External Threat and Collective Action

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Abstract

In this paper, we develop a model to study how players allocate their endowed resources between productive and conflictual efforts in the context of rivalry between two groups. Our analysis shows that by endogenizing external threat the validity of the suboptimality and exploitation hypotheses established by Olson holds true for members of a defensive group, but not for members of an offensive group. When both groups are offensive, the two hypotheses are only valid for members of the resource-laden group, but not for the resource-poor group. We also show that it does not always pay to take an offensive stance. When competing with an offensive group, it might be better off for members of a defensive group to remain defensive. Another of our model’s interesting findings is that, in the context of rivalry between two groups, free riding can actually benefit everyone in the system.
1 Introduction

Collective action problems are often studied as phenomena within a single group. In *The Logic of Collective Action*, for example, Olson (1965) shows that: a.) the larger the group, the smaller the provision level of the collective good (the *suboptimality* hypothesis); and b.) there is a systematic tendency of the small to exploit the large (the *exploitation* hypothesis). In his analysis, external threat to a group either does not exist or is treated as a constant.\(^1\) In many contexts, however, two or more groups compete against each other. In this case, a group’s provision of collective goods might be affected not only by its own configuration in terms of the number of members of a group and the resource distribution among its members, but also by the competing group’s configuration and the two groups’ relative strengths.

To illustrate, assume that two alliances, say A and B, are competing for supremacy. An increase in A’s military spending poses a greater threat to B’s security. In response to the heightened threat, states in B might feel the need to increase their military spending, which in turn could affect the military spending decisions of members of A.\(^2\) Interest group lobbying is another example. To understand a group’s lobbying efforts, scholars often focus only on the number and the size of firms within the group and their geographical distribution. However, the configuration of a competing interest group and the two groups’ relative strengths should also figure into an individual firm’s decision as to the appropriate contribution to its group’s lobbying efforts. The same conceptual framework is applicable to studies of party discipline, voter turnout (Led-


\(^2\)Bruce(1990) develops a model that endogenizes external threat in a three-country model with two allies and one adversary.
yard, 1981, 1984; Palfrey and Rosenthal, 1983), electoral coalitional politics (Tsebelis, 1990; Hausken, 1995a, 1995b), negative campaigning in primary elections (Newman and Niou, 2000), factional rivalry in civil wars, and joint ventures for research and development in industrial organizations. What unifies these examples is that they all point to the importance of external threat in explaining the logic of collective action.

In this paper, to study simultaneously the way inter-group competition and intra-group collective action interact, we develop a collective good model that endogenizes external threat. Once external threat is endogenized, two issues arise. First, in the context of competition between two groups, a group can be defensive or offensive in nature. We define a group as defensive if its fighting capacity can be used only to defend its own resources, and a group as offensive if it can use its fighting capacity to expropriate resources from its rivals as well as to protect its own resources. We realize that empirically it is difficult to determine whether a group is offensive or defensive. But theoretically it is useful to know how the offensive or defensive nature of a group can affect the collective action problem. We consider the following two cases: (i) group A is defensive but group B is offensive, and (ii) both A and B are offensive.

Second, if a group is offensive and if conquests are economically profitable, then the way spoils of victory are divided among group members can also affect collective action. To study the effects of different division rules on collective action, we consider proportional and equal division rules.

The organization of this paper is as follows. In the following section, we develop a collective good model which endogenizes external threat, assuming that one of the groups is offensive whereas the other is defensive. In Section 3, we study the collective
action problem when both groups are offensive. In Section 4, we analyze the effects of proportional and equal division rules on collective action. Section 5 summarizes our theoretical findings and compares them to the exploitation and suboptimality hypotheses established by Olson (1965). Section 6 concludes.

2 Defensive versus Offensive

Suppose groups A and B have $m$ and $n$ members, respectively. Let $i$ index members of A, $i = 1, \ldots, m$, and $j$ index members of B, $j = 1, \ldots, n$. Furthermore, suppose that: (a) each member $i$ of A has endowed resources $w_{iA}$, where $w_{iA} > 0$; (b) each $i$ divides his endowed resources between conflictual ($q_{iA}$) and productive ($w_{iA} - q_{iA}$) efforts; (c) depending on the technologies of production and conflict, endowed resources are converted into income and fighting force. For simplicity, we assume that the conversion rate is one to one. These assumptions are also applied to every $j$ in B. Furthermore, we assume that (d) every member in the system seeks to maximize his expected net income; and (e) fighting capabilities are not productive, and can only be used to defend members against aggression or to commit aggression against other groups (Viner, 1948; Hirshleifer 1991a, 1991b, 1995; Skaperdas, 1992, 1998).

Notationally, let

$$W_A = \sum_{i=1}^{m} w_{iA}, \quad W_B = \sum_{j=1}^{n} w_{jB}$$

be the total endowed resources groups A and B possess, respectively. Let
be the total amount of endowed resources A and B devote to fighting, respectively.\textsuperscript{3}

Suppose that A is defensive and that each member \(i\) in A has the following expected net income,

\[
I^i_A(q^i_A, Q_A, Q_B) = \frac{Q_A}{Q_A + Q_B}(w^i_A - q^i_A)
\]

(1) if \(Q_A > 0\) and \(Q_B \geq 0\). In addition \(I^i_A = w^i_A\) if \(Q_A = Q_B = 0\), and \(I^i_A = 0\) if \(Q_A = 0\) and \(Q_B > 0\).

Suppose that group B is offensive and that each member \(j\) in B has the following expected net income,

\[
I^j_B(q^j_B, Q_A, Q_B) = w^j_B - q^j_B + \left(\frac{Q_B}{Q_A + Q_B}\right)(q^j_B)(W_A - Q_A).
\]

(2)

The payoff functions (1) and (2) have a number of distinct features which need further discussion.

(i) In this paper, we assume that conflict between two groups can be resolved through either fighting or bargaining, and that each group member’s expected net income is positively associated with his group’s total fighting capabilities but nega-

\textsuperscript{3}In the literature on collective-good, other methods of aggregating individual members’ contributions have been proposed: the weakest-link technology, the best-shot technology, and the constant elasticity of substitution technology (see Hirshleifer, 1983).
tively associated with its adversary’s fighting capabilities. The specific function we use in this paper takes the following ratio form,

\[
\frac{Q_A}{Q_A + Q_B}.
\]

(ii) We assume that the resources devoted to fighting are sunk costs (Hirshleifer, 1991b and Skaperdas, 1992). A more general specification of the utility function would be to assume that only a fraction, \(\gamma\), \(0 \leq \gamma \leq 1\), of the resources devoted to fighting are sunk. To simplify our presentation we assume that \(\gamma = 1\). The theoretical results we establish in the following sections, however, hold for any \(0 < \gamma \leq 1\). If \(\gamma = 0\), then \(Q_A\) and \(Q_B\) are fully recoverable fixed costs. Then, trivially, all of the members in B allocate all their resources to fighting.

(iii) Equation (2) assumes that when two groups are competing, the spoil of victory for group B is the amount of resources group A devotes to productive purpose, \((W_A - Q_A)\). So the prize of conflict is endogenously determined. Alternatively, groups might be competing for a fixed prize (Katz, Titan, and Rosenberg, 1990; Esteban and Ray, 2001). The appropriateness of each assumption depends on the contexts of applications. In this paper, we adopt the former assumption and leave the latter one for future research.

(iv) Equation (2) specifies that the portion of A’s endowed resources \((W_A - Q_A)\) that member \(j\) expects to receive if B wins is determined by \(j\)’s share of B’s total spending on fighting, \(q_B^i/Q_B\). This assumption can be justified by the following argument that, during competition, group members compete with each other to acquire

\[\text{Other functional forms have been discussed in the literature on conflicts and rent-seeking. See Hirshleifer (1989, 1991b), Skaperdas (1992), and Neary (1997a, 1997b).}\]
the adversary’s resources, and their relative contributions to the conflict determine their relative successes. Similarly, if a conflict is settled through negotiation, we assume that relative fighting efforts determine the portion of \((W_A - Q_A)\) that \(j\) expects to gain. An alternative to the proportional division rule is to divide the spoils of victory equally among individual members regardless of their contribution to the group’s fighting effort (Esteban and Ray, 2001). In Section 4, we will compare the effects of different division rules on collective good provision.

(v) For each member \(i\) in the defensive group, equation (1) implies that an increase in \(q^i_A\) has two effects: it improves group A’s chance of winning and it reduces the amount of resources that \(i\) can allocate to production. The trade-offs between these two effects determine \(i\)’s optimal allocation of resources between production and fighting. But for each member \(j\) in the offensive group B, an increase in the amount of resources \(j\) devotes to fighting has an additional effect: it increases \(j\)’s share of the spoils of victory. In general, the trade-offs between these three effects determine \(j\)’s optimal resource allocation between production and fighting.

(vi) Players in our model simultaneously make their optimal once-and-for-all decisions between productive and conflictual allocations. In reality, players often can adjust their decisions in response to interaction outcomes. The task of determining the equilibrium in such a dynamic setting, for members of both groups, poses an extremely challenging analytic problem that we do not attempt to address in this paper.\(^5\)

We are now ready to determine the game’s Nash equilibrium. Each player chooses

\(^5\)Powell (1993) develops a dynamic game of this nature studying countries’ choices between guns and butter in two-country systems. See Hirshleifer (1995) for an exposition of different choices in modeling conflicts.
his fighting effort to maximize his net income in equation (1) or (2), subject to the resource constraint, and given the choices of other players. We focus on the interior solution for which resource constraints are not binding.

Taking the derivative of $I_A^i$ with respect to $q_A^i$ and setting the derivative equal to zero, we obtain the following first-order condition for an interior solution,

$$\frac{Q_B}{(Q_A + Q_B)^2} (w_A^i - q_A^i) - \frac{Q_A}{Q_A + Q_B} = 0. \tag{3}$$

It can be verified that the second-order condition for player $i$ is satisfied, and as such equation (3) is sufficient for player $i$’ optimization problem. Equation (3) implies that all of the members of group A keep the same amount of resources for production. An immediate implication of (1) and (3) is that all of the members of A receive the same payoff and that the wealthy members allocate more resources to fighting.

Aggregating equation (3) over $i = 1, \ldots, m$, we obtain

$$Q_B(W_A - Q_A) = mQ_A(Q_A + Q_B) \tag{4}$$

from which we can solve for $Q_A$ as $Q_A = R(Q_B, W_A, m)$, where

$$R(Q_B, W_A, m) = \sqrt{(1 + 1/m)^2Q_B^2/4 + (W_A/m)Q_B} - (1 + 1/m)Q_B/2.$$ 

This is the aggregate best-reply function for group A in response to the aggregate spending by group B. It can be verified that this best-reply curve is upward-sloping and concave and intersects the 45 degree line at $Q = W_A/(2m + 1)$. Moreover, the
best-reply function increases with $W_A$ and decreases with $m$.

Similarly, the first-order condition for members of the offensive group B is

$$\frac{Q_A + Q_B - q_B^j}{(Q_A + Q_B)^2}(W_A - Q_A) = 1,$$

which implies that all of the members of B spend the same amount on conflict. Since we assume the spoils of victory are distributed in proportion to a member’s contribution to the group’s fighting effort, this result also implies that all of the members of B expect to receive equal portions of the victory spoils.

Aggregating over $j$, we obtain

$$n(Q_A + Q_B)^2 = (nQ_A + (n-1)Q_B)(W_A - Q_A),$$  \hspace{1cm} (5)

which determines the aggregate best-reply function for the offensive group B. Equation (5) can be equivalently written as the following quadratic equation

$$Q_B^2 + [(3 - 1/n)Q_A - (1 - 1/n)W_A]Q_B + (2Q_A - W_A)Q_A = 0.$$  \hspace{1cm} (6)

Note that when $Q_A = W_A/2$, $Q_B = 0$, and that when $Q_A > W_A/2$, equation (6) does not have a positive solution. Thus, we restrict $Q_A \in [0, W_A/2]$, for which the quadratic equation has only one positive solution.

There are several properties of this positive solution. First, as $Q_A$ increases, the positive solution of (6) decreases, so that the aggregate best-reply curve for the offensive group B is downward-sloping. Second, as a.) the number of members of group B, $n$, increases; or b.) A’s total resources, $w_A$, increase, the quadratic function on the left-hand side of (6) decreases so that the positive solution of (6) increases,
and hence the best-reply curve for B shifts upward. Third, the best-reply curve for B intersects the 45 degree line at $Q = W_A(2n - 1)/(6n - 1)$.

In Figure I, we illustrate the aggregate best-reply curves for groups A and B, labeled as $R_A$ and $R_B$, for values $W_A = 100$ and $W_B = 60$, where the horizontal axis represents $Q_A$ and the vertical axis represents $Q_B$. $E_0$ is the equilibrium point for $m = n = 2$. As $m$ increases from 2 to 4, $R_A$ shifts toward left, but $R_B$ does not change, so the new equilibrium point (located at $E_1$) constitutes a drop in A’s aggregate fighting effort while B’s aggregate fighting effort rises. On the other hand, as $n$ goes up from 2 to 4, the $R_A$ curve does not change while $R_B$ shifts upward. The new equilibrium point is located at $E_2$ where both groups’ fighting efforts rise. When $m = n = 4$, the new equilibrium is located at $E_3$.

[Figure I here].

To summarize, the aggregate best-reply curve for the offensive group is downward-sloping and moves up as either $n$ or $W_A$ increases. On the other hand, the aggregate best-reply curve for the defensive group is upward-sloping, decreases with $m$, and increases with $W_A$. The intersection of these two curves determines the equilibrium levels resources devoted to conflictual efforts. Proposition 1 summarizes these general properties of the equilibrium in words.

**Proposition 1:** Suppose that i) only one of the groups is offensive; ii) external threat is endogenously determined; iii) a proportional division rule is used by members of the offensive group; and iv) resource constraints are not binding. Then the following results hold:
a) As the number of members of the defensive group increases, the defensive group spends less on fighting but the offensive group spends more;

b) all of the members of the defensive group receive the same payoff, but wealthy members allocate more resources to fighting;

c) as the number of members of the offensive group increases, both groups increase their spending on fighting;

d) all of the members of the offensive group devote the same amount of resources to fighting and expect to receive equal portions of the victory spoils.

The difference in spending on fighting for members of the defensive and offensive groups is determined by whether or not fighting capability is a pure public good. When $i$ in A unilaterally increases his fighting effort, it increases A’s chance of prevailing in the conflict. Thus, since spending on fighting is a pure public good, group members of A’s incentives to free-ride rise with $m$. But for each member $j$ of B, an increase in $j$’s spending on fighting not only increases B’s chance of winning, but also increases $j$’s own share of the spoils of victory. So $j$’s spending on fighting has a private good component which gives $j$ a stronger incentive to contribute to $B$’s fighting effort.

Another interesting feature of the equilibrium is that the defensive group spends less on fighting than does the offensive group whenever $m > 2n/(2n - 1)$. This can be easily demonstrated by comparing how each best-reply curve intersects with the 45 degree line. As long as the defensive group is not too small in size (i.e., $m > 2$ and $n > 1$), it always spends less than does the offensive group.
3 Offensive versus Offensive

In the previous section, we analyze collective action problems when only one of the groups is offensive. In this section, we investigate whether the properties established in Proposition 1 continue to hold when both groups are offensive in nature.

We assume that each member $i$ in $A$ has the following payoff function,

$$I^i_A(q^i_A, Q_A, Q_B) = \frac{Q_A}{Q_A + Q_B} \left( w^j_A - q^i_A + \frac{q^i_A}{Q_A} (W_B - Q_B) \right)$$  \hspace{1cm} (7)

if $Q_A > 0$ and $Q_B \geq 0$. In addition $I^i_A = w^j_A$ if $Q_A = Q_B = 0$, and $I^i_A = 0$ if $Q_A = 0$ and $Q_B > 0$. The payoff function for each member $j$ in $B$ can be specified analogously,

$$I^j_B(q^j_B, Q_A, Q_B) = \frac{Q_B}{Q_A + Q_B} \left( w^j_B - q^j_B + \frac{q^j_B}{Q_B} (W_A - Q_A) \right)$$  \hspace{1cm} (8)

if $Q_A \geq 0$ and $Q_B > 0$. Furthermore, $I^j_B = w^j_B$ if $Q_A = Q_B = 0$, and $I^j_B = 0$ if $Q_A > 0$ and $Q_B = 0$.

Taking the derivative of $I^i_A$ with respect to $q^i_A$ and setting the derivative equal to zero, we obtain the following first-order condition for an interior solution,

$$\frac{Q_B}{(Q_A + Q_B)^2} \left( w^j_A - q^i_A + \frac{q^i_A}{Q_A} (W_B - Q_B) \right) - \frac{Q_A}{Q_A + Q_B} + \frac{Q_A - q^i_A}{(Q_A + Q_B)Q_A} (W_B - Q_B) = 0.$$  \hspace{1cm} (9)

It can be verified that $I^i_A$ is strictly concave in $q^i_A$ so that (9) is sufficient for player $i$’s optimality.

Aggregating equation (9) over $i = 1, \ldots, m$, we obtain
\[
\frac{Q_B}{(Q_A + Q_B)^2} (W_A + W_B - Q_A - Q_B) - \frac{mQ_A}{Q_A + Q_B} + \frac{m - 1}{Q_A + Q_B} (W_B - Q_B) = 0,
\]

or equivalently,

\[
-m(Q_A + Q_B)^2 + (m - 1)W_B(Q_A + Q_B) + (W_A + W_B)Q_B = 0. \quad (10)
\]

This equation implicitly determines the aggregate spending by group A, \( Q_A \), in response to the aggregate spending by group B, \( Q_B \).

Analogously, the first-order conditions for members of B yield

\[
-n(Q_A + Q_B)^2 + (n - 1)W_A(Q_A + Q_B) + (W_A + W_B)Q_A = 0, \quad (11)
\]

which determines the aggregate spending by group B in response to the aggregate spending by group A.

If there exists an interior solution for which all resource constraints are not binding, then the equilibrium levels of conflictual spending, \( \hat{Q}_A \) and \( \hat{Q}_B \), are simultaneously determined by equations (10) and (11) as follows:

\[
\hat{Q}_A = \frac{(W_A/m + W_B/n)(W_B - \beta W_A)}{(1 - \beta)^2(W_A + W_B)}, \quad \hat{Q}_B = \frac{(W_A/m + W_B/n)(W_A - \beta W_B)}{(1 - \beta)^2(W_A + W_B)}, \quad (12)
\]

where \( \beta = 1 - 1/m - 1/n \). Notice that when the resource constraints are not binding,
\( \hat{Q}_A \) and \( \hat{Q}_B \) depend on \( m, n, W_A, \) and \( W_B, \) but not on the internal distribution of resources within each group. Furthermore, \( \hat{Q}_A \leq \hat{Q}_B \) if and only if \( W_A \geq W_B. \) This implies that the resource-laden group spends less on fighting than does the resource-poor group. From (11), the total spending by the two groups can be written as

\[
\hat{Q}_A + \hat{Q}_B = \frac{nW_A + mW_B}{m + n},
\]

which is a linear combination of \( W_A \) and \( W_B. \)

An immediate implication of (12) is that if \( m = n = 1, \) then \( \hat{Q}_A = \hat{Q}_B = (W_A + W_B)/4. \) In this case, both groups spend the same amount of resources on fighting, independent of the initial resource distribution and conditional on the resource constraints being nonbinding. This is consistent with the result obtained by Tullock (1980) in a rent-seeking model where two individuals compete for a single, private good prize. Hirshleifer (1989, 1991b) discusses a similar model of conflicts and rent-seeking.

When a group has more than one member, the collective action problem within the group affects members’ incentives to spend on fighting. As implied by equation (12), when A and B have equal initial resources, the two groups’ total marginal gains and marginal costs are symmetric. Thus, in equilibrium, both groups’ fighting efforts are the same and independent of group size.\(^6\) When the endowed resources of the two group are unequal, however, we show that the resource-laden group actually spends less on fighting than does the resource-poor group. Furthermore, as the number of members of the resource-laden group increases, its total spending on fighting

\(^6\)See Katz, Titan, and Rosenberg (1990) for a similar result in a rent-seeking model where two groups compete for pure public goods.
decreases, while the resource-poor group's spending increases.

To illustrate, in Figure II, we show the two groups’ aggregate best-reply curves for values of $W_A = 100$, $W_B = 60$, and $m = n = 2$. Both curves are initially increasing and then downward-sloping. They intersect at $E_0$, which determines the aggregate equilibrium spending by the two groups. As group A’s size increases from 2 to 4, the new equilibrium point $E_1$ indicates that group A spends less while group B spends more than before. But as the less endowed group, B, increases its size from 2 to 4, the new equilibrium point $E_2$ indicates that group B’s aggregate spending on conflict increases while A’s spending decreases.

[Figure II here].

Thus far, we have shown that when both groups are offensive, members of the better endowed group behave differently than members of the less endowed group. To understand individual members’ strategic thinking, in the following, we compute individual members’ spending on fighting and expected net income in equilibrium.

Using the first-order condition (9), we can calculate member $i$’s fighting effort as

$$\hat{q}_{iA} = w_i^A \hat{Q}_B/W_B + (\hat{Q}_A + \hat{Q}_B)(W_B - \hat{Q}_A - \hat{Q}_B)/W_B. \quad (13)$$

When resource constraints are nonbinding, the equilibrium expected net income for member $i$ in A can be calculated from equations (7) and (13) as

$$\hat{U}_{iA} = (W_B - \hat{Q}_A - \hat{Q}_B)^2/W_B + (W_B - \hat{Q}_B)w_i^A/W_B. \quad (14)$$
A similar solution can be derived for members of group B.

It follows from equation (13) that an individual member’s equilibrium spending on conflict is a linear function of his initial resources. Thus, in general, the wealthy in a group allocate more resources to fighting than the poor. But in terms of the percentage of resources that a country allocates to fighting, the wealthy in the better endowed group spend more than the poor while the opposite is true for the less endowed group.

In Proposition 2, we summarize the theoretical findings established in this section.

**Proposition 2**: Suppose that i) both groups are offensive; ii) external threat is endogenously determined; and iii) a proportional division rule is used to distribute the spoils of victory. The following conclusions hold:

(a) If the two competing groups have equal resource endowments, their fighting efforts are also the same and independent of group size. Furthermore, members within each group spend the same percentage of their resources on fighting.

(b) If the two competing groups’ endowed resources are unequal, the resource-laden group actually spends less on fighting than does the resource-poor group. Furthermore, wealth members of the better endowed group allocate a higher percentage of their resources to fighting than the poor, while the wealthy in the less endowed group allocate a lower percentage of their resources to fighting.

Before we turn our attentions to the effects of division rules on collective action problems in the next section, we present an interesting and unexpected finding based
on the results in Sections 2 and 3. Intuitively, it seems that it would always benefit a group to be offensive when it faces an offensive opponent. The following example shows that this claim is not always true. Suppose that \((w_A^1, w_A^2) = (55, 45)\) and \((w_B^1, w_B^2) = (35, 25)\).

**Case 1: Defensive versus Offensive:** We use the result in Section 2 to calculate the equilibrium fighting efforts and net incomes as follows.

A: Defensive : \(\left( Q_A = 22.65, (q_A^1 = 16.32, I_A^1 = 16.01), (q_A^2 = 6.32, I_A^2 = 16.01) \right)\)

B: Offensive : \(\left( Q_B = 32.04, (q_B^1 = 16.02, I_B^1 = 41.64), (q_B^2 = 16.02, I_B^2 = 31.64) \right)\)

**Case 2: Offensive versus Offensive:** We use the results in this section to calculate the equilibrium fighting efforts and net incomes.

A: Offensive : \(\left( Q_A = 30, (q_A^1 = 19.17, I_A^1 = 15.83), (q_A^2 = 10.83, I_A^2 = 14.17) \right)\)

B: Offensive : \(\left( Q_B = 50, (q_B^1 = 26.50, I_B^1 = 28.50), (q_B^2 = 23.50, I_B^2 = 21.50) \right)\)

This example shows that, for members of the defensive group, it is not to their benefit to become offensive even if their opponent is offensive. To endogenize group type, however, is beyond the scope of this paper.

# 4 Effects of the Division of the Spoils of Victory

Olson and Zeckhauser (1966) argue that “The problem of disproportionality and suboptimality in international organizations ... should be met instead through insti-
tutional changes that alter the pattern of incentives” (pp.278-9). In this section, we pursue this suggestion by analyzing the effects of different division rules on fighting efforts.

An alternative to the proportional division rule employed in the previous section would be to divide the spoils of victory equally among individual members regardless of their contribution to the group’s fighting effort. The imperial partition of China at the end of the 19th century provides an illustrative example of this equal division rule in action. The larger imperial powers (Britain and Russia) were to get larger shares, while smaller shares would go to Japan, Germany, France, and possibly the United States. As a late comer to China, the United States did not support the proportional division rule. Instead, the United States advocated an “open door” policy which allowed that each imperial power should have equal access to China’s market and resources. Eventually, the other imperial powers accepted the “open door” policy.

In this section, we first derive the equilibrium results when both groups use equal division; then, we derive the equilibrium results when one of the groups uses proportional division while the other group uses equal division. Lastly, we study the conditions under which a group would prefer the proportional to the equal division rule.

Under the equal division rule, the expected net income for $i$ in $A$ is

$$I^i_A(q^i_A, Q_A, Q_B) = \frac{Q_A}{Q_A + Q_B} \left( w^i_A - q^i_A + \frac{W_B - Q_B}{m} \right).$$

(15)

Analogously, the expected net income for $j$ in $B$ is

$^7$See Moulin (1996) for other specifications of division rules.
\[ I_B(q_B^i, Q_A, Q_B) = \frac{Q_B}{Q_A + Q_B} \left( w_B^i - q_B^i + \frac{W_A - Q_A}{n} \right). \]  

(16)

In equilibrium, \( i \) chooses \( q_A^i \) to maximize his payoff in (15) subject to resource constraints \( 0 \leq q_A^i \leq w_A^i \), and given the choices of other group members. Analogously, \( j \) chooses \( q_B^j \) to maximize his payoff in (16) subject to resource constraints \( 0 \leq q_B^j \leq w_B^j \), and given the choices of other group members. We focus on the interior solution in which resource constraints are not binding. Taking the derivative of \( I_A^i \) with respect to \( q_A^i \) and setting the derivative equal to zero, we obtain the following first-order condition for an interior solution

\[
\frac{Q_B}{(Q_A + Q_B)^2} \left( w_A^i - q_A^i + \frac{W_B - Q_B}{m} \right) - \frac{Q_A}{Q_A + Q_B} = 0. \]  

(17)

It can be easily verified that the second-order condition is satisfied. Aggregating equation (17) over \( i = 1, \ldots, m \), we obtain

\[
-m(Q_A + Q_B)^2 + (m - 1)Q_B(Q_A + Q_B) + (W_A + W_B)Q_B = 0. \]  

(18)

Analogously, the first-order condition for members of B yields

\[
-n(Q_A + Q_B)^2 + (n - 1)Q_A(Q_A + Q_B) + (W_A + W_B)Q_A = 0. \]  

(19)

Figure III illustrates the two aggregate best-reply curves for values of \( W_A = 100 \), \( W_B = 60 \), and \( m = n = 2 \), which are symmetric and upward-sloping. The equilibrium point is located at \( E_0 \). Note that each group’s best reply curve falls with an increase
in its own size. For instance, as $m$ goes up from 2 to 4, the $R_A$ curve shifts toward left while $R_B$ remains the same, and the new equilibrium point is located at $E_1$. The new equilibrium point shows that both groups spend less on fighting than before, and the group with more members, group A, spends less than group B.

[Figure III here].

In the interior solution, $\tilde{Q}_A$ and $\tilde{Q}_B$, are determined by (18) and (19) and computed as follows

$$
\tilde{Q}_A = \frac{\sqrt{n}(W_A + W_B)}{(\sqrt{n} + \sqrt{m})(\sqrt{mn} + 1)}, \quad \tilde{Q}_B = \frac{\sqrt{m}(W_A + W_B)}{(\sqrt{m} + \sqrt{n})(\sqrt{mn} + 1)}.
$$

(20)

From equations (17) and (20), each member’s fighting effort, $\tilde{q}^i_A$, can be computed as

$$
\tilde{q}^i_A = w^i_A - \frac{(2\sqrt{mn} + 1)W_A - \sqrt{mn}W_B}{m(\sqrt{mn} + 1)}.
$$

(21)

The equilibrium payoffs can be calculated from (15)-(16) and (20)-(21), and are given by

$$
\tilde{U}_A^i = \frac{n(W_A + W_B)}{(\sqrt{mn} + m)(\sqrt{mn} + 1)}, \quad \tilde{U}_B^j = \frac{m(W_A + W_B)}{(\sqrt{mn} + n)(\sqrt{mn} + 1)},
$$

(22)

which are independent of $i$ and $j$, respectively.

In Proposition 3, we summarize the implications of these equilibrium results.

**Proposition 3:** Suppose that i) both groups are offensive; ii) external threat is endogenously determined; and iii) both use an equal division rule to divide the
spoils of victory. Then for any m, n, Wa, and Wb, the following conclusions hold:

a) Wealthy group members allocate a higher percentage of their resources to fighting, but receive the same expected net incomes as less fortunate group members.

b) Both groups’ fighting efforts decrease as either group’s size increases.

c) The group with more members allocates less resources to fighting, and its members receive lower expected net incomes than members of the smaller group.

d) A group member’s expected net income is negatively associated with the size of its own group, but positively associated with the size of the competing group.

Proposition 3 characterizes the equilibrium results when both groups adopt the equal division rule. And in the previous section, Proposition 2 characterizes the equilibrium results when both groups adopt the proportional division rule. If we want to study group members’ preferences over different division rules, we need to establish the equilibrium results when one of the groups uses the proportional division rule while the other adopts the equal division rule. Suppose A is defensive and B offensive. The equilibrium fighting efforts for A and B are determined by (10) and (19). Accordingly, we can calculate the equilibrium payoffs. To simplify the analysis, we consider only the case in which \( m = n = 2 \).

The equilibrium expected net income for a member with resource-endowment \( w \) in A is
Table 1: Proportional versus Equal Division Rules: Symmetric Resource Distribution

\[ w_A^1 = w_A^2 = 50 \text{ and } w_B^1 = w_B^2 = 30 \]

<table>
<thead>
<tr>
<th>( \backslash )</th>
<th>e</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>(26.67, 26.67); (26.67, 26.67)</td>
<td>(14.25, 14.25); (26.90, 26.90)</td>
</tr>
<tr>
<td>p</td>
<td>(27.43, 27.43); (20.47, 20.47)</td>
<td>(15.00, 15.00); (25.00, 25.00)</td>
</tr>
</tbody>
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\[
I_{pe}(W_A, W_B, w) = \frac{(3W_A + 6W_B - \Delta)^2}{16W_B} + \frac{(-13W_A - 10W_B + 3\Delta)w}{4W_B}
\]

and the equilibrium expected net income for a member of B is

\[
I_{ep}(W_A, W_B) = \frac{(13W_A + 14W_B - 3\Delta)^2}{16(-4W_A - 4W_B + \Delta)}.
\]

Given the equilibrium payoffs for individual members under each pair of division rules, we are now ready to study group members’ preferences over the two division rules. Consider the following example: \( w_A^1 = w_A^2 = 50 \) and \( w_B^1 = w_B^2 = 30 \). The payoffs are presented in Table 1.

Notice that this is a prisoners’ dilemma game: \( I_{pe} > I_{ee} > I_{pp} > I_{ep} \) for members of both groups and \((p, p)\) is the game’s unique pure strategy Nash equilibrium.

As our example illustrates when the distribution of resources is relatively symmetric, proportional division remains the dominant strategy for members of both groups. But when resource distribution becomes relatively asymmetric, proportional division...
Table 2: Proportional versus Equal Division Rules: Asymmetric Resource Distribution

\[(w_A^1, w_A^2) = (60, 40) \text{ and } (w_B^1, w_B^2) = (35, 25)\]

<table>
<thead>
<tr>
<th>(w_A^1)</th>
<th>(w_A^2)</th>
<th>(w_B^1)</th>
<th>(w_B^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group B</td>
<td>60</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>Group A</td>
<td>40</td>
<td>60</td>
<td>25</td>
</tr>
</tbody>
</table>

In Olson (1965), the number of members of a group and the distribution of resources among them together explain the provision (or lack thereof) of collective goods, stylized as the “suboptimality” and “exploitation” hypotheses. In the context of rivalry between two groups we show that, in addition to these two factors, the configuration of the competing group and the relative strength of the two groups may also affect the provision of collective goods. In this section, we highlight five results and compare them to the suboptimality and exploitation hypotheses.
Result 1: The exploitation and the suboptimality hypotheses are valid for the defensive, but not the offensive, group. All members of the offensive group spend the same amount on fighting and expect to receive equal portions of the victory spoils. So, neither the wealthy nor the poor group members are exploited. Furthermore, as the number of members increases in the offensive group, its total fighting effort increases. Though Result 1 is consistent with the empirical finding on budget sharing in NATO conducted by Olson and Zeckhauser (1966), since NATO is considered a defensive alliance, this result, however, highlights the need to conduct empirical tests of collective action problems within the offensive group.

Result 2: It does not always pay to be offensive. When competing with an offensive group, members of the defensive group do not necessarily benefit from becoming offensive. This is due to the fact that, unlike members of an offensive group, members of a defensive group do not expect to receive any income from victory. If the group decides to become offensive, the total amount of resources that each group must devote to fighting increases, which lowers the amount of resources available for redistribution at the conflict’s end. Thus, members of a defensive group might receive a lower payoff if they become offensive.

Result 3: Olson’s results do not hold for the resource-laden group when both groups use the proportional division rule. If both groups are offensive and both use equal division, individual group members’ spending on conflict is a pure public good, and the exploitation hypothesis continues to hold for members of both groups. But if they use proportional division rule, the incentives of members of resource-poor group change. Instead of free riding on the wealthy, they have incentives to devote a higher percentage of their resources to fighting in order to earn a larger
portion of the potential victory spoils. Likewise, the suboptimality hypothesis is no longer valid for members of the resource-poor group if the proportional division rule is used.

**Result 4: Free riding can be Pareto improving.** In Section 4, we show that the equal division rule induces more severe free riding than does the proportional division rule, which results in lower spending on fighting. Lower conflict spending by one group implies a lesser external threat to the other group. In response, the other group can reduce its own spending on conflict. In equilibrium, equal division leads to lower spending on fighting and higher payoffs for members of both groups.8

**Result 5: Paradox of power.** If the proportional division rule is used, in equilibrium, the resource-poor group allocates more resources to fighting, has a higher probability of winning, and higher total payoffs than the resource-laden group. This finding is consistent with Hirshleifer’s finding that “No. 2 tries harder.” (Hirshleifer, 1991b) However, our paradox of power is even more paradoxical than the one discovered by Hirshleifer (1991a). Hirshleifer shows that contending parties can end up with identical incomes regardless of the two players’ initial resource ratio (p.182). Our result shows that the resource-poor group may get even more expected net income than the resource-laden group. Since the two contending groups are not unitary actors and since the spoils of victory are divided proportionally to each member’s contribution to the group’s fighting efforts, members of the poor group have stronger incentives to allocate resources to fighting because the spoils of victory are relatively greater than

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8This result is consistent with the finding in Bruce (1990). He shows that “a cooperative treaty to increase defense spending in an alliance may actually make it worse off. Further, even where the treaty makes an alliance better off, ceteris paribus, such treaty making may constitute a form of the prisoner’s dilemma for the world as a whole with both sides worse off than in the absence of cooperative treaties.”
for members of the rich group. For members of the resource-laden group to improve their payoffs, they must increase the degree of intra-group cooperation in order to curtail free riding. The group’s total payoff reaches its maximum when its members behave as a unitary actor.

6 Conclusion

In this paper, we study intra-group collective action problems when two groups are competing. Conceptually, our model fits into the two-level game framework proposed by Putnam (1988) or Tsebelis’ nested game approach (1990). Our model greatly enriches understanding of collective action problems because it expands the domain of the theory to include factors such as the configuration of the competing group, the relative strength of the two groups, the defensive or offensive nature of the groups, and the division rules groups use to divide the spoils of victory. It is our hope that these theoretical findings provide new directions to future empirical research on collective action.
References


Figure I: The Aggregate Best-replies: Defensive (A) vs. Offensive (B)
Figure II: The Aggregate Best-replies: Offensive vs Offensive
Figure III: The Aggregate Best-replies: The Case of Equal Sharing