An Optimization and Comparison of Internally and Externally-Valved Magnetorheological Dampers

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Abstract

Magnetorheological (MR) dampers can be optimized for power usage and then compared to evaluate the relative benefits of different designs. MR fluids have promising applications to earthquake response suppression devices because of their rapidly (ms) controllable properties when exposed to a magnetic field (≈ 1 T). The semi-active dampers considered in this study consist of a hydraulic cylinder, filled with MR fluid which is forced to flow through magnetizable ducts in the device. In an internally-valved design, the magnetic field is generated by an electromagnet in the piston itself. In the externally-valved design, magnetic circuits are placed external to the hydraulic cylinder. In the analysis of both systems the non-Newtonian material behavior of the MR fluid is modeled by the Bingham equation. The magnetic Kirchoff circuit equation with non-linear models for material magnetization are used to analyze the magnetostatics. Design parameters for both systems were defined and used in numerical optimizations. The designs were optimized to minimize power usage for three different nominal force capacities: 5 kN, 20 kN and 100 kN. Relative power usages and scaling effects are then compared. The results will lead to future research which has the capability of creating viable devices that would be economically beneficial and save lives.

1 Introduction

Earthquakes are challenging to structural engineers because of the level of uncertainty involved in earthquake-resistant design. Earthquake loads result in significant deformations and accelerations in a structural system, which have negative effects on the safety of the structure and on the objects and people within it. The purpose of damping devices for earthquake mitigation is not only to keep the deformations low, but also to limit the accelerations in the higher floors. For this reason, it is important that damping devices accurately and quickly respond to the forces applied at any particular moment. Three types of earthquake mitigation devices are currently being studied for their abilities to achieve the desired effects. These are passive dampers, semi-active dampers and active control systems. The devices examined in this study are semi-active and utilize magnetorheological materials, which demonstrate a change in visco-elasticity and yield stress with exposure to magnetic flux. This property is desirable because it allows MR fluids to be used in a hydraulic cylinder where the forces applied to the structure can vary with the magnetic flux. In this paper, internally and externally-valved devices are designed and compared. The internally-valved device has the magnetic circuitry within the piston of the hydraulic cylinder. The externally-valved device has a magnetizable external bypass for the MR fluid. Both magnetic circuits consist of a set of wire spools which are spun around a low-permeability magnetic material. Device design parameters are optimized with the goal of creating devices that use the least amount...
of power, but still fulfill all other system requirements. Minimizing power usage by the devices is important because loss of electricity is quite common during earthquakes.

In order for these devices to be useful for seismic resistance, it is imperative that they be battery-operated. The optimization of these devices involves a large number of variables which have non-linear relationships. This makes for a challenging design problem. However, some of these variables, such as the wire thickness, the number of layers of wire and the number of spools, have only a finite number of possibilities. By altering these values, a set of data can be compiled, using a full parameter-search method, to find the best combination of design variables for that particular combination of wire size, layers and spools. The optimization process uses the coupled analysis of the magnetic circuit and Non-Newtonian fluid mechanics. The relationships between fluid pressure and flow used in the design are approximations to the solutions of partial differential equations describing fluid flow and magnetic flux (Gavin 2001). Magnetic Kirchoff circuit equations and non-linear magnetization curves are used to describe the magnetic flux relationships.

There has been some question as to whether internally or externally-valved devices have the greatest promise for future implementation. Both devices have their own distinct advantages. This paper describes optimal designs for both types of devices for several nominal force capacities. They will then be compared to see which design is more appropriate with respect to the amount of power used.

2 Background

The three categories of vibration control dampers are passive, active and semi-active. Passive dampers are used to resist velocities, but the drawback is that they cannot adjust for different loading characteristics, such as one might find in an earthquake. Some examples of passive control systems are viscous shear walls, base isolators and tuned mass or liquid dampers (Koh 1995). An active control system, on the other-hand, uses sensors in order to actively respond to external loadings with appropriate reaction forces. There are several issues which make the use of active control systems difficult. The control law, which is a function that describes the control force as a function of the building response, can be linear or non-linear and its implementation is influenced by the time delay required to run the calculations. It is impossible to apply the control force at exactly the same time the motion of the building is measured, because the system has to first sense the motion and then perform the calculations. If not properly compensated for, the time delay can cause dynamic instability. For this same reason, it is not feasible to include a complicated model of the inelasticity of a particular structure. The more involved the control law is, the more time it will take to run and the less effective the device will be. There will always be some uncertainties in the modeling of the structure because there is the chance that defects within the materials or the structural connections could alter the behavior. There is also the problem of incorrect signals being sent to the device due to electronic problems that could occur during an earthquake. Sensing errors can be addressed through the use of digital signal processing methods (Wong 1997). The devices in this study use semi-active control in the desire to incorporate the positive aspects of both active and passive systems. Forces in semi-active dampers can be controlled only within time which ensure that the dampers will always absorb energy. A certain amount of pressure is available at all times depending on the viscosity and yielding strength of the MR fluid. In addition to this, when the magnetic circuit is turned on, the properties of the MR fluid can be altered in order to change the control forces that the device will use to mitigate seismic response.

The fluid that is used in the devices is a magnetorheological suspension. MR fluids are a colloidal suspension of iron alloy particles coated with a surfactant in order to stabilize them in a hydraulic
solvent (Kormann 1996). When a magnetic field is applied to the fluid, the iron particles arrange into a fibrated microstructure that resists shearing with yielding and viscous stresses. The fluid can attain a yield stress of approximately 100 kPa and it can be controlled by a magnetic field up to approximately 1 T. The effects of this magnetic field are completely reversible on a rapid time scale (milliseconds).

Iron and cobalt are some of the metals used in MR fluids and they are both ferromagnetic, which means that their atomic dipoles will align, even under a weak field. The dipoles align with the field in units called domains, and if the field changes direction the domains will reorient themselves. This results in a non-linear pattern of reversals in the direction of the field. The magnetic field, if large enough, will eventually saturate the material and no further domain-reorientation can be gained. This is the limiting point in the design of the devices because utilizing a stronger field would be a waste of energy. Iron is a soft metal and has a narrow magnetic hysteresis loop, which means that it will have a small remnant magnetization. Ideally, in order to make the material as efficient as possible, we would like to have no remnant magnetization. Iron also exhibits a relatively small energy loss per cycle of magnetization which makes it an appropriate material for this application (Campbell 1996, Serway 2000).

In the design of the devices, the material properties of the MR fluid are very important. If an increase in the maximum yield stresses are desired, this can be reached by increasing the saturation magnetization of the fluid. Findings by Phule and Ginder indicate that the yield stress increases quadratically with the saturation magnetization (Phule 1999). The materials with the highest saturation levels are alloys of iron and cobalt, $\mu_0 M_s \approx 2.4T$ (Ginder 1996).

There have been many MR devices patented since the 1990's by companies such as the Lord Corporation. The Lord Corporation developed the first internally-valved hydraulic MR device (Carlson 1994). In 1998, they licensed the technology to Sanwa-Tekki Corporation of Tokyo, Japan, which developed the externally-valved design, where the magnetics are outside of the hydraulic cylinder. This paper evaluates both devices using a set of equivalent design guidelines so that they can be compared on an equal level.

Another group of researchers in Nevada has also patented their devices. The difference between the two devices being examined here and the patent by Gordaninejad is that the direction of the magnetic field through the MR fluid is perpendicular to the direction of the fluid flow. MR fluids align in the direction of the magnetic field, forming chains. These chains, given the opportunity will aggregate into a body-centered tetragonal lattice structure (Tao 2001). These chain structures are particularly strong when sheared, as would occur if the flow was perpendicular to them. Gordaninejad’s design would have the chain structures aligned in the parallel direction, with no wall to attach to, and thus floating in the fluid. This is why the force levels attained by the design are not high even though the piston velocities used are similar to those used in the present designs (1997).

3 Device Design

The design process for both the internally and externally-valved devices begins with the selection of a set of design variables which describe the dimensions and material properties of the system. The objective of the design process is to minimize the electrical power consumption of the device such that:

- a specified level of force is achieved;
- a specified force range (the ratio of the ‘ON’ to ‘OFF’ forces) is achieved;
A candidate design is evaluated by determining its feasibility and by comparing its performance to other designs. The ‘optimal’ design is the feasible design with the best performance. This section details the parameters and constraints which were used to define the design problem. The design process involves evaluating models of the electrical and mechanical device behavior. The fluid mechanics and magnetics which determine the behavior are discussed in later sections. Three minimum force systems were designed, 5 kN, 20 kN and 100 kN, in order to evaluate the scaling effects for both types of devices.

Certain design variables are defined for both devices: the number of spools \( N_s \), the number of turns of wire on each spool \( N \), the number of layers of wire \( N_{\text{layer}} \), the diameter of the piston \( D_p \), the strength of the magnetic field \( B_g \), the thickness of the gap \( t_g \) and the volume fraction...
of iron in the MR fluid ($\phi$). For the externally-valved device there were also the parameters of the width ($w_c$) and length ($L_c$) of the magnet core and the thickness of the bar ($t_b$). For the internally-valved device, the diameter of the magnet core ($D_c$) and the diameter of the hydraulic cylinder ($D_b$) were also parameters.

A pair of computer programs were developed to design the devices. The first program uses the design variables and the design relationships explained below in order to determine the fluid properties, the circuit properties, the forces that the device will be able to produce, the time constant and the power requirement. Forces are found by multiplying the pressure across the piston by the area of the piston. The second program is a full parameter search in which discrete combinations of the design variables are evaluated to find the ‘optimal’ solution. This was implemented in MATLAB using a series of nested for-loops. The drawback to this method is that there are possibly, between any two values, an infinite number of solutions which could yield a better result. Other optimizing techniques were tried but failed to produce any results because the relationships between the variables are non-linear and any change in one variable will have far reaching effects. There are many local minima, which makes the search for the absolute minimum very difficult. There are certain variables for which this approximation is accurate because they are only commercially available in discrete sizes, an example being the wire gage. For the other variables, this is a very good approximation since a device which will be created in reality will have so many other unknowns.

The combination of parameters that is chosen as ‘optimal’ is the one that is able to satisfy all the design constraints with the lowest power usage. The constraints on both devices are the same. At a slow piston velocity of 2 cm/s, with the magnetic circuit electricity ‘ON’, the force exerted should be at least 5 kN, 20 kN or 10 kN, depending on the size being tested. The force level achieved when the velocity of the piston is 50 cm/s also has to be increased by a factor of at least 10 when the electricity is ‘ON’. The magnetic field in the steel parts has to be maintained at a level lower than 1.5 T to avoid saturation. The maximum allowable inductive time constant was 0.2 sec. The voltage and current in the system were not allowed to be higher than 24 V and 10 A, respectively, to avoid burning out the wire coils.

4 Fluid Mechanics

MR fluid can be analyzed using the Bingham model of Non-Newtonian fluid mechanics. This is a convenient and simple constitutive relationship that is used in the modeling of magnetic field dependent material behavior. It does a good job of representing the stresses in the fluid. The governing stress equation is,

$$
\tau(\phi, B_g, \dot{\gamma}) = \tau_y(\phi, B_g)\text{sgn}\dot{\gamma} + \eta(\phi)\dot{\gamma},
$$

where $\tau_y$ is the yielding shear stress and is dependent upon $\phi$, the particulate volume fraction, and $B_g$, the magnetic flux density in the fluid. The relationship of the magnetic field to the magnetic flux density in the fluid is seen in Figure 2. In this study the yield stress, $\tau_y$, and the plastic viscosity, $\eta$, are taken to be linear with the particulate volume fraction, $\phi$. The yield stress is also related by a power-law to the magnetic flux. The plastic viscosity is assumed to be unrelated to the shearing rate, $\dot{\gamma}$. In units of Tesla, Pascals, and seconds the equations are,

$$
\tau_y(\phi, B_g) = 15 + 2.5 \times 10^5 \phi B_g^{3/2}
$$

and

$$
\eta(\phi) = 0.5\phi
$$
These equations are based upon a typical MR fluid and therefore can be used to represent it no matter what particulate volume fraction is chosen in optimization. The equation used to describe the viscosity, \( \eta \), with respect to particulate volume fraction is an approximation used because there is very little research in this area. In the externally-valved design (see Figure 1), the MR fluid is pumped by the hydraulic cylinder through an external by-pass valve. A tube of cross section \( w_c \times t_g, t_g < w_c \), carries the fluid past a set of \( N_s \) magnetic spools, which are oriented with opposite polarity. The total length of the gap the fluid flows through is \( N_s \times L_s \) and the length of the region of magnetization is \( N_s \times L_c \), where \( L_c \) is the width of the core of the spool and \( L_s \) is the center to center spacing of the spools. The pressure drop between the ends of the piston, \( \Delta p \), is approximately equal to the viscous pressure drop, \( \Delta p_N \), plus a constant times the ratio of the shear stress in the fluid to the thickness of the gap it is flowing through.

\[
\Delta p \approx \Delta p_N + 2.1N_sL_c\frac{\tau_g(\phi, B_g)}{t_g} \tag{4}
\]

\[
\Delta p_N \approx \frac{12Q\eta(\phi)N_sL_s}{Wc t_g^3} \tag{5}
\]

The MR fluid is assumed to be incompressible and therefore the flow, \( Q \), is the same out of one end of the piston and into the other. The flow is therefore directly related to the velocity of the piston, \( V_p \),

\[
Q = \frac{\pi}{4}V_p(D_p^2 - D_r^2), \tag{6}
\]

where \( D_p \) is the diameter of the piston and \( D_r \) is the diameter of the rod. In the internally-valved device (see Figure 1), the viscous component \( \Delta p_N \) of the force is modified because the path that the fluid takes is different. The fluid is pushed back and forth by the piston inside the hydraulic cylinder. The portion of the fluid between the pistons and the wall of the cylinder is magnetized.

\[
\Delta p_N \approx \frac{12Q\eta(\phi)2N_sL_p}{\pi(D_p + t_g)t_g^3} + \frac{12Q\eta(\phi)N_sL_c}{\pi(D_c + d_w)(D_o/2 - t_w - D_c/2 - N_{layer}d_w)^3} \tag{7}
\]

The thickness of the wire is represented by \( d_w \), the number of layers of wire by \( N_{layer} \) and the outer wall diameter of the cylinder is represented by \( D_o \), where \( D_o = D_b + 2t_w \). The effects of friction are ignored in this study and they will need to be examined in further detail in later studies.

5 Magneto-statics

The devices are designed to be battery-powered and therefore the circuits are DC. The magnetic circuits contain a set of iron spools that are wound with a selected wire at least 2 layers thick, creating a solenoid. Ampere’s Law can be used to describe the systems because magnetic flux travels in a closed loop path which can be determined. The flux will follow the path of least resistance, and therefore through the material with the highest permeability. The gap between the spools and metal cylinder or bar is where the flux is forced to pass through the MR fluid. The magnetic flux density that is created in the fluid is determined from the magnetic circuit equation.

For the externally-valved design,

\[
Ni = 2H_g t_g + 2H_c(h_c + T_b) + 2H_b L_s \tag{8},
\]

and for the internally-valved design,

\[
Ni = 2H_g t_g + H_c(L_c + L_p) + H_p(D_p + t_w) + H_w(L_c + L_p) \tag{9},
\]
where \( N \) is the number of turns of wire, \( H_g \) is the magnetic field in the gap, \( H_c \) is the magnetic field in the steel core, \( H_b \) is the magnetic field in the base bar and \( H_p \) is the field in the piston. Due to the conservation of magnetic flux, the flux within the circuit is constant. The magnetic field generated in the steel and MR fluid is described by Figure 2. It is related to the strength of the flux created by the solenoids. This allows the flux density at any point within the circuit to be calculated. The permeability of the MR material increases with the particulate volume fraction and affects the magnetization of the material.

\[
\Phi_B = \frac{N \Phi_B}{i},
\]

Figure 2. Magnetization Curves for Steel and the MR Fluids with High and Low Volume Fractions

In the externally-valved device, the spools are electrically in parallel. The inductance of the coils in parallel, neglecting the effects of the iron cores, is:

\[
L = \frac{1}{N_i} \frac{N \Phi_B}{i},
\]  

where \( \Phi_B \) is the magnetic flux. The iron cores of the solenoids will cause the generated field to actually be greater than if it was not there, due to their ferromagnetic properties. This will depend on exactly what material is chosen and will have to be adjusted for in the experimentation of the devices. The overall result would be to lower the necessary power usage to attain an equivalent field, thereby increasing the efficiency of the system. The resistance of the coils is found using the linear resistivity in the wire, \( r \), and the length of wire used in the number of windings in each coil,

\[
R = \frac{1}{N_i} 2rN \left( L_e + w_e \right).
\]  

In the internally-valved device, the spools are electrically in series. Therefore the inductance and the resistance are:

\[
L = N_i \frac{N \Phi_B}{i},
\]  

and

\[
R = N_i rN \pi D_e.
\]
The inductive time constant, \( L/R \), of the system affects its ability to respond to the control system and impart the required changes on the MR fluid. This can be shortened by the addition of resistance outside of the system, if necessary, but it will lead to an increased power usage. The voltage used given by \( V = iR \). The objective that is being minimized in the optimization process is the electrical power usage, \( P = Vi = i^2R \). It is important that the maximum current carrying capacity of the wire is not exceeded, the saturation magnetizations of all the components of the magnetic circuit are not broached and the \( L/R \) time constant is very small.

6 Results

The above determination of design variables and system behavior equations allows computer models of the internally and externally-valved designs to be created. For each force level, an appropriate hydraulic cylinder diameter and rod diameter was selected that would keep the working stress to within allowable limits. The strength of steel used in the designs was assumed to be about 1000 psi, which is much lower than the yield stress of 50,000 psi, in order to account for fatigue. For the internally-valved device, the piston size does not directly correlate to the hydraulic cylinder interior diameter in the same way that it does for the externally-valved device. This can be seen in Figure 3.

The results for the internally and externally-valved devices are included in Table 1. The largest available volume fraction, \( \phi \), was always found to entail the lowest power usage. The range of allowable volume fractions was limited by those found in commercially available fluids. Other than this, the largest allowable number of spools and number of layers was always the ‘optimal’ choice. These values were limited through practical concerns for the sizes of the devices. It is interesting to note that the smallest wire gage was used for the internally-valved device, while the largest gage was used in the externally-valved device. This is most likely due to the size constraints in the internally-valved device since all the materials have to fit inside of the chosen hydraulic cylinder.

The most surprising of all the findings is that the strength of the magnetic field does not need to be as high as possible in either device in order to reach the optimal solution. The values show that it is most appropriate to use lower field strengths to achieve the desired force level. The advantage of a lower magnetic field is that less power is consumed. The most difficult constraint to achieve was the need for at least 10 times increase in the force level achieved between the ‘ON’ and ‘OFF’ cases at high piston velocities (50 cm/s). This difficulty arises from the non-linear relationships between the design variables and the performance functions. The relationship for the externally-valved device is simpler than the internally-valved one and can be used as an example of these challenges.

\[
\frac{F_{\text{on-hi}}}{F_{\text{off-hi}}} = 1 + \frac{525000B_g^{3/2}\phi L_c}{756(D_p^2-D_f^2)L_s} + 31.5L_c > 10
\]  

(14)

As can be seen, there is an inverse variation of this ratio with the piston size, \( D_p \), but if the piston size were to decrease, the minimum force levels would not be achieved at all. The ways to satisfy this constraint would be to increase the thickness of the gap, \( t_g \), the width of the core, \( W_c \), or the strength of the magnetic field, \( B_g \). All of these changes would have impacts on the other constraints, possibly causing them not to be satisfied anymore. This is just one example of the difficulties encountered when trying to design such a complicated system. There are many designs that were found from the analyses that could lead to feasible designs for a semi-active MR device. This is desirable because it allows the designer to choose the materials that are the most economical and not have to sacrifice much in the way of additional power usage. The power usage for the externally-valved devices is so low (approximately 1 Watt) that they can be run off of small
batteries. This is was the objective of the design process because as was discussed earlier, low power usage is very important during an actual earthquake when battery power is the most accessible source. The internally-valved device, while it achieved all the necessary force levels using the same size hydraulic cylinder, it did not compete in terms of power usage. The drastic difference is clearly displayed in Figure 3.

Another noteworthy aspect seen in Figure 3 is the scaling effects of the power usage with increasing nominal force levels. The externally-valved device actually shows a greater percentage increase between levels than the internally-valved device. This is offset by the fact that the power usage is so low that it is inconsequential. Even though the power usage increases at a fractionally faster rate in the externally-valved device, it is unlikely that the two systems would become equal in the realm of useful devices. These finding are important because there has not been research done on device design in the past. There are currently no other studies with which to compare these results.

These findings do not mean, however, that the internally-valved device is not worthy of further attention. This device has a major advantage in that it is much simpler than the externally-valved device and therefore there are fewer modes of failure. There is also a tendency for MR devices to wear out their seals and there are fewer places in this design where fluid leakage could be a problem.

The advantage of the externally-valved device, besides the power savings, is that the magnetic circuitry is exterior to the cylinder and therefore is accessible for maintenance. In the internally-valved device, in order to fix one of the spools, the entire system would need to be drained of MR fluid and disassembled. This is especially noteworthy since in the internally-valved device the spools are connected in series. If one of these were to fail, it would cause the whole system to shut down. In the externally-valved device there is the advantage of having the spools in parallel so that if one of the parts should fail, the system would still work to a lesser degree. Earthquakes are very unpredictable and it is likely that some part of the device might be damaged during use. This
quality is therefore very desirable.

7 Conclusion

Overall, the externally-valved device enables a more economical usage of power, which is desirable in an earthquake mitigation device. These results are supported by Figure 3 and Table 1. The externally-valved design also displays more significant scaling effects than the internally-valved design, but its overall power usage starts out two orders of magnitude smaller and therefore this is not an issue. These findings suggest that further research should be concentrated on the improvement of the externally-valved design and not the internally-valved design. Models are needed to confirm the numerical results. A sensitivity analysis of the design variables is very difficult due to the non-linearity of the constitutive relationships of both the MR fluid and the steel. Future research in this area would be useful because then the importance of how precisely each of the pieces needs to be machined would be known.

The most economical decision, in terms of power usage, is to implement many small externally-valved devices instead of one larger device. Using twenty 5 kN devices would use 1.778 W, while using one 100 kN device uses 2.5762 W. When using the internally-valved design, however, it is more economical to use one larger device than to use many smaller ones. The use of one 100 kN device would use 119.13 W, while twenty 5 kN devices would use 162.898 W. These findings are important in the implementation of the devices in a building. It is useful for the engineer to know what kind of savings will be achieved when different nominal force size dampers are used.

Additionally, there is a question as to whether the MR fluid that was used in the design of this device is the optimal fluid. In order to create the strongest damper, the fluid should have the highest yield strength possible. The two ways to increase this are to increase the magnetic field and to increase the particulate volume of iron based particles (Margida 1996). Since all of the optimal designs using MR fluid with the greatest volume fraction of particulates, one of the ways in which to improve the results would be to increase the allowable volume fraction. This would have to be done through more experimentation in the field of MR fluid design, since there is not yet such a commercially available fluid. The best choice might not be to increase the particulate volume, but by varying the iron-alloy components, the fluid could be made stronger. The strongest iron alloy is one that is iron-cobalt based, and it displays a yield strength about 70 percent higher than the more commonly used carbonyl iron-based MR fluid (Margida 1996). There are also other types of MR fluids currently under research which might help to increase the force capacity of the device. Ferrofluids that are altered by decreasing the particle size and reducing the dispersing layer are promising. They have particles on the range of 0.01 \( \mu \text{m} \) while those in the standard MR fluid are on the range of 1-3 \( \mu \text{m} \). This reduction in particle size has the effect of reducing the size of the field that it takes in order to create the same degree of magnetization in the fluid. They also dissipate less energy while rotating to align with the field. This fluid is also able to resist large changes in temperature which would be useful during an earthquake, where fires are likely to occur (Kormann 1996). More research needs to be done in the field of MR fluid properties and design in order for the most efficient dampers to be created.

8 Acknowledgments

The author would like to thank Professor Henri Gavin who made this entire project possible and helped every step of the way.
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9 References


