Preferences with Taste Shock Representations: Price Volatility and the Liquidity Premium*

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Abstract

If price volatility is caused in some part by taste shocks, then it should be positively correlated with the liquidity premium. Our argument is based on Krishna and Sadowski (2014), who provide foundations for a representation of dynamic choice with taste shocks, and show that volatility in tastes corresponds to a desire to maintain flexibility. To formally connect volatile tastes to price volatility and preference for flexibility to the liquidity premium, we analyze a modified simple Lucas tree economy, where the representative agent is uncertain about his degree of future risk aversion, and where the productive asset cannot be traded in every period, while rights to output can. We show that a representative agent with a higher degree of uncertainty about his future risk aversion implies a higher liquidity premium (ie a lower price for the illiquid asset) and more price volatility.

1. Introduction

Following Black (1987), a growing literature has argued that taste shocks are important for our understanding of business cycles and asset prices. For example, Smith and Whitelaw (2009) find evidence that the largest component of changes in the equity risk premium is variation in risk aversion, rather that the amount of risk, and

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Bekaert, Engstrom and Grenadier (2010) show that stochastic risk aversion that is not driven by, or perfectly correlated with, the fundamentals of the economy can simultaneously explain a range of asset pricing phenomena as well as the behavior of bond and stock markets.

At the same time, a powerful critique of the use of taste shocks in empirical work notes that taste shocks are typically not directly observable, so that they become free parameters. As Nason (1997) writes, 'for taste shocks to have economically meaningful content, they must be grounded solidly in economic theory and tied to observable phenomena, which is not always easy.'

In this paper we build on the axiomatically founded model of taste shocks in Krishna and Sadowski (2014) (henceforth KS), in which those shocks can be uniquely identified from observable behavior. Based on this identification, KS provide comparative statics that link more volatile tastes to more preference for flexibility, that is, a desire not to commit to future choice ahead of time. We argue that more volatile tastes should correspond to higher price volatility, and a desire to maintain flexibility should lead to a higher liquidity premium.

To formally make this argument in a very simple model, we consider a representative agent, who is modeled as in KS and receives iid shocks to his aversion to risk in current consumption. We then analyze a small Lucas tree economy with closed asset markets but open goods markets, that is, we enrich the most basic Lucas tree economy by a market stage in which only goods (or output), but not shares in the productive asset can be traded. Our main result establishes that, indeed, greater preference for flexibility corresponds to higher price volatility and drives a larger liquidity premium (ie, a lower price of the productive asset). This direct link between price volatility (which the taste shock models help explain) and the liquidity premium could provide discipline for the use of taste shock models in applied work.

The basic intuition for the relationship between volatility in tastes and in prices is as follows. An agent with iid shocks values his shares in the productive asset independently of his current realized risk aversion. At the same time, current realized risk aversion obviously affects his valuation of uncertain current output. The price for trading shares of the asset against rights to current output should be determined by the difference in these valuations. Hence, more variation in risk aversion should imply more variation in prices. This intuition is incomplete, because the equilibrium price in the Lucas model depends on the derivative of the representative agent's utility in the realized output of the productive asset (from the first order condition that ensures that at the equlibrium price all output is consumed). We solve this discrepancy between intuition and model with a simple trick: We interpret shares as probabilistic rights to the entire output, so that utility becomes linear in the share for any given level of output.[1]

Now suppose that ownership of the productive asset constrains liquidity as suggested above. We expect preference for flexibility to be associated with a tendency to invest less in the productive asset, as such investment reduces liquidity. In order to capture this intuition in our simple Lucas tree economy, there must be periods where the productive asset, the tree, is not tradeable. Consider the following environment: The tree produces a perishable good twice a day. The first output happens during the daytime, when *domestic* asset markets are open and the probabilistic right to current output can be traded for future shares in the tree as in the basic Lucas tree economy.

The market structure of the economy during the night is more complicated: Production happens with probability 1/2 before and with probability 1/2 after midnight (when ownership rights change). That is, rights to the tree's output are equally likely to be determined by the day's right to output as by shares in the tree. This specification ensures that, in terms of expected received output during the night, it is as good to own rights to the day's output as it is to own shares. In addition, we assume that *international* goods markets are open only until midnight, when ownership rights have not yet changed. On those markets, domestic assets cannot be traded, but probabilistic rights to different levels of output can be traded at a fixed price. Therefore, ownership rights to output provide the agent with more flexibility or 'liquidity' than shares of the productive asset do.

The remainder of the paper is structured as follows. Section 2 introduces the representation as well as the comparative statics from KS. Section 3 analyzes the standard Lucas tree economy without liquidity concerns and relates volatility in tastes to volatility in prices. Section 4 adds liquidity concerns to this economy and presents our main theorem.

2. Taste Shock representation

Let K be a finite set of prizes with typical member k. We follow Gul and Pesendorfer (2004) (henceforth GP) in defining an infinite horizon consumption problem (IHCP) as a collection of lotteries that yield a consumption prize in the present period and a

(1) Since our focus is the interplay between taste shocks and liquidity concerns, we chose to keep the model as simple as possible in all other respects. A less immediate, more common way to ensure that higher utility correspond to a higher price would be an appropriate single crossing property on the slope of u.

new infinite horizon problem starting in the next period. Let Z be the collection of all IHCPs.[2] GP show that Z is a compact metric space, and that each $z \in Z$ can be identified with a compact set of probability measures over $K \times Z$. For the compact metric space $K \times Z$, let $\mathcal{P}(K \times Z)$ denote the space of probability measures endowed with the topology of weak convergence, so that $\mathcal{P}(K \times Z)$ is compact and metrizable. Let $\mathcal{F}(\mathcal{P}(K \times Z))$ denote the space of closed subsets of $\mathcal{P}(K \times Z)$, endowed with the Hausdorff metric, such that $\mathcal{F}(\mathcal{P}(K \times Z))$ is a compact metric space. It can be shown that Z is linearly homeomorphic to $\mathcal{F}\big(\mathcal{P}(K \times Z)\big)$. We shall denote this linear homeomorphism as $Z \simeq \mathcal{F}\big(\mathcal{P}(K \times Z)\big)$. Typical elements $x, y, z \in Z$ are interpreted as ***menus*** of lotteries over consumption and continuation problems.

KS provide representations of choice over Z with the understanding that DM will choose from the IHCP he faces in every subsequent period. Take p, q to be typical lotteries in $\mathcal{P}(K \times Z)$. By the recursive nature of Z, continuation problems are members of Z. When there is no risk of confusion, we identify prizes and continuation problems with degenerate lotteries and lotteries with singleton menus.

As in KS, let $\mathcal{U} := \{u \in \mathbb{R}^K : \sum u_i = 0\}$ be the set of all vN-M utility functions over instantaneous consumption (ie, over K) that are identified up to a constant. The subjective state space relevant for the taste shock representation is $\mathcal{U}$. To ensure that expected consumption utility under a measure μ is well defined, the measure μ must be ***nice***, ie, must satisfy $\mu u_k := \int_{\mathcal{U}} \mu u_k \,\mathrm{d}\mu(u)$ is finite for each $k \in K$, which is to say the expected utility from each prize is finite.

Subjective states $u \in \mathcal{U}$ are naturally interpreted as consumption utilities, and the two terms are treated as synonyms. Similarly, in what follows, all probability measures are interpreted as subjective beliefs, and the two terms are used interchangeably. For $p \in \mathcal{P}(K \times Z)$, let p_k and p_z denote the marginal distributions on K and Z respectively.

Definition 2.1. Let $\mathcal{U}$ be defined as above, μ a nice probability measure on (the Borel σ-algebra of) $\mathcal{U}$, and $\delta \in (0, 1)$. We say that $\succsim$ has a ***taste shock*** representation, (μ, δ), if there exists a continuous function $V : Z \to \mathbb{R}$, linear on Z, that satisfies

$$V(x) = \int_{\mathcal{U}} \max_{p \in x} \big[u(p_k) + \delta V(p_z)\big] \,\mathrm{d}\mu(u) \tag{2.1}$$

and represents $\succsim$.

In the presentation above, $u(p_k) = \sum_{k' \in K} p_k(k')u(k')$, and $V(p_z)$ is the extension of V to $\mathcal{P}(Z)$ by linearity (and continuity), ie $V(p_z) = \int_Z V(z') \,\mathrm{d}p_z(z')$. KS

(2) See GP for the recursive construction of Z.

provide axioms on a preference relation, $\succsim$, that are equivalent to the existence of a taste shock representation, (μ, δ), of $\succsim$. Moreover, they establish that δ is unique and μ is unique up to scaling (Theorem 1 in KS).

Definition 2.2. Two probability measures μ and μ' on $\mathcal{U}$ are ***identical up to scaling,*** if there is $\lambda > 0$ such that $\mu(E) = \mu'(\lambda E)$ for all measurable $E \subset \mathcal{U}$, where $\lambda E := \{\lambda u : u \in E\}$.

Preference for flexibility is the preference for non degenerate menus over singletons. Intuitively, one DM has more preference for flexibility than another if she has a stronger preference for menus over singletons. To make this precise requires us to consider the restricted domain $L \subset Z$ of ***Infinite Horizon Consumption Streams (IHCSs)*** . This domain consists of lotteries that deliver consumption for the present period and an IHCS for the next period. It is easy to show that L is a closed and convex subset of Z. In a manner analogous to the characterisation of risk aversion where lotteries are compared to certain amounts of money, characterizing preference for flexibility requires a comparison between IHCPs and IHCSs. This comparison is meaningful only if the preference restricted to L is non-trivial, ie, there exist $\ell, \ell' \in L$ such that $\ell \succ \ell'$. We refer to this property of $\succsim$ as ***Consumption non-triviality***. If $\succsim$ has a taste shock representation, KS show that $\succsim$ satisfies Consumption non-triviality if, and only if, $\mu u \neq \mathbf{0}$.

Definition 2.3. $\succsim^*$ has a ***greater preference for flexibility*** than $\succsim$ if

$$x \succsim \ell \quad \text{implies} \quad x \succsim^* \ell$$

for all $\ell \in L$ and $x \in Z$.

Note that the comparison in the definition requires that $\succsim$ and $\succsim^*$ rank IHCSs the same; that is, $\ell \succsim \ell'$ if, and only if, $\ell \succsim^* \ell'$.

Definition 2.4 (Dilation)**.** Let $Q(u, D)$ be a Markov kernel from $\mathcal{U}$ to itself. Then $Q(u, D)$ is a ***dilation*** if it is expectation preserving, ie, for each $u \in \mathcal{U}$, $\int_{\mathcal{U}} u' \, Q(u, \mathrm{d}u') = u$. If μ and μ^* are probability measures on $\mathcal{U}$, then μ^* is a ***dilation*** of μ if there exists a dilation Q, such that $\mu^* = Q\mu$, ie, $\mu^*(\mathrm{d}u') := \int Q(u, \mathrm{d}u') \, \mu(\mathrm{d}u)$.

The taste shock representation (μ, δ) only identifies the measure μ up to scaling. In order to facilitate a comparison of measures, we shall say that a taste shock representation (μ, δ) is ***canonical*** if $\|\mu u\|_2 = 1$. Obviously, $\succsim$ admits a canonical taste shock representation if, and only if, $\mu u \neq \mathbf{0}$ if, and only if, $\succsim$ satisfies Consumption non-triviality.

Theorem 1 (Theorem 2 in KS). *Let $\succsim$ and $\succsim^*$ have canonical taste shock representations (μ, δ) and (μ^*, δ^*), respectively. Then, the following are equivalent:*
(a) *$\succsim^*$ has a greater preference for flexibility than $\succsim$.*
(b) *$\delta = \delta^*$ and μ^* is a dilation of μ.*

3. A Lucas Tree Economy

Consider a version of the Lucas tree after Lucas (1978). There is an economy with a representative agent and one productive asset. The asset produces $\omega \geqslant 0$ units of perishable output, or dividends, in each period. For simplicity, we assume that output is distributed identically and independently over time, according to the distribution $F(\omega)$ with finite support. In period 0 there is no production and no trade. Consider an agent whose period 0 preferences have a taste shock representation of choice over IHCPs, where the support of F is in K.

The agent has $z \in [0, 1]$ shares in the asset, which gives him a proportional right to the output. Specifically, with probability z he gets all the output, and with complementary probability, he gets none of the output. Let $\mathcal{U}^\dagger$ be the set of states he considers possible, ie, states in the support of μ, where we normalize $u(0) = 0$ for all $u \in \mathcal{U}^\dagger$.

There is a market where the agent can trade the probability q of getting all of the output and shares of the right to future output. The price of a unit of q is normalized to 1 in each state (ω, u), while the price of a share is $p(\omega, u)$. We assume that the agent has a taste shock representation, such that the value of holding z shares in the asset is

$$v(z, \omega, u) = \max_{q,x} \Big[u\big([q; \omega]\big) + \delta \iint v(x, \omega', u') \,\mathrm{d}F(\omega') \,\mathrm{d}\mu(u') \Big]$$

subject to

$$q + p(\omega, u)x \leqslant z + p(\omega, u)z$$

where $[q; \omega]$ is the the lottery that gives ω with probability q and 0 with probability $1 - q$. Then, $u\big([q; \omega]\big) = qu(\omega) + (1 - q)u(0)$.

By following the arguments in Lucas, we can show that for each continuous $p(\omega, u)$, there exists a unique continuous, bounded, nonnegative function $v(z, \omega, u)$ that satisfies the Bellman equation above, and which is concave in z.

We know that in equilibrium, we must have $q = z + p(\omega, u)z - p(\omega, u)x$, so that $u\big([q; \omega]\big) = u(\omega)\big[z + p(\omega, u)z - p(\omega, u)x\big]$. Still following Lucas, we can show that the pricing function $p(\omega, u)$ takes the form

$$\begin{aligned} p(\omega, u)u(\omega) &= \frac{\delta}{1-\delta} \iint u(\omega')\, \mathrm{d}F(\omega')\, \mathrm{d}\mu(u') \\ &= \frac{\delta}{1-\delta}\, \mathbf{E}[u; F, \mu] =: \Lambda(F, \mu). \end{aligned}$$

Note that one can just as easily normalize the price of a share to be 1 in each state (ω, u), in which case the price of a unit of q (the probability for immediate consumption) becomes $\psi(\omega, u) = 1/p(\omega, u)$. In what follows, it is more natural to work with the price $\psi(\omega, u)$. The solution to the pricing equation can thus be rewritten as

$$\psi(\omega, u) = \frac{u(\omega)}{\Lambda}$$

Suppose that there are only three levels of output, 0, 1/2 and 1. We are now in a position to relate the distribution of prices to the distribution of utilities. Consider two exchange economies, A and B, with representative agents A and B respectively. We assume that both agents have period 0 preferences with a taste shock representation based on state spaces $\mathcal{U}_i^\dagger$, where $u(0) = 0 \leqslant u\,(1/2) \leqslant u(1) = 1$ for all $u \in \mathcal{U}_i^\dagger$, $i = A, B$. Thus, agents are uncertain about their risk aversion, which is captured by $u(1/2)$. We also assume that $\succsim^A$ and $\succsim^B$ agree on the intertemporal tradeoff for getting 1 instead of 0, which implies $\delta_A = \delta_B$.

Thus, for $\omega \in \{0, 1/2, 1\}$, prices are given by

$$\begin{aligned} \psi_i(0, u) &= 0 \\ \psi_i(1, u) &= \frac{1}{\Lambda_i} \\ \psi_i(1/2, u) &= \frac{u\,(1/2)}{\Lambda_i} \end{aligned}$$

where $u\,(1/2) \in [0, 1]$. In both economies the price of a unit (in probability) of consumption is constant across states if output is either 0 or 1. Let $\psi_i(0)$ and $\psi_i(1)$ denote these prices. Since utilities are stochastic, we can say something about the distribution of prices in the two economies for the case where output is $\omega = 1/2$. We let $H_i(\lambda) = \mathbf{P}\left(\psi(1/2, u) \leqslant \lambda\right)$ denote this distribution in economy i.

Proposition 3.1. In the two economies above, (a) and (b) are equivalent and imply (c).

(a) Agent B has a greater preference for flexibility than agent A.

(b) H_A second order stochastically dominates H_B.

(c) $\psi_A(1) = \psi_B(1)$.

Proof. (a) implies (c): We must establish $\Lambda_A = \Lambda_B = \Lambda$. Let f_j be the probability that output is $j \in \{0, 1/2, 1\}$. Then we require that $\mathbf{E}\left[u; F, \mu_i\right] = \int \left[0 \cdot f_0 + u\,(1/2) \cdot f_{1/2} + 1 \cdot f_1\right] \mathrm{d}\mu_i(u) = f_1 + f_{1/2} \int u\,(1/2)\,\mathrm{d}\mu_i(u)$ is independent of i, which holds if and only if $\int u\,(1/2)\,\mathrm{d}\mu_A(u) = \int u\,(1/2)\,\mathrm{d}\mu_B(u)$ if, and only if, singletons are ranked the same by both agents, which follows from (i).

(a) iff (b): First note that

$$\mathbf{E}\left[\psi_i(1/2, u); F, \mu_i\right] = \frac{\int u\,(1/2)\,\mathrm{d}\mu_i(u)}{f_1 + f_{1/2} \int u\,(1/2)\,\mathrm{d}\mu_i(u)}$$

Hence, $\mathbf{E}\left[\psi_A(1/2, u); F, \mu_A\right] = \mathbf{E}\left[\psi_B(1/2, u); F, \mu_B\right]$ if, and only if, $\int u\,(1/2)\,\mathrm{d}\mu_A(u) = \int u\,(1/2)\,\mathrm{d}\mu_B(u)$ if, and only if, $\Lambda_A = \Lambda_B = \Lambda$. It is then immediate to see that H_A second order stochastically dominates H_B if, and only if, μ_A second order stochastically dominates μ_B. Applying Theorem 1 to the one dimensional case implies (a) iff (b), which concludes the proof. □

The proposition says that in the context of our asset pricing model, where preference for flexibility stems from uncertainty about future risk aversion, a greater preference for flexibility corresponds to higher price volatility.

Now consider a setting where investment in the productive asset reduces liquidity. Since preference for flexibility manifests itself as preference for liquidity, preference for flexibility is associated with a tendency to underinvest in the productive asset. Proposition 3.1 suggests that this tendency to underinvest should correlate with price volatility. In order to capture this intuition in a simple asset pricing model like our Lucas tree economy, there must be periods where the productive asset, the tree, is not tradeable. We now consider this case.

4. A Lucas Tree Economy with Investment

Intuitively, preference for flexibility is associated with a tendency to invest less in the productive asset, as such investment reduces liquidity. In order to capture this intuition in a simple asset pricing model, we will consider a Lucas tree economy with closed asset markets (that operate periodically), but where the agent also has access to an international market for goods. We assume that the domestic market is small in that it doesn't affect international prices.

Consider the following environment: Odd periods ($t = 1, 3, 5, \dots$) are days, and even periods ($t = 2, 4, 6, \dots$) are nights. The tree produces a perishable good every period according to the stationary distribution $F(\omega)$. Entering period t, the representative agent is endowed with shares z_t and proportional probabilistic rights to the tree's output, ω_t. In the day (t odd) domestic asset markets are open and the probabilistic right, q_t, to current output, ω_t, can be traded for future shares in the tree, z_{t+1}.

The market structure of the economy at night is different. First, ownership rights change at midnight and the tree is equally likely to provide the night's output before or after midnight. Because of this timing of production, ownership rights to the tree's output, ω_{t+1}, are determined by q_t with probability $1/2$ and by z_{t+1} with probability $1/2$. This specification ensures that, in terms of expected received output in period $t+1$, it is as good to own q_t as it is to own z_{t+1}. Second, we assume that domestic asset markets are closed at night, but international goods markets are open until midnight. On those markets, domestic assets cannot be traded, but probabilistic rights to output of $\omega = 1/2$ can be traded at the fixed price $\kappa > 1$ for probabilistic rights to the output of $\omega = 1$.[3] Thus, the tree's (nighttime) output can only be traded on international markets if it is realized before midnight. In expectation (from the daytime perspective), therefore, ownership rights to output, q_t, provide the agent with more flexibility or 'liquidity' at night than do shares of the productive asset z_{t+1}.

First we establish the pricing function $\psi(\omega, u)$ for a representative agent whose value function in odd periods, that is, at the time asset markets are open, takes the form

$$
\begin{aligned}
v(z,\omega,u) = \max_{q,x} \Big[& q u(\omega) + \tfrac{1}{2}\delta x \int u'(\omega')\, \mathrm{d}F(\omega')\, \mathrm{d}\mu(u') \\
& + \tfrac{1}{2}\delta q\big(f_{\frac{1}{2}} + \kappa f_1\big) \int \max[u'(\tfrac{1}{2}), \tfrac{1}{\kappa}u'(1)]\, \mathrm{d}\mu(u') \\
& + \delta^2 \int v(x,\omega',u')\, \mathrm{d}F(\omega')\, \mathrm{d}\mu(u') \Big]
\end{aligned}
$$

subject to $q + p(\omega,u)x \leqslant z + p(\omega,u)z$.

Let us define $u^*(\mu) := \int \max[u(\tfrac{1}{2}), \tfrac{1}{\kappa}u(1)]\, \mathrm{d}\mu(s) = \int \max[u\,(1/2)\,, \tfrac{1}{\kappa}]\, \mathrm{d}\mu(u)$ and $\mathbf{E}[u; F, \mu] := \int u'(y')\, \mathrm{d}F(\omega')\, \mathrm{d}\mu(u')$. Define

$$
\phi(\omega,u) := p(\omega,u)\big(u(\omega,u) + \tfrac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu)\big)
$$

(3) It would be straightforward to allow the international goods market to be open during the day as well. We choose not to do so to keep the value function as simple as possible.

and

$$\gamma := \tfrac{1}{2}\delta\, \mathbf{E}[u; F, \mu] + \delta^2 \iint (u'(\omega') + \tfrac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu))\ \mathrm{d}F(\omega')\,\mathrm{d}\mu(u')$$

Following the standard arguments in Lucas, we find

$$\phi(\omega, u) = \gamma + \delta^2 \iint f(\omega', u')\ \mathrm{d}F(\omega')\,\mathrm{d}\mu(u')$$

It is easy to show that the unique solution is $\phi(\omega, u) = \gamma/(1-\delta^2)$. Let $\Lambda := \gamma/(1-\delta^2)$ and $\Gamma := \frac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu)/\Lambda$, so that

$$\psi(\omega, u) = \frac{u(\omega)}{\Lambda} + \Gamma$$

We expect to verify the standard intuition that more preference for flexibility implies more demand for liquidity, or a lower willingness to invest in the productive asset, which will be reflected in a higher price for q_t in terms of z_{t+1}. Note that Γ increases in the expected utility the agent derives from being able to access international markets.

We now confine attention to the example where there are only three levels of output, 0, 1/2 and 1. Consider two exchange economies, A and B, with representative agents A and B respectively. We assume that both agents have period 0 preferences with a taste shock representation based on state spaces $\mathcal{U}_i^\dagger$, where $u(0) = 0 \leqslant u(1/2) \leqslant u(1) = 1$ for all $u \in \mathcal{U}_i^\dagger$, $i = A, B$. Thus, agents are uncertain about their risk aversion, which is captured by $u(1/2)$. We also assume that $\succsim^A$ and $\succsim^B$ agree on the intertemporal tradeoff for getting 1 instead of 0, which implies $\delta_A = \delta_B$.

Thus, for $\omega \in \{0, 1/2, 1\}$, prices are given by

$$\begin{aligned}
\psi_i(0, u) &= \Gamma_i \\
\psi_i(1, u) &= \frac{1}{\Lambda_i} + \Gamma_i \\
\psi_i(\tfrac{1}{2}, u) &= \frac{u\,(1/2)}{\Lambda_i} + \Gamma_i
\end{aligned}$$

Let $H_i(\lambda) := \mathbf{P}(\frac{u(1/2)}{\Lambda_i} < \lambda)$ be the distribution of prices in the two economies for the case where output is $\omega = 1/2$, renormalized such that the two distributions can be compared in terms of second order stochastic dominance. Let $\overline{\psi}_i := \mathrm{E}\left[\psi_i(\frac{1}{2}, u), \mu_i\right]$ be the average domestic price in the case where output is $\omega = 1/2$.

Theorem 2. *In the two economies above, (a) and (b) below are equivalent and imply (c).*

(a) *Agent B has greater preference for flexibility than agent A.*
(b) *H_A second order stochastically dominates H_B.*
(c) $\overline{\psi}_B \geqslant \overline{\psi}_A$ *and* $\psi_B(1) - \psi_A(1) = \psi_B(0) - \psi_A(0)$
Furthermore, if (c) holds for all prices $\kappa > 1$, then (a) and (b) are also implied.

The condition $\psi_B(1) - \psi_A(1) = \psi_B(0) - \psi_A(0)$ is the manifestation of the fact that singletons are ranked identically by both agents in terms of prices. Theorem 2 tells us to expect correlation between the price fluctuations and the liquidity premium in small economies that have open goods markets but closed asset markets: the average price of current dividends (which are more liquid than the productive asset) is higher in economy B, ie, $\overline{\psi}_B \geqslant \overline{\psi}_A$. The model suggests that the determinant of these two effects might be the level of uncertainty about future risk aversion in the economy.

Proof of Theorem 2. To investigate the effect of the liquidity provided by holding the right to output rather than shares, consider

$$\Gamma^{-1} = \frac{1}{1-\delta^2}\left(\frac{(1+2\delta)\,\mathbf{E}[u; F, \mu]}{(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu)} + \delta^2\right)$$

Observe that $\frac{\partial \Gamma^{-1}}{\partial u^*(\mu)} < 0$ and therefore $\frac{\partial \Gamma}{\partial u^*(\mu)} > 0$. The proof rests on the following claim.
Claim: If μ_A SOSD μ_B, then $u^*(\mu_B) > u^*(\mu_A)$. Furthermore, if $u^*(\mu_B) > u^*(\mu_A)$ for all $\kappa > 1$, then μ_A SOSD μ_B.
Proof of Claim: Notice that $u^*(\mu) = \int \max[u(\frac{1}{2}), \frac{1}{\kappa}]\, \mathrm{d}\mu\big(u(\frac{1}{2})\big)$. By definition μ_A SOSD μ_B if, and only if, $\int \max[u(\frac{1}{2}), \frac{1}{\kappa}]\, \mathrm{d}\mu_A\big(u(\frac{1}{2})\big) > \int \max[u(\frac{1}{2}), \frac{1}{\kappa}]\, \mathrm{d}\mu_B\big(u(\frac{1}{2})\big)$ for all $\kappa > 1$ (eg, page 33 in Laffont 1989).

We can now establish the proposition. Equivalence of (i) and (ii) follows as in the case without investment. To establish that (ii) implies (iii), note $\psi_B(0) - \psi_A(0) = \psi_B(1) - \psi_A(1)$ if, and only if, $\Lambda_A = \Lambda_B = \Lambda$, if, and only if, $\mathbf{E}[u(\frac{1}{2}); F, \mu_A] = \mathbf{E}[u(\frac{1}{2}); F, \mu_B] = \overline{u}$. By the claim, $u^*(\mu_B) > u^*(\mu_A)$ for all $\kappa > 1$ if, and only if, μ_A SOSD μ_B, which is obviously the case if, and only if, H_A SOSD H_B. By the observation above $\Gamma_B > \Gamma_A$, if, and only if, $u^*(\mu_B) > u^*(\mu_A)$. Hence, $\overline{\psi}_B = \overline{u}/\Lambda + \Gamma_B \geq \overline{u}/\Lambda + \Gamma_A$.

Conversely, if (iii) holds for all $\kappa > 1$, then $\overline{u}_A = \overline{u}_B$, and $\Lambda_A = \Lambda_B$, and hence $\Gamma_B \geq \Gamma_A$, which implies $u^*(\mu_B) \geq u^*(\mu_A)$, and hence by the claim, μ_A SOSD μ_B. $\square$

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