x, \zeta), \omega^\ast(x, \zeta))e^{i(\sigma t + b y)})], \quad (2.6)
\]

\[
(v', \pi', b') = \text{Re}[((v^\ast(x, \zeta), \pi^\ast(x, \zeta), b^\ast(x, \zeta))e^{i(\sigma t + b y)})], \quad (2.7)
\]

where \(\sigma\) is the complex frequency \((\sigma = \sigma_r + i\sigma_i), l\) is the alongfront wavenumber, the starred variables are the structure functions in \(x\) and \(\zeta\) and \(\text{Re}[\ ]\) denotes the real part of the expression inside the brackets. Substitution of Eqs. (2.6) and (2.7) into (2.1)–(2.5) yields

\[
(\sigma + R_o V)u^\ast - v^\ast = -h\pi^\ast + (\zeta - 1)h b^\ast \quad (2.8)
\]

\[
(\sigma + R_o V)u^\ast - u^\ast(1 - R_o V_o) - R_o V_t \omega^\ast = -h\pi^\ast \quad (2.9)
\]
\[\begin{align*}
-b^* + \pi^* &= 0 \\
\sigma + R_i V b^* - S B v^* - S B^* \omega^* &= 0,
\end{align*}\]

where $Ro = V^o /fL_o$ is the Rossby number for the background flow, and $S$, the Burger number, is defined as $[N_x H_x /fL_o]^2$. For the numerical calculation of Eqs. (2.8)–(2.12) the background flow and the perturbations are confined to a domain bounded by $x = 0, x = 100$ km, the sea surface ($\zeta = 1$) and sea floor ($\zeta = 0$) (Fig. 2a). The size of the domain was chosen to balance the need for computational affordability with the need for all perturbations to be vanishingly small at the channel walls. Additionally, to provide for no normal flow, $u^*$ is set to 0 at $x = 100$ km. With these boundary conditions, Eqs. (2.8)–(2.12) define the problem with a complex frequency given by $\sigma$ and functional forms given by Eqs. (2.6) and (2.7). This set of equations is transformed into spectral space via a combination of Galerkin and Fourier colocation schemes. To obtain a solution, structure functions, for example, $u^*(x, z)$, are spectrally decomposed into orthogonal sets of trigonometric basis functions in the vertical ($\zeta$) and cross-shelf ($x$) directions, as detailed in Xue and Mellor (1993). For numerical solution these expansions are truncated at a finite number of spectral modes, $M$. These expansions are substituted into (2.8)–(2.12) and manipulated to produce a standard matrix eigenvalue equation, which is solved by standard linear algebra methods for a specified background field, truncation level, and wave-number. Model solutions yield growth rate, $-\sigma$, phase speed, $-\sigma/l_1$, and modal structure of the instabilities at each wavenumber.

b. Background fields

The background velocity field, modeled as a cross-shelf Gaussian wave form that exponentially decays with depth, is expressed as

\[V(x, z) = V_o \exp[-\sigma_{x}(x - x_o)^2 - \sigma_{z}(z - z_o)],\]

where $\sigma_x = 4\ln(V^o/0.1 \text{ m s}^{-1})/x_w^2$ and $\sigma_z = \ln(V^o/0.1 \text{ m s}^{-1})/z_d$ are formulated such that the 10 cm s\(^{-1}\) isolatch will fall at a distance $x_w/2$ from $x_o$ and $z_d$ from $z_o$. For all model runs $x_o = 50$ km, that is, the center of the jet is placed at $x = 50$ km, as explained earlier and shown in Fig. 2a, and $z_o = 0$ m, that is, the maximum jet velocity is at the surface. The width and depth of the 10 cm s\(^{-1}\) isolatch are specified for each model run by the variables $x_w$ and $z_d$, respectively. These variables, along with changes to $V_o$, are adjusted to produce a range of jet structures that are generally consistent with the frontal jet in the Middle Atlantic Bight. Because the basic state is in thermal wind balance, the background density field, $B(x, z)$, is set by the velocity field and the specification of density at the offshore boundary. At the offshore boundary the stratification is assumed to be linear ($B_z$ is constant) for all model runs, with the deepest isopycnal set at $\rho = 1.0275$ g cm\(^{-3}\) (Linder 1996).

To create changes in the background stratification, the uppermost isopycnal, $\rho_{\text{shallow}}$, is varied at the eastern boundary.

The governing equations for the perturbation [Eqs. (2.8)–(2.12)] depend on the horizontal and vertical shear of the background density and velocity fields, as well as on the strength of the velocity field. Since the horizontal shear of the buoyancy field ($B_y$) is linearly related to the vertical shear of the velocity field ($V_z$) via the thermal wind relationship, there are four background conditions to vary for the parameter study ($V_x$, $V_y$, $V_z$, and $B_z$). In order to produce a manageable number of jet variations, we have chosen to approach this parameter study by varying separately the jet width (which for a constant $V_o$ will principally vary $V_x$, but also $B_z$), the jet depth (which for a constant $V_o$ will principally vary $V_z$, but also $B_z$), and the background stratification ($B_z$), each for two basic states, one with $V_o = 30$ cm s\(^{-1}\) and the other with $V_o = 60$ cm s\(^{-1}\). Each basic state has a jet width of 20 km, a jet depth of 70 m (both measured by the location of the 10 cm s\(^{-1}\) isolatch), and $\rho_{\text{shallow}} = 1.022$ g cm\(^{-3}\). For each basic state, the jet width is varied from 15 to 40 km, while holding the jet depth. The offshore boundary stratification constant, the jet depth is varied from 30 to 120 m, while holding the jet width and the offshore boundary stratification constant, and $\rho_{\text{shallow}}$ is varied from 1.020 to 1.026 g cm\(^{-3}\), while holding the jet width and depth constant. This parameter range, dictated by synoptic and climatological data in the Middle Atlantic Bight, results in 34 test cases, which will be presented in section 3. The envelope of jet widths and depths used in this study is shown in Fig. 2b. It is important to note that for our present study we have chosen to keep the frontal jet’s centerline position fixed, to maintain constant stratification at the offshore boundary and to maintain symmetrical horizontal velocity shear. Each of these constraints is made for the purpose of simplification and is expected to be varied in our ongoing study of this frontal system.

Finally, the bathymetry used in our model study was determined from the fit of a hyperbolic tangent functional form (Xue and Mellor 1993) to the bathymetric measurements in the Nantucket Shoals region (Linder 1996). This fit is given by

\[h(x) = H_s + 0.5(H_d - H_s)(1 + \tanh[(x - x_m)/\alpha]),\]

where $H_s$ (shelf depth) = 60 m, $H_d$ (maximum domain depth) = 200 m, $x_m$ (location of maximum slope) = 50 km, and $\alpha$ (lateral extent of slope) = 15 km. While the offshore asymptotic depth in the Middle Atlantic Bight is on the order of 2000 m, we have used 200 m in our model study in order to reduce the computational grid. A sensitivity study to offshore depth found negligible differences between model runs using 200 m as the offshore depth and those using depths in excess of 200 m.
c. Model convergence and computation

Past studies using this spectral model have found that $M = 28$ was necessary for model convergence (Xue and Mellor 1993; Barth 1994; Samelson 1993). However, these studies were restricted to relatively small wavenumbers. Our choice of the wavenumber domain to explore was dictated by the observed spatial variability in the Middle Atlantic Bight, which is on the order of 10–50 km. In order to ascertain the number of spectral modes required to adequately resolve the physical modes at these wavelengths, we conducted a convergence study for a representative baroclinic jet. We ran this convergence test from $l_1 = 1$ to $l_2 = 100$, corresponding to wavelengths ($\lambda$) from $\sim 600$ to 6 km ($\lambda = 2\pi L_0/l$), using $M = 20$ to $M = 60$, in increments of 4. Results from this convergence test are shown for selected wavenumbers in Fig. 3. As expected, convergence for the low wavenumber modes was achieved for a relatively small $M$, while convergence at the high wavenumber end required considerably more modes. In order to reach convergence for wavenumbers up to 60 (\lambda \sim 10$ km), it was necessary to use 44 spectral modes. Beyond this wavenumber, convergence was approached with $M = 60$, but not achieved. Due to the high computational costs associated with high spectral mode computations, we decided to use $M = 44$ for all computations and limit our analysis to the wavenumber range from 1 to 60. With this cutoff we exclude perturbations with wavelengths less than 10 km. Since our emphasis is principally on spatial scales on the order of 10–50 km this restriction does not significantly hamper our study.

The computational cost of running this model is significant, mainly because the size of the eigenvalue matrix becomes quite large for the high truncation level necessary to investigate the stability of large wavenumbers. The choice of $M = 44$ results in significantly longer run times than those of previous studies since the run-time scales as approximately $M^6$ (Moore and Peltier 1987). To efficiently investigate the instabilities at large wavenumbers the performance of the model was improved by implementing a parallel version of the model code, which uses the Message Passing Interface (MPI) library to perform communication between processors. The resultant code scales well with processor number and is highly portable, running on either shared or distributed memory architectures. Finally, verification tests reported in Xue and Mellor (1993) were repeated using the topography and stratification specific to our study. In all cases, modes that arise due to computational errors have growth rates several orders of magnitude smaller than those associated with the physical modes of the system. Therefore, errors in the computational scheme do not significantly affect our model results.

3. Parameter study of an idealized jet

In this section the growth rate and phase speed of the most unstable mode will be presented as a function of wavenumber for each of the 34 background configurations. Representative modal structures will also be shown. As mentioned earlier, each background configuration is determined by a jet width, jet depth, $V_0$, and $\rho_{\text{shallow}}$. We have chosen to characterize the resultant two-
dimensional $V$, $V_x$, $V_z$, and $B_z$ fields by two nondimensional numbers, namely, a characteristic Rossby number, $\text{Ro} = \left| \frac{V}{f L_0} \right|_{\text{max}}$ and Burger number, $\text{S} = \langle \frac{F^2 H^2}{(f L_0)^2} \rangle_{\text{max}}$, as well as by $\left| \frac{V_z}{V_{x}} \right|_{\text{max}}$ and $V_{x}$. The brackets are used to indicate depth averaging. (Note that we now use a definition for Ro that distinguishes between jets with differing horizontal shears.) These parameters provide a meaningful, if not complete, description of the background field.

In addition to a classification of the basic state, each resultant perturbation can be characterized by the degree to which it obtains energy for its growth. Basically, modes can derive energy from the available potential energy created by the sloping isopycnals, the horizontal shear of the mean flow (a Rayleigh instability), and/or the vertical shear across the isopycnals (a Kelvin–Helmholtz instability). The former is a baroclinic instability, while the latter two are barotropic instabilities. In order to classify the perturbations, it is customary to analyze the kinetic and potential energy budgets for both the mean and perturbation flows. Previous studies (Barth 1994; Xue and Mellor 1993) using this model have conducted such energy analyses to quantify the energy source for the unstable perturbations. Though our focus is not specifically on the energy sources for these perturbations, we do note that, based on these past studies and on the parameter range of our study, we expect that the unstable perturbations in this study will derive their energy from both the baroclinic and barotropic fields. A measure of the importance of available potential energy as the energy source relative to kinetic energy is given by the ratio of the internal Rossby radius of deformation, $r_i$ (where $r_i = \frac{N_s D}{f V_{x}}$, with $D$ the vertical scale of the background flow defined as $V_{x} / \left| V_{z} \right|_{\text{max}}$) to the horizontal length scale of the basic flow field, $L_h$ (where $L_h = \frac{V_{x}}{\left| V_{z} \right|_{\text{max}}}$. If $L_h \gg r_i$, $L_h \ll r_i$, then it is expected that baroclinic (barotropic) instability will predominate. In prior studies the reciprocal of this ratio, $L_h/r_i$, has been referred to as the internal Kelvin number (Krauss 1973). For the 34 cases presented here, the range for $r_i/L_h$ is from 0.43 ($L_h = 22.4$ km, $r_i = 9.6$ km) to 1.62 ($L_h = 11.3$ km, $r_i = 18.3$ km). Thus, it is expected that neither baroclinic nor barotropic instabilities will exclusively govern the perturbations’ growth. Furthermore, it is expected that the barotropic instabilities will result primarily from a Rayleigh-type instability since Kelvin–Helmholtz instabilities are likely to occur when the Richardson number of the flow $\left( \frac{f L_0}{B^2} \right)$ is $< 1$. In all but one of the 34 cases in this study the Richardson number exceeds 1, reflecting the relatively weak vertical shear of these background states.

Finally, as mentioned earlier, the use of primitive equation dynamics allows for the expression of a nongeostrophic (or ageostrophic) mode, which Moore and Peltier (1987) referred to as a cyclogenesis mode. In a study of coastal jets and fronts, Barth (1994) found the oceanic equivalent of this mode, which he termed a frontal mode, to derive its energy primarily from the potential energy of the basic state. Borrowing Barth’s nomenclature, we will refer to this nongeostrophic mode as a frontal instability in our subsequent discussions.

### a. Horizontal shear variations

1) $V_o = 60$ cm s$^{-1}$

In order to test the sensitivity of the stability characteristics to the horizontal shear of the background jet, model runs were made with varying jet widths. The width of a $V_o = 60$ cm s$^{-1}$ jet was varied from 15 to 20 to 30 to 40 km, yielding Rossby numbers ranging from 0.96 to 0.366. The growth rates and phase speeds for each of the four jets are shown in Fig. 4a as a function of wavenumber. Overall, it is evident that each of these jets is unstable over the entire range of the chosen wavenumber space. For each of the four curves, the growth rates generally increase with increasing wavenumber. The fastest growth rates over this wavenumber range are on the order of 1 day for perturbations with wavelengths of approximately 10–20 km. Since the horizontal scale of the background jet ($L_h = 7, 9, 13,$ and 18 km for the 15-, 20-, 30-, and 40-km jets, respectively) is smaller than the Rossby deformation radii ($r_i = 9, 9.7$, 11, and 12 km), it is expected that barotropic instabilities will contribute to the growth of the perturbation energy. In fact, as the Rossby number increases, the growth rate increases over the entire range of wavenumber space (Fig. 4a), confirming the importance of horizontal shear as an energy source for this instability.

The phase speeds in Fig. 4a are bounded from 0 to $V_o$, in accordance with the semicircle theorem (Pedlosky 1979). Also evident from Fig. 4a is the overall increase in the magnitude of the phase speed as wavenumber increases. In other words, the shorter waves tend to travel faster downstream. Such a pattern is consistent with the behavior of baroclinic (Eady) modes where the phase speed of the instability is approximately the velocity of the background jet at the depth of the Rossby height, $H_R$, which is given by $f_n/L_n$ (Gill 1982). Essentially, the steering level of the perturbation [where $V(z) = c$] is located at the Rossby height. As the Rossby height decreases with increasing $l$, the phase speed increases in magnitude since the background speed increases toward $z = 0$. A quantification of this relationship will be given in the following paragraph. Finally, it is noted that the relationship between phase speed and wavenumber is characterized by stepwise changes, which correspond to changes in the modal structure of the perturbation, as discussed next.

The amplitude of the structure functions for $u$, $v$, and $b$ were analyzed for each wavenumber to determine the modal structures for the dominant perturbations. For the jet with a width of 15 km, there were five identifiable modal structures. Each mode was characterized by nearly uniform structure for $u$, $v$, and $b$ and by a nearly
Fig. 4. (a) Growth rate and phase speed as a function of wavenumber for jets with varying widths. Each jet has $V_o = 60$ cm s$^{-1}$. As shown in the gray inset, the four cases principally differ in their Rossby number. (b) As above but for jets with $V_o = 30$ cm s$^{-1}$. 
constant phase speed over a range of wavenumbers. A switch in modes as the wavenumber changed was noted by abrupt changes in the structure of $u$, $v$, and $b$, by a stepwise change in the phase speed for the perturbation and, at times, by changes in the slope of the growth rate curve. Because the five modes for the 15-km jet have fairly similar structures, only three of them are shown in Fig. 5. Although the $u$, $v$, and $b$ structures were analyzed, for the sake of brevity only the $v$ perturbation is reproduced here. The $u$ and $b$ structure functions are generally similar to the $v$ structure function. As evident from an examination of these modal structures in Fig. 5, all modes are approximately centered at 50 km, the center of the background jet. Additionally, all modes are surface-intensified, with the degree of surface trapping increasing as the wavenumber increases.

From all 34 model runs a consistent pattern for the modal structures emerged. At low wavenumbers the modal structure has horizontal scales commensurate with the jet width, and the vertical decay scale of the

Fig. 5. The modal structure for a jet with a width of 15 km, depth of 70 m, $V_0 = 60$ cm s$^{-1}$, and $\rho_{\text{salt}} = 1.022$ gm cm$^{-3}$. The amplitude of the perturbation downstream velocity, $v$, is shown for (a) $l = 11$ ($\sim 57$ km), (b) $l = 23$ ($\sim 27$ km), and (c) $l = 59$ ($\sim 11$ km). The growth rate and phase speed for each of these wavenumbers is shown in Fig. 4a. The lowest contour and the contour interval for this and all subsequent modal structures is 0.05 (in nondimensional units). (d) The cross-shelf flux of buoyancy, $\nu b$, associated with the $l = 11$ case. Shown in (a)–(d) and all subsequent plots of the background jet are the background isopycnals, denoted in sigma units.
perturbation is approximately the same as the depth of the basic flow field. Following this mode, at higher wavenumbers, is a series of modes that are all quite similar in that they are center and surface trapped (i.e., quite narrow and shallow). These characteristics agree with Barth’s (1994) description of the modal structures that resulted from his analysis. Basically, the low wavenumber mode exhibits the behavior of a classical ageostrophic instability, while the high wavenumber modes exhibit the characteristics of the ageostrophic or frontal instability. The dominant wavelength for the geostrophic mode is approximated by \( 2\pi r_p \), while the shorter wavelengths of the frontal modes are much closer in horizontal scale to \( L_a \) (Barth 1994).

Although the high wavenumber modes are nongeostrophic modes, it is interesting that the decreasing vertical scale with increasing wavenumber is consistent with the behavior of a classical baroclinic instability (the Eady mode), where the \( e \)-folding depth of the perturbation is given by the Rossby height, \( H_p \) (Gill 1982), as defined above. Calculation of the Rossby heights for the modes shown in Fig. 5 yields \( H_p = 40, 19, \) and 8 m for the \( l = 11, 23, \) and 59 cases, respectively. From an inspection of the modes in Fig. 5 it is apparent that the decay scale for the perturbation is in close agreement with these estimated Rossby heights, with a sharp decrease in height as the perturbation wavenumber increases. Furthermore, the phase speeds corresponding to these \( l = 11, 23, \) and 59 cases approximately match the magnitude of the background jet velocity at the Rossby heights, consistent with baroclinic modes, as discussed above. As an example, the phase speed for the \( l = 11 \) perturbation (with \( H_p = 40 \) m) is expected to be \( \sim -22 \) cm s\(^{-1} \) [calculated using the expression for \( V(x, z) \) in section 2b with \( x = 0 \) and \( z = -40 \) m]. As seen in Fig. 4a, this estimation is quite close to the computed phase speed of \( \sim -25 \) cm s\(^{-1} \).

As there is no mean cross-shelf flow, it is the perturbations that are responsible for cross-frontal exchange of both buoyancy and momentum. The cross-shelf buoyancy flux and the cross-shelf momentum flux are calculated as \( \langle ub \rangle \) and \( \langle uv \rangle \), respectively, where the angle brackets denote an average over one wavelength. As this analysis is restricted by the linearization of the governing dynamics, the magnitude of the cross-shelf fluxes cannot be assessed. However, the utility of this computation lies in the ability to assess the structure and sign of the fluxes. Not surprisingly, given the surface-trapped modal structures in Figs. 5a–c, the calculated buoyancy flux for the \( l = 11 \) case is also surface-trapped (as shown in Fig. 5d). As with the modal structures, this flux becomes increasingly surface-intensified as the wavelength increases. The sign of the flux is positive, indicating a net offshore downgradient flux of buoyancy. (While the density gradient, \( \partial \rho / \partial x \), is positive, the buoyancy gradient, \( \partial \beta / \partial x \), is negative.) This downgradient flux is consistent with a baroclinic instability where the perturbations extract their energy from the mean buoyancy field. The cross-shelf momentum flux (not shown) has a structure quite similar to that for the buoyancy flux (Fig. 5d). However, this flux does not show exclusively downgradient structure relative to the background velocity field, \( V \), suggesting a lesser role for barotropic instability for this mode. A counter case, where it appears that the barotropic instability dominates, will be discussed in section 3a(2). As mentioned earlier, an energetics analysis could ascertain the relative importance of these instabilities. The concentration of these model fluxes over a limited depth range has implications for the measurement of shelfbreak fluxes. Garvine et al. (1989) found that heat and salt fluxes from summer measurements were concentrated within a depth range of 20 to 50 m, immediately beneath the seasonal pycnocline. The model similarly suggests that buoyancy fluxes may be limited in vertical extent, although the model basic state creates surface-trapped fluxes. Further work with a model configuration containing realistic summer stratification might reveal concentration of cross-shelf fluxes below the surface.

For the background jets with greater widths (and correspondingly smaller Rossby numbers), the same trend as with the narrow jets was observed for the modal structures. As the wavenumber of the perturbation increases, its structure becomes increasingly surface trapped. Only the structure of the first mode (i.e., the geostrophic mode found at the low wavenumber end of the range) is noticeably sensitive to changes in the width of the jet, although the growth rates at all wavenumbers are quite sensitive to jet width changes. Figure 6 shows the change in the low wavenumber structure as the jet width increases from 20 to 30 to 40 km. These structures are to be compared with the low wavenumber modal structure for the 15-km jet, shown in Fig. 5a. It is evident from a comparison of these modal structures that as the jet widens, the perturbation widens proportionally (consistent with the increase in \( L_a \)), but, more surprisingly, the perturbation maximum shifts strongly offshore. While the perturbation structure itself remains centered over the background jet axis (at 50 km offshore) the perturbation \( \nu \) becomes increasingly asymmetric as the frontal width increases. When the jet is 40 km wide, its perturbation maximum has shifted approximately 10 km offshore from its position at a jet width of 15 km. From an inspection of Fig. 6, it is apparent that the center of the perturbation tracks the offshore outcropping of isopycnals, or the density front (Barth 1994). Such a perturbation would tend to alter any symmetry in the background frontal jet and perhaps lead to asymmetrical cross-frontal exchange. This would particularly apply to relatively wide frontal jets, where the density front is significantly displaced from the velocity center.

2) \( V_o = 30 \) cm s\(^{-1} \)

Using \( V_o = 30 \) cm s\(^{-1} \), the set of runs discussed in the preceding section was repeated. Jet widths from 15 to...
40 km now yield Rossby numbers ranging from 0.379 to 0.144. (This contrasts to the Rossby number range from 0.96 to 0.366 for the $V_o = 60$ cm s$^{-1}$ runs.) Overall, as seen by comparing Fig. 4b to Fig. 4a, there is a significant drop in growth rates as $V_o$ decreases. The maximum growth rates for the slow jet, which occur at the high end of the wavenumber range, yield $e$-folding times of 3 days. Even though these rates are approximately five times slower than the growth rates for the $V_o = 60$ cm s$^{-1}$ jet, they are still of sufficient magnitude to lead to observable instabilities. While these results generally show an increase in growth rate with increasing Rossby number, it is apparent that the correlation between growth rate and Rossby number is not as strong for the low Rossby number cases as it is with the high Rossby number cases. A comparison of the 30-km jet to the 40-km jet in Fig. 4b illustrates this point. At high wavenumbers ($l > 45$) the higher Rossby number jet (30 km) has smaller growth rates. Though the offshore stratification is held constant in each of these runs, the Burger number, $S$, does vary with jet width since, according to the thermal wind balance, variations in the velocity field create a shifting of the isopycnals from their nearly horizontal plane at the offshore boundary. Associated with increasing Rossby number is a decreasing Burger number since the isopycnal slope increases to accommodate the increased shear. As will be shown in section 3b, as $S$ decreases, with all else constant, the perturbation growth rate generally increases. Thus, the two effects (an increasing Ro and a decreasing S) should
act in tandem to produce increasing growth rates as the jet narrows. While this holds true at the low wave-number end, it does not hold for high wavenumbers. This discrepancy may result from our use of scalars to characterize the two-dimensional background fields when clearly their structure is more complex. Additionally, it may be that the frontal instability is sensitive only to the baroclinic structure in the near vicinity of the density front and our choice of parameters fails to account for these changes. However, as will be demonstrated in subsequent sections, the use of scalars is sufficient to explain the trends for other variations in this study, so their utility is not discounted because of this unexplained behavior at high wavenumbers.

It is interesting to note that the differences in the growth rate curves between the slower background jet and its faster counterpart cannot solely be explained in terms of the generally higher Rossby number changes associated with the faster jet. A comparison of the $V_o = 60 \text{ cm s}^{-1}$ jet with a width of 40 km (Fig. 4a) to the $V_o = 30 \text{ cm s}^{-1}$ jet with a width of 15 km (Fig. 4b) highlights this point. For the former the Rossby number is 0.360, while for the latter the Rossby number is 0.379. This ~4% change in Rossby number cannot account for the large difference in the growth rates of these two jets. Overall, the $V_o = 60 \text{ cm s}^{-1}$ jets have much greater growth rates than the slower jets, a testimony to the importance of the background velocity’s advection of the perturbation density and velocity, as expressed in Eqs. (2.2) and (2.5).

The phase speeds for the perturbations (Fig. 4b) are reduced by half for the slow jet, compared to the fast jet, in accordance with the phase speeds being bounded by 0 and $V_o$. Again, the phase speeds in Fig. 4b show the stepwise change from one physical mode to the next. The trend for the faster growing modes to have smaller phase speeds generally holds for these $V_o = 30 \text{ cm s}^{-1}$ cases, except in the vicinity of wavenumber 10 where a mode with a particularly slow phase speed appears to be dominant for the 15- and 20-km jets and near wavenumber 18, where there is a local decrease in the phase speed for the 30- and 40-km jets. As will be shown below, this local decrease is associated with a mode that was not found in the faster jet runs.

The modal structures for the $V_o = 30 \text{ cm s}^{-1}$ jets show a similar pattern to the $V_o = 60 \text{ cm s}^{-1}$ perturbation structures in that changes in the background configuration bring significant changes to only the low wavenumber, or geostrophic, mode. While changes in the background horizontal shear bring discernible and significant changes to the growth rates and phase speeds for all wavenumbers, the surface-trapped modal structures at high wavenumbers are apparently insensitive to the parameter changes made in this study. This insensitivity may result from the fact that the changes to the background field are on scales that exceed the perturbation length scales, particularly at high wavenumbers. Shown in Fig. 7 are the changes in the low wavenumber perturbation structure as the width of the $V_o = 30 \text{ cm s}^{-1}$ jet changes. It is apparent that the perturbation structures associated with the slower jet are considerably wider and deeper than those associated with the faster jet (Figs. 5 and 6). While the width of the perturbation does increase in tandem with the increase in $L_o$, the relationship is only approximate. The deeper penetration of the modes that result from the slower jets is consistent with the increase in Rossby height associated with the decrease in $N_o$ (and thus in Burger number). The Rossby heights for the 15-, 20-, 30-, and 40-km jets are calculated to be 48, 46, 44, and 42 m, respectively. While these depths roughly approximate the decay scales observed in Fig. 7, it is not at all clear that the decay scale actually decreases with increasing jet width, as suggested by the calculated Rossby heights. Thus, while the dependence of the Rossby height on wavenumber, $l$, appears to be robust, the dependence on $N_o$ does not hold for most, if not all, of our model runs. However, this discrepancy could result from the fact that the Rossby height is based on a depth-averaged $N_o$. Finally, it is noted that, in addition to increases in the width and depth of the perturbation, it is evident that the modes for the slower jet have structure along the sloping bathymetry. The magnitude of this perturbation is small relative to the surface perturbation, but it appears to be a robust signal over each of the jet widths.

The trend for the wider jets to have their perturbation maximum shifted offshore with the density front holds also for these $V_o = 30 \text{ cm s}^{-1}$ jets. In fact, this trend is even more pronounced for the slow jet cases, as seen by the shift in the 40-km jet. In Fig. 7d the perturbation maximum is located at approximately 70 km offshore, or 20 km seaward of the jet’s center. At this distance the perturbation would be at the offshore edge of the background jet.

As mentioned above, a new modal structure appears for this slow jet and is associated with very slow phase speeds. The modal structure shown in Fig. 8a is representative of the mode present in each of the 15-, 20-, 30-, and 40-km jets in the wavenumber range of 10–25. This double lobe mode is centered at the midpoint of the background jet, yet its magnitude is stronger on the offshore side. Runs made with a flat bottom show that this asymmetry is solely the result of the sloping bottom, implying a possible contribution from topographic wave motion. This mode is also interesting in that its cross-shelf momentum flux (Fig. 8b) shows a nice signature of downgradient behavior. The two lobes, split at the $V_{max}$ location, are of opposite sign, since the mean velocity gradient changes sign at $V_{max}$. Such a structure implies the increasing importance of the barotropic contribution to the instability. Morgan (1997, his Fig. 3.22) shows that, as the relative vorticity increases on the offshore side of the front, the horizontal Reynolds stress contribution to the buoyancy flux becomes increasingly important. The asymmetry of the buoyancy flux implies a preference for offshore exchange, which was noted...
b. Stratification variations

From Rossby height considerations, increasing (decreasing) $N_o$ should reduce (increase) the vertical scale of the modal structures and increase (decrease) the phase speeds [since the Rossby height will be located higher (lower) in the water column]. To explore these effects, a change in the background stratification was achieved without attendant changes in the horizontal or vertical gradient of velocity. Thus, the influence of stratification changes on the stability of the background jet is more straightforward than the other parameter changes. For each $V_o$ case, changes were made to $\rho_{\text{shallow}}$ for both a high Rossby number (jet width = 15 km) and a low Rossby number (jet width = 40 km) case. Since there was such a dependence of the stability characteristics on Rossby number (section 3a), we felt it necessary to explore stratification changes for the high and low end of this parameter. While our study included both the $V_o = 60$ cm s$^{-1}$ and $V_o = 30$ cm s$^{-1}$ cases, only the former will be reported here since the latter cases differ substantially from the $V_o = 60$ cm s$^{-1}$ cases only in the magnitude of the growth rates, as explained in the prior section.

The growth rate and phase speed curves for the 15-km and 40-km jet are shown in Figs. 9a and 9b, respectively. As the stratification is decreased, there is an
increase in the growth rate of the dominant mode at each wavenumber. However, this destabilizing effect is only substantial for the higher wavenumber modes for the high Rossby number jet (Fig. 9a). At wavenumbers greater than ~14 for the fast jet (where there is a modal shift for all four cases), the growth rate curves for the various stratifications diverge, with the least stratified jet showing a remarkable increase in growth rate. Interestingly, this divergence occurs approximately where the perturbation wavelength matches $2\pi r_s$, the scale that separates the geostrophic mode from the frontal modes.

At the high wavenumber end in Fig. 9a, all growth rates are considerably less than 1 day, even those associated with a sharply stratified water column (where $S = 0.249$). It is apparent that these rapid growth rates are attributable to the high Rossby number associated with each of these cases. This influence is evident from an inspection of Fig. 9b, where the growth rates are less than a day over the entire Rossby number range, yet the offshore specification of $\rho_{\text{shallow}}$ is the same as it is in Fig. 9a. It is noted that since the density structure away from the offshore boundary is dictated by the thermal wind relation, our measure of $S$ does vary in each case. However, the $S$ variation alone cannot account for the differences in the growth rate curves. For instance, a comparison of the $S = 0.249$ case in Fig. 9a to the $S = 0.260$ case in Fig. 9b shows that the high Rossby number jet (in Fig. 9a) always has the larger growth rate, particularly at low wavenumbers. That the growth rates for these two cases ($S = 0.249$ and $S = 0.260$) become more similar at higher wavenumbers reflects the increasing baroclinicity of the higher wavenumber modes and, as a result, their strong dependence on the background stratification. Likewise, the weak dependence on stratification for the low wavenumber mode (Fig. 9a) suggests that a primary source of energy for this mode is the horizontal shear.

The weak dependence of the growth rates on the background stratification in Fig. 9b is interesting to note. While this weak dependence is evident only at the low wavenumber end in Fig. 9a, it is in place for the entire wavenumber range in Fig. 9b. Such a feature may indicate that the lateral velocity shear fields are more important in establishing the stability characteristics at these relatively high Burger numbers.

It is apparent from an inspection of Figs. 9a and 9b that stratification has very little effect on the phase speeds. As with the previous model runs, the magnitude of the phase speed increases as wavenumber increases. Thus, consistent with our earlier assessment, the phase speed shows the expected dependence on $l$, according to Rossby height considerations, but does not display the expected dependence on stratification. Finally, an inspection of the modes for the cases shown in Figs. 9a and 9b reveals essentially the same pattern for modal structures as explained in section 3a. An inspection of the perturbation $u$, $v$, and $b$ fields shows that the structure functions for the geostrophic mode deepen as $N_r$ increases, but the frontal modes are essentially insensitive to stratification changes.

c. Vertical shear variations

To complete our parameter study, the runs made by varying the depth of the background jet are presented in this section. With $V_o$ at either 60 or 30 cm s$^{-1}$ a variation in depth creates variation in $V_o$, which has not been explicitly tested in the previous runs.
1) $V_z = 60 \text{ cm s}^{-1}$

Changing the depth from 30 to 120 m creates significant differences in the growth rates of the dominant instabilities, as seen in Fig. 10a. For these runs, the Rossby number of the background flow remains a constant, yet $S$ varies significantly as $V_z$ is changed. In fact, $S$ varies over a range comparable to the range tested in section 3b, where the stratification effect was explicitly tested. In this set of runs (Fig. 10a) a decrease in growth rate as the jet deepens is apparent despite the fact that the stratification decreases as the jet deepens (which would tend to increase the growth rate). Apparently, the decrease in $V_z$, which is in tandem with $\rho$, changes, overrides the destabilizing effect of a weakly stratified water column. Since this parameter changes the baroclinicity of the background jet, it is no surprise that the surface-trapped modes, at high wavenumbers, are the most affected. It is important to note that for all jets (with depths from 30 to 120 m) the model growth rates are less than $\sim2$ days for wavelengths smaller than approximately 60 km ($l = 10$).

Phase speed does show some sensitivity to changes in the jet depth (Fig. 10a), yet the relationship is not consistent over the entire wavenumber range. At some wavenumbers a large $V_z$ corresponds to small phase speeds, while at other wavenumbers a large $V_z$ corresponds to large phase speeds. Modal structures follow the same trend as discussed with the previous parameter runs, with a tendency for deeper jets to have deeper perturbations for the low wavenumber mode and for the shallow jets to have shallower perturbations. These changes are expected for the geostrophic mode. Additionally, the increase in the depth of the perturbation is consistent with a decreasing $S$, which results in a larger $H_n$. Shown in Fig. 11a is a low wavenumber mode for the deepest jet studied (120 m). Its center is situated at $x = 50 \text{ km}$, as before, yet now its structure extends to $\sim50 \text{ m}$ and its width is greater than the mode shown in Fig. 7, to which it should be compared.

2) $V_z = 30 \text{ cm s}^{-1}$

Shown in Fig. 10b is the sensitivity of a slow jet to changes in $V_z$. The pattern is the same as for the fast jet, yet the overall growth rates and phase speeds are reduced in magnitude. For this slow jet the modal structure for a low wavenumber mode with a deep jet is shown in Fig. 11b. Here the feature of interest is the structure near the shelf break. While most of the modes found in this parameter study have their structure concentrated near the surface, it is interesting to note that certain configurations can give significant structure else-where in the water column, consistent with observations.

d. Summary of parameter study

To better understand the parameter space spanned by the Middle Atlantic Bight shelfbreak flow field, a lower resolution parameter study was performed. For the purposes of this study, the simulation was performed with $M = 32$ instead of the $M = 44$ that was used in the rest of this work. The shortened simulation run time allows a more complete exploration of parameter space and gives results that are qualitatively similar. In order to illustrate the simultaneous dependence of the growth rates on $Ro$, $S$, and $V_z$ (with $V_o$ fixed at $60 \text{ cm s}^{-1}$), a three-dimensional plot is employed (Fig. 12). Our aim with this analysis is to give a more complete and accurate picture of the relationship between the variables. Some general trends are noticeable in Fig. 12. For example, higher Rossby numbers uniformly show higher growth rates. A general trend of decreasing growth rates for increased stratification can be seen, as evidenced by the several sets of data for fixed horizontal (Rossby) and vertical ($V_z$) shear. By looking at planes of fixed Rossby number, it can be seen that for a given Burger number, increasing vertical shear is associated with more unstable disturbances. Note that for low Rossby number cases there does not appear to be enough energy in the system to drive high growth rates, even when the vertical shear is large. It should be pointed out that, although the physical characteristics of the background state, namely width and depth and initial background stratification, were varied uniformly, this did not result in a uniform distribution of points in parameter space. However, the sampled areas of parameter space represent those regions associated with observed Middle Atlantic Bight fronts.

4. Instability of the shelfbreak frontal jet in the Middle Atlantic Bight

The intent of the parameter study in section 3 was to establish the stability for an envelope of possible shelfbreak frontal jets in the Middle Atlantic Bight. In this section we seek to establish the stability characteristics for a model jet based on the climatological velocity and density fields (Linder 1996) and for a model jet based on a composite of synoptic sections that have been stream-coordinate averaged (Fratantoni et al. 2001). To facilitate a comparison with our parameter runs, we have chosen to use the same analytical formulation for the velocity field (section 2b) for the two jets in this section.
(a)

Growth Rate

Phase Speed (cm/s)

Wavenumber

(b)

Growth Rate

Phase Speed (cm/s)

Wavenumber
We have selected a jet width (20 km), jet depth (70 m), and $V_o$ (25 cm s$^{-1}$) from a composite of the winter and summer climatological fields given by Linder (1996). Based on the climatological density fields the offshore density profile varies linearly from 1.024 to 1.0275 gm cm$^{-3}$. The resultant stability profile for a jet with these characteristics (labeled CLIM) is shown in Fig. 13. Also shown are the results for a model jet whose width (40 km), depth (90 m), and $V_o$ (40 cm s$^{-1}$) are based on the stream-coordinate averaged jet that Fratantoni et al. (2001) constructed from eight synoptic sections across the shelfbreak frontal jet south of Nantucket Shoals from 1997 to 1998. Based on the synoptic density fields the offshore density profile was set to vary linearly from 1.0244 to 1.027 gm cm$^{-3}$. As seen in Fig. 13, both jets are unstable over the range of wavenumbers tested, with $e$-folding scales of between 2 and 3 days for the most unstable perturbations of the stream-coordinate averaged jet and between 3 and 4 days for the climatological jet. The faster growth rates for the stream-coordinate-averaged jet are attributed almost entirely to the faster speed of this jet. While one would expect the stream-coordinate-averaged jet to have a higher Rossby number, the width of this jet (40 km) precludes such a characteristic. In fact, the Rossby numbers for the two jets are nearly identical, thus precluding the possibility that Rossby number differences could account for the differences in the stability results. While it is arguable which of these jets, if either, is representative of an averaged basic state in the Middle Atlantic Bight, these model results show that, even for an averaged profile, the frontal jet in the Middle Atlantic Bight is unstable to perturbations that have substantial growth rates.

5. Summary and conclusions
A stability analysis of a two-dimensional geostrophic jet overlying shelfbreak topography has found the jet to be unstable to perturbations for a wide range of background conditions. While earlier studies of shelfbreak frontal instabilities found growth rates to be prohibitively small to be of consequence in the energetic shelfbreak region, the model growth rates from this study are on the order of 1 day. Such rapid growth would clearly lead to substantial temporal and spatial variability of the shelfbreak front. The inclusion of continuous horizontal and vertical shear for the background density and velocity field, as well as the use of primitive equation dynamics, are believed to be responsible for the capture of physical modes with rapid growth rates. While the perturbations with the shortest wavelengths generally had higher growth rates, often the growth rate curves were relatively flat at high wavenumbers, suggesting that a range of wavelengths might be present in the shelfbreak vicinity rather than one dominant wavelength. Indeed, past observations of spatial variability...
Fig. 12. Summary of the parameter runs for a $V_o = 60$ cm s$^{-1}$ jet. The dependence of growth rate on Rossby number, Burger number, and $|V_{z_{\text{max}}}|$ are shown. The position of the sphere represents the point in three-dimensional parameter space, while the size and color of the sphere (from blue, smallest, to red, largest) represent the relative magnitude of the maximum growth rate for the single fastest growing wavenumber (e.g., small spheres denote small growth rates). These simulations were performed with a modal cutoff of 32 instead of the 44 that was used in the rest of this work. Additionally, wavenumber space was explored from 1 to 59, for every odd wavenumber.

Fig. 13. Growth rate and phase speed as a function of wavenumber for an idealized climatological frontal jet (Linder and Gawarkiewicz 1996) and an idealized stream-coordinate-averaged frontal jet (Fratantoni et al. 2001).
in the Middle Atlantic Bight have not narrowly defined the dominant spatial scale of the eddy motions. Collectively, these past studies have reported a range of from 10 to 75 km for the spatial scale, a range that is consistent with our model results.

Because the shelfbreak frontal jet in the Middle Atlantic Bight exhibits a large degree of variability about its climatological mean winter and summer states, an envelope of possible synoptic jet structures was tested in a parameter study. Jet width, depth, and velocity maximum were varied, along with stratification, in an attempt to isolate the relative importance of the Rossby number, the vertical velocity shear, the maximum velocity, and the Burger number to the stability of the background jet. Changes to the background fields were based on the range of conditions present in the Middle Atlantic Bight. Overall, growth rates increased as the jet’s maximum velocity increased, as the Rossby number increased, and as the jet’s velocity shear in the vertical increased. A decrease in growth rate resulted from an increase in the jet’s stratification. Thus, a fast, narrow, shallow jet with relatively weak stratification maximizes the growth rate of the unstable perturbations. Of all the parameters tested, the model growth rates were the least sensitive to changes in the background stratification. This result runs somewhat counter to the notion that a summer jet is more stable than a winter jet because of the strong stratification provided by seasonal heating. All things being constant, these model results do show this to be true but, if the summer jet also happens to be faster or narrower or shallower than the winter jet, it is likely to be more unstable. Thus, seasonal differences in stability may not be controlled solely by seasonal changes in stratification if the seasons also bring significant changes in the width and depth and strength of the jet. Considering the strong sensitivity of the stability characteristics to the structure of the velocity field, it is likely that changes in the stability of the frontal jet on the order of days may occur if the background jet is altered due to ring interactions, slope water variability, riverine input, and/or local wind events. It seems likely that the stability may change more rapidly on synoptic timescales than on seasonal timescales. This suggestion is made with the caveat that we have conducted our model study with the constraint of uniform stratification. Nonconstant stratification might alter this generalization.

While the results presented here provide a consistent framework for anticipating stability characteristics of shelfbreak fronts, it is important to recognize some of the limitations of the model in light of recent observations in the Middle Atlantic Bight. The basic state used in our study assumes a purely geostrophic balance, without frictional boundary layers, and hence does not contain significant convergences or divergences. Recent observational work (e.g., Houghton and Visbeck 1998) suggests that there are significant convergences within the bottom boundary layer that result in upwelling along frontal isopycnals. These effects are not included in the present basic state, nor are they easily accommodated within linear eigenvalue problems. In his study on the stability of coastal upwelling jets, Barth (1989) included the effects of bottom friction on the evolution of instabilities, but the friction was introduced into the perturbation quantities and not the basic state. Furthermore, with this formulation, Barth found that the perturbations were not damped much by friction since the e-folding times for dissipation exceeded the perturbation growth rates. Our present model also lacks a mean cross-shelf flow, which is commonly observed in the Middle Atlantic Bight. The structure of the cross-shelf flow would be dependent on the details of the vertical mixing within the boundary layer, as well as the lateral shear of the overlying (geostrophic) alongshelf flow.

Another interesting issue relative to observations is the possible existence of bottom-trapped modes that might lead to variability of the foot of the front, where the frontal isopycnals intersect the bottom. Gawarkiewicz (1991) found bottom-trapped modes in a two-layer model, but these were rarely the most unstable modes. Given the surface-trapped jet considered in this study, it is not surprising that the most unstable modes were also surface-trapped. Basic states with stronger shelf flows in the vicinity of the foot of the front or with nonconstant stratification might lead to more energetic bottom-trapped modes. However, these modes would, in the actual front, likely be affected by the convergence patterns and dissipative processes near the foot of the front, neither of which are included within the present model.

The extension of this work to include realistic frontal features such as shear asymmetry, variations in the onshore/offshore location of the front, nonuniform stratification, and a subsurface jet maximum are essential to further the applicability of these results to the observed fields. Finally, it is noted that, while the range of conditions tested in this study was based on observations in the Middle Atlantic Bight, we believe this specificity does not limit the general application of this work to other geographical regions.

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