Disruption Risk and Optimal Sourcing in Multi-tier Supply Networks

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Abstract

We study sourcing in a supply chain with three levels: a manufacturer, Tier 1 suppliers, and Tier 2 suppliers prone to disruption from, e.g., natural disasters like earthquakes or floods. The manufacturer may not directly dictate which Tier 2 suppliers are used, but may influence the sourcing decisions of Tier 1 suppliers via contract parameters. The manufacturer’s optimal strategy depends critically on the degree of overlap in the supply chain: if Tier 1 suppliers share Tier 2 suppliers, the manufacturer relies less on direct mitigation (procuring excess inventory and multisourcing in Tier 1) and more on indirect mitigation (inducing Tier 1 suppliers to mitigate disruption risk). We also show that while the manufacturer always prefers less overlap, Tier 1 suppliers may prefer a more overlapped supply chain; however, penalty contracts—in which the manufacturer penalizes Tier 1 suppliers for a failure to deliver ordered units—alleviate this coordination problem.

Keywords: disruption risk; multisourcing; supply chains; multiple tiers

1 Introduction

In the wake of the March 11, 2011 Tōhoku earthquake off the eastern coast of Japan, many companies faced massive disruptions in their supply chains. Automotive manufacturers—particularly Toyota—were especially affected, as a number of key suppliers were located in the impacted region and were disabled due to a combination of earthquake damage, tsunami damage, and radiation exposure resulting from the meltdown of the Fukushima Daiichi nuclear power plant. In addition, some automotive suppliers that were not directly damaged were forced to cease production due to power shortages that followed in the weeks after the disaster. The consequences for manufacturers were severe: Toyota faced immediate shortages on over 400 parts, and production capacity was reduced for 6 months following the disaster (Tabuchi 2011).

Prior to the disaster, Toyota had primarily concerned themselves with their Tier 1 (immediate) suppliers, and had left the management of higher tiers in the supply chain to their direct partners. In

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most cases, Toyota did not even know who its Tier 2 suppliers were, much less their risk profiles or how Tier 1 suppliers managed that risk via, for instance, sourcing decisions or holding safety stocks. As part of the recovery process, Toyota created a systematic map of the upper levels of their supply chain and the disruption management strategies of their Tier 1 suppliers for the first time. During this investigation, a fundamental problem emerged (Greimel 2012, Masui and Nishi 2012): much to their surprise, Toyota discovered that while they may have attempted to mitigate disruption risk by multi-sourcing with Tier 1 suppliers, many Tier 1 suppliers shared some or all of their Tier 2 suppliers, leading to a significant degree of correlation in risk for supply coming from Tier 1. This problem manifested itself with a wide variety of Tier 2 suppliers, ranging from large firms with specific expertise, such as Renesas, an automotive semiconductor manufacturer, to smaller firms with generic capabilities, such as Fujikura, a rubber manufacturer (Masui and Nishi 2012). The substantial overlap in the upper levels of the network, which Toyota internally referred to as a “diamond shaped” supply chain, clearly called for the firm to rethink its disruption management and sourcing decisions.

The findings of Toyota and other major manufacturers quickly led firms to realize that a more comprehensive disruption management strategy, extending beyond a firm’s immediate suppliers and into the entire supply network, is necessary to build a robust enterprise (Greimel 2012, Masui and Nishi 2012, Tucker 2012). Motivated by this example and numerous recent instances of disruption caused by natural disasters (e.g., flooding in Thailand, Soble 2011), financial distress of suppliers (e.g., in the automotive supply chain, Hoffman 2012), or industrial accidents (e.g., at apparel suppliers in Bangladesh, Greenhouse 2012), in this paper we consider the issue of optimal sourcing when disruption risk originates in the upper levels of a multi-tier supply chain. Specifically, we examine a stylized model in which a downstream manufacturer (such as Toyota) sources identical critical components from Tier 1 suppliers. Those Tier 1 suppliers in turn source subassemblies or raw materials from Tier 2 suppliers. We assume that disruption may occur in Tier 2, which in turn will cause a shortage impacting Tier 1, and ultimately the manufacturer; see Figure 1.

Following the classic supply chain disruption approach (Tomlin 2006, Aydin et al 2010), we assume that Tier 2 suppliers have heterogeneous cost and disruption risk profiles, and hence the choice of Tier

![Figure 1. Supply chain overview.](image-url)
2 suppliers is critical in determining the risk profile of the supply chain. However, unlike the existing disruption risk literature, in our model the manufacturer cannot directly dictate that risk profile by choosing Tier 2 suppliers. Instead, the manufacturer acts as a sequential leader, first deciding contract terms with its Tier 1 suppliers; those Tier 1 suppliers then choose sourcing quantities from one or more Tier 2 suppliers to maximize their own expected profits, given the contract terms offered by the manufacturer. Consequently, the manufacturer’s contract decisions with its Tier 1 suppliers will induce some sourcing arrangement from Tier 1 to Tier 2, ultimately determining both the probability of disruption, as well as the correlation of disruption risk for the Tier 1 suppliers. Prompted by Toyota’s findings after the Tōhoku earthquake, we pay particular attention to the impact of Tier 2 configuration—the number of Tier 2 suppliers and the manner in which they are connected to Tier 1 suppliers—on the manufacturer’s optimal risk management strategy, focusing on three key questions. First, how should a manufacturer manage disruption risk when both the probabilities and the correlations in the disruption profile of its immediate suppliers are endogenously determined by the contracts that the manufacturer offers? How does the manufacturer’s optimal sourcing strategy depend on the configuration of Tier 2 (§3-5)? Second, if Tier 1 suppliers were capable of choosing the Tier 2 configuration, would they choose the optimal configuration from the manufacturer’s point of view (§6)? And third, if the manufacturer and Tier 1 suppliers have different preferences for the Tier 2 configuration, does a simple mechanism exist to align those preferences and coordinate the supply chain (§7)?

2 Literature Review

The existing disruption management literature has extensively explored mitigation strategies employable by a firm—primarily safety stocks and supplier diversification—to manage the risk of random supply at its own facilities or from its immediate Tier 1 suppliers. Examples include Anupindi & Akella (1993), Yano & Lee (1995), Elmaghraby (2000), Minner (2003), Tomlin (2006), Chopra et al. (2007), Dada et al. (2007), Federgruen & Yang (2008), Tomlin (2009), Yang et al (2009), Aydin et al. (2010), Tomlin and Wang (2011), Hu and Kostamis (2012), Wadecki et al. (2012), and many others. As we consider the issue of endogenous correlation in the supply network, our paper is particularly related to models with correlated risk and endogenous risk.

From the former category (correlated risk), Babich et al. (2007) find that a monopolistic firm might prefer positively correlated supplier defaults as this increases price competition among its suppliers,
while Tang & Kouvelis (2011) show that firms that compete in a common consumer market and share a supplier base will earn lower profits when supplier defaults are highly correlated. Our model differs from these in that, instead of focusing on the tension between competition and diversification in a firm’s Tier 1 supplier base, we introduce supply correlation between Tier 1 suppliers through their common suppliers in Tier 2. In this case, the firm can use diversification as a strategy to induce its Tier 1 suppliers to source from different Tier 2 suppliers.

From the latter category (endogenous risk), Swinney & Netessine (2009) consider how to contract with a supplier when that supplier’s (financial) default risk depends on the contract price. Wang et al. (2010) explore how a firm can invest to improve the reliability of its suppliers and how this can be used along side dual sourcing as a two-pronged strategy against yield uncertainty. Babich (2010) considers how financial subsidies can be used to mitigate the risk of supplier default. Hwang et al. (2014) explore how contracts and delegation can be employed to induce an immediate supplier to improve its reliability. In contrast to these models, in which risk is determined by firm actions or financial subsidies, we consider how the choice of Tier 1 and Tier 2 suppliers impacts the shape, and subsequently the risk, of the supply network. Bimpikis et al. (2013) also consider how the endogenous choice of risky Tier 2 suppliers impacts the supply chain; however, they focus on how non-convexities of the production function affect supply chain risk, while we focus on how supplier asymmetries and access to different sets of Tier 2 suppliers can influence the optimal sourcing strategy of the downstream manufacturer.

To summarize, our model is, to the best of our knowledge, among the first to explore disruption risk management in supply chains with more than two tiers, and the first to consider how supplier selection and contracting impacts the shape and disruption risk level of the supply chain.

3 Model

We analyze a supply chain consisting of three tiers: Tier 0, Tier 1 and Tier 2. Tier 0 contains a single firm, the “manufacturer,” which assembles finished goods and sells to a consumer market. The manufacturer sources a critical component from Tier 1, which consists of two identical suppliers, denoted A and B, that provide fully substitutable products. Each Tier 1 supplier, in turn, sources a critical intermediate component from Tier 2. Using an example from our introduction, the manufacturer in this supply chain might be Toyota, the Tier 1 suppliers might be manufacturers of automotive air conditioning systems, and the Tier 2 suppliers might be semiconductor manufacturers that provide
the electronics used in the air conditioning units.

3.1 Tier 2

Disruption in the supply chain originates in Tier 2. Following Tomlin (2006) and others, we assume there are two “classes” of Tier 2 suppliers: reliable suppliers (denoted by the subscript \( r \)), which always deliver the requested quantity, and unreliable suppliers (denoted \( u \)) that are prone to disruption, but are less costly than reliable suppliers. This may be the case if, for example, some suppliers are located in an earthquake-prone region (e.g., Japan) that is close to the downstream manufacturing facilities, while other suppliers are located in more geologically stable but distant regions, requiring greater transportation costs (e.g., North America or Europe). Specifically, we assume that reliable suppliers have infinite capacity, while unreliable suppliers have random capacity that follows a two point disruption distribution: with probability \( 1 - \lambda \), there is no disruption and effective capacity is infinite, while with probability \( \lambda \) a disruption occurs, and production capacity is reduced to some finite \( K \geq 0 \) (Yano and Lee 1995, Aydin et al 2010).

The Tier 2 sourcing cost is exogenous (e.g., because suppliers at this level tend to offer more commoditized goods, with prices determined by the market), and the per unit sourcing cost from a supplier of type \( t = r, u \) is \( c_t \), where \( c_r > c_u \). Aside from their reliability in delivering production orders and their sourcing cost, the Tier 2 suppliers are otherwise identical, i.e., they offer products of identical quality that are fully substitutable, and all Tier 2 suppliers have sufficient capacity to fulfill any downstream order if they are not disrupted.

3.2 Tier 1

Suppliers in Tier 1, after receiving a contract from the manufacturer (described below), select which Tier 2 suppliers to use, with the goal of maximizing their expected profit. If a Tier 1 supplier selects a Tier 2 supplier that disrupts, then all parts from that supplier in excess of the disrupted capacity \( K \) are lost, and the Tier 1 supplier potentially faces a shortage of components, which may in turn lead to a failure to deliver product to the manufacturer. We assume that both Tier 1 suppliers have access to one reliable and one unreliable Tier 2 supplier. Tier 1 suppliers may choose to single source from either supplier type or to dual source, with an arbitrary quantity split between the suppliers.

Motivated by Toyota’s discovery of substantial overlap in the upper levels of its supply chain, we assume that there are two possible configurations of Tier 2, as shown in Figure 2: either Tier 2 suppliers are independent (i.e., Tier 1 suppliers share no Tier 2 suppliers in common) or Tier 2 suppliers are
shared (i.e., Tier 1 suppliers source from the exact same set of Tier 2 suppliers)\[1\]. There are several ways to interpret the scenarios depicted in the figure. The first is that Tier 1 suppliers have existing relationships with some Tier 2 suppliers (of which there are potentially many), and these exogenous existing relationships determine the Tier 2 configuration. Such relationships may be formed, e.g., from previous business between the parties or from Tier 2 suppliers that have passed internal qualification hurdles to earn a place on an approved supplier list.

![Diagram of Tier 2 configurations]

**Figure 2.** The two possible Tier 2 configurations. Dashed lines represent potential sources of supply for downstream supply chain members.

Second, it may be the case that Tier 2 suppliers are chosen after Tier 1 suppliers contract with the manufacturer, but the degree of concentration of the Tier 2 supplier base determines whether the supply chain is likely to end up with shared or independent Tier 2 suppliers. For instance, if there are only two Tier 2 suppliers in the world that manufacture the product, one reliable and one unreliable, then the Tier 2 supply base is very “concentrated”, and the Tier 1 suppliers necessarily share those two Tier 2 suppliers. This may be the case for highly specialized Tier 2 suppliers, such as semiconductor manufacturers like Renesas. Conversely, this may be driven by industry trends outside of our model: for example, Sasaki (2013) describes how consolidation in the Japanese automotive supply chain due to cost pressure throughout the 1990s and 2000s has generally lead to a much more concentrated supply base than in the past. By contrast, if there are many suppliers (of both reliable and unreliable type), and the likelihood that the Tier 1 suppliers randomly select identical Tier 2 suppliers is negligible, then the Tier 2 supply base is very “diffuse”, and it is likely that Tier 2 suppliers will be independent; this can occur particularly with Tier 2 suppliers that offer commoditized products with generic capabilities, such as rubber component manufacturers like Fujikura.

Third, if risk is primarily determined by geographic location, the different scenarios may represent

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\[1\] Since the “reliable” Tier 2 suppliers are perfectly reliable, the key differentiating feature is whether the unreliable Tier 2 suppliers are shared.
the degree of geographic concentration in the distribution of Tier 2 suppliers, rather than simply the concentration in the numbers or capabilities of such suppliers. For instance, if there are only two regions (one risky and one safe, e.g., Japan and Canada) that supply the Tier 2 component, and disruption risk amongst all suppliers within a region is correlated, the Tier 1 suppliers necessarily must select from a common pool of reliable and unreliable suppliers; in that case, Tier 2 suppliers are shared. If there are many regions throughout the world with Tier 2 suppliers that produce the necessary component, and the chance that the Tier 1 suppliers choose to source from the same region is low, then Tier 2 suppliers are independent.

For most of our analysis, the Tier 2 configuration is assumed to be exogenously specified. In practice, this is frequently the case: given the high cost and long term nature of establishing a list of approved suppliers, Tier 1 suppliers operate over the short term with an effectively fixed set of Tier 2 suppliers, around which the manufacturer optimizes its day-to-day procurement decisions. In addition, Tier 2 suppliers are often chosen for reasons outside of our model, such as capability or quality, meaning that a key problem is how to optimally manage disruption risk given a fixed Tier 2 configuration; once this is understood, a longer term strategic issue is how the Tier 2 configuration might be chosen by Tier 1 suppliers, and how the manufacturer’s sourcing strategy impacts this choice, an extension that we analyze in §6.

In the independent Tier 2 supplier configuration, the risk of disruption from the unreliable Tier 2 suppliers is assumed to be statistically independent. If both Tier 2 suppliers source from the same unreliable Tier 2 supplier who disrupts, both Tier 1 suppliers are impacted by reduced capacity at the disrupted supplier. There is no possibility of disruption originating in Tiers 0 or 1.

3.3 The Manufacturer

The manufacturer faces deterministic market demand $D$ (Zimmer 2002, Babich et al. 2012) and makes fixed per unit revenue of $\pi$ for every unit it sells up to $D$. We assume that $\pi \geq c_u$, so that it is possible for the supply chain to profitably sell the product; all manufacturer costs outside of procurement from Tier 1 suppliers are normalized to zero.

The sequence of events is as follows. In the first period, the manufacturer offers its Tier 1 suppliers A and B a price and quantity contract of the form $(p_i, Q_i)$, $i \in \{A, B\}$. Such contracts are commonly observed in practice and in the literature, and stipulate that the manufacturer pays the supplier $p_i$
for every unit delivered up to the specified amount $Q_i$. The manufacturer makes no payment for ordered units that are not delivered. We assume that the choice of Tier 2 suppliers is not directly contractible. Even manufacturers as powerful as Toyota are typically unable to unilaterally dictate supplier choice; after the 2011 earthquake, citing competitive reasons, approximately 50% of Toyota’s Tier 1 suppliers refused to even share the identities of their Tier 2 suppliers, let alone to allow Toyota to unilaterally dictate those suppliers (Masui and Nishi 2012). In addition, while other tools may be available to influence the choice of Tier 2 suppliers by Tier 1 suppliers, such as manufacturer involvement in the development of the Tier 2 supply base, these tools are longer term mechanisms, leaving procurement contracts as the most obvious immediate short- and medium-term tool available to influence the sourcing decisions of Tier 1 suppliers.

After receiving the manufacturer’s contract, each Tier 1 supplier then places orders $q_i^u$ and $q_i^r$ with its Tier 2 unreliable and reliable suppliers, to maximize expected profit. We call the pair $(q_i^u, q_i^r)$ the sourcing strategy of Tier 1 supplier $i$. In the second period, the Tier 2 suppliers either disrupt or fully fulfill their deliveries to the Tier 1 suppliers. After collecting all deliveries from Tier 2 suppliers, Tier 1 suppliers then attempt to fulfill the order submitted by the manufacturer. Any excess inventory delivered by Tier 2 suppliers to Tier 1 suppliers has zero value. In turn, after collecting deliveries from Tier 1 suppliers, the manufacturer attempts to fulfill consumer demand $D$, and if supply is less than demand, lost sales occur. Any excess components delivered by Tier 1 suppliers to the manufacturer have zero value.

The manufacturer seeks to maximize its expected profit from the consumer market at the end of the second period, and its decision variables are the price and quantity in the contracts it gives to its Tier 1 suppliers, $(p_A, Q_A)$ and $(p_B, Q_B)$; hence, we define the manufacturer’s sourcing strategy to be $\{ (p_A, Q_A), (p_B, Q_B) \}$, the pair of contract offers to its two Tier 1 suppliers. We assume that the disruption probabilities, sourcing costs, and Tier 2 configuration are known to all parties in the supply chain: there is no private information in the model.

4 Optimal Sourcing with Independent Tier 2 Suppliers

4.1 Tier 1’s Sourcing Decision

We first examine the manufacturer’s optimal sourcing strategy when facing a supply chain with independent Tier 2 suppliers. This resembles the type of supply network that Toyota (mistakenly) believed they possessed prior to the Tōhoku earthquake, and as such will serve as a suitable benchmark for
understanding how an increased overlap in Tier 2 impacts the manufacturer’s sourcing strategy. We begin our analysis by characterizing the optimal sourcing strategy for each Tier 1 supplier \(i \in \{A, B\}\) in response to a contractual offer \((p_i, Q_i)\) from the manufacturer. Given a sourcing strategy \((q_{iu}^i, q_{ir}^i)\), the expected profit of supplier \(i\) is:

\[
\Pi^i(q_{iu}^i, q_{ir}^i) = \lambda \left[ p_i \min\{q_{ir}^i + \min\{K, q_{iu}^i\}, Q_i\} - c_r q_{ir}^i - c_u \min\{K, q_{iu}^i\}\right] \\
+ (1 - \lambda) \left[ p_i \min\{q_{iu}^i + q_{ir}^i, Q_i\} - c_u q_{iu}^i - c_r q_{ir}^i\right].
\]

Let the superscript \(\ast\) denote the optimal sourcing strategy. Optimizing the Tier 1 supplier’s expected profit over the quantity sourced from each Tier 2 supplier leads to the following result:

**Theorem 1.** With independent Tier 2 suppliers, the optimal sourcing strategy of Tier 1 supplier \(i \in \{A, B\}\) is:

\[
(q_{iu}^i, q_{ir}^i)^\ast = \begin{cases} 
(Q_i, 0) & \text{if } c_u \leq p_i < \frac{c_r - (1 - \lambda)c_u}{\lambda}, \\
(K, Q_i - K) & \text{if } p_i \geq \frac{c_r - (1 - \lambda)c_u}{\lambda}.
\end{cases}
\]

**Proof.** All proofs appear in the appendix. \(\square\)

Observe that it is never optimal for the supplier to single source from the reliable Tier 2 supplier. This is due to the fact that the supplier may completely eliminate disruption risk by sourcing only the disrupted output \(K\) from the unreliable supplier and sourcing the remainder, \(Q_i - K\), from the reliable supplier, achieving the same outcome (at a lower cost) as sourcing everything from the reliable supplier. Moreover, sourcing inventory in excess of the manufacturer’s contracted quantity \(Q_i\) is never optimal, due to the fact that disruptions are manifested as random supplier capacity (as opposed to random yield, see Hwang et al 2014), combined with the presence of a perfectly reliable Tier 2 supplier. This implies that a Tier 1 supplier effectively chooses from two possible sourcing strategies: sourcing from the unreliable supplier, or sourcing from both suppliers. The former strategy is also known as passive acceptance (i.e., paying a low cost and merely accepting the risk of disruption), while the latter strategy is known as dual sourcing (Tomlin 2006). Intuitively, for low contract prices \(p_i\), the supplier follows a passive acceptance strategy, and single sources from the unreliable supplier, while for high prices, the supplier employs dual sourcing to completely eliminate the risk of disruption.  

\[3\] The price
necessary to induce the supplier to dual source is greater than the marginal cost of the reliable supplier, an agency cost arising from the moral hazard feature inherent in a decentralized supply chain.

4.2 The Manufacturer’s Optimal Sourcing Strategy

Having derived the optimal sourcing strategy of a Tier 1 supplier in response to a contract \((p_i, Q_i)\) from the manufacturer, we may now determine the optimal contracts offered by the manufacturer to the two Tier 1 suppliers. In what follows, we use the term supply chain structure to refer to an induced set of sourcing relationships between Tiers 0, 1, and 2. In theory, the manufacturer could induce four sourcing strategies at each Tier 1 supplier (no participation, single sourcing from either Tier 2 supplier, or dual sourcing from both Tier 2 suppliers), implying a total of 16 possible induced structures. However, from Theorem \(1\), the manufacturer can only feasibly induce three strategies for each of its Tier 1 suppliers: no participation, single sourcing from the unreliable supplier (by offering a low price), or dual sourcing (by offering a high price). This implies that via its contractual offers to Tier 1 suppliers, the manufacturer is capable of generating a total of five unique supply chain structures (excluding the case of no production and all symmetric cases). We denote these Structures 1-5, and summarize them in Figure 3. To determine the manufacturer’s optimal sourcing strategy, we must consider the performance of each of these possible induced structures. In any structure, the manufacturer’s expected profit as a function of its sourcing strategy \(\{(p_A, Q_A), (p_B, Q_B)\}\) is:

\[
\Pi^M(\{(p_A, Q_A), (p_B, Q_B)\}) = f_1(p_A, p_B) \left[ \pi \min\{Q_A + Q_B, D\} - p_A Q_A - p_B Q_B \right] \\
+ f_2(p_A, p_B) \left[ \pi \min\{\min\{K, Q_A\} + Q_B, D\} - p_A \min\{K, Q_A\} - p_B Q_B \right] \\
+ f_3(p_A, p_B) \left[ \pi \min\{Q_A + \min\{K, Q_B\}, D\} - p_A Q_A - p_B \min\{K, Q_B\} \right] \\
+ f_4(p_A, p_B) \left[ \pi \min\{\min\{K, Q_A\} + \min\{K, Q_B\}, D\} - p_A \min\{K, Q_A\} - p_B \min\{K, Q_B\} \right].
\]

manufacturer clearly prefers this outcome to a passive acceptance strategy, and indeed this outcome could be achieved if the manufacturer merely raises the price an infinitesimal amount above \(\frac{c_r - (1 - \lambda)c_u}{\lambda}\).
In the above equation, \( f_1, f_2, f_3 \) and \( f_4 \) denote the probability that the orders are fulfilled by both Tier 1 suppliers, only supplier A, only supplier B, or neither Tier 1 supplier, respectively. Note that this is similar in form to the Tier 1 supplier’s profit function in equation (1), except that (2) allows for the disruption of both immediate suppliers, and the probability terms in (2) are endogenously determined by the manufacturer’s offered prices.

Each structure in Figure 3 corresponds to a particular set of prices offered to Tier 1 suppliers, and this in turn determines the disruption probabilities \( f_1-f_4 \). For instance, Structure 4 is induced by offering supplier A a high price and supplier B a low price, and the resultant probabilities are \( f_1 = 1 - \lambda, f_2 = 0, f_3 = \lambda, \) and \( f_4 = 0 \), because only supplier B may fail to deliver the manufacturer’s order, as supplier A dual sources and achieves perfect reliability. To determine the manufacturer’s optimal strategy, we must find the optimal sourcing quantities in each structure given the prices and probabilities associated with the structures, and then compare the optimal expected profit across the five structures. For the sake of brevity, we relegate the derivation of these profit expressions to Lemma 1 in the appendix, and focus our subsequent discussion on the outcome of this analysis: the determination of the manufacturer’s optimal sourcing strategy and induced supply chain structure.

We begin by considering the case of minor disruptions.

**Theorem 2.** If Tier 2 suppliers are independent and disruptions are minor \((K \geq D/2)\), the manufacturer’s optimal sourcing strategy is \( \{(p_A, Q_A), (p_B, Q_B)\}^* = \{(c_u, \theta D), (c_u, (1 - \theta)D)\} \), for any \( \theta \in \left[\frac{D-K}{D}, \frac{K}{D}\right] \).

The theorem shows if that the disrupted capacity is high (i.e., if sufficient disrupted capacity exists across the entire unreliable Tier 2 supply base for the manufacturer to cover all of its demand), the manufacturer should induce structure 3 in Figure 5: single sourcing from the unreliable Tier 2 suppliers by both Tier 1 suppliers. As a result, Tier 1 suppliers do nothing to mitigate disruption risk in the supply chain, and all risk mitigation is pursued directly by the manufacturer, who is able to completely eliminate risk via its own sourcing actions. We thus refer to this approach as a manufacturer dual sourcing (DS) strategy—the manufacturer mitigates risk by dual sourcing, while the Tier 1 suppliers employ passive acceptance, by single sourcing from a risky Tier 2 supplier. The only restriction on the manufacturer’s sourcing strategy is that no more than \( K \) units should be sourced from each Tier 1 supplier, to ensure that Tier 2 suppliers are always able to deliver in the event of a disruption; this determines the feasible range of \( \theta \), the division of quantities between Tier 1 suppliers, which the manufacturer can employ.
Next, we consider the case when disruptions are severe, meaning disrupted capacity is small. We first define the following cost threshold for the reliable supplier:

\[ \bar{c}_r \equiv c_u \left( \frac{D - 2K}{D - K} \lambda (1 - \lambda) + 1 \right). \]  

(3)

Observe that this cost threshold varies depending on the severity and likelihood of disruptions, and lies between \( c_u \) and \( 5c_u/4 \). We say that reliable Tier 2 suppliers with a marginal cost below \( \bar{c}_r \) are “cheap,” while those with a cost above \( \bar{c}_r \) are “costly.” As the following theorem shows, the manufacturer’s optimal sourcing strategy depends on whether reliable suppliers are costly or cheap:

**Theorem 3.** Define \( \bar{\pi} \equiv c_u + \frac{D - K}{D - 2K} \left( \frac{\pi - c_u}{\lambda} \right) \). If Tier 2 suppliers are independent and disruptions are severe (\( K < D/2 \)), then:

(i) If reliable Tier 2 suppliers are cheap (\( c_r \leq \bar{c}_r \)), the manufacturer’s optimal sourcing strategy is

\[
\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} 
(c_u, \theta D), (c_u, (1 - \theta)D) \text{ for any } \theta \in \left[ \frac{K}{D}, \frac{D - K}{D} \right] & \text{if } c_u \leq \pi < \bar{\pi} \\
\left\{ \frac{c_r - (1-\lambda)c_u}{\lambda}, D - K \right\}, (c_u, K) & \text{if } \pi \geq \bar{\pi}
\end{cases}
\]

(ii) If reliable Tier 2 suppliers are costly (\( c_r > \bar{c}_r \)), the manufacturer’s optimal sourcing strategy is

\[
\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} 
(c_u, \theta D), (c_u, (1 - \theta)D) \text{ for any } \theta \in \left[ \frac{K}{D}, \frac{D - K}{D} \right] & \text{if } c_u \leq \pi < \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2} \\
(c_u, D - K), (c_u, D - K) & \text{if } \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2} \leq \pi < \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2} \\
\left\{ \frac{c_r - (1-\lambda)c_u}{\lambda}, D - K \right\}, (c_u, K) & \text{if } \pi \geq \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2}.
\end{cases}
\]

The theorem shows that for small unit revenues (\( \pi \)), regardless of whether the reliable Tier 2 suppliers are costly or cheap, the manufacturer’s approach is similar to the case when the magnitude of disruptions is small: the optimal strategy is to induce both Tier 1 suppliers to single source from unreliable Tier 2 suppliers, dividing total demand between the two suppliers with the restriction that at least \( K \) units (the “risk free” supply) are purchased from each Tier 1 supplier. Hence, manufacturer dual sourcing (DS) is optimal for low unit revenues. As the unit revenue increases, the cost of reliable Tier 2 suppliers begins to play a role. If they are cheap, the manufacturer moves from DS to inducing one Tier 1 supplier to dual source by paying a high price, while paying the other Tier 1 supplier a low price to induce single sourcing. We call this a manufacturer dual sourcing + supplier mitigation (DS+SM) strategy, since it combines dual sourcing in Tier 1 with induced supplier mitigation via dual sourcing in Tier 2. Note that this strategy completely eliminates disruption risk for the manufacturer,
but the manufacturer must pay a higher price to achieve this outcome.

If, on the other hand, reliable Tier 2 suppliers are costly, at intermediate unit revenues, the manufacturer continues to pay Tier 1 suppliers a low price, but increases the quantity sourced from each Tier 1 supplier to \( D - K \), meaning the total amount of inventory procured exceeds demand. Tomlin (2006) refers to this strategy of excess procurement—in essence, holding safety stock—as “inventory mitigation.” Hence, when reliable Tier 2 suppliers are costly, the optimal sourcing strategy switches from DS to \textit{manufacturer dual sourcing + inventory mitigation} (DS+IM), while still inducing Tier 1 suppliers to follow a passive acceptance strategy by paying a low unit price. This strategy results in the manufacturer having more inventory than is necessary to satisfy demand if no disruption occurs; however, because the reliable Tier 2 suppliers are “costly,” the excess inventory cost is preferred to paying a high marginal procurement cost for reliable supply. Manufacturer efforts alone, however, cannot completely eliminate the risk of disruption, and even with the DS+IM strategy, there is still a chance that the manufacturer will have insufficient inventory to satisfy demand (i.e., if both Tier 2 unreliable suppliers disrupt). Hence, as the unit revenue continues to increase, the manufacturer eventually switches to a DS+SM strategy, which eliminates disruption risk by engaging Tier 1 suppliers in mitigation efforts. The optimal strategies when disruptions are severe are summarized in Table 1.

Table 1 illustrates that when disruptions are severe, the manufacturer’s optimal strategy builds risk mitigation from the bottom of the supply chain up. Put differently, it is optimal to begin with manufacturer dual sourcing and, as the unit revenue increases, add manufacturer inventory mitigation (if reliable Tier 2 suppliers are costly) and finally supplier mitigation. Consequently, for small and moderate unit revenues, Tier 1 is not involved with disruption risk mitigation: the manufacturer bears all such responsibility. For large unit revenues, however, it is optimal to engage Tier 1 in disruption risk mitigation efforts, by paying a higher price and inducing supplier mitigation. Comparing Theorems 2 and 3, we see that the manufacturer’s optimal sourcing strategy is qualitatively different for minor and severe disruptions. For minor disruptions, the manufacturer bears all mitigation responsibility, following a strategy of pure manufacturer mitigation implemented via dual sourcing. For severe
disruptions, the manufacturer may also employ inventory mitigation (at moderate unit revenues if reliable suppliers are costly) or induce supplier mitigation (at high unit revenues), sharing the burden of disruption risk mitigation with its suppliers. This optimal strategy as a function of both unit revenue and disrupted capacity is depicted graphically in Figure 4 for the case when reliable suppliers are costly. In the figure, darker shading corresponds to greater disruption risk mitigation efforts. As the figure demonstrates, an increase either in unit revenues or the severity of disruptions leads the manufacturer to increase mitigation efforts, either directly (via inventory mitigation) or indirectly (via induced supplier mitigation).

Figure 4. The manufacturer’s optimal sourcing strategy with independent Tier 2 suppliers, when reliable Tier 2 suppliers are costly.

After the 2011 Tōhoku earthquake, a key determination of Toyota executives was that they had placed too much emphasis on direct mitigation efforts, and too little on supplier mitigation. Given that our model shows inducing supplier mitigation is indeed optimal for high unit revenues and severe disruptions, a natural question is: precisely how much does the manufacturer suffer when using the wrong strategy? Our final result in this section answers this question, by comparing manufacturer profit under direct dual sourcing (with or without inventory mitigation) to the profit under the optimal sourcing strategy (using supplier mitigation) for large unit revenues.

Corollary 1. Let \( \Pi^M(X) \) be the manufacturer’s expected profit under sourcing strategy \( X \in \{DS, DS + IM, DS + SM\} \), and define \( L(X) \equiv 1 - \Pi^M(X)/\Pi^M(DS + SM) \) to be the percentage loss in expected profit of strategy \( X \) relative to a \( DS + SM \) strategy. When Tier 2 suppliers are independent and disruptions are severe:

(i) The percentage profit loss from employing manufacturer dual sourcing is \( L(DS) \leq \lambda (1 - \frac{2K}{D}) \).
(ii) The percentage profit loss from employing manufacturer dual sourcing and inventory mitigation is 
\[ L(DS + IM) \leq \lambda^2(1 - \frac{2K}{D}) \].

For example, if the probability of disruption in Tier 2 is 10\%, the result shows that using an “incorrect” strategy that ignores inducements to Tier 1 suppliers and solely uses manufacturer dual sourcing results in at most a 10\% loss in profit compared to the optimal strategy. By adding inventory mitigation, the manufacturer’s profit is substantially improved, achieving a loss of at most 1\% compared to the optimal profit. Note that the loss under either strategy depends similarly (i.e., linearly) on the severity of the disruption, as measured by the demand fill rate achievable when all unreliable Tier 2 suppliers disrupt (i.e., \( 2K/D \)). As one would expect, the performance of an incorrect strategy becomes increasingly poor as disruptions become very likely or very severe (i.e., high \( \lambda \) or low \( K \)), and all strategies perform equally well when disruptions are highly unlikely or not impactful (i.e., \( \lambda \) close to zero or \( K \) close to \( 2D \)). Importantly, however, the result shows that with independent Tier 2 suppliers, manufacturer performance is substantially worse under a standalone dual sourcing strategy than under a dual sourcing and inventory mitigation strategy. Thus, focusing on dual sourcing alone in Tier 1, as Toyota and many other lean manufacturers did prior to the Tōhoku earthquake, is potentially quite costly. This implies that, if the manufacturer cannot engage suppliers in disruption mitigation efforts when that would otherwise be optimal, a strategy that performs well is to employ manufacturer dual sourcing and inventory mitigation. While this cannot achieve the optimal profit earned under supplier mitigation, it results in an order of magnitude smaller loss than using dual sourcing alone. Indeed, Toyota has begun to implement inventory mitigation in instances where suppliers either refuse to engage in mitigation or find mitigation impractical (Greimel 2012; Masui and Nishi 2012).

5 Optimal Sourcing with Shared Tier 2 Suppliers

5.1 Tier 1’s Sourcing Decision

We now consider the case of shared Tier 2 suppliers. Before deriving the optimal strategies for the Tier 1 suppliers and the manufacturer, there are two issues we must discuss: how disrupted capacity compares to the independent Tier 2 case, and how Tier 1 suppliers are allocated scarce capacity in the event of a disruption from their common Tier 2 supplier. First, we note that in practice, the disrupted capacity of the single unreliable Tier 2 supplier, which we define to be \( K_s \) (where \( s \) stands for “shared”) may or may not be the same as \( K \), the disrupted capacity of each unreliable supplier
when Tier 2 suppliers are independent.\footnote{For example, if the shared configuration arises from consolidation of unreliable Tier 2 suppliers, we might expect that the total capacity of the shared unreliable Tier 2 supplier is twice that of each independent unreliable Tier 2 supplier; if disrupted capacity is proportional to nominal capacity, this would imply that disrupted output in the shared configuration is twice the disrupted output of each independent Tier 2 supplier.}

Second, observe that whenever the Tier 1 suppliers $A$ and $B$ are induced to source from their common unreliable Tier 2 supplier, they potentially compete for limited capacity, i.e., when they respectively order $q_u^A$ and $q_u^B$ units, and the disrupted output is $K_s < q_u^A + q_u^B$. Hence, we must discuss how scarce capacity in Tier 2 is allocated in the event of a disruption. While there are many possible allocations, we select the uniform rule, which has the desirable property of simultaneously minimizing shortage gaming and maximizing total supply chain profit in a number of settings (Sprumont 1991, Cachon and Lariviere 1999). A uniform allocation splits the disrupted capacity of the unreliable Tier 2 supplier equally between Tier 1 suppliers, unless one of the Tier 1 suppliers requested less than half the disrupted capacity, in which case the other Tier 1 supplier is allotted all unused capacity. In other words, supplier $i$’s allocation in the event of a disruption is $\min\{q_u^i, \hat{K}_i\}$, where

$$\hat{K}_i \equiv \frac{K_s}{2} + \left(\frac{K_s}{2} - q_u^i\right)^+$$

is the effective disrupted capacity. Under this allocation rule, the unique Nash equilibrium of the shortage game between Tier 1 suppliers is, essentially, for both suppliers to operate as if they have a dedicated Tier 2 unreliable supplier with exogenously fixed capacity equal to $\hat{K}_i$ (see the appendix for details), leading to the following result:

**Theorem 4.** With shared Tier 2 suppliers, the optimal sourcing strategy of Tier 1 supplier $i \in \{A, B\}$ is:

$$(q_u^i, q_r^i)^* = \begin{cases} (Q_i, 0) & \text{if } c_u \leq p_i < \frac{c_r - (1-\lambda)c_u}{\lambda}, \\ (\hat{K}_i, Q_i - \hat{K}_i) & \text{if } p_i \geq \frac{c_r - (1-\lambda)c_u}{\lambda}. \end{cases}$$

Note that this is nearly identical to the suppliers’ optimal strategy with independent Tier 2 suppliers derived in Theorem\footnote{For example, if the shared configuration arises from consolidation of unreliable Tier 2 suppliers, we might expect that the total capacity of the shared unreliable Tier 2 supplier is twice that of each independent unreliable Tier 2 supplier; if disrupted capacity is proportional to nominal capacity, this would imply that disrupted output in the shared configuration is twice the disrupted output of each independent Tier 2 supplier.} except for the disrupted capacity $\hat{K}_i$; in particular, the manufacturer may still only induce single sourcing from the unreliable Tier 2 supplier or dual sourcing from both Tier 2 suppliers, and the prices necessary to achieve each outcome are identical to the case with independent Tier 2 suppliers.
5.2 The Manufacturer’s Optimal Sourcing Strategy

As a result of Theorem 4, there are once again five possible unique structures that the manufacturer can induce, graphically depicted in Figure 5. We thus derive the manufacturer’s optimal sourcing strategy by following the same procedure as in §4, beginning with the case of minor disruptions.

**Theorem 5.** If Tier 2 suppliers are shared and disruptions are minor \((K_s \geq D)\), the manufacturer’s optimal sourcing strategy is \(\{(p_A, Q_A), (p_B, Q_B)\}^* = \{(c_u, \theta D), (c_u, (1 - \theta)D)\}\) for any \(\theta \in [0, 1]\).

This is similar to the optimal sourcing strategy for minor disruptions with independent Tier 2 suppliers (Theorem 2), with two key differences. First, with shared Tier 2 suppliers, what qualifies as a “minor” disruption is a greater capacity than with independent Tier 2 suppliers; in fact, the disrupted capacity must be greater than the manufacturer’s demand to achieve this case, implying disruptions are effectively meaningless. Note that if industry-wide output in the event of a disruption is held constant across the two cases (i.e., \(K_s = 2K\)), as might occur if the shared Tier 2 case emerges from industry consolidation, then the conditions in Theorems 2 and 5 are the same, i.e., \(K \geq D/2\).

Second, with shared Tier 2 suppliers, any division of sourcing quantities between Tier 1 suppliers will result in the same manufacturer profit, including single sourcing. Thus, while minor disruptions can be completely overcome by the manufacturer with either independent or shared Tier 2 suppliers, dual sourcing with a specific division of sourced quantities is necessary in the former case, while single sourcing suffices in the latter. As a result, we say that a manufacturer single sourcing (SS) strategy is optimal in this case.

Moving next to the case of severe disruptions:

**Theorem 6.** Define \(\tilde{\pi}_s \equiv c_u + \left(\frac{D - c_u}{D - K_s}\right) \frac{(D - K_s/2)}{(D - K_s)}\). If Tier 2 suppliers are shared and disruptions are...
severe ($K_s < D$), the manufacturer’s optimal sourcing strategy is:

\[
\{(p_A, Q_A), (p_B, Q_B)\}^* = \begin{cases} 
\{(c_u, \theta D), (c_u, (1-\theta) D)\} & \text{for any } \theta \in [0,1] \text{ if } c_u \leq \pi < \bar{\pi}_s, \\
\left\{ \left( \frac{c_u-(1-\lambda)c_u}{\lambda}, D - \frac{K_s}{2} \right), (c_u, \frac{K_s}{2}) \right\} & \text{if } \pi \geq \bar{\pi}_s.
\end{cases}
\]

Note that, once again, while dual sourcing is an optimal strategy at low unit revenues, single sourcing performs just as well. In other words, in a fully shared supply chain, manufacturer dual sourcing has no explicit value at low margins—regardless of the severity of disruptions—because Tier 1 suppliers have access to identical Tier 2 suppliers. There is no Tier 2 supplier that Supplier A can access that Supplier B cannot; hence, there is no compelling reason to dual source in Tier 1. Particularly if there are economies of scale with Tier 1 suppliers or other reasons to value single sourcing, the manufacturer may wish to adopt a sourcing strategy employing just one Tier 1 supplier. Moreover, when Tier 2 suppliers are shared, inventory mitigation—sourcing more from Tier 1 than total demand $D$—is also not valuable, since this strategy is only beneficial if it is possible that one Tier 1 supplier delivers while the other does not. Hence, the manufacturer never needs to adopt direct disruption mitigation efforts in a fully shared supply chain structure at low unit revenues. At high unit revenues, however, the manufacturer does find dual sourcing to be optimal, as it provides a way to lower the average unit procurement cost when supplier mitigation is also employed.

Thus, comparing Theorems 3 and 6, we see that increased overlap in Tier 2 leads the manufacturer to value direct mitigation efforts (dual sourcing and inventory mitigation) less. In addition, we have the following result:

**Theorem 7.** If $K_s \leq 2K$, the manufacturer finds supplier mitigation optimal at lower unit revenues with shared Tier 2 suppliers than with independent Tier 2 suppliers.

This implies that overlap in Tier 2 causes the manufacturer to value supplier mitigation more, provided that the disrupted capacity of the shared Tier 2 unreliable supplier does not exceed the aggregate disrupted industry capacity from independent unreliable Tier 2 suppliers. In other words, overlap in the upper tiers of the supply chain simultaneously *increases* the manufacturer’s reliance on supplier mitigation and *decreases* its reliance on manufacturer mitigation. Figure 6 illustrates this fact by plotting the manufacturer’s optimal sourcing strategy as a function of disrupted capacity and the manufacturer’s unit revenues, using the same parameters that were used to generate Figure 4, with

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5If disruption can occur in Tier 1, direct manufacturer mitigation—including dual sourcing and inventory mitigation—will likely have some value. Nevertheless, the qualitative impact of increased overlap in Tier 2 reducing the value of manufacturer mitigation should remain even if disruption can occur in Tier 1.
the shared disrupted capacity $K_s = 2K$. Compared to Figure 4, the region of DS+SM optimality is larger in Figure 6 and there are no regions of DS+IM or DS optimality (in the strict sense)—single sourcing (SS) performs equally well.

![Figure 6. The manufacturer's optimal sourcing strategy with shared Tier 2 suppliers.](image)

Returning to our motivating example of an unexpectedly high degree of overlap in the automotive supply chain discovered following the 2011 Tōhoku earthquake, our results suggest that an appropriate reaction from the downstream manufacturer in this scenario is to reduce the emphasis placed on its own direct mitigation efforts, and instead offer higher prices to its Tier 1 suppliers; these suppliers, facing higher margins and potentially greater losses in the event of a disruption, will in turn invest more in their own risk mitigation efforts, thereby lowering the overall risk to the downstream manufacturer. This implies that awareness of the degree of overlap of the supply chain and the presence of disruption risk in Tier 2 leads the manufacturer to offer higher prices to Tier 1 suppliers than would otherwise be optimal; hence, it is crucial for the manufacturer to consider disruption risk originating throughout the supply chain, even if it is not optimal to directly mitigate that risk.

Our final result in this section considers the loss in profit that the manufacturer suffers from failing to use supplier mitigation:

**Corollary 2.** Let $L(X)$ be as defined in Corollary 1. When Tier 2 suppliers are shared and disruptions are severe, the percentage profit loss from employing single sourcing is $L(SS) \leq \lambda(1 - \frac{K_s}{D})$.

As with the case of independent Tier 2 suppliers, the profit loss depends on both the likelihood, as well as the severity of the disruption. Continuing with the same example as before, if there is a 10% chance of disruption, the worst case profit loss from ignoring supplier mitigation is 10% of the optimal
profit. However, unlike the independent Tier 2 supplier case, with shared Tier 2 suppliers, there is no intermediate strategy that performs well, i.e., achieving the $\lambda^2$ profit loss that dual sourcing and inventory mitigation would yield with independent suppliers. This once again illustrates the critical importance of inducing Tier 1 suppliers to mitigate disruption when there is overlap in Tier 2. When there is substantial overlap in the upper tiers of the supply, the manufacturer must engage Tier 1 suppliers in disruption mitigation efforts, or else suffer potentially large losses in expected profit.

6 Choosing the Tier 2 Configuration

As the results in the previous sections show, the optimal sourcing strategy may vary substantially depending on the degree of overlap in the Tier 2 supplier base. Aside from the observations regarding the behavior of the manufacturer’s optimal sourcing strategy, this fact has three additional implications: first, the manufacturer may prefer one Tier 2 configuration over the other; second, the Tier 1 suppliers may prefer one Tier 2 configuration over the other; and third, these preferences may not align. As a result, if Tier 2 suppliers are chosen strategically by Tier 1 suppliers, there is a potential for the supply chain to be uncoordinated—for the Tier 1 suppliers to select Tier 2 suppliers that do not maximize the manufacturer’s profit—an issue that we investigate in this section.

We first note that, depending on the relative values of $K$ and $K_s$, either Tier 2 configuration may be preferred by the manufacturer or the Tier 1 suppliers. Hence, to facilitate a fair comparison between the configurations, we focus here on the case where the two Tier 2 configurations have precisely the same industry-wide disrupted capacity from unreliable suppliers, i.e., $K_s = 2K$. Let $\Pi^M_I$ and $\Pi^M_S$ denote the manufacturer’s optimal expected profit with independent and shared Tier 2 suppliers, respectively, and let $\Pi^T_1I$ and $\Pi^T_1S$ denote the average expected profit of the Tier 1 suppliers with independent and shared Tier 2 suppliers, respectively. In this setting, it is possible to show that both the manufacturer and Tier 1 suppliers are indifferent between the Tier 2 configurations in all cases except those described in the following theorem:

Theorem 8. If disruptions are severe and the reliable supplier is costly:

(i) If $\frac{2\pi}{\lambda} < \pi < \bar{\pi}$, the manufacturer strictly prefers independent Tier 2 suppliers, while Tier 1 suppliers are indifferent ($\Pi^M_I > \Pi^M_S$ and $\Pi^T_1I = \Pi^T_1S$).

(ii) If $\bar{\pi} < \pi < \frac{2\pi \bar{\pi}}{\lambda} - \frac{c_u}{\lambda}$, the manufacturer strictly prefers independent Tier 2 suppliers, while Tier 1 suppliers strictly prefer shared Tier 2 suppliers ($\Pi^M_I > \Pi^M_S$ and $\Pi^T_1I < \Pi^T_1S$).

For example, if $K_s > D$ and $K = 0$, the manufacturer trivially prefers the shared Tier 2 configuration, while if $K_s = 0$ and $K > D/2$, the manufacturer trivially prefers the independent Tier 2 configuration.
In both parts of the theorem, the manufacturer has a strict preference for an independent Tier 2 configuration. This is because, in this range of unit revenues, the manufacturer would choose to pursue dual sourcing and inventory mitigation with independent Tier 2 suppliers, a strategy that is not valuable with shared Tier 2 suppliers; put differently, as one would expect, the manufacturer prefers the most independent Tier 2 configuration because it is the most diversified and allows the manufacturer to pursue the largest set of different sourcing strategies. Tier 1 suppliers are indifferent between Tier 2 configurations in part (i) of the theorem, because Tier 1 suppliers are paid a low price ($c_u$) and left with zero profit irrespective of whether Tier 2 suppliers are independent or shared. However, in part (ii) of the theorem, the manufacturer would pursue supplier mitigation with a shared Tier 2 configuration, but not with an independent Tier 2—because Tier 1 suppliers have (on average) strictly positive profit when supplier mitigation is pursued, this means that Tier 1 suppliers prefer a shared Tier 2 over an independent Tier 2 for moderate manufacturer unit revenues.

As a result, Theorem 8 highlights a potential coordination problem that can arise between Tier 1 suppliers and the manufacturer. This may have serious implications for the manufacturer if Tier 1 suppliers strategically choose Tier 2 suppliers, anticipating the manufacturer’s subsequent contractual offers, a decision that we now explore more formally. Specifically, we analyze a model in which, prior to contractual offers from the manufacturer, the Tier 1 suppliers engage in a simultaneous move supplier selection game, in which they choose which Tier 2 suppliers (one reliable and one unreliable, from a set of two identical suppliers of each type) to establish relationships with, determining the supply chain configuration (either shared or independent Tier 2).

After their preferred Tier 2 suppliers have been selected, the manufacturer offers contracts to Tier 1 suppliers and induces a particular supply chain structure, and the remainder of the game proceeds as in our base model. This sequence of events is reflective of our motivating example of the automotive supply chain, in which supplier selection and approval is a long term decision, while the manufacturer’s day-to-day procurement contracts are short or medium term decisions; hence, Tier 1 suppliers are sequential leaders in determining the set of approved Tier 2 suppliers, and the manufacturer chooses a sourcing strategy in response to the chosen Tier 2 configuration. If the manufacturer treats Tier 1 suppliers asymmetrically—i.e., by offering one a high price to induce supplier mitigation and the other

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7We restrict Tier 1 suppliers to selecting exactly one reliable and one unreliable supplier to keep the model parsimonious and consistent with our preceding analysis. Note that, in order to make positive profit, a Tier 1 supplier should always select at least (and, without loss of generality, exactly) one reliable supplier, and at least one unreliable supplier. While the cost of qualification and vetting makes it reasonable to assume that at most one unreliable supplier is selected in most cases, we also suspect that allowing two or more does not impact our qualitative results (from the manufacturer’s perspective, it would still generate a fixed—but possibly considerably more complex—disruption profile in Tier 2, hence only significantly complicating the analysis of the optimal sourcing decisions in the various tiers).
a low price to induce passive acceptance—then we assume the manufacturer randomly selects Tier 1 suppliers to fulfill each role; hence, during the supplier selection game, the \textit{ex ante} expected profit of a Tier 1 supplier is simply the average profit of the suppliers in Tier 1. We assume all information and parameters in the game are public knowledge.

Because reliable Tier 2 suppliers are identical and perfectly reliable, whether the Tier 1 suppliers share a reliable supplier or not is irrelevant; hence, the supplier selection game reduces to Tier 1 suppliers choosing an unreliable supplier from a set of two \textit{ex ante} identical unreliable suppliers, which we label $U_1$ and $U_2$. If the Tier 1 suppliers select the same unreliable Tier 2 supplier, then the resultant supply chain configuration is the shared Tier 2 scenario. If Tier 2 suppliers select different unreliable Tier 2 suppliers, then the supply chain possesses an independent Tier 2 configuration. Consequently, the Tier 1 suppliers play a two-by-two normal form game, depicted in Table 2 which fits the profile of a coordination game (Osborne and Rubinstein 1994). The pure strategy equilibria to the game in Table 2 will thus determine the supply chain configuration. These equilibria are described in Corollary 3:

**Corollary 3.** If disruptions are severe, the reliable supplier is costly, and $\bar{\pi} < \pi < \frac{c_u + \bar{\pi}}{1 - \lambda} - \frac{c_u}{\lambda^2}$, there are two pure strategy equilibria to the Tier 2 supplier selection game: either both Tier 1 suppliers select $U_1$ or both select $U_2$, resulting in a shared supply chain configuration. Otherwise, any supply chain configuration is an equilibrium to the Tier 2 supplier selection game.

The corollary shows when the manufacturer’s unit revenues are intermediate, disruptions are severe, and the reliable supplier is costly, the only equilibria to the supplier selection game both involve the Tier 1 suppliers selecting the shared configuration, precisely the wrong supply chain configuration from the manufacturer’s point of view, as Theorem 8 shows. This is because, in this region, the diagonal entries in the game in Table 2 have positive payoffs (since the manufacturer uses supplier mitigation under a shared Tier 2), while the off diagonal entries have zero payoff (since the manufacturer does not use supplier mitigation under an independent Tier 2). We note that this result was derived despite controlling for a number of factors, such as any cost or capability asymmetry in the Tier 2 suppliers, which obviously may exacerbate the effect (e.g., if one unreliable Tier 2 supplier is particularly cheap, we focus on pure strategy equilibria, as it is well known that mixed strategy equilibria in coordination games are Pareto inferior to pure strategy equilibria, and are also evolutionarily unstable.

<table>
<thead>
<tr>
<th>Supplier B $U_1$</th>
<th>Supplier B $U_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier A $U_1$</td>
<td>Shared Tier 2</td>
</tr>
<tr>
<td>Supplier A $U_2$</td>
<td>Independent Tier 2</td>
</tr>
</tbody>
</table>

Table 2. The Tier 2 Supplier Selection Game
then Tier 1 suppliers may have even more incentive to select the shared Tier 2 configuration).

Interestingly, in all other cases, any selection of Tier 2 suppliers is an equilibrium in the supplier selection game. This is because the manufacturer’s contractual offers are the same in either supply chain configuration, meaning all cells in the game in Table 2 have the same payoff, leaving Tier 1 suppliers indifferent. As such, when the manufacturer strictly prefers an independent Tier 2 configuration, this can be easily induced via a small side payment, contingent on Tier 1’s choice.

Unfortunately, as the following corollary demonstrates, this is not the case when the Tier 1 suppliers have a strict preference for a shared Tier 2 configuration—when this happens, it is impossible for the manufacturer to profitably induce an independent Tier 2 configuration:

**Corollary 4.** When \( \bar{\pi} < \pi < \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{2\lambda} \), the manufacturer can never increase its profit by paying Tier 1 suppliers to select an independent Tier 2 configuration.

This occurs because the side payment necessary to induce Tier 1 suppliers to select an independent Tier 2 is greater than the profit gain enjoyed by the manufacturer when sourcing from an independent Tier 2. As a result, the coordination problem appears to be especially severe in this region, and the manufacturer is presented with a particularly challenging problem: if Tier 1 suppliers can strategically choose Tier 2 suppliers, what mechanism could it use to ensure that the independent configuration is induced? In the following section, we investigate one such mechanism, with an appealing and simple form: penalty contracts.

### 7 Penalty Contracts and Supply Chain Coordination

In §§2-6, we restricted our attention to price and quantity contracts of the form \((p, Q)\). While these are commonly observed in practice, they expose the manufacturer to a moral hazard problem: when the manufacturer attempts to induce sourcing from a reliable Tier 2 supplier, Tier 1 suppliers may shirk and source from an unreliable Tier 2 supplier, necessitating a high payment from the manufacturer to ensure the desired behavior. It is easy to see that this makes supplier mitigation quite costly for the manufacturer and, as a result, may cause the manufacturer to avoid supplier mitigation even when it would be profitable for the supply chain as a whole to pursue this strategy, resulting in a coordination problem. Furthermore, an additional layer of mis-coordination occurs if Tier 1 suppliers endogenously choose Tier 2 suppliers: Tier 1 may prefer a shared Tier 2, while the manufacturer prefers an independent Tier 2. Thus, three questions immediately arise: first, is it possible to overcome the moral hazard problem using a simple coordinating contract; second, do the results derived in §§3-5.
continue to hold qualitatively if a coordinating contract is used; and third, can this contract overcome the coordination problem from §6 when the Tier 2 configuration is endogenous? In this section, we demonstrate that the answer to all of these questions is yes, if the coordination mechanism used is a penalty contract.

Specifically, we consider contracts of the form \((p, Q, f)\), where \(f\) is a per-unit fee for each unit ordered by the manufacturer that the contracted Tier 1 supplier fails to deliver. The following theorem demonstrates that these contracts are effective both at eliminating moral hazard and at solving the endogenous Tier 2 configuration coordination problem discussed in §6.

**Theorem 9.** When the manufacturer uses an optimal penalty contract,

(i) With either independent or shared Tier 2 suppliers, the manufacturer’s optimal sourcing strategy is the same as in the price-quantity contract case, except for the fact that supplier mitigation is adopted at lower unit revenues.

(ii) The manufacturer extracts all profit and Tier 1 suppliers always receive zero profit, regardless of the Tier 2 configuration, and hence Tier 1 suppliers are indifferent between Tier 2 configurations.

Part (i) of Theorem 9 shows that if penalty contracts are employed, the manufacturer adopts supplier mitigation at lower unit revenues than under simple price and quantity contracts; otherwise, the manufacturer’s preferences between the various mitigation strategies and induced supply chain structures are qualitatively identical to our earlier results. This increased reliance on supplier mitigation stems from the fact that penalty contracts enable the manufacturer to extract all surplus from Tier 1 suppliers even when inducing them to dual source, making supplier mitigation relatively more attractive than in our earlier setting. Beyond this effect, however, all the qualitative observations made in §§3-5 continue to hold, confirming that our basic insights persist even in the absence of a moral hazard problem.

There is, however, one critical difference when penalty contracts are used, as shown in part (ii) of the theorem. Since the manufacturer can extract the entire profit in all induced structures by using an appropriate penalty, Tier 1 suppliers become indifferent between all Tier 2 configurations. In the context of the supplier selection game analyzed in §6, this implies that a manufacturer who prefers an independent Tier 2 configuration can (profitably) make a small side payment to Tier 1 suppliers.

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9-It is also straightforward to show that with the optimal penalty contracts, the manufacturer makes the same sourcing choices as a centralized system (i.e., a system in which the manufacturer sources directly from Tier 2). For a discussion of this issue, please see the Supplemental Appendix of the paper.

10-For instance, the result that increased overlap in Tier 2 leads the manufacturer to favor supplier mitigation more, and direct manufacturer mitigation less.
to induce them to make this selection, thus eliminating the perverse incentives generating the moral hazard problem, and effectively coordinating the supply chain.

We note that penalty contracts are commonly discussed as mechanisms to induce increased risk mitigation efforts by suppliers (Kleindorfer and Saad 2005, Yang et al 2009, Hwang et al 2014). However, it is important to recognize that such contracts may face challenges to implementation in practice. First, while buyers and suppliers may be willing to accept penalty contracts when disruptions arise endogenously (e.g., due to supplier malfeasance or negligence, such as from industrial accidents or poor quality control), buyers are often loathe to penalize suppliers because their supply chain experienced an exogenous natural disaster. This is especially true when buyers and suppliers have long-term relationships, as many of the firms in our example of the Japanese automotive industry do. Second, our motivating example of the 2011 Tōhoku earthquake shows precisely the opposite behavior of penalties occurred in practice: rather than punish suppliers for failing to deliver after the earthquake, Toyota took an active role in assisting them to quickly recover, providing manpower and financial assistance that subsidized the recovery efforts of the suppliers—in effect a payment from the manufacturer to the suppliers in the wake of a disruption. Thus, penalties may come with a credibility problem: once a disruption has occurred, it is in the manufacturer’s best interests to help suppliers quickly recover via direct financial and resource assistance (or else the manufacturer will experience even greater lost sales), meaning a punishment in the form of a financial penalty is not credible. This dynamic is not captured in our stylized single period model, but nonetheless it suggests that penalty contracts may have limitations and might not be easily implemented in practice.

8 Conclusion

In this paper, we have analyzed the sourcing strategy of a manufacturer managing disruption risk in a multi-tier supply chain. In contrast to the existing disruption risk literature, we have focused on the impact of sourcing decisions on risk originating in Tier 2 of the supply chain, rather than Tier 1. We find that the degree of overlap in Tier 2 (whether Tier 2 suppliers are shared by Tier 1 suppliers) has a substantial impact on a manufacturer’s optimal sourcing strategy, as greater overlap causes the manufacturer to rely less on direct mitigation efforts (dual sourcing and procuring excess inventory from Tier 1) and more on indirect methods (inducing Tier 1 suppliers to mitigate risk via contract terms). Moreover, we showed that the manufacturer and Tier 1 suppliers may have conflicting preferences regarding the configuration of the supply network, potentially leading to a coordination
problem in the supply chain if Tier 1 suppliers strategically choose Tier 2 suppliers. The manufacturer can rectify this coordination problem by using penalty contracts to eliminate moral hazard and hence the perverse incentives that Tier 1 suppliers have to select a shared Tier 2 configuration.

There are several assumptions of our model that bear discussion. First, we focused on a single period model rather than one in which demand (and disruptions) occur over multiple periods (Tomlin 2006). In a multiperiod model, inventory mitigation—essentially holding safety stocks—becomes a more valuable strategy, as excess inventory from one period can be held to use in subsequent periods. Nevertheless, we expect that the same sort of progression of the optimal sourcing strategy persists even over multiple periods, i.e., the manufacturer moves from direct mitigation to supplier mitigation as overlap increases. Indeed, Toyota itself is following the strategy of greater supplier mitigation following the Tohoku earthquake, as it has worked with several key Tier 1 suppliers to have them dual source and hold inventories, despite the fact that the latter strategy is a significant departure from the Toyota Production System philosophy of zero inventory (Greimel 2012). Our model thus supports the optimality of Toyota’s increased reliance on supplier mitigation in the wake of the 2011 disaster.

Second, we have assumed that no emergency backup supply is available. In practice, it may be possible for Tier 1 suppliers to find secondary sources of component supply if Tier 2 suppliers disrupt, an extension that we have analyzed but omit for the sake of brevity; as one might expect, the availability of such an option means that the manufacturer is less likely to use supplier mitigation, but otherwise the same progression of optimal mitigation strategies holds. Interestingly, if the cost of emergency supply is sufficiently low, the manufacturer induces single sourcing from each Tier 1 supplier for any unit revenues, regardless of the Tier 2 configuration; this implies that for components where backup supply is readily available (e.g., commodity products) the optimal manufacturer strategy is independent of the configuration of Tier 2. Conversely, knowing the Tier 2 configuration and engaging Tier 1 suppliers in risk mitigation are critically important if emergency supply is costly, e.g., for specialized or custom products like semiconductor components. This result illustrates that it is important for manufacturers to focus their limited resources on understanding and managing disruption risk in the upper tiers of their supply chain primarily for specialized, non-commodity components, while a more traditional manufacturer-only mitigation approach may be employed for more standard products with readily available backup supply.

Lastly, we have assumed that the manufacturer knows the configuration of Tier 2 when making its optimal sourcing decision. In practice, this may not be the case—even Toyota, one of the most powerful manufacturers in the world, was unable to persuade almost half of its Tier 1 suppliers to
share the identities of their Tier 2 suppliers. Thus, an interesting question remains unsolved by our analysis: how can a manufacturer manage disruption risk if it does not know the configuration of its supply network? Future work might explore this question, using our analysis of the optimal sourcing strategy under public information as a baseline for comparison.

Viewed as a whole, our results illustrate the significant impact that an extended supply chain can have on optimal sourcing and disruption mitigation efforts. While much of the existing work on disruption risk management has focused on immediate suppliers (Aydin et al 2010), the trend towards longer and longer global supply chains has exposed firms to risk at multiple levels, necessitating an understanding of how Tier 1 sourcing decisions impact risk originating at higher levels of the supply chain. Taken in that light, our model provides a first step at understanding how to manage disruption risk and optimal sourcing in multi-tier supply networks.

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References


A Supporting Results

Lemma 1. With independent Tier 2 suppliers,

(i) In Structure 1, the optimal contract is \( \{ (c_u, D) \} \) and the optimal expected profit is \( \Pi^M = \lambda(\pi - c_u)K + (1 - \lambda)(\pi - c_u)D \)
(ii) In Structure 2, the optimal contract is \( \left\{ \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, D \right) \right\} \) and the optimal expected profit is \( \Pi^M = (\pi - c_u) D \).

(iii) In Structure 3, if \( K \geq D/2 \), the optimal contracts are \( \{(c_u, \theta D), (c_u, (1 - \theta)D)\} \) for any \( \theta \in [0, 1] \) such that neither quantity exceeds \( K \), and the optimal expected profit is \( \Pi^M = (\pi - c_u) D \). If \( K < D/2 \) and \( \pi < c_u/\lambda \), the optimal contracts are \( \{(c_u, \theta D), (c_u, (1 - \theta)D)\} \) for any \( \theta \in [0, 1] \) such that both quantities are at least \( K \), and the optimal expected profit is \( \Pi^M = (\pi - c_u) \left( (1 - \lambda)D + \lambda 2K \right) \). Otherwise, the optimal contract is \( \{(c_u, D - K), (c_u, D - K)\} \) and the optimal expected profit is \( \Pi^M = (\pi - c_u) D - (\lambda^2 \pi + (1 - 2\lambda)c_u) (D - 2K) \).

(iv) In structure 4, the optimal contract is \( \left\{ \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, D - K \right), (c_u, K) \right\} \) and the optimal expected profit is \( \Pi^M = \pi D - \frac{c_r - (1 - \lambda)c_u}{\lambda} (D - K) - c_u K \).

(v) In structure 5, the optimal contracts are \( \left\{ \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, \theta D \right), \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, (1 - \theta)D \right) \right\} \) for any \( \theta \in [0, 1] \) and the optimal expected profit is \( \Pi^M = \left( \pi - \frac{c_r - (1 - \lambda)c_u}{\lambda} \right) D \).

Proof. (i) and (ii) follow immediately from Theorem 1 and equation (2). (iii) Structure 3 is induced by offering a price \( c_u \) to a both Tier 1 suppliers, implying that in equation (2), \( f_1 = (1 - \lambda)^2 \), \( f_2 = f_3 = \lambda(1 - \lambda) \), and \( f_4 = \lambda^2 \). Two observations can be made to simplify equation (2): at optimality, \( Q_A + Q_B \geq D \), and \( Q_i + K \leq D \), for \( i = A, B \). Further, observe that if \( K > D/2 \), the manufacturer can simply source \( D/2 \) from each Tier 1 supplier and fully eliminate disruption risk. More generally, any sourcing strategy in which \( Q_A = \theta D, Q_B = (1 - \theta)D \), for some \( \theta \in [0, 1] \), and in which neither quantity exceeds \( K \), results in profit equal to \( (\pi - c_u) D \). If, on the other hand, \( K < D/2 \), it’s clearly optimal for the manufacturer to source at least \( K \) from each supplier (because this is the “risk free” quantity which each supplier delivers for certain), resulting in expected profit

\[
\Pi^M \left( \{(c_u, Q_A), (c_u, Q_B)\} \right) = \lambda^2 (\pi - c_u) 2K + \lambda(1 - \lambda)(\pi - c_u) (K + Q_B + K + Q_A) + (1 - \lambda)^2 (\pi D - c_u (Q_A + Q_B)).
\]

It can be checked that \( \frac{\partial \Pi^M}{\partial Q_i} = (1 - \lambda)(\lambda \pi - c_u) \), for \( i = A, B \). Hence, if \( \lambda \pi > c_u \), the manufacturer will source as much as possible from each supplier, satisfying the constraints: \( Q_A + Q_B \geq D \), \( K \leq Q_A \), \( K \leq Q_B \), \( D - K \geq Q_A \), and \( D - K \geq Q_B \). This implies \( Q_A^* = Q_B^* = D - K \), resulting in the optimal expected profit given in the lemma. Conversely, if \( \lambda \pi < c_u \), the manufacturer wants to source as little as possible while satisfying all the constraints. This implies any \( Q_A = \theta D, Q_B = (1 - \theta)D \), for some \( \theta \in [0, 1] \) so long as each quantity is at least \( K \), is optimal yielding the expected profit given above.

(iv) Structure 4 is induced by offering a price of \( \frac{c_r - (1 - \lambda)c_u}{\lambda} \) to one Tier 1 supplier, and a price of \( c_u \) to
the other Tier 1 supplier. Manufacturer profit is

\[ \Pi^M \left( \left\{ \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, Q_A \right), (c_u, Q_B) \right\} \right) = \lambda \left( \pi \min\{D, Q_A + \min\{K, Q_B\}\} - c_u \min\{K, Q_B\} \right) + (1 - \lambda) \left( \pi D - c_u Q_B \right) - \frac{c_r - (1 - \lambda)c_u}{\lambda} Q_A. \]

Clearly, at optimality \( Q_A + Q_B \geq D, Q_B \geq K, \) and \( Q_A + K \leq D, \) hence

\[ \Pi^M \left( \left\{ \left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, Q_A \right), (c_u, Q_B) \right\} \right) = \lambda (\pi (Q_A + K) - c_u K) + (1 - \lambda) (\pi D - c_u Q_B) - \frac{c_r - (1 - \lambda)c_u}{\lambda} Q_A. \]

Given the linearity of this profit function, the manufacturer will either find it optimal to source nothing from supplier A (in which case the structure is degenerate with Structure 1; we ignore this without loss of generality) or source \( Q - K \) from A and K from B, yielding the profit given in the lemma.

(v) Structure 5 is induced by offering both suppliers a high price and inducing them to dual source, resulting in perfectly reliable supply; the manufacturer is thus indifferent between any division of sourcing quantities between these two suppliers so long as the total quantity sourced is \( D. \)

Lemma 2. With shared Tier 2 suppliers:

(i) In structures 1 and 2, the optimal sourcing quantities and profits are equal to the expressions in Lemma 7 substituting \( K_s \) for \( K. \)

(ii) In structure 3, if \( D \leq K_s \), the optimal contract is \( \{ (c_u, \theta D), (c_u, (1 - \theta)D) \} \) for any \( \theta \in [0, 1] \), and the optimal expected profit is \( \Pi^M = (\pi - c_u) D. \) If \( D > K_s \), the optimal contract is \( \{ (c_u, \theta D), (c_u, (1 - \theta)D) \} \) for any \( \theta \in [0, 1] \) and the optimal expected profit is \( \Pi^M = \lambda (\pi - c_u) K_s + (1 - \lambda) (\pi - c_u) D. \)

(iii) In structure 4, the optimal contract is \( \{ (c_r - (1 - \lambda)c_u, D - K_s/2), (c_u, K_s/2) \} \) and the optimal expected profit is \( \Pi^M = \left( \pi - \frac{c_r - (1 - \lambda)c_u}{\lambda} \right) (D - K_s/2) + (\pi - c_u) K_s/2. \)

(iv) Structure 5 is degenerate with structure 2.

Proof. (i) Follows immediately. (ii) In structure 3, there are two cases. If \( D \leq K_s \), the manufacturer can achieve risk-free sourcing by splitting his order between the two Tier 1 suppliers in any way. Conversely, if \( D > K_s \), under the uniform allocation rule the manufacturer can divide its sourcing
quantities between Tier 1 suppliers in any way and achieve expected profit

\[ \Pi^M(\{(c_u, Q_A), (c_u, Q_B)\}) = \lambda (\pi - c_u)K_s + (1 - \lambda)(\pi \min(D, Q_A + Q_B) - c_u(Q_A + Q_B)) \]

In that case, any \( Q_A + Q_B = D \) is optimal, and manufacturer profit is as given in the lemma. (iii)

Under the uniform allocation rule, to induce supplier A to source from the reliable Tier 2 supplier, the manufacturer must allocate at least \( K_s/2 \) units of inventory to supplier A (see Theorem 4). If the manufacturer allocates such that \( Q_i \geq K_s/2 \) for \( i = A, B \), then supplier A will source \( K_s/2 \) units from the unreliable Tier 2 supplier, and \( Q_A - K_s/2 \) from the reliable Tier 2 supplier, while supplier B will source \( Q_B \) from the unreliable Tier 2 supplier (see Theorem 4). The manufacturer’s profit will be

\[ \Pi^M(\left\{\left\{\frac{c_r - (1 - \lambda)c_u}{\lambda}, Q_A\right\}, (c_u, Q_B)\right\}) = \lambda (\pi - c_u)K_s + (1 - \lambda)(\pi \min(D, Q_A + Q_B) - c_uQ_B) - \frac{c_r - (1 - \lambda)c_u}{\lambda}Q_A. \]

It is straightforward to see that \( \frac{\partial \Pi^M}{\partial Q_B} < 0 \), and \( Q_A + Q_B \leq D \) at optimality, suggesting the solution \( Q_A = D - K_s/2, Q_B = K_s/2 \). Conversely, if the manufacturer allocates \( Q_B < K_s/2 \), profit will be

\[ \Pi^M(\left\{\left\{\frac{c_r - (1 - \lambda)c_u}{\lambda}, Q_A\right\}, (c_u, Q_B)\right\}) = \pi D - c_uQ_B - \frac{c_r - (1 - \lambda)c_u}{\lambda}(D - Q_B). \]

Since \( c_u < \frac{c_r - (1 - \lambda)c_u}{\lambda} \), the optimal allocation is clearly \( Q_B = K_s/2 \), leading to the optimal contracts and profits as stated in the lemma. (iv) Follows immediately.

\[ \square \]

B Proofs of the Main Results

**Proof of Theorem 1** From equation (1), we may make several observations. First, the supplier will always source enough inventory to cover demand in the non-disrupted state, i.e., \( q_u + q_r \geq Q \). Second, the supplier will never source more inventory than covers demand in the disrupted state, i.e., \( q_r + \min\{K, q_u\} \leq Q \). Third, the supplier always sources at least \( q_u \geq K \) units from the unreliable Tier 2 supplier, as this quantity is risk free and less expensive than sourcing from the reliable Tier 2 supplier.

Hence, profit may be written \( \Pi^S(q_u, q_r) = \lambda \left[p(q_r + K) - c_rq_r - c_uK\right] + (1 - \lambda)\left[pQ - c_uq_u - c_rq_r\right] \). Due to the linear nature of this function, it must be true that either \( q_u = K \) and \( q_r = Q - K \), or \( q_u = Q \) and \( q_r = 0 \) (or some convex combination of the two) maximizes profit. Under the former
strategy, profit is \( \Pi^S(K, Q - K) = (p - c_r)(Q - K) + (p - c_u)K \), while under the latter strategy, profit is \( \Pi^S(Q, 0) = (1 - \lambda)(p - c_u)Q + \lambda(p - c_u)K \). Comparing the profit functions yields the optimal strategy in the theorem.

**Proof of Theorem 2**: Follows directly from Lemma 1. When \( K \geq D/2 \), structure 3 can achieve riskless supply at a unit cost of \( c_u \), which is the lowest cost and highest revenue outcome, and is hence optimal.

**Proof of Theorem 3**: Observe that structure 4 dominates 2 and 5, because all three structures have perfectly reliable supply but structure 4 achieves this at a lower average unit cost. Structure 3 dominates structure 1, since the strategy of dual sourcing outperforms single sourcing by ensuring a greater level of minimum supply. Since all profit functions are linearly increasing in \( \pi \), determining the preference between structures 3 and 4 reduces to determining the threshold \( \pi \) for which these two structures are equivalent. First, structure 4 dominates structure 3 without inventory mitigation if

\[
\pi D - \frac{c_r - (1 - \lambda)c_u}{\lambda}(D - K) - c_uK > (\pi - c_u)((1 - \lambda)D + \lambda 2K).
\]

This reduces to \( \pi > \frac{c_r}{\lambda} \left( \frac{D-K}{D-2D} \left( \frac{c_r/c_u-1}{\lambda} \right) + 1 \right) \equiv \bar{c} \). In addition, structure 4 dominates structure 3 with inventory mitigation if

\[
\pi D - \frac{c_r - (1 - \lambda)c_u}{\lambda}(D - K) - c_uK > \lambda^2 (\pi - c_u) 2K + (1 - \lambda^2) (\pi - c_u) D - (1 - \lambda)^2(D - 2K)c_u.
\]

This reduces to \( \pi > \frac{c_u}{\lambda} \left( \frac{D-K}{D-2D} \left( \frac{c_r/c_u-1}{\lambda} \right) + (2\lambda - 1) \right) = \frac{1}{\lambda^2} (\lambda \bar{c} - (1 - \lambda)c_u) \). There are thus two cases. If \( c_r < c_u \left( \frac{D-2K}{D-K} \lambda (1 - \lambda) + 1 \right) \), structure 4 becomes optimal before inventory mitigation becomes optimal within structure 3, and hence the manufacturer’s optimal sourcing strategies are:

\[
\{(c_u, Q_A), (c_u, Q_B)\} = \begin{cases} 
(c_u, \theta D), (c_u, (1 - \theta)D) \quad \text{if } \pi \leq \bar{c} \\
\left( \frac{c_r - (1 - \lambda)c_u}{\lambda}, D - K \right), (c_u, K) \quad \text{otherwise}
\end{cases}
\]

while if \( c_r > c_u \left( \frac{D-2K}{D-K} \lambda (1 - \lambda) + 1 \right) \), structure 4 becomes optimal after inventory mitigation is opti-
mal in structure 3, and the optimal strategies are thus

\[
(c_u, Q_A), (c_u, Q_B) = \begin{cases} 
(c_u, \theta D), (c_u, (1 - \theta)D) & \text{if } \pi \leq \frac{c_u}{\lambda} \\
(c_u, D - K), (c_u, D - K) & \text{if } \frac{c_u}{\lambda} < \pi \leq \frac{1}{\lambda} (\lambda \bar{c} - (1 - \lambda)c_u). \\
\left(\frac{c_u - (1 - \lambda)c_u}{\lambda}, D - K\right), (c_u, K) & \text{otherwise}
\end{cases}
\]

**Proof of Corollary 1** (i) Let \( \Pi^M(X) \) be the manufacturer’s expected profit under strategy \( X \in \{DS, DS + IM, DS + SM\} \), such that the maximum percentage profit loss from using strategy \( X \) is \( L(X) = 1 - \lim_{\pi \to \infty} \Pi^M(X)/\Pi^M(DS + SM) \). Then, from Lemma 1

\[
L(DS) = 1 - \lim_{\pi \to \infty} \frac{(\pi - c_u)((1 - \lambda)D + \lambda 2K)}{\pi D - \frac{c_u(1 - \lambda)c_u}{\lambda}(D - K) - c_u K} = \lambda \left(1 - \frac{2K}{D}\right),
\]

which is bounded below by zero and above by \( \lambda \), since \( D > 2K \).

(ii) When using strategy \( DS + IM \),

\[
L(DS + IM) = 1 - \lim_{\pi \to \infty} \frac{\lambda^2 (\pi - c_u)2K + 2\lambda(1 - \lambda)(\pi - c_u)D + (1 - \lambda)^2(\pi D - c_u2(D - K))}{\pi D - \frac{c_u(1 - \lambda)c_u}{\lambda}(D - K) - c_u K} = \lambda^2 \left(1 - \frac{2K}{D}\right),
\]

which is bounded below by zero and above by \( \lambda^2 \).

**Proof of Theorem 4** With a uniform allocation rule, supplier \( i \)'s allocation in the event of a disruption from the shared unreliable Tier 2 supplier is \( \min \left\{ q_u^i, K_s + \left(\frac{K_s}{2} - q_u^i\right)^+ \right\} \). This implies that supplier \( i \)'s profit is

\[
\pi_i(q_u^i, q_r^i) = \lambda \left[ p_i \min\left( Q_i, q_r^i + \min\left\{ q_u^i, \frac{K_s}{2} + \left(\frac{K_s}{2} - q_u^i\right)^+ \right\}\right) - c_u \min\left\{ q_u^i, \frac{K_s}{2} + \left(\frac{K_s}{2} - q_u^i\right)^+ \right\} \right]
\]

\[
+(1 - \lambda) \left[ p_i \min\left( Q_i, q_r^i + q_u^i - c_u q_u^i\right) - c_r q_r^i \right] - c_r q_r^i.
\]

The following must be true: \( q_r^i + q_u^i \geq Q_i, q_u^i \geq \frac{K_s}{2} + (\frac{K_s}{2} - q_u^i)^+ \equiv \bar{K}, \) and \( q_r^i + \min\left\{ q_u^i, \frac{K_s}{2} + (\frac{K_s}{2} - q_u^i)^+ \right\} \leq Q_i \). Using these facts, profit is

\[
\pi_i(q_u^i, q_r^i) = \lambda \left[ p_i q_r^i + p\bar{K} - c_u \bar{K}\right] + (1 - \lambda) \left[ p_i Q_i - c_u q_u^i\right] - c_r q_r^i.
\]

Observing that the profit function is linear in supplier \( i \)'s sourcing quantities, we have two cases: either
the supplier will source \( q_u^* = Q \) and \( q_r^* = 0 \), yielding profit \( \pi_i(q_u^*, q_r^*) = \lambda [p_i - c_u] \hat{K} + (1 - \lambda) [p_i - c_u] Q_i \), or the supplier will source \( q_u^* = \hat{K} \) and \( q_r^* = Q_i - \hat{K} \), yielding profit \( \pi_i(q_u^*, q_r^*) = p_i Q_i - c_u \hat{K} - c_r (Q_i - \hat{K}) \). The latter is preferred if \( p_i \geq \frac{c_r - (1 - \lambda) c_u}{\lambda} \), which is identical to the condition derived with independent Tier 2 suppliers (Theorem 1) and is independent of \( \hat{K} \) and hence \( q_u^{-1} \). Consequently, the supplier has two sourcing strategies (single source or dual source), with the conditions given in the theorem.

**Proof of Theorem 5.** Follows directly from Lemma 2.

**Proof of Theorem 6.** From Lemma 2, Structures 1 and 3 are equivalent, and Structures 2 and 5 are clearly dominated by structure 4, so the manufacturer’s choice is effectively between structures 1/3 and structure 4. The latter is preferred if \( \pi \geq c_u + \left( \frac{c_r - c_u}{\lambda} \right) \frac{D - K_s/2}{(D - K_s)} = \bar{\pi}_s \).

**Proof of Theorem 7.** If the reliable supplier is cheap, then \( \bar{\pi}_s \leq \bar{\pi} \) if and only if \( K_s \leq 2K \). If the reliable supplier is costly, observe that supplier mitigation is only employed in the independent Tier 2 case if \( \bar{\pi} > \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda} \geq \bar{\pi} \geq \bar{\pi}_s \) if \( K_s \leq 2K \), proving the theorem.

**Proof of Corollary 2.** Let \( \Pi^M(X) \) be the manufacturer’s expected profit under strategy \( X \in \{DS, DS + SM\} \), such that the maximum percentage profit loss from using strategy \( DS \) is \( L(DS) = 1 - \lim_{\pi \to \infty} \Pi^M(DS)/\Pi^M(DS + SM) \). Then, from Lemma 2:

\[
L(DS) = 1 - \lim_{\pi \to \infty} \frac{\lambda(\pi - c_u)K_s + (1 - \lambda)(\pi - c_u)D}{(\pi - \frac{c_r - (1 - \lambda) c_u}{\lambda})(D - K_s/2) + (\pi - c_u)K_s/2} = \lambda \left( 1 - \frac{K_s}{D} \right).
\]

This is bounded below zero and above by \( \lambda \), since \( D > K_s \).

**Proof of Theorem 8.** From Lemmas 1 and 2 if disruptions are minor, the manufacturer can eliminate all risk at unit cost \( c_u \) under both configurations, meaning they are equivalent. We now focus on severe disruptions. When the reliable supplier is cheap, the manufacturer’s and the Tier 1 suppliers’ optimal profits are identical with independent and shared Tier 2 suppliers. When the reliable supplier is costly, we have:

\[
\Pi^M_I = \begin{cases} 
\lambda(\pi - c_u)2K + (1 - \lambda)(\pi - c_u)D, & \text{if } c_u \leq \pi < \frac{c_u}{\lambda} \\
(\pi - c_u)D - (\lambda^2\pi + (1 - 2\lambda)c_u)(D - 2K), & \text{if } \frac{c_u}{\lambda} \leq \pi < \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2} \\
(\pi - \frac{c_r - (1 - \lambda) c_u}{\lambda})(D - K) + (\pi - c_u)K, & \text{if } \pi \geq \frac{c_u + \bar{\pi}}{\lambda} - \frac{c_u}{\lambda^2},
\end{cases}
\]

\[
\Pi^M_S = \begin{cases} 
\lambda(\pi - c_u)2K + (1 - \lambda)(\pi - c_u)D, & \text{if } c_u \leq \pi < \bar{\pi} \\
(\pi - \frac{c_r - (1 - \lambda) c_u}{\lambda})(D - K) + (\pi - c_u)K, & \text{if } \pi \geq \bar{\pi}.
\end{cases}
\]
Since \( \frac{c_u}{\lambda} < \bar{\pi} < \frac{c_u + \bar{\pi}}{\lambda} \), note that \( \Pi_I^M = \Pi_S^M \) for \( \pi < c_u/\lambda \) and for \( \pi > \frac{c_u + \bar{\pi}}{\lambda} \). However, in the intermediate region, \( \Pi_I^M \) is given by a contract that yields a strictly larger profit than the contracts for the other two regions, and thus \( \Pi_I^M > \Pi_S^M \). Furthermore, in this intermediate region, both Tier 1 suppliers make zero profit when Tier 2 suppliers are independent, while they make a strictly positive average profit when Tier 2 suppliers are shared.

**Proof of Corollary 3.** Follows immediately from Theorem 8.

**Proof of Corollary 4.** From Theorem 8, it can be readily seen that, when \( \frac{c_u}{\lambda} < \pi < \bar{\pi} \), \( \Pi_I^M > \Pi_S^M \), while \( \Pi_{TI}^I = \Pi_{TS}^I = 0 \). When \( \bar{\pi} < \pi < c_u + \bar{\pi} - c_u/\lambda \), we have

\[
\Pi_I^M - \Pi_S^M - \sum_{i \in \{A,B\}} (\Pi_i^S - \Pi_i^M) = (D - 2K)(c_r - \lambda^2 \pi - 2(1 - \lambda) c_u).
\]

This expression is positive if and only if \( \pi < \frac{c_r - 2(1 - \lambda) c_u}{\lambda^2} \). Since the latter bound is always smaller than \( \bar{\pi} \), we see that the gain in manufacturer profit from an independent Tier 2 is strictly smaller than the loss in profit the Tier 1 suppliers experience.

**Proof of Theorem 9.** (i) We show the result only for the case of independent Tier 2 suppliers; the proof with shared Tier 2 suppliers is similar and is omitted. With a contract \((p, Q, f)\), the Tier 1 supplier profit with independent Tier 2 suppliers is

\[
\Pi_i(q_u^i, q_r^i) = \lambda \left[ (p_i + f) \min \{q_r^i, q_u^i\} + \min \{K, q_u^i\} - c_u q_u^i - c_r q_r^i \right] + (1 - \lambda) \left[ (p_i + f) \min \{q_u^i + q_r^i, Q_i\} - c_u q_u^i - c_r q_r^i \right] - f Q_i.
\]

The last term is a constant; the rest of the expression is identical to the \((p, Q)\) contract case with \( p \) replaced by \( p + f \). Hence, the Tier 1 supplier’s optimal sourcing strategy is

\[
(q_u^i, q_r^i)^* = \begin{cases} 
(Q_i, 0) & \text{if } c_u \leq p_i + f < \frac{c_r - (1 - \lambda) c_u}{\lambda}, \\
(K, Q_i - K) & \text{if } p_i + f \geq \frac{c_r - (1 - \lambda) c_u}{\lambda}.
\end{cases}
\]

Consequently, the manufacturer has two strategies: it can induce passive acceptance by a Tier 1 supplier, by offering a contract price \( p_i = c_u \) and no penalty \( (f = 0) \), or it can induce dual sourcing, by offering a contract price plus penalty \( (p_i + f) \) equal to \( \frac{c_r - (1 - \lambda) c_u}{\lambda} \). To ensure participation of the tier 1 supplier, the manufacturer must also ensure non-negative expected profit, meaning \( p \geq c_u + \left(1 - \frac{K}{Q_i}\right)(c_r - c_u) \). The optimal price is clearly the lowest price that achieves this outcome, i.e.,
\[ p = c_u + \left(1 - \frac{K}{Q}\right) (c_r - c_u), \text{ with a fee equal to } f = \frac{c_r-c_u}{\lambda} \left(1 - \frac{\lambda}{1 - \frac{K}{Q}}\right), \text{ such that the sum of the price and fee is } \frac{c_r-(1-\lambda)c_u}{\lambda}. \] This implies that inducing passive acceptance is identical to the \((p, Q)\) case, but inducing supplier mitigation is less expensive for the manufacturer than in the \((p, Q)\) case. Hence, manufacturer profit when inducing structure 3 is unchanged, but profit when inducing structure 4 is increased. Specifically, the profit in structure 4 is \(\Pi^M = \pi D - c_r(D - 2K) - 2c_uK\). This is greater than the profit in structure 3 with no inventory mitigation if \(\pi > c_u + \frac{c_r-c_u}{\lambda} < \bar{\pi}\), and is greater than the profit in structure 3 with inventory mitigation if \(\pi > \frac{c_r-2(1-\lambda)c_u}{\lambda^2} < \frac{c_u+\bar{\pi}}{\lambda^2} - \frac{c_u}{\lambda^2}\), hence the manufacturer pursues supplier mitigation at lower unit revenues than in Theorem 3, but otherwise follows an identical strategy.

(ii) Because the manufacturer either offers a unit price \(c_u\) or \(c_u + \left(1 - \frac{K}{Q}\right) (c_r - c_u)\), it leaves the Tier 1 suppliers with zero expected profit in any structure, yielding the result.
Supplemental Appendix

Emergency Sourcing in Tier 2

In practice, a disruption in Tier 2 may not result in a shortage of components and, ultimately, lost sales; in many cases, firms have access to outside sources of emergency supply that can be leveraged after a disruption to satisfy demand. In these cases, disruptions result in an increase in costs rather than lost sales. In this section, we analyze the impact of such a source of emergency supply on the manufacturer’s optimal sourcing strategy. Specifically, we assume that in addition to the reliable and unreliable Tier 2 suppliers, all Tier 1 suppliers have access to an “emergency” supplier selling at unit cost $c_e$ that is capable of fulfilling an order of any size even after a disruption has occurred. Following disruptions caused by unpredictable natural disasters such as earthquakes or floods, downstream manufacturers often help subsidize emergency sourcing; for example, Toyota and other members of JAMA, the Japanese Automobile Manufacturer Association, used joint funds following the Tohoku earthquake to help stabilize supply and set up emergency capacity for key components (Tabuchi 2011). Thus, disrupted Tier 1 suppliers are frequently left with only a portion of the cost burden. To model this, we assume that the Tier 1 suppliers pay a fraction $\gamma$ of the emergency procurement cost, while the downstream manufacturer pays the remaining fraction $1 - \gamma$. This parameter $\gamma$ is a decision variable of the manufacturer, whose contractual offer to Tier 1 supplier $i$ is denoted by the triple $(p_i, Q_i, \gamma_i)$.

Rather than rederive all of our previous results with the addition of emergency supply, we focus on how an emergency supplier impacts the manufacturer’s overall choice of sourcing strategy, as illustrated in the following theorem:

**Theorem 10.** With emergency supply available at unit cost $c_e$,

(i) With either independent or shared Tier 2 suppliers, profit in structure 3 is greater than in the case of no emergency supply, while profit in structure 4 is unchanged.

(ii) There exists a critical value $\bar{c}_e > c_u$ such that the manufacturer’s optimal sourcing strategy with both independent and shared Tier 2 suppliers is to induce structure 3 whenever the cost of emergency supply does not exceed $\bar{c}_e$.

The theorem shows two key results. Part (i) demonstrates that, as one would expect, the presence of emergency supply makes it favorable for the manufacturer to induce single sourcing by Tier 1 suppliers—

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11This emergency supplier may be an outside party or may be one of the existing suppliers, e.g. the reliable supplier, producing at a higher than normal cost to deliver units in an expedited fashion, or even the disrupted unreliable supplier leasing facilities or equipment from third parties.
in other words, the manufacturer is less likely to invest in supplier mitigation when emergency supply is available, either with independent or shared Tier 2 suppliers. Part (ii) shows that, when emergency supply is cheap, both the independent and shared Tier 2 cases have the same optimal sourcing strategy for the manufacturer: always induce single sourcing from each Tier 1 supplier, i.e., induce structure 3. Conversely, it is clear that when emergency supply is very expensive ($c_e > \pi$), it is never utilized, leading to the same results that we derived in §4.5 and in particular leading to different optimal sourcing strategies for the manufacturer, depending on the Tier 2 configuration. In other words, the differences in the manufacturer's sourcing strategy between the two configurations are minor if emergency supply is inexpensive, but can be significant if emergency supply is expensive.

Returning to our motivating example of Toyota's experiences following the Tōhoku earthquake, recall that Toyota experienced disruptions from both commodity suppliers (like Fujikura, a rubber manufacturer), and specialized suppliers (like Renesas, a semiconductor manufacturer). Theorem 10 implies that for the commodity supplier Fujikura, for which emergency supply is likely to be relatively inexpensive, the optimal sourcing strategy does not depend heavily on the configuration of Tier 2, and investing effort to learn or influence the configuration of upper tiers of the supply chain may be a fruitless activity for the downstream manufacturer. Moreover, inducing passive acceptance in Tier 1 is likely to be favored over a more costly strategy of induced supplier mitigation.

In contrast, engaging Tier 1 suppliers in disruption mitigation efforts may be critical for Tier 2 suppliers like Renesas, for which emergency supply is practically non-existent, given the very long leadtimes of semiconductor manufacturing, and the small number of specialized automotive semiconductor suppliers. In addition, the optimal sourcing strategy depends heavily on the configuration of Tier 2, and hence knowing—and perhaps influencing—this configuration is potentially quite valuable to the manufacturer, an observation that we explore further in the next section.

**Centralized System**

We now consider a centralized system, in which the manufacturer sources directly from the Tier 2 suppliers. All other assumptions remain identical to our original setting. In particular, the manufacturer faces a deterministic demand of $D$ units, with unit revenues of $\pi$. Tier 2 consists of a single perfectly reliable supplier, with unit cost $c_r$, and either (a) two unreliable suppliers that disrupt independently, with probability $\lambda$, and have disrupted capacity of $K$ each, or (b) a single unreliable supplier that disrupts with probability $\lambda$, and has disrupted capacity $K_s$. 
We first consider the case of minor disruptions, i.e., $D \leq 2K$ when two independent unreliable suppliers exist, and $D \leq K_s$ when a single unreliable supplier exists, respectively. Under these circumstances, the manufacturer can always source the entire quantity $D$ from the unreliable supplier(s). This strategy and the resulting profit exactly correspond to those under a multi-tier supply chain, when Tier 1 suppliers are offered contracts with price $c_u$ (see §2 and §5, respectively). As such, the manufacturer’s optimal sourcing strategy and profit are identical in the centralized and decentralized settings. This argument is formalized in the following corollary.

**Corollary 5.** When disruptions are minor, i.e., $D \leq 2K$ ($D \leq K_s$) when two (one) independent unreliable suppliers exist(s), the manufacturer’s optimal sourcing strategy and profit under a centralized setting are identical to those under a multi-tier (i.e., decentralized) setting.

We now consider the case of severe disruptions, when two unreliable Tier 2 suppliers (denoted by A and B) exist. The manufacturer’s sourcing strategy can be summarized with the triple $(q^A_u, q^B_u, q_r)$, corresponding to the quantities sourced from the two unreliable suppliers and the reliable supplier, respectively. The following lemma characterizes the optimal strategy.

**Lemma 3.** If two unreliable Tier 2 suppliers exist and disruptions are severe ($K < D/2$), then:
(i) If the reliable Tier 2 supplier is “cheap” ($c_r \leq (2 - \lambda)c_u$), the manufacturer’s optimal strategy is:

$$(q^A_u, q^B_u, q_r)^* = \begin{cases} 
(\theta D, (1 - \theta)D, 0) \text{ for any } \theta \in [\frac{K}{D}, \frac{D-K}{D}], & \text{if } c_u \leq \pi < c_u + \frac{c_r - c_u}{1 - \lambda} \\
(K, K, D - 2K), & \text{otherwise.} 
\end{cases}$$

(ii) If the reliable Tier 2 supplier is costly ($c_r > (2 - \lambda)c_u$), the manufacturer’s optimal strategy is:

$$(q^A_u, q^B_u, q_r)^* = \begin{cases} 
(\theta D, (1 - \theta)D, 0) \text{ for any } \theta \in [\frac{K}{D}, \frac{D-K}{D}], & \text{if } c_u \leq \pi \leq \frac{c_u}{1 - \lambda} \\
(D - K, D - K, 0), & \text{if } \frac{c_u}{1 - \lambda} \leq \pi < \frac{c_r - c_u(2 - 2\lambda)}{\lambda^2} \\
(K, K, D - 2K), & \text{otherwise.} 
\end{cases}$$

The lemma confirms that the manufacturer’s optimal strategy is qualitatively similar to the one under a multi-tier (decentralized) supply chain. In particular, at low unit revenues ($\pi$), the manufacturer always finds it optimal to employ dual sourcing (DS) from the two unreliable suppliers, splitting the total quantity $D$ between the two suppliers so as to ensure that the “risk free” capacity $K$ is utilized at each.
As unit revenues increase, the manufacturer’s strategy critically depends on the marginal cost of reliable supply. When reliable supply is cheap \((c_r \leq (2 - \lambda)c_u)\), the manufacturer switches to a triple-sourcing (TS) strategy, which makes minimal use of the unreliable suppliers (by sourcing only the “risk free” quantity \(K\)), and switches the focus to reliable supply. Note that the manufacturer’s use on reliable supply is directly related to the severity of disruptions—when these are extreme (e.g., \(K = 0\)), the strategy collapses to sourcing exclusively from the reliable supplier.

When reliable supply is costly \((c_r > (2 - \lambda)c_u)\), the manufacturer avoids using reliable supply even at intermediate profit margins, i.e., \(\frac{c_u}{\lambda} < \pi < \frac{c_r - c_u(2 - 2\lambda)}{\lambda^2}\). Instead, the DS strategy is complemented with inventory mitigation, by increasing the amount sourced from each unreliable supplier to \(D - K\). At sufficiently high unit revenues, however, the manufacturer eventually switches to the same TS strategy, shifting the focus to reliable supply. As intuition would dictate, having costly reliable supply causes the manufacturer to rely more heavily on unreliable suppliers (through dual-sourcing and possibly over-sourcing), restricting the use of the TS strategy and reliable supply to only higher unit revenues. It is interesting to note that the per-unit threshold governing this switch in the strategy, i.e., \(\frac{c_r - c_u(2 - 2\lambda)}{\lambda^2}\), is the same as when penalty contracts are used in a decentralized setting (see the proof of Theorem 9), confirming our observation that the latter mechanism can successfully induce the first-best sourcing strategy from Tier 2.

The strategies and insights derived here are qualitatively similar to those in a multi-tier supply chain (see §2). It can be checked that \(\bar{c}_r < c_u(2 - \lambda)\), so that “cheap” reliable supply in a centralized setting may appear as “costly” in a decentralized one. Furthermore, even when reliable suppliers are “costly” for both centralized and decentralized settings, the threshold under which the manufacturer switches from a DS + IM strategy is larger in a decentralized system. Both of these observations imply that having a multi-tier supply chain causes the manufacturer to rely more on direct dual sourcing and inventory mitigation, and less on an alternative strategy involving reliable supply. This is intuitive, and a direct manifestation of the agency costs inherent in a decentralized setting, where having to contract for (instead of having direct access to) reliable supply leads to less reliance on it.

We now consider the case when there is a unique unreliable Tier 2 supplier. In this case, the manufacturer’s strategy is characterized by the pair \((q_u, q_r)\) of quantities sourced from the unreliable and the reliable supplier, respectively. The next result characterizes the optimal strategy.

**Lemma 4.** If there is a unique unreliable Tier 2 supplier with disrupted capacity \(K_s\), and disruptions

\[\text{Mathematically, } \frac{c_r - c_u(2 - 2\lambda)}{\lambda^2} < \frac{c_u + \phi}{\lambda} + \frac{c_u}{\lambda^2}.\]
are severe ($K_s < D$), the manufacturer’s optimal strategy is

\[
(q_u, q_r)^* = \begin{cases} 
(D, 0) & \text{if } c_u \leq \pi < \frac{c_e - (1-\lambda)c_u}{\lambda}, \\
(K_s, D - K_s) & \text{if } \pi \geq \frac{c_e - (1-\lambda)c_u}{\lambda}.
\end{cases}
\]  

(5)

The result confirms that single-sourcing from the unreliable supplier is optimal at sufficiently low profit margins, followed by dual-sourcing, with a higher quantity sourced from the reliable supplier. As in the case of a decentralized supply chain, the latter strategy occurs at sufficiently high profit margins (i.e., $\pi > \frac{c_r - (1-\lambda)c_u}{\lambda}$), and completely eliminates the risk.

These insights again closely parallel those in a decentralized, multi-tier setting (see §5). It can be checked that $\frac{c_r - (1-\lambda)c_u}{\lambda} < \bar{\pi}_s$, which suggests that a manufacturer operating in a multi-tier supply chain relies more heavily on (inducing) single sourcing from unreliable suppliers, and less on (inducing) strategies that involve reliable supply, when compared with a centralized system. This confirms the earlier intuition that the agency problems inherent in a decentralized supply chain cause the manufacturer to make less use of reliable supply.

Technical Results for Supplemental Appendix

**Lemma 5.** With an emergency source of supply at unit cost $c_e$, when $D > 2K$ and $D > K_s$,

1. In structure 1, the manufacturer’s optimal expected profit is $\Pi^M = (\pi - c_u)D - \lambda(c_e - c_u)(D - K)$.
2. In structure 2, the manufacturer’s optimal expected profit is $\Pi^M = \left(\pi - \frac{c_r - c_u(1-\lambda)}{\lambda}\right)D$.
3a. In structure 3 with independent Tier 2 suppliers, the manufacturer’s optimal expected profit is

\[
\Pi^M = \begin{cases} 
(\pi - c_u)D - \left(\lambda^2\pi + (1 - 2\lambda)c_u\right)(D - 2K), & \text{if } c_e > \frac{c_u}{\lambda} \text{ and } \pi < \frac{\lambda c_e - (1-\lambda)c_u}{\lambda^2} \\
(\pi - c_u)D - \lambda(c_e - c_u)(D - 2K), & \text{otherwise}.
\end{cases}
\]

In particular, the profit is always at least as large as the profit when no emergency supply is available.

3b. In structure 3 with shared Tier 2 suppliers, the manufacturer’s optimal expected profit is $\Pi^M = (\pi - c_u)D - \lambda(c_e - c_u)(D - K_s)$, and always exceeds the profit achievable when no emergency supply is available.

4a. In structure 4 with independent Tier 2 suppliers, the manufacturer’s optimal expected profit is the same as without emergency sourcing, i.e., $\Pi^M = \pi D - \frac{c_r - (1-\lambda)c_u}{\lambda} (D - K) - c_u K$. 

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4b. In structure 4 with shared Tier 2 suppliers, the manufacturer’s optimal expected profit is the same as without any emergency sourcing, i.e., $$\Pi^M = \left( \pi - \frac{c_r - (1-\lambda)c_u}{\lambda} \right) (D - K_s/2) + (\pi - c_u) K_s/2$$.

5. In structure 5, the manufacturer’s optimal expected profit is $$\Pi^M = \left( \pi - \frac{c_r - c_u(1-\lambda)}{\lambda} \right) D$$.

If $$\frac{c_r - c_u}{c_r - c_u} \leq \lambda$$ the manufacturer can induce any of the five structures; if $$\frac{c_r - c_u}{c_r - c_u} > \lambda$$, the manufacturer can only induce single sourcing with its Tier 1 suppliers, and hence structures 2, 4, and 5 are unavailable.

Proof. 1. If $$p \geq c_u, \gamma c_e$$, supplier profit in structure 1 is $$\Pi^S(Q, 0) = (p - c_u)Q + \lambda(c_u - \gamma c_e)(Q - K)$$, and the manufacturer’s profit is $$\Pi^M = (\pi - p)D - \lambda(1-\gamma)c_e(D - K)$$. The manufacturer clearly wants the lowest price that will induce this outcome, which is $$p = \gamma c_e$$ (meaning the supplier has incentive to use emergency supply if necessary), resulting in profit $$\Pi^M = (\pi - \gamma c_e)D - \lambda(1-\gamma)c_e(D - K)$$.

We note that $$\frac{d\Pi^M}{dp} = -c_eD(1 - \lambda) - \lambda c_e K < 0$$, meaning the manufacturer wants the smallest $$\gamma$$ that can achieve this outcome, $$\gamma = c_u/c_e$$. This implies that the offered price is $$p = c_u$$ and the Tier 1 supplier is “fully subsidized” when purchasing units from the emergency supplier, i.e., its effective cost of emergency sourcing is $$c_u$$. Moreover, because $$p < c_r$$, the Tier 1 supplier will indeed single source from the unreliable Tier 2 supplier (rather than use the reliable supplier in any way). Manufacturer profit is $$\Pi^M = (\pi - c_u)D - \lambda(c_u - c_e)(D - K)$$.

2. Note that structure 2 with a non-zero quantity sourced from the reliable Tier 2 supplier implies that the Tier 1 supplier never uses the emergency source. This is because either the emergency supplier is more expensive than the reliable supplier on expected cost basis (meaning the problem is effectively the same as in the no emergency source case), or vice versa (implying that the effective structure is structure 1). Thus the Tier 1 supplier’s profit in structure 2 is $$\Pi^S(K, Q - K) = (p - c_r)Q + (c_r - c_u)K$$, and in order to induce this outcome the manufacturer must set $$p$$ and $$\gamma$$ such that (i) $$\Pi^S(K, Q - K) \geq 0$$ and (ii) $$\Pi^S(K, Q - K) \geq \Pi^S(Q, 0)$$.

Comparing the supplier profit expressions, dual sourcing is preferred if $$\gamma \geq \frac{(c_r - (1-\lambda)c_u)}{\lambda c_e}$$, satisfying condition (ii) for any $$p \geq \gamma c_e$$. The manufacturer clearly prefers the smallest price that can induce the supplier to dual source, which implies both the smallest $$\gamma$$ that can accomplish this, and a price equal to $$p = \gamma c_e$$, i.e., $$\gamma = \frac{c_r - c_u(1-\lambda)}{\lambda c_e}$$ and $$p = \frac{c_r - c_u(1-\lambda)}{\lambda}$$. This price is the same as the price that can induce dual sourcing in the absence of an emergency supply option, and manufacturer profit is $$\Pi^M = \left( \pi - \frac{c_r - c_u(1-\lambda)}{\lambda} \right) D$$. The requirement that $$\gamma = \frac{c_r - c_u(1-\lambda)}{\lambda c_e} \leq 1$$ implies that we require that $$\frac{c_r - c_u}{c_r - c_u} \leq \lambda$$ for this solution to be interior; otherwise, the supplier can never be induced to dual source.

3a. From part (1), to induce single sourcing with the use of emergency supply by a Tier 1 supplier (say, A), the manufacturer should offer a price $$c_u$$, and charge the respective supplier a fraction
\( \gamma_A = c_u / c_e \) of the emergency supply cost. This would immediately generate an additional manufacturer cost burden of \( c_e - c_u \) when using emergency supply through that Tier 1 supplier.

Note that the manufacturer has three alternatives, depending on the number of Tier 1 suppliers induced to use emergency sourcing: both, only one, or none. Inducing both Tier 1 suppliers to use emergency sourcing is never optimal. When no Tier 1 suppliers use emergency sourcing, the manufacturer’s profits are identical with those in part (iii) of Lemma 1. When only Tier 1 supplier A is induced to use emergency sourcing, the manufacturer’s profit becomes

\[
\Pi^M(Q_A, Q_B) = (1 - \lambda) [\pi D - c_u(Q_A + Q_B)] + \lambda(\pi - c_u)(Q_A + K) - \lambda(c_e - c_u)(Q_A - K),
\]

where \( Q_A, Q_B \geq K, Q_A + Q_B \geq D, D \geq K + Q_B, D \geq Q_A + K \). It can be checked that \( \partial \Pi^M / \partial Q_B < \partial \Pi^M / \partial Q_A \), so that the optimal contract for the manufacturer is \( Q_A^* = D - K, Q_B^* = K \), resulting in a profit of \( (\pi - c_u)D - \lambda(c_e - c_u)(D - 2K) \). This is less than the profit obtained without emergency sourcing (in part (iii) of Lemma 1) only when \( \pi > \frac{c_e}{\lambda} \) and \( \pi < \frac{\lambda c_e - (1 - \lambda)c_u}{\lambda^2} \), which is possible only if \( c_e > \frac{c_u}{\lambda} \). Thus, the manufacturer’s optimal profit in structure 3 with independent Tier 2 suppliers is given by

\[
\Pi^M = \begin{cases} 
(\pi - c_u)D - (\lambda^2\pi + (1 - 2\lambda)c_u) (D - 2K), & \text{if } c_e > \frac{c_u}{\lambda} \text{ and } \pi < \frac{\lambda c_e - (1 - \lambda)c_u}{\lambda^2} \\
(\pi - c_u)D - \lambda(c_e - c_u)(D - 2K), & \text{otherwise.}
\end{cases}
\]

The corresponding optimal contracts are \( \{(c_u, D-K, \bar{\gamma}), (c_u, D-K, \tilde{\gamma})\} \), and \( \{(c_u, D-K, \frac{c_u}{\lambda}), (c_u, K, \bar{\gamma})\} \), respectively, where \( \bar{\gamma} > \frac{c_e}{c_u} \) is arbitrary.

3b. If Tier 2 suppliers are shared, structure 3 is degenerate with structure 1, as in the case with no emergency supply. Following the same reasoning as in part 1, the manufacturer will source \( D \) from one of its Tier 1 suppliers, and induce that supplier to use emergency supply, resulting in an optimal profit of \( \Pi^M = (\pi - c_u)D - \lambda(c_e - c_u)(D - K) \).

4a. By the same argument as for structure 2, emergency sourcing should be induced only for the Tier 1 supplier B, who does not have access to a reliable Tier 2 supplier, yielding a profit of

\[
\Pi^M(Q_A, Q_B) = \pi D - c_e - (1 - \lambda)c_u Q_A - c_u Q_B - \lambda(c_e - c_u)(Q_B - K).
\]

It can be readily checked that \( \partial \Pi^M / \partial Q_A < 0 \) and \( \partial \Pi^M / \partial Q_B < 0 \), and \( \partial \Pi^M / \partial Q_A > \partial \Pi^M / \partial Q_B \) if and only if \( c_e > c_u + \frac{c_e - c_u}{\lambda^2} \). When the latter condition holds, no emergency sourcing is used, and the profit is the same as in part (iv) of Lemma 1. When the condition is false, it is optimal to not source anything from Tier 1.
supplier A, which would mean that structure 4 essentially degenerates into structure 1.

4b. The argument follows similarly to that in part (4a), and is omitted.

5. Structure 5 is degenerate with structure 2, as in the case with no emergency supply. \qed

**Proof of Theorem 10.**

(i) By Lemma 5, note that the manufacturer’s optimal profit in structure 3 is at least as large when emergency sourcing is possible (see parts (3a) and (3b)), while it remains the same in structure 4 (see parts (4a) and (4b)). The result is immediate.

(ii) By parts (3a) and (3b) of Lemma 5, note that the manufacturer’s optimal profit in structure 3 increases to \((\pi - c_u)D\) as \(c_e \leq c_u\), irrespective of whether Tier 2 suppliers are independent or shared. The latter expression is the maximal profit available to the manufacturer, achievable under direct access to Tier 2 suppliers, and is always strictly greater than the profit obtained by inducing structure 4. Since the profit expressions are continuous in \(c_e\), the result is immediate.

**Proof of Lemma 3.** Let \(\Pi(q_u^A, q_u^B, q_r)\) denote the resulting expected profit for the manufacturer. First, note that when \(\frac{\partial \Pi}{\partial q_r} \leq 0\), it is optimal to source only from the unreliable suppliers, and the manufacturer’s profit expression becomes identical to that achieved in Structure 3 under independent Tier 2 unreliable suppliers (recall Figure 3, and the discussion in §4.2). As such, by part (iii) of Lemma 1, the optimal quantities will be

\[
(q_u^A, q_u^B, q_r)^* = \begin{cases} 
(\theta D, (1 - \theta)D, 0), & \text{if } \pi \leq \frac{c_u}{\lambda} \\
(D - K, D - K, 0), & \text{otherwise},
\end{cases}
\]

where \(\theta \in \left[\frac{K}{D}, \frac{D - K}{D}\right]\) is arbitrary.

When \(\frac{\partial \Pi}{\partial q_r} > 0\), we can make similar observations to those in the Proof of Theorem 10. In particular, it is always optimal to have \(q_u^A \geq K\), \(q_u^B \geq K\), \(q_u^A + q_u^B + q_r \geq D\), and \(2K + q_r \leq D\). Furthermore, since \(\Pi\) is concave and symmetric in \(q_u^A\) and \(q_u^B\), we must have

\[
\Pi(q_u^A, q_u^B, q_r) = \Pi(q_u^B, q_u^A, q_r) \leq \Pi\left(\frac{q_u^A + q_u^B}{2}, \frac{q_u^A + q_u^B}{2}, q_r\right),
\]

so that it is always optimal for the manufacturer to source the same quantity from A and B, which we henceforth denote by \(q_u\). In this case, all optimal sourcing strategies \((q_u, q_r)\) must satisfy the constraints \(q_u \geq K\) and \(D - 2q_u \leq q_r \leq D - 2K\). In particular, note that \(q_r + q_u \leq D - 2K + q_u \leq D - K\), and if
$q_u = K$, then $q_r = D - 2K$. With these observations, the profit function becomes:

$$
\Pi(q_u, q_r) = \lambda^2 \left[ \pi \cdot (q_r + 2K) - c_r q_r - 2c_u K \right] + 2\lambda(1 - \lambda) \left[ \pi \cdot (q_r + q_u + K) - c_r q_r - c_u (q_u + K) \right] + (1 - \lambda)^2 \left( \pi \cdot D - c_r q_r - 2c_u q_u \right).
$$

Using $\Pi(D/2, 0)$ as a proxy for any sourcing strategy $(\theta D, (1 - \theta)D, 0)$, it can be readily checked that

$$
\Pi(D/2, 0) = (\pi - c_u) [\lambda 2K + D(1 - \lambda)]
$$

$$
\Pi(D - K, 0) = (\pi - c_u) [\lambda^2 2K + 2D\lambda(1 - \lambda)] + (1 - \lambda)^2 [\pi D - 2c_u (D - K)]
$$

$$
\Pi(K, D - 2K) = \pi D - c_r (D - 2K) - c_u 2K
$$

Comparing these three expressions, we see that if $c_r < c_u (2 - \lambda)$, we can express the overall optimal sourcing strategy as follows:

$$
(q_u^A, q_u^B, q_r) = \begin{cases} 
(\theta D, (1 - \theta)D, 0), & \text{if } \pi < c_u + \frac{c_r - c_u}{\lambda} \\
(K, K, D - 2K), & \text{otherwise.}
\end{cases}
$$

If $c_r > c_u (2 - \lambda)$, then the optimal strategy becomes:

$$
(q_u^A, q_u^B, q_r) = \begin{cases} 
(\theta D, (1 - \theta)D, 0), & \text{if } \pi < \frac{c_u}{\lambda} \\
(D - K, D - K, 0), & \text{if } \frac{c_u}{\lambda} \leq \pi < \frac{c_r - c_u (2 - 2\lambda)}{\lambda^2} \\
(K, K, D - 2K), & \text{otherwise.}
\end{cases}
$$

Proof of Lemma 4: In this case, the manufacturer’s decision problem is effectively identical to that of a Tier 1 supplier in our original model, when facing independent Tier 2 suppliers (each with disrupted capacity $K_s$). As such, the optimal strategy is characterized by Theorem 1 with $\pi$, $D$, and $K_s$ replacing $p_i$, $Q$ and $K$, respectively.