Turbulence in Natural Environments

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Environment in the Graduate School of Duke University

2015
ABSTRACT

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Abstract

Problems in the area of land/biosphere-atmosphere interaction, hydrology, climate modeling etc. deal with mass, energy and momentum exchange with the land surface and hence can be systematically organized as a study of turbulent flow in presence of boundary conditions in an increasing order of complexity. The present work is an attempt to study a few subsets of this general problem of turbulence in natural environments - in the context of neutral and thermally stratified atmospheric surface layer, the presence of a heterogeneous vegetation canopy and the interaction between air flow and a static water body in presence of flexible protruding vegetation. The main issue addressed in the context of turbulence in the atmospheric surface layer is whether it is possible to describe the macro-states of turbulence such as mean velocity and turbulent velocity variance in terms of the micro-states of the turbulent flow, i.e., a statistical distribution of turbulent kinetic energy across a multitude of scales. This has been achieved by a ‘spectral budget approach’ which is extended for thermal stratification scenarios as well, in the process unifying the seemingly different and unrelated theories of turbulence such as Kolmogorov’s hypothesis, Heisenberg’s eddy viscosity, Monin Obukhov Similarity Theory (MOST) etc. under a common framework. In the case of a more complex scenario such as presence of a vegetation canopy with edges and gaps, the question that is addressed is in what detail the turbulence is needed to be resolved to capture the bulk flow features such as recirculation patterns. This issue is addressed by a simple numerical framework and
it has been found out that an explicit prescription of turbulence is not necessary in presence of heterogeneities such as edges and gaps where the interplay between advection, pressure gradients and drag forces are sufficient to capture the first order dynamics. This result can be of significance for eddy-covariance flux calibration strategies in non-ideal environments and the developed numerical model can be used in related dispersion studies and coupled land atmosphere interaction models. For other more complex biosphere atmosphere interactions such as greenhouse gas emissions from wetlands, the interplay between air and water, often in presence of flexible aquatic vegetation, controls turbulence in water, which in turn affect the gas transfer processes. This process of wind shear induced wave-turbulent-vegetation interaction is studied for the first time in the laboratory and the state of turbulence as well as the bulk flow is found to be highly sensitive to environmental controls such as water height, wind speed, vegetation density and flexibility. This dissertation describes and gradually develops these concepts in an increasing order of complexity of boundary conditions. The first three chapters address the neutral and thermally stratified boundary layers and the last two chapters address the canopy edge problem and the air-water-vegetation experiments respectively.
Dedicated to my parents, Rina and Tarasankar Banerjee.
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List of Abbreviations and Symbols

Symbols

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<td>$U$</td>
<td>Mean longitudinal velocity.</td>
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<td>$W$</td>
<td>Mean vertical velocity.</td>
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<tr>
<td>$u_*$</td>
<td>Friction velocity.</td>
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<td>$\sigma_u^2$</td>
<td>Longitudinal velocity variance, also streamwise turbulent intensity.</td>
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<tr>
<td>$\sigma_w^2$</td>
<td>Vertical velocity variance.</td>
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<tr>
<td>$\delta$</td>
<td>Boundary layer height.</td>
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<tr>
<td>$\kappa$</td>
<td>Von Kármán constant.</td>
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<tr>
<td>$k$</td>
<td>Wavenumber.</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic viscosity.</td>
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<tr>
<td>$\tau$</td>
<td>Turbulent kinetic energy dissipation rate.</td>
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<td>$Re$</td>
<td>Reynolds number.</td>
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<td>$C_d$</td>
<td>Drag Coefficient.</td>
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Abbreviations

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<th>Abbreviation</th>
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<tr>
<td>MOST</td>
<td>Monin Obukhov Similarity Theory.</td>
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<td>TKE</td>
<td>Turbulent Kinetic Energy.</td>
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<td>K41</td>
<td>Kolmogorov’s famous paper in 1941 on homogeneous isotropic turbulence.</td>
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LAI  Leaf Area Index.
LAD  Leaf Area Density.
RMSE Root Mean Square Error.
ABL  Atmospheric Boundary Layer.
ASL  Atmospheric Surface Layer.
CSL  Canopy Sub Layer.
LES  Large Eddy Simulation.
RANS Reynolds Averaged Navier Stokes.
AHATS Advection Horizontal Array Turbulence Study.
HW   Hardwood Canopy.
PIV  Particle Imaging Velocimetry.
FFT  Fast Fourier Transform.
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This work addresses the fundamental nature of turbulent fluid flow in nature by means of analytical, numerical and experimental methods. The problem of turbulence is an active area of research because of the large and diverse clientele it enjoys— including mathematicians, astrophysicists, atmospheric physicists, environmental scientists, hydrologists, ecologists, oceanographers, aeronautical, mechanical, and chemical engineers. All these areas of interest in the natural sciences and engineering have different needs and approach the problem of turbulence with correspondingly different emphasis and tools. Moreover, the aspect of resolving turbulent flows is a key component in quantifying the interaction between land/biosphere and atmosphere which is central to a number of disciplines such as prediction of local and global weather, climate and ecosystems, hydrology, water resource management, air and water pollution monitoring etc. However, from the perspective of an applied physicist, this vast multitude of seemingly different problems arising in different natural environments can be systematically organized into a study of turbulent flows under different boundary conditions of increasing order of complexity. Hence, it is safe to state that there exists a common need of describing turbulent flow statis-
tics close to interfaces such as solid, vegetated, water, or multi-phase boundaries (air-water- vegetated systems). This encapsulates the scope of the present work.

There can be different approaches to pursue this objective of studying turbulent flows under different complex boundary conditions occurring in nature. Since a complete theoretical solution of the Navier Stokes equations (the equation describing the behavior of a turbulent flow) is not possible, either idealized or approximated solutions are sought. There are two extremes of this approach. The simplest and least expensive approach uses dimensional analysis in conjunction with experimental data to formulate similarity arguments. These similarity arguments such as Monin Obukhov Similarity Theory (MOST) are very easy to implement operationally, although they fail under non idealized conditions since they are hypothesized for simple ideal assumptions. On the other extreme, extensive computational schemes such as direct numerical simulations (DNS) can resolve complexities in high resolutions but they are highly expensive. The present work provides an intermediate viewpoint of studying wall-bounded turbulence by means of a phenomenological approach so that the bulk flow properties or macro-states are connected to the statistical properties of the micro-states of the flow albeit for neutral and thermally stratified boundary layers. For a more complex boundary condition, such as presence of vegetation, a more fundamental question to ask is to what degree the turbulence needs to be resolved to capture the perturbation on the bulk flow under non-idealized conditions such as presence of a canopy edge or a gap. To answer this question, a simple numerical model is sought which can explore the role of different processes responsible for generating those perturbations. For an even more complex boundary condition such as the presence of an interface between different phases, such as air, water and solids at the same time, much is still to be learnt and thus lab experiments are conducted to study the first order of dynamics of how the state of turbulence is dependent on different environmental controls.
The present work attempts to address these questions in a systematic manner. Figure 1.1 graphically illustrates the order in which the complexity of the boundary conditions are enhanced. Chapter 2 presents the development of the aforementioned phenomenological theory based on the manuscript Banerjee and Katul (2013) in the context of the neutral atmospheric surface layer. Chapter 3 and chapter 4 extends this framework in the context of unstable and stable stratifications in the atmospheric surface layer, based on the manuscripts Banerjee et al. (2014) and Banerjee et al. (2015b) respectively. Chapter 5 addresses the canopy edge problem using a simple numerical framework based on Banerjee et al. (2013). Finally, the experimental work on air-water-vegetation is presented in chapter 6 based on the manuscript Banerjee et al. (2015a). It is interesting to note that that even such as a complex problem of multi-phase fluids and vegetation interaction can be studied with the tools that are used to study the more idealized cases like wall bounded turbulence in the atmospheric surface layer (hence the cyclic nature as demonstrated in figure 1.1.

The issue most common to many applied fields such as ecology, hydrology, hydraulics, and engineering is a problem of scaling - or the uncovering of relations between motion on different scales in a problem that contains many scales and the consequences of those relations on bulk flow properties. These scaling relations are examples of general expressions that can be applied even without a detailed solution to the fundamental equations of motion (i.e. the Navier-Stokes). A case in point is the logarithmic scaling of the mean velocity profile above a boundary routinely used in hydrology (and employed to explain Manning’s formula for open channel flow), aerodynamic resistances in gas exchange between plants and their environment (in biosphere-atmosphere studies), or seed dispersal in wind, among others. The scaling laws of the root-mean squared longitudinal turbulent velocity component with distance from a solid boundary also appear to inherit quasi-universal behavior as confirmed by recent and unprecedented high resolution experiments conducted in
Figure 1.1: A general graphical outline of the dissertation showing the different orders of complexity of the boundary conditions. Chapter 2, 3 and 4 describe the phenomenological model in the context of the neutral and thermally stratified atmospheric surface layer. Chapter 5 describes the problems involving canopy edges and gaps. A further complexity of canopy edges and gaps situated over complex topography will be attempted in future studies. In chapter 6, an even more complex problem of air-water-vegetation interaction is attempted.

wind tunnels, open channels, pipes and field observations. This parameter is a direct measure of the turbulent intensity and this scaling law is thus relevant for a myriad of industrial and environmental applications such as modeling wind power generation, air pollution modeling, and studies on seed and pollen dispersion, aerosol deposition etc. These experiments were viewed as support for the so-called ‘Townsend’s attached eddy hypothesis’, a hypothesis that remains to date based on a conjecture and dimensional analysis of how eddy sizes scale with distance from a boundary.

Chapter 2 presents a novel framework explaining how this quasi-universal character of the root-mean squared velocity is inherited based on theories predicting the distributional properties of turbulent kinetic energy associated with the sizes of eddying motion. In the process - it links established theories of fine-scaled turbulence
such as Kolmogorov’s hypothesis and Heisenberg’s eddy viscosity (energy flow) to the empirical argument of Townsend’s attached eddy hypothesis, thereby providing - for the first time - a physical basis to the associated constants with these theories.

In presence of thermal stratification, mechanical generation of turbulence is either enhanced (for unstable) or destroyed (stable) by buoyant production/destruction due to the effect of a hot surface in daytime or a cool surface in the night-time. Under those conditions, Monin Obukhov Similarity Theory (MOST) has traditionally been employed as a correction for first order turbulent statistics like the mean velocity or potential temperature profiles. These similarity theories have been developed based on dimensional arguments and experimental data, without any connection to the underlying physics. Moreover, they fail to explain the variations of second order statistics like the longitudinal turbulent velocity variance as observed in experiments.

The present work has described the scaling behavior of this turbulent intensity under unstable and stable (chapter 3 and chapter 4) atmospheric stratification using a spectral budget and has suggested modifications to MOST when applied to the longitudinal velocity variance under thermal stratifications, apart from explaining and connecting the various constants with fundamental turbulent processes. These results can be directly included in models of air quality and dispersion in the context of both urban and forest regions among others and also coupled land-atmosphere interaction models.

For studies describing coupled biosphere-atmosphere interaction, the most common practice is to employ eddy-covariance measurement systems. In practice, most of these stations are often situated inside or near a vegetation canopy. The fundamental assumption in measuring fluxes by these measurements is a planar, homogeneous flow which results in a constant flux with height and provides a practical measurement approach, so that the flux at the measurement height is assumed to be representative of the ground flux or the flux (momentum or scalar such as carbon-
dioxide or water vapor etc.) emanating from the canopy top. Unfortunately, in reality, these measurement stations are often located in non-ideal environments such as near a forest edge or a gap or a complex topography, which disturbs the flat land-constant flux assumption. This results in a very important aspect or requirement of calibrating the measurements, often by a separate model which can quantify the fluxes or the perturbations from the constant flux with height. A most fundamental question that arises in addressing this issue of calibrating fluxes is in how much details the turbulent flow is needed to be modeled- which is addressed in chapter 5 in the context of canopy edges and gaps. A streamfunction vorticity model has been developed with a Reynolds Averaged Navier Stokes (RANS) scheme and first order turbulence closure models. In presence of edges and gaps, it has been found out the the mean flow features are a manifestation of the interplay among the advection terms, mean pressure gradient, turbulent stress and drag force (inside canopy) in the momentum budget equation. Interestingly, it has been found out that even without explicit prescription of the turbulent (Reynolds) stresses (called a turbulently inviscid scheme), the advection, pressure gradient and drag forces can capture the bulk features like recirculation patterns and the signature length scales associated with a canopy transition problem, which are often comparable with a much expensive Large eddy simulation (LES). This framework can thus also be included as a sub-grid scale model in global climate models and also in the context of modeling seed dispersion, aerosol deposition etc. in non-ideal realistic scenarios like edges and gaps in a canopy.

Biosphere atmosphere interaction problems are not only limited to land and forests, but also include water-atmosphere exchange processes. Lakes and wetlands are an important source of greenhouse gases like carbon dioxide and methane and their emissions are controlled by the interaction of air flow over the water surface and the resulting state of turbulence in the water body, among other factors such as soil temperature etc. To make things more complicated, static water bodies are often
habitat by aquatic vegetation which are flexible and can protrude out of the water, subjected to air induced oscillations, resulting in a complex interaction between air water and vegetations. Hence in order to eventually study gas transfer from such a wetland, the first order of business is to study this interaction in a controlled environment since no instance of such interactions with the full complexity has been found in the literature. Chapter 6 presents the first experiment of this kind using Particle Imaging Velocimetry (PIV) conducted at the Institute of Hydroscience of Engineering (IIHR), University of Iowa. The experiment has suggested that there exists a complex interaction between wave and turbulence in such a scenario which vary with different environmental factors like water height, wind speed, vegetation density and flexibility. Depending upon certain combinations of these parameters, there can be complete reversal in the flow direction, which can be also useful for other important problems such as nutrient and sediment transport in wetlands, vegetation health modeling etc.
Logarithmic scaling in the longitudinal velocity variance explained by a spectral budget

2.1 Introduction

Scaling-laws and self-similar states remain the cornerstone of turbulence research, especially in the so-called intermediate region of wall bounded flows where production and dissipation of turbulent kinetic energy (TKE) are in balance and spectrally separated, the Kolmogorov inertial subrange theory describes the local structure of the velocity statistics, and the von Kármán-Prandtl logarithmic law describes the mean velocity ($\bar{U}$) given by

$$U^+ = \frac{1}{\kappa} \log(z^+) + A_w,$$  \hspace{1cm} (2.1)

where $U^+ = \bar{U}/u_*$ is the normalized mean longitudinal velocity, $z$ and $z^+ = z u_* / \nu$ are respectively the distance and normalized distance from the wall, $u_* = \sqrt{\tau_t / \rho_f}$ is the friction (or shear) velocity, $\kappa$ is the von Kármán constant, $A_w$ is a wall constant, $\tau_t$ is the turbulent stress assumed independent of $z$ in the intermediate region, $\nu$ and $\rho_f$ are the fluid kinematic viscosity and density, respectively, and over-line
designates time-averaging. The logarithmic profile shape for $U^+$ in the intermediate region has been supported by a myriad of studies (Prandtl, 1925; Von Karman, 1930; Millikan, 1938; Rotta, 1962; Townsend, 1976; Perry and Chong, 1982; Perry and Li, 1990; Marusic et al., 2013), including Townsend’s attached eddy hypothesis. Besides propounding the log-law for $U^+$, Townsend’s attached eddy hypothesis has also predicted a logarithmic scaling for the streamwise turbulence intensity ($u'^2$), defined as the mean squared quantity of the streamwise turbulent velocity fluctuations of the following form (Marusic et al., 2013; Meneveau and Marusic, 2013; Perry and Chong, 1982)

$$u'^2 = B_1 - A_1 \log(z/\delta)$$

(2.2)

where $u'^2 = u'^2/u_*^2$, the normalized longitudinal velocity variance in wall units and $\delta$ is the thickness of the turbulent boundary layer. This result has lacked robust experimental support except from a few studies (Perry and Abell, 1977; Perry et al., 1986; Perry and Li, 1990; Perry and Marusic, 1995; Jimenez and Hoyas, 2008) until recently. Significant interest has been noticed about the topic in recent years following publications by Smits et al. (2011), Marusic et al. (2013) and Smits and Marusic (2013) where four different experiments at very high Reynolds numbers were compiled to illustrate the universality of Eq. 2.2 across a wide range of bulk Reynolds number (Hultmark et al., 2012; Hutchins et al., 2012; Kulandaivelu, 2012; Marusic et al., 2013; Winkel et al., 2012). The compass of this work is to provide a phenomenological explanation to Eq. 2.2 based on a spectral budget of the longitudinal velocity thereby offering another perspective on the origin of its logarithmic (or power-law) character. Specifically, a $k^{-1}$ scaling at low wavenumbers ($k$) for the streamwise turbulent velocity spectrum ($E_u(k)$) has been prevalent in turbulence literature for some time (Tchen, 1953, 1954; Klebanoff, 1954; Hinze, 1959; Pond et al., 1966; Bremhorst and Bullock, 1970; Panchev, 1971; Bremhorst and Walker, 1973; Perry and Abell, 1990; Marusic et al., 2013).
1975, 1977; Korotkov, 1976; Bullock et al., 1978; Hunt and Joubert, 1979; Kader and Yaglom, 1984; Perry et al., 1986, 1987; Turan et al., 1987; Erm et al., 1987; Perry and Li, 1990; Erm and Joubert, 1991; Kader and Yaglom, 1991; Yaglom, 1994; Katul et al., 1996; Katul and Chu, 1998; Jimenez, 1999; Nikora, 1999) both for wall bounded flows and atmospheric surface layer turbulence. Those studies differ in predicting the extent of the $k^{-1}$ scaling at low $k$. One study even found a prevalent $k^{-1}$ scaling and a limited inertial subrange for $z^+ \in [220 - 890]$ (George and Tutkun, 2011). The arguments resulting in a $k^{-1}$ scaling range from a theoretical spectral budget analysis (Tchen, 1953, 1954; Panchev, 1971) to dimensional analysis (Kader and Yaglom, 1984) to laboratory and field experiments (Korotkov, 1976; Katul et al., 1996). Townsend’s attached eddy hypothesis has also been used in the interpretation of the aforesaid $k^{-1}$ scaling in a few studies (Hunt and Joubert, 1979). In Nikora (1999), a $k^{-1}$ scaling has been explained in the range $1/H \leq k \leq 1/z$ as a result of superimposition of eddy cascades (Kolmogorov, 1941) generated at all possible distances from the wall, where $H = \alpha \delta$ is the external length scale proportional to the boundary layer thickness via a coefficient $\alpha$. A few studies have not observed a clear $k^{-1}$ scaling (Kaimal, 1978; Antonia and Raupach, 1993; Morrison et al., 2002) and others have found the existence only under certain constraints (Nickels et al., 2005). Another phenomenological explanation has been proposed by Katul et al. (2012) that bridges some of the mechanisms explaining the $k^{-1}$ power law scaling at low $k$ in the turbulent kinetic energy spectrum $E_{tke}(k)$ using simplifications to Tchen’s spectral budget (Tchen, 1953, 1954), Nikora’s phenomenological scaling for the dissipation rate spectrum (Nikora, 1999), and Heisenberg’s eddy viscosity (Heisenberg, 1948). However, a spectral budget approach lacking any accounting for a rigid boundary remains problematic (Kunkel and Marusic, 2006; Marusic et al., 2010; Smits et al., 2011). In the present study, an explicit production term that accounts for the mean velocity gradient and a break-point in the vertical velocity spectra at $z$ are used.
to supplement a spectral budget leading to the $k^{-1}$ scaling, which is then used to explain the log-law of $\overline{u'^2}$, including a link between the Kolmogorov constant, the parameters $A_1$ and $B_1$, and the proportionality coefficient $\alpha$.

2.2 Theory

2.2.1 Definitions and general considerations

The Reynolds-averaged TKE budget equation (without buoyancy or rotational acceleration terms) is given by (Stull, 1988)

$$\frac{\partial \bar{e}}{\partial t} + U_j \frac{\partial \bar{e}}{\partial x_j} = -w_i u_j' \frac{\partial \bar{U}_i}{\partial x_j} - \frac{\partial (u_j' \bar{e})}{\partial x_j} - \frac{1}{\rho f} \frac{\partial (u_j' P)}{\partial x_i} - \bar{\epsilon} $$

(2.3)

where $x_1 = x$, $x_2 = y$, and $x_3 = z$ are the longitudinal, lateral, and vertical directions, respectively, $U_1 = U$, $U_2 = V$, and $U_3 = W$ are the mean longitudinal, lateral, and vertical velocity components, $u_i'$ are turbulent velocity excursions around $U_i$, $\bar{e} = \frac{1}{2} (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ is the TKE, $\sigma_u^2 = \overline{u'^2}$, $\sigma_v^2 = \overline{v'^2}$, and $\sigma_w^2 = \overline{w'^2}$ are the root-mean-squared velocity component along directions $x_i$, respectively, and the coordinate system $x_i$ is aligned so that $x_1$ is along the $\bar{U}$ direction with $\bar{W} = \bar{V} = 0$. The first and second terms on the left hand side of Eq. 2.3 represent local storage and advection of $\bar{e}$ by the mean flow. On the right hand side, the first term indicates mechanical or shear production of TKE due to a finite mean velocity gradient, the second and third terms represent transport of TKE by turbulence and pressure-velocity interactions respectively, and the last term indicates viscous dissipation of TKE. As is the case with laboratory studies, assuming stationary and planar-homogeneous flow results in (Pope, 2000; Stull, 1988)

$$\frac{\partial \bar{e}}{\partial t} = 0 = -u'w' \frac{d\bar{U}}{dz} - \frac{\partial}{\partial z} (w' \bar{e} + u' \bar{P}') - \bar{\epsilon}, $$

(2.4)
where \( \bar{u'}u' \) is the mean momentum flux. It is to be noted that \( p' \) in Eq. 2.4 is normalized by \( \bar{p'}f \). In the intermediate region of boundary-layers, the transport terms are usually small resulting in a near-balance between production and dissipation of TKE as assumed by Townsend and many others (Townsend, 1976; Pope, 2000).

2.2.2 A spectral budget

If \( \bar{\epsilon} \) is a conservative quantity in the turbulent energy cascade, then a simplified spectral budget representing the interplay between the same terms in Eq. 2.4 can be derived for any wavenumber \( k \) as (Hinze, 1959; Panchev, 1971):

\[
\bar{\epsilon} = -\frac{d\bar{\cal U}}{dz} \int_k^\infty F_{wu}(p)dp + F_{TR}(k) + 2\nu \int_0^k p^2 E_{tke}(p)dp, \tag{2.5}
\]

where the first, second, and third terms represent the production of TKE in the range of \([k, \infty]\), the transfer of TKE in the range \([k, \infty]\), and the viscous dissipation in the range of \([0, k]\). Two asymptotic conditions must be satisfied so that this spectral budget recovers its Reynolds-averaged TKE counterpart. The first is that at \( k = 0 \), \( F_{TR}(0) = 0 \), and

\[
\bar{\epsilon} = -\frac{d\bar{U}}{dz} \int_0^\infty F_{wu}(p)dp = -\frac{d\bar{U}}{dz} \left( \bar{u'}u' \right), \tag{2.6}
\]

so that \( \int_0^\infty F_{wu}(p)dp = \bar{u'}u' \) to ensure a balance between mechanical production and \( \bar{\epsilon} \) is maintained in the intermediate region. The second is that as \( k \to \infty \), \( F_{TR}(\infty) \to 0 \), and

\[
\bar{\epsilon} \approx 2\nu \int_0^\infty p^2 E_{tke}(p)dp, \tag{2.7}
\]

or \( \bar{\epsilon} \) is primarily explained via the viscous term at very large \( k \). The \( F_{wu}(k) \) that is related to the production term, and the \( F_{TR}(k) \) that is related to the action of the
triple moments and pressure-velocity interactions both require closure.

In deriving closure expressions for $F_{wu}(k)$ and $F_{TR}(k)$, $E_{tke}(k)$ is related to the spectra of the individual velocity components by

$$E_{tke}(k) = \frac{1}{2} [E_u(k) + E_v(k) + E_w(k)], \quad (2.8)$$

and for $kz > 1$, these component-wise velocity spectra can be described by the Kolmogorov (Kolmogorov, 1941) scaling (hereafter referred to as K41) given as

$$E_{tke}(k) = C_0 \epsilon^{2/3} k^{-5/3}; E_w(k) = C'_K \epsilon^{2/3} k^{-5/3}; E_v(k) = C''_K \epsilon^{2/3} k^{-5/3}; E_u(k) = C''_K \epsilon^{2/3} k^{-5/3}, \quad (2.9)$$

where $C'_K = (24/55)C_K$, $C''_K = (18/55)C_K$, $C_K \approx 1.55$ is the Kolmogorov constant associated with 3-dimensional wavenumbers, and $C_o = (33/55)C_K$. Exponential (or Pao type) adjustments Pope (2000) to these individual spectra as $k \eta \to 1$ are momentarily ignored (to be discussed later), where $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov micro-scale, and $\nu$ is the kinematic viscosity of the fluid. Throughout, $k$ is interpreted as one-dimensional cut along the streamwise direction $x$ and this interpretation is adopted hereafter as invoked when converting the time domain to the wavenumber domain using Taylor’s frozen turbulence hypothesis (Taylor, 1938) in experiments. Last, because the TKE and the component-wise velocity spectra are known for $kz > 1$ given by their K41 scaling, it is convenient to consider the spectral budget in Eq. 2.5 at $k_a = 1/z$ instead of any arbitrary $k$.

2.2.3 Modeling the production term $F_{wu}(k)$

The $F_{wu}(k)$ can be obtained via a co-spectral budget similar in form to the spectral budget above given by Katul et al. (2013a)

$$\frac{\partial F_{wu}(k)}{\partial t} + 2\nu k^2 F_{wu}(k) = P_{wu}(k) + T_{wu}(k) + \pi(k), \quad (2.10)$$
where $P_{wu}(k) = (d\bar{U}/dz)E_w(k)$ is the production term, $T_{wu}(k)$ is the co-spectral flux-transport term, and $\pi(k)$ is the velocity-pressure interaction term. Considering the co-spectral transfer term $T_{wu}(k)$, it is reasonable to assume that it may be small compared to the other terms given that its integral over $k$ is $\bar{\partial}w^yu_yw^f/\partial z$ which is known to be minor in the intermediate region Kader and Yaglom (1990); Raupach (1981). With these assumption, the dominant terms in the co-spectral budget that remain in stationary flows are

$$\frac{d\bar{U}}{dz}E_w(k) + \pi(k) - 2\nu k^2 F_{wu}(k) \approx 0. \quad (2.11)$$

In second-order closure modeling, the integral of $\pi(k)$, $R_{uw}$, is expressed as

$$R_{uw} = -C_R \frac{1}{\tau} \left( \bar{u}d^\prime \bar{u} \right) + C_2 \sigma_w^2 \frac{d\bar{U}}{dz}, \quad (2.12)$$

where $\tau = \bar{\epsilon} / \bar{\tau}$ is a relaxation time scale, $C_R \approx 1.8$ is the Rotta constant, and $C_2 = 3/5$ is a constant associated with isotropization of the production term correcting the original Rotta model Pope (2000); Choi and Lumley (2001) and shown to be consistent with Rapid Distortion Theory Pope (2000). If the Rotta model is further invoked for the spectral version of this term, then it follows that Katul et al. (2013a)

$$\pi(k) = -C_R \frac{F_{wu}(k)}{\tau(k)} + C_2 P_{wu}(k), \quad (2.13)$$

where $\tau(k) = \bar{\tau}^{-1/3}k^{-2/3}$ becomes a wavenumber dependent relaxation time scale Bos et al. (2004) assumed to vary only with $k$ and $\bar{\epsilon}$ for consistency with K41. With this approximation for $\pi(k)$, the co-spectral budget reduces to Katul et al. (2013a)

$$2\nu k^2 F_{wu}(k) = (1 - C_2) \frac{d\bar{U}}{dz}E_w(k) - C_R \frac{F_{wu}(k)}{\tau(k)}. \quad (2.14)$$
The relative importance of the Rotta component and the viscous term $2\nu k^2 F_{wu}(k)$ can be evaluated and is given as Katul et al. (2013a)

$$\frac{2\nu k^2 F_{wu}(k)}{C_R F_{wu}(k)/\tau(k)} = \frac{2}{C_R} \left( \frac{\nu^3 k^4}{\epsilon} \right)^{1/3} \approx (k\eta)^{4/3}, \quad (2.15)$$

Provided $k\eta \ll 1$, de-correlation between $u'$ and $w'$ due to viscous effects can be ignored relative to the Rotta term. However, it is to be noted that as $k\eta \gg 1$, the two terms become comparable in magnitude, but at such fine scales, $|F_{wu}(k)|$ is sufficiently small so that ignoring contributions to the turbulent stress can be justified. With the two remaining terms describing a balance between production and pressure-velocity interaction (i.e. destruction), the co-spectral budget is now given by Katul et al. (2013a)

$$F_{wu}(k) = \frac{1}{A} \frac{d\bar{U}}{dz} \frac{\sqrt{3}}{k^2} E_{w}(k) \frac{1}{k^{2/3}}, \quad (2.16)$$

where $A = C_R/(1 - C_2) \approx 4.5$. For $\eta \ll k^{-1} \ll z$, then

$$F_{wu}(k) = \frac{C_K^\prime}{A} \frac{1}{\epsilon^{1/3}} k^{-7/3}. \quad (2.17)$$

This expression agrees with $F_{wu}(k) = C_{uw}(d\bar{U}/dz)\sqrt{3/\epsilon} k^{-7/3}$ derived from dimensional considerations Lumley (1967); Bos et al. (2004) and supported by a large corpus of measurements in high Reynolds number pipe, boundary layer and atmospheric flows Pope (2000); Saddoughi and Veeravalli (1994); Cava and Katul (2012). Also, the $C_{uw} = C_K^\prime/A \approx 0.65/4.5 = 0.15$, agrees with the accepted value Pope (2000); Ishihara et al. (2002). With such a closure model for $F_{wu}(k)$,

$$\frac{d\bar{U}}{dz} \int_{k_a}^{\infty} F_{wu}(p) dp = -\frac{3}{4} \left( \frac{d\bar{U}}{dz} \right)^2 C_{uw}\epsilon^{1/3} k^{-4/3}. \quad (2.18)$$

This completes the estimation of the production term in the spectral budget.
2.2.4 Modeling the transport term $F_{TR}(k)$

The Heisenberg model (Heisenberg, 1948) can be used to achieve a closure to $F_{TR}(k)$ and is given by

$$F_{TR}(k) = \nu_t(k) |\text{curl } \tilde{u}|^2 \approx 2\nu_t(k) \int_0^k p^2 E_{tke}(p) dp,$$  \hspace{1cm} (2.19)

where $\tilde{u}$ is a 'macro-scale' component of the velocity, and $\nu_t(k)$ is referred to as the Heisenberg (Heisenberg, 1948) eddy viscosity. It is produced by the motion of eddies with wavenumbers greater than $k$ and is given by

$$\nu_t(k) = C_H \int_k^\infty \frac{E_{tke}(p)}{p^3} dp,$$ \hspace{1cm} (2.20)

where $C_H$ is the Heisenberg constant. The turbulent viscosity can be evaluated as

$$\nu_t(k_a) = C_H \int_{k_a}^\infty \sqrt{\frac{C_o \epsilon^{2/3} p^{-5/3}}{p^3}} dp = \frac{3C_H C_o^{1/2} \epsilon^{1/3}}{4k_a^{4/3}},$$ \hspace{1cm} (2.21)

so that the ratio of turbulent to molecular viscosity is given by

$$\frac{\nu_t(k_a)}{\nu} = \frac{3C_H C_o^{1/2}}{4(k_a \eta)^{4/3}}.$$ \hspace{1cm} (2.22)

Because it is assumed that $k_a \eta \ll 1$, $\nu \ll \nu_t(k_a)$, and molecular effects can be ignored (as was the case in the co-spectral budget).

2.2.5 Solving the spectral budget

If $\overline{U}$ follows Eq. 2.1, then $d\overline{U}/dz = u_s/(\kappa z)$ and $\bar{\epsilon} = u_s^2 d\overline{U}/dz = u_s^3/(\kappa z)$, and using $F_{wu}(k)$ from Eq. 2.18, $\nu_t$ from Eq. 2.21, with $\nu_t(k_a) \gg \nu$ simplifies the spectral
budget in Eq. 2.5 to

$$\int_0^{k_a} p^2 E_{tke}(p) dp = \frac{u_s^2}{z^2} C_b,$$  

(2.23)

where

$$C_b = \frac{2}{3C_H C_o^{1/2}} \frac{-(3/4) C_{uv} + \kappa^{4/3}}{\kappa^2}.$$  

(2.24)

To solve for $E_{tke}(k)$ in Eq. 2.23 for $kz < 1$, assume $E_{tke}(k) = a_1 k^{b_1}$ thereby reducing Eq. 2.23 to

$$\frac{a_1 z^{-3-b_1}}{3 + b_1} = \frac{u_s^2}{z^2} C_b.$$  

(2.25)

Upon using polynomial matching, $-3 - b_1 = -2$ or $b_1 = -1$, and $a_1 = 2C_b u_s^2$. This analysis suggests that $E_{tke}(k) = C_{TKE} u_s^2 k^{-1}$, thereby recovering the $-1$ power-law in the spectrum of TKE, where $C_{TKE} = 2C_b$. To link $E_{tke}(k)$ to $E_u(k)$, consider its definition in Eq. 2.8. For $kz << 1, E_u(k) + E_v(k) >> E_w(k)$ (generally, $E_w(k) \sim k^0$ for $kz < 1$ as discussed elsewhere Pope (2000); Katul et al. (2013a)).

Figure 2.1 shows measured $E_u(k), E_v(k)$ and $E_w(k)$ (in regular and pre-multiplied form) computed using orthonormal wavelet transforms (OWT) providing some experimental support to the simplification $E_u(k), E_v(k) >> E_w(k)$ for $kz < 1$. Also at low wavenumbers, $E_u(k)$ scales with $E_v(k)$ so that $E_u(k) = \beta E_{tke}(k) = \beta C_{TKE} u_s^2 k^{-1} = C'_{TKE} u_s^2 k^{-1}$, where $\beta$ is a proportionality constant the role of which is discussed in the appendix A and $C'_{TKE} = \beta C_{TKE}$. The OWT is used here (instead of Fourier spectra) because the wavelet coefficients are less sensitive to possible non-stationarity in the component-wise velocity series. These measurements were collected at 10 Hz using a triaxial sonic anemometer positioned at $z = 39.5m$ above the ground surface on a meteorological tower situated in a 28m tall southern hardwood forest at maximum leaf area index ($LAI = 6m^2m^{-2}$). The figure shows three long ($\sim 6$ hours) and near-stationary periods collected over 3 different days ($u_s = 0.63, 0.56, 0.57 \text{ ms}^{-1}$) and at heights
well above any viscous layer \((z+ = 7.4, 6.6, 6.7 \times 10^5)\). These data sets, described elsewhere (Katul et al., 2012), are also shown to illustrate the scaling laws of \(E_u(k)\), \(E_v(k)\) and \(E_w(k)\). The \(E_u(k)\) and \(E_v(k)\) follow the same scaling trends as assumed, which is \(-5/3\) scaling for \(kz > 1\) and a near \(k^{-1}\) scaling for \(kz < 1\). On the other hand, \(E_w(k)\) follows the \(-5/3\) scaling for \(kz > 1\) and attains a near-uniform value for \(kz < 1\) consistent with numerous studies Pope (2000); Katul et al. (2013a)). Now, instead of a monotonic \(k^{-1}\) behavior for \(E_u(k)\), it is assumed that \(E_u(k)\) attains a uniform value at \(k \leq 1/H\) as already hinted at in Figure 2.1 and also depicted in Figure 2.2, where \(H = \alpha \delta\), \(\alpha\) is a non-dimensional measure of the largest eddies in the system, and \(\delta\) is the boundary layer height. From continuity requirement, \(E_u(k) = C'_{TKE} u_*^2 (1/H)^{-1}\) for \(kH \leq 1\). To summarize,

\[
E_u(k) = \begin{cases} 
C'_{TKE} u_*^2 H^{-1}, & \text{if } kH \leq 1 \\
C'_{TKE} u_*^2 k^{-1}, & \text{if } kH > 1, kz \leq 1 \\
C_K \epsilon^{2/3} k^{-5/3}, & \text{otherwise}.
\end{cases}
\tag{2.26}
\]

The numerical value of \(C'_{TKE}\) can be inferred in multiple ways. One requires numerical estimates of \(\kappa\), \(C_o\), \(C_{uw}\), \(C_H\) and \(\beta\) as discussed in the appendix A. A simpler one assumes continuity of \(E_u(k)\) at \(k_a\). As evidenced from Figure 2.1, the transition from \(k^{-5/3}\) to \(k^{-1}\) is sufficiently narrow in \(k\) to permit using this continuity constraint to determine \(C'_{TKE}\) by matching the two spectral results in Eq. 2.26 at \(k_a = 1/z\). This continuity condition leads to a \(C'_{TKE} = C_K' \kappa^{2/3}\). Using \(\kappa = 0.4\), and \(C_K' = (18/55) \times 1.55 = 0.55\), the \(C'_{TKE} \approx 1.0\). This value agrees with several atmospheric surface layer experiments that suggest \(C'_{TKE} = 0.9 - 1.1\) Kader and Yaglom (1991); Katul et al. (1995, 1996); Katul and Chu (1998).
Figure 2.1: Measured $E_u(k)$, $E_v(k)$ and $E_w(k)$ (in regular and pre-multiplied form) computed using orthonormal wavelet transforms (OWT) for atmospheric flows over a hardwood forest and for 3 separate runs where stationary conditions prevailed over extended periods of time ($u_* = 0.63, 0.56, 0.57 \text{ ms}^{-1}$) and $(z_+ = 7.4, 6.6, 6.7 \times 10^5)$ (Katul et al., 2012). For these runs, $\sigma_u/u_* = 2.0, 1.9, 1.9$, $\sigma_v/u_* = 1.7, 1.7, 1.8$, and $\sigma_w/u_* = 1.0, 1.0, 1.0$.

2.2.6 The longitudinal velocity variance

Integrating the $E_u(k)$ from the largest scale $H = \alpha \delta$ following the idea of Perry and Chong (1982), the variance is

$$\sigma_u^2 = \int_0^{1/H} C'_{TKE} u_*^2 H^{-1} \, dk + \int_{1/H}^{k_a} C'_{TKE} u_*^2 k^{-1} \, dk + \int_{k_a}^{\infty} C''_K \bar{\epsilon}^{2/3} k^{-5/3} \, dk \quad (2.27)$$

Substituting $\bar{\epsilon} = u_*^3/ (\kappa z)$ and performing the integration,

$$\sigma_u^2 = C'_{TKE} u_*^2 + C'_{TKE} u_*^2 \ln \left( \frac{H}{z} \right) + \frac{3}{2} \frac{C''_K u_*^2}{\kappa^{2/3}}. \quad (2.28)$$

Normalizing $\sigma_u^2$ by $u_*^2$, Eq. 2.29 recovers Townsend’s attached eddy hypothesis form (Marusic et al., 2013)

$$\frac{\sigma_u^2}{u_*^2} = B_1 - A_1 \ln \left( \frac{z}{\delta} \right), \quad (2.29)$$
where

\[ B_1 = C'_{TKE}(1 + \ln(\alpha)) + \frac{3}{2} C'_K \kappa^{2/3} \]  

(2.30)

and

\[ A_1 = C'_{TKE} \]  

(2.31)

are the intercept and slope, respectively. These estimates are now compared to the range of estimates reported in Marusic et al. (2013) across the four very high Reynolds number experiments \((B_1 = 1.56 - 2.3, \text{ and } A_1 = 1.21 - 1.33)\). As earlier noted, for a \(C'_{TKE} \approx 1.0\) estimated from the continuity in \(E_u(k)\) at \(k_a\) as discussed in the previous section, a \(C'_K = 0.55\) and \(\kappa = 0.4\) result in \(A_1 = 1.01\). If \(\alpha = 1.0\), i.e., \(\log(\alpha) = 0\), then \(B_1 = 2.5\) is consistent with the upper limit reported by Marusic et al. (2013). These numbers are also close to the empirical estimates of \(A_1 = 1.03\) and \(B_1 = 2.39\) provided in Smits et al. (2011). The other estimates of \(B_1\) and \(A_1\) using the alternate estimate of \(C'_{TKE}\) is discussed in the appendix A. Also, it is interesting to note that for the three runs collected above the hardwood forest reported in Figure 2.1, \(\sigma_u/u_* = 2.0, 1.9, 1.9\). For a typical daytime neutral atmospheric boundary \(\delta = 1000m\), a measurement height of about \(40m\), a zero-plane displacement height of \(2/3\) canopy height, \(A_1 = C'_K/\kappa^{2/3} = 1.01\) and a \(B_1 = (1 + 3/2)A_1\), result in \(\sigma_u/u_* = 2.4\), reasonably close to the field measurements here \((\sim 2.0)\).

2.2.7 Effect of very large scale motion

So far, the largest length scale discussed in the problem is \(\alpha \delta\), the presence of which indicates effects of very large scale motions (VLSM), where \(\alpha \geq 1\). The four experiments from Marusic et al. (2013) are digitized and fitted to Eq. 2.29 so as to determine \(\alpha\). As mentioned before, \(A_1\) is set at 1.01, and a nonlinear least squares method is used to fit the other parameter \(\alpha\). Table 2.1 reports the values of \(\alpha\) along with the coefficient of determination \((R^2)\) and root mean square error \((RMSE)\) in

20
the first four rows. With such a three-regime spectral shape, the range of $\alpha$ (close to 1.0) indicates that the large scales contributing $\sigma_u/u_*$ are commensurate to the expected boundary layer height. The shape of $E_u(k)$ is schematically shown in Figure 2.2. The four different datasets—Melbourne (Kulandaivelu, 2012), Superpipe (Hultmark et al., 2012), LCC (Winkel et al., 2012) and SLTEST (Hutchins et al., 2012) are also plotted against the predicted log-law $B_1 - A_1 \ln(z/\delta)$ in Figure 2.3 for three different values of $\alpha = 1.0, 1.5, 0.5$, where $B_1$ and $A_1$ are predicted by Eq. 2.30 and Eq. 2.31 respectively. It is however, noted that a $k^{-1}$ regime is not observed in the Superpipe data in the $E_u(k)$ spectra as reported by Rosenberg et al. (2013). The reason for showing the predicted log-law for three different $\alpha$ is worth discussing here. It has to be noted that the Superpipe data is different from the other three datasets as the flow in case of pipe is confined (wall-bounded) in all sides, while the channel or field data are roughly bounded on one side only. The confining effect of the walls from all the sides might constrain the largest eddies in the system, and this effect can be picked up by a reduced $\alpha$, say 0.5 ($\alpha = 1.0$ would imply the largest eddy size equal to the boundary layer size). As evident from Fig. 2.3, a smaller $\alpha = 0.5$ places the predicted log-law close to the Superpipe data by lowering the intercept $B_1$ but keeping the slope $A_1$ undisturbed. For the sake of completeness, the log-law is also shown for a higher $\alpha = 1.5$, which would imply largest eddies larger than the boundary layer height. As expected, this places the log-law line higher up by increasing the intercept $B_1$. It is also interesting to note that the experimental data fall between these two limits of $\alpha = 0.5$ and 1.5, while the $\alpha = 1.0$ line falls in the middle of the datasets, thereby providing insight into uncertainties associated with the intercept $B_1$ mentioned in Marusic et al. (2013).
Figure 2.2: Schematic depicting different scaling laws of $E_u(k)$ and $E_w(k)$ along with their ranges. In $E_u(k)$ (left) the solid black line depicts the $k^{-5/3}$ scaling at $kz > 1$ and the solid grey line depicts the $k^{-1}$ scaling at $1/\alpha \delta < kz < 1$. The dotted grey line describes the imperfect scaling in the same range ($1/\alpha \delta < kz < 1$). The dashed grey and dash-dotted grey lines indicate the uniform $E_u(k)$ assumed at $k < 1/\alpha \delta$ discussed in subsections 2.2.7 and 2.2.8. In $E_w(k)$ (right) the solid black line depicts the $k^{-5/3}$ scaling at $kz > 1$ and the solid grey line depicts the uniform spectrum at $kz < 1$.

2.2.8 Imperfect scaling and deviations from a $k^{-1}$ law in the longitudinal velocity spectrum

As discussed in the first section, some studies have already reported a deviation from the $k^{-1}$ power-law in $E_u(k)$ at low wavenumbers $kz \leq 1$ (Katul et al., 2012) as $E_u(k) = C_{TKE}^2 \delta^2 (k\delta)^p$. Now assuming a uniform $E_u(k)$ at $k \leq 1/H$, the uniform value $E_u(k)$ is derived from continuity condition at $kH = 1$ where $H = \alpha \delta$ as $C_{TKE}^2 \delta^2 (1/\alpha)^{p-1}$. This shape of $E_u(k)$ is also depicted in Figure 2.2. In summary,
Figure 2.3: Four different datasets presented by Marusic et al. (2013), namely the Melbourne, Superpipe, LCC and SLTEST data, against the predicted log-law $B_1 - A_1 \ln(z/\delta)$ for three different values of $\alpha = 1.0, 1.5, 0.5$, where $B_1$ and $A_1$ are predicted by Eq. 2.30 and Eq. 2.31 respectively.

The imperfect scaling can be written as

$$E_u(k) = \begin{cases} C_{TKE}^\alpha u_\ast^2 \delta (1/\alpha)^{p-1}, & \text{if } kH \leq 1 \\ C_{TKE}^\alpha u_\ast^2 k^{-1} (k\delta)^p, & \text{if } kH > 1, kz \leq 1 \\ C_K^\alpha \epsilon^{2/3} k^{-5/3}, & \text{otherwise} \end{cases} \quad (2.32)$$

where $p$ can be any real number, and $\delta$ is the height of the boundary layer as before.

As before, continuity requirement at $kz = 1$ leads to $C_{TKE}^\alpha = (C_K^\epsilon/\kappa^{2/3})(z/\delta)^p$.

Table 2.1: The values of $\alpha$ (95% confidence level with intervals) computed using nonlinear least squares method along with the coefficient of determination ($R^2$) and the root mean square error $RMSE$ using Eq. 2.29. The spectral shape associated with the model is depicted in Figure 2.2 using the dotted blue, solid blue and solid and dotted black lines.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\alpha$</th>
<th>95% Confidence Interval</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>Fitted Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melbourne</td>
<td>1.56</td>
<td>(1.41, 1.72)</td>
<td>0.96</td>
<td>0.16</td>
<td>Eq.2.29</td>
</tr>
<tr>
<td>Superpipe</td>
<td>0.76</td>
<td>(0.68, 0.83)</td>
<td>0.96</td>
<td>0.19</td>
<td>Eq.2.29</td>
</tr>
<tr>
<td>LCC</td>
<td>1.36</td>
<td>(1.24, 1.48)</td>
<td>0.96</td>
<td>0.19</td>
<td>Eq.2.29</td>
</tr>
<tr>
<td>SLTEST</td>
<td>1.58</td>
<td>(0.98, 2.19)</td>
<td>0.87</td>
<td>0.37</td>
<td>Eq.2.29</td>
</tr>
</tbody>
</table>
Performing the integration as in Eq. 2.27 and using this value of $C'_{TKE}$,

$$
\sigma_w^2 = \int_0^{1/H} C'_{TKE} u_*^2 \delta (1/\alpha)^{p-1} dk + \int_{1/H}^{k_1} C'_{TKE} u_*^2 \delta^p k^{m-1} dk + \int_{k_1=1/z}^{\infty} C''_K \varepsilon (k_1)^{2/3} k^{-5/3} dk.
$$

(2.33)

Simplifying, and normalizing by $u_*^2$,

$$\frac{\sigma_w^2}{u_*^2} = \left( \frac{C''_K}{\kappa^{2/3}} \right) \left( \frac{z}{\delta} \right)^p \alpha^{-p} + \frac{3}{2} \frac{C''_K}{\kappa^{2/3}} + \frac{1}{p} \frac{C''_K}{\kappa^{2/3}} \left( 1 - \left( \frac{z}{\delta} \right)^p \left( \frac{1}{\alpha} \right)^p \right).$$

(2.34)

The datasets are fitted using a non-linear least squares method with Eq. 2.34 and values of $p$ and $\alpha$ are shown in the first four rows of table 2.2.

From Table 2.2, a power law deviated from $-1$ by $p$ may be a plausible description to the $E_u(k)$ scaling at $kz \leq 1$ for the data reported in Marusic et al. (2013). Also, a constant $E_u(k)$ at $kH \leq 1$ seems to be a plausible representation if a near unity in $\alpha$ is used as an evaluation metric. However, it should be emphasized that a finite $p$ leads to power-law dependence of $\overline{u^2}$ on $z/\delta$ instead of logarithmic. The ranges of values of $p$ is to be noted in the table 2.2, indicating roughly a power-law of $k^{-1.08}$ instead of a $k^{-1}$ scaling. The pre-multiplied spectra presented in Rosenberg et al. (2013) (arguing against a $k^{-1}$ scaling) and Nickels et al. (2005) (supporting a limited $k^{-1}$ scaling), when digitized, also indicate a deviation in the pre-multiplied spectral ordinate of about 0.3, which strongly suggests the presence of a power law $k^{-1.08}$ instead of a $k^{-1}$ scaling.

### 2.2.9 The vertical velocity variance

While Townsend’s hypothesis predicts a logarithmic decrease in $\sigma_w^2/u_*^2$ with increasing $z$, it predicts a constant $\sigma_w^2/u_*^2$ in the intermediate region. A possible explanation for the $z$ independence of $\sigma_w/u_*$ is now considered. This explanation begins with the turbulent vertical velocity difference between two points separated by a distance $r$
given as

\[ \Delta w(r) = w'(x + r) - w'(x) \tag{2.35} \]

whose root-mean squared value is

\[ \Delta w(r)^2 = \overline{w'^2 (x + r)} + \overline{w'^2 (x)} - 2 \overline{w'(x + r) \cdot w'(x)}. \tag{2.36} \]

For homogeneous turbulence, \( \overline{w'^2 (x + r)} = \overline{w'^2 (x)} \), and this expression simplifies to

\[ \Delta w(r)^2 = 2 \sigma_w^2 \left[ 1 - \rho(r) \right]. \tag{2.37} \]

Here, \( \rho (r) \) is the vertical velocity autocorrelation function that varies with \( r \). As \( r \to z \), \( \rho (r) \ll 1 \) and in a first order analysis, Eq. 2.37 suggests that \( \Delta w(z)^2 \approx 2 \sigma_w^2 \).

Adopting inertial subrange structure function scaling for \( \Delta w(z)^2 = C'_K \epsilon^{2/3} z^{2/3} \), equating \( 2 \sigma_w^2 \) to \( C'_K \epsilon^{2/3} z^{2/3} \) and noting that in the intermediate region \( \epsilon = u_*^3 / \kappa z \),

\[ \sigma_w^2 = \frac{C'_K}{2} \frac{u_*^2}{\kappa^{2/3}}, \tag{2.38} \]

so that in normalized form,

\[ \frac{\sigma_w^2}{u_*^2} = \frac{C'_K}{2} \frac{1}{\kappa^{2/3}} = B_3. \tag{2.39} \]

Eq. 2.39 demonstrates the \( z \) independence of \( \sigma_w^2 \) noted in many studies (Perry and Chong, 1982). It also suggests an ‘upper-limit’ on the constant \( B_3 \approx 2.7 \) when \( C'_K \times 4 \) and \( \kappa = 0.4 \). The factor 4 is needed to convert the conventional constant from a

Table 2.2: The values of \( \alpha \) and \( p \) along with \( R^2 \) and \( RMSE \) when fitting Eq. 2.34. The spectral shape associated with the model is depicted in Figure 2.2 using the dotted red, solid red and solid and dotted black lines.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \alpha )</th>
<th>( p )</th>
<th>( R^2 )</th>
<th>( RMSE )</th>
<th>Fitted Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melbourne</td>
<td>1.03</td>
<td>-0.06</td>
<td>0.99</td>
<td>0.06</td>
<td>Eq.2.34</td>
</tr>
<tr>
<td>Superpipe</td>
<td>0.54</td>
<td>-0.05</td>
<td>0.99</td>
<td>0.10</td>
<td>Eq.2.34</td>
</tr>
<tr>
<td>LCC</td>
<td>0.96</td>
<td>-0.04</td>
<td>0.98</td>
<td>0.14</td>
<td>Eq.2.34</td>
</tr>
<tr>
<td>SLTEST</td>
<td>0.71</td>
<td>-0.12</td>
<td>0.99</td>
<td>0.12</td>
<td>Eq.2.34</td>
</tr>
</tbody>
</table>
spectral to structure function. In practice, as \( r \rightarrow z \), \( \rho(z) \sim e^{-1} \) and \( B_3 \) in Eq. 2.39 is reduced by a factor \( \sim 1 - 1/e \). This finite \( \rho(z) = 1/e \) correction was estimated by assuming an exponentially decaying autocorrelation function \( (\rho(r) = \exp(-r/z)) \) whose integral length scale (i.e. \( \int_0^\infty \rho(r)dr \)) is \( z \), as expected. The co-location of the integral length scale with \( z \) is support by a myriad of experiments (including Fig. 2.1) reporting \( E_w(k) \) a constant up to \( kz = 1 \), and then \( E_w(k) \) follows its Kolmogorov scaling for \( kz > 1 \). The schematic in the right panel of Figure 2.2 depicts the expected shape of the spectrum. Hence, the peak in \( kE_w(k) \), co-located with the integral length scale, is situated at \( kz = 1 \). The finite \( \rho(z) \) adjustment leads to \( B_3^{1/2} = 1.3 \), which is in good agreement with numerous experiments, including those reported by Marusic et al. (2013). As discussed in subsection 2.2.3, the clear separation of the spectrum at \( kz = 1 \) also relates to attached eddies, thereby demonstrating interconnectivity between spectral and co-spectral budgets and attached eddies.

2.3 Conclusion

A phenomenological explanation to the log-law scaling in the streamwise turbulent intensity was provided based on a solution to the spectral budget derived for the intermediate region of boundary layers. Linking of a co-spectral budget to the spectral budget by means of a production term whose main source is \( E_w(k) \) (Katul et al., 2013a) is noted. Dividing the \( E_w(k) \) spectrum in two different zones at \( k_a = z^{-1} \) reveals underlying connections to Townsend’s attached eddy hypothesis. The breakpoint at \( k_a = z^{-1} \) is clearly observed in many \( E_w(k) \) spectra within the intermediate region, dividing the eddies into attached manifesting a \( k^{-5/3} \) scaling at \( kz \geq 1 \) and detached eddies often indicated by a flat spectrum at \( kz \leq 1 \), though this low-wavenumber portion is not used here beyond its consequence on \( E_{tke}(k) \approx E_w(k) \) for this range of wavenumbers. Thus the co-spectral budget, the spectral budget, and
Townsend’s attached eddy hypothesis encoded in the shape of $E_w(k)$ are all brought under a common framework to predict the logarithmic scaling in the streamwise turbulent intensity. Because inertial subrange scaling for the $u'$ spectrum was assumed for $k > 1/z$, the coefficients associated with the logarithmic scaling in the streamwise turbulent intensity were then linked to the Kolmogorov and von Kármán constants. When the $k^{-1}$ scaling was extended to all $k < 1/z$, the predicted constants describing the log-law scaling differed by some 20% from measured values reported using laboratory experiments at high Reynolds numbers. Deviation from the $k^{-1}$ scaling in the $u'$ spectrum for $kz < 1$ have also been discussed, along with the effects of very large scale motions (VLSM) on these constants. It was demonstrated via calculations here that these spectral exponent deviations can be commensurate to $k^{-1.06}$. If so, then $\sigma_u^2/u_*^2$ exhibits power-law instead of logarithmic scaling in $z/\delta$, though the difference between the two in terms of statistical fitting to the data may be too small to discern.
Revisiting the formulations for the longitudinal velocity variance in the unstable atmospheric surface layer

3.1 Introduction

Scaling laws of the root-mean squared longitudinal turbulent velocity component ($\sigma_u$) with distance from a solid boundary ($z$) in high Reynolds number turbulent flows are receiving renewed interest (Alfredsson et al., 2011; Smits and Marusic, 2013) given their relevance to a myriad of meteorological problems regarding wind power generation, dispersion of pollutants, footprint estimation among others (Poggi et al., 2006; Cai et al., 2008; Hsieh and Katul, 2009; Hansen et al., 2012; McKeon, 2013; Yang et al., 2014). A logarithmic scaling has been proposed as $\sigma_u^2/u_\ast^2 = B_1 - A_1 \log(z/\delta)$ by Townsend (1976), where $u_\ast$ is the friction velocity, $A_1$, $B_1$ are constants, and $\delta$ is the boundary layer height. Recent laboratory experiments confirmed the universal character of a log-law scaling in $\sigma_u/u_\ast$ within a region where the normalized mean velocity profile also exhibits a log-law scaling (Marusic et al., 2013; McKeon, 2013; Smits and Marusic, 2013) expressed as $U/u_\ast = \kappa^{-1} \log(u_\ast z/\nu) + C_w$, where $\kappa$ is the
von Kármán constant and $C_w$ is a surface roughness coefficient.

In the atmospheric surface layer (ASL), distortions to the $U/u_*$ log-law due to the presence of thermal stratification can be accommodated using Monin-Obukhov similarity theory (MOST) via a stability correction function that varies with the atmospheric stability parameter $\zeta = z/L$, where $L$ is the Obukhov length (Obukhov, 1946; Monin and Obukhov, 1954). Recent theoretical and phenomenological arguments suggest that the stability correction function to $U/u_*$ appears to inherit its quasi-universal character from the shape of the turbulent spectrum (Katul et al., 2011b, 2013b) and the variations of the integral length scale of the flow with atmospheric stability (Salesky et al., 2013).

However, the effects of thermal stratification on $\sigma_u/u_*$ remain a thorny issue within the MOST framework. Earliest attempts identified acceptable scaling with $\zeta$ for the vertical turbulent intensity $\sigma_w/u_*$ but mixed success with $\sigma_u/u_*$ was already noted (Lumley and Panofsky, 1964). Panofsky et al. (1977) even precluded the possibility of a $\zeta$ scaling and proposed a $(\delta/L)^{1/3}$ power-law scaling for $\sigma_u/u_*$ for unstable stratification. The lack of a universal similarity behavior of $\sigma_u/u_*$ was also discussed by Townsend (1961) and supported by Bradshaw (1967), Bradshaw (1978), Kaimal (1978) and Yaglom (1994). While the Panofsky et al. (1977) scaling has been used (Liu et al., 2011) and included in standard micro-meteorology literature (Sorbjan, 1989; Kaimal and Finnigan, 1994), other experiments did report a $1/3$ scaling with $\zeta$ for $\sigma_u/u_*$, especially for tower measurements close to the ground (Hicks, 1981; Hsieh and Katul, 1997). A modification was further suggested by Wilson (2008) who appended a multiplicative $z/\delta$ dependence to Panofsky’s scaling (Panofsky et al., 1977) similar to Rodean (1996), which is constrained by the condition $z \ll \delta$.

The goal of this work is to explain the onset of these divergent results for $\sigma_u/u_*$ under a common framework using a turbulent kinetic energy spectral budget. It has been shown elsewhere (Banerjee and Katul, 2013) that a spectral budget method
(Hinze, 1959; Panchev, 1971) can recover the logarithmic scaling in \( \sigma_u^2 / u^2_a \) in the absence of thermal stratification and provide a theoretical basis for linking \( A_1 \) and \( B_1 \) to the Kolmogorov constant describing the turbulent kinetic energy spectrum within the inertial subrange. This spectral budget is expanded here to explicitly include the effects of thermal stratification using a new source term attributed to the presence of a finite sensible heat flux. The main theoretical contribution is an analytical expression linking \( \sigma_u^2 / u^2_a \) to both \( \zeta \) and \( z/\delta \), at least when \(|\zeta|\) is small (< 0.5) and varies primarily due to variations in land-surface fluxes (i.e. \( L \)).

The newly derived expression can be further simplified to show under what conditions \( A_1 \) and \( B_1 \) actually vary with \( \zeta \) thereby generalizing Townsend’s attached eddy hypothesis for neutral flows to mildly unstable ASL via MOST. For large \( z \) variations, other considerations must be accommodated in the spectral budget that precludes a complete solution. However, under some restrictive assumptions, the scaling proposed by Panofsky and its variant (Wilson, 2008) may be recovered.

Before considering the behavior of \( \sigma_u / u_a \) for such unstably stratified flow, the model prediction in the near-neutral stability limit (\( \zeta \approx 0 \)) is briefly reviewed and compared to results from recent laboratory experiments. The large scatter in the near-neutral \( \sigma_u / u_a \) value reported in several ASL studies, often varying between 2 and 3 (McBean, 1971; Panofsky et al., 1977; Panofsky and Dutton, 1984; Kader and Yaglom, 1990; Hsieh and Katul, 1997; Pahlow et al., 2001; Liu et al., 2011) are shown to be explicitly related to differences in \( \delta \) across experiments and can be explained by Townsend’s hypothesis. A recent dataset is also employed to explore assumptions and provide support for the proposed analysis. As will be seen, the present analysis provides a basis to bridging different concepts such as Townsend’s attached eddy hypothesis, the \( k^{-1} \) and \( k^{-5/3} \) spectral laws in low wavenumbers and the similarity arguments for unstably stratified ASL.
3.2 Theory

3.2.1 Background and Definitions

In the ASL, the time-averaged longitudinal momentum balance and heat budget equations are given by

\[
\frac{\partial U}{\partial t} + U_j \frac{\partial U}{\partial x_j} = \nu \frac{\partial^2 U}{\partial x_j \partial x_j} - \frac{\partial u'_j u'_j}{\partial x_j} - \frac{1}{\rho} \frac{\partial P}{\partial x_j}, \tag{3.1}
\]

\[
\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = D_m \frac{\partial^2 T}{\partial x_j \partial x_j} - \frac{\partial u'_j T'}{\partial x_j}, \tag{3.2}
\]

where \( t \) is time, \( U_j \) are the time-averaged velocity components along direction \( x_j \), \( x_j = (x, y, z) \) are the longitudinal (= \( x \)), lateral (= \( y \)), and vertical (= \( z \)) directions with the longitudinal direction aligned so that \( U_2 = 0 \). Here, \( U \) (or \( U_1 \)), \( T \), and \( P \) represent the time-averaged longitudinal velocity, temperature, and pressure, respectively, \( \rho \) is the mean air density, \( \nu \) is the mean air kinematic viscosity, \( D_m \) is the molecular diffusivity of heat in air, \( u'_i = (u', v', w') \) are the component-wise turbulent velocity excursions in direction \( x_i \), \( T' \) is the turbulent temperature fluctuation, and unless otherwise stated, primed quantities represent turbulent excursions from the time-averaged state represented by overbar or capital letter symbols. Hence, the instantaneous velocity and temperature can be expressed as \( U_i + u'_i \) and \( T + T' \), respectively. For an idealized ASL, the flow can be simplified so that it can be (i) characterized by high Reynolds and Peclet numbers (i.e. neglect molecular viscosity and diffusivity relative to their turbulent counterparts in the mean momentum and heat budget equations), (ii) stationary (i.e. \( \frac{\partial (\cdot)}{\partial t} = 0 \)) and planar-homogeneous (i.e. \( \frac{\partial (\cdot)}{\partial x} = \frac{\partial (\cdot)}{\partial y} = 0 \)), (iii) lacking any subsidence (i.e. \( U_3 = W = 0 \)) or significant mean horizontal pressure gradients (i.e. \( \frac{\partial P}{\partial x} = 0 \)). For these idealized conditions, the mean longitudinal momentum balance and the mean heat budget equations in the ASL reduce to \( \frac{\partial w'u'}{\partial z} = 0 \) and \( \frac{\partial w'T'}{\partial z} = 0 \), suggesting that the
turbulent stress (i.e. $\overline{w'w'}$) and heat flux (i.e. $\overline{w'T'}$) are constant with $z$. It is for this reason that the ASL subjected to such idealized assumptions is labeled as the constant-stress or constant-flux layer (Brutsaert, 1982).

The time-averaged turbulent kinetic energy (TKE) budget in this idealized ASL is

$$\frac{\partial e}{\partial t} = 0 = -\overline{w'w'} \frac{d\overline{U}}{dz} + \frac{g}{T} \overline{w'T'} - \frac{\partial}{\partial z} \left( \frac{1}{2} \overline{w'(u'^2 + v'^2 + w'^2) + \frac{1}{\rho} \overline{w'T'}} - \overline{\epsilon}, \right. \tag{3.3}$$

where $e = (1/2) (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ is the TKE, $\sigma_u^2 = \overline{u'^2}$, $\sigma_v^2 = \overline{v'^2}$, and $\sigma_w^2 = \overline{w'^2}$. The first, second, third and fourth terms on the RHS of Eq. 3.3 are respectively the mechanical and buoyant production (or dissipation) of TKE, the TKE transport by turbulence and pressure-velocity interactions, and the viscous dissipation of TKE. In the ASL, the transport terms have opposite signs, often producing small net TKE transport and a near-balance between production and dissipation of TKE even though some studies report a significant imbalance. The sign and magnitude of this imbalance remains uncertain and will be neglected here. However, it is to be noted that unstable cases often report superior balance between TKE production and dissipation than stable cases (Townsend, 1976; Pope, 2000; Charuchittipan and Wilson, 2009). As evident from Eq.3.3, when $\overline{w'T'} > 0$ (e.g. during daytime conditions over land), the second term is a source of TKE and the ASL flow is labeled as unstable. Near-neutral conditions prevail when $\overline{w'T'} = 0$, and stable conditions occur when $\overline{w'T'} < 0$ (e.g. nocturnal conditions over land). For the mean states, MOST (Monin and Obukhov, 1954) yields $d\overline{U}/dz = \phi_m(\zeta) u_3/(\kappa z)$ and $\overline{\epsilon} = (\phi_m(\zeta) - (\zeta)) u_3^3/(\kappa z)$ where $L = -u_3^3/(\kappa (g/T) \overline{w'T'})$, $g$ is gravitational acceleration, and $\phi_m(\zeta)$ is the stability correction function for momentum. The function $\phi_m(\zeta)$ can be described by the widely used empirical Businger-Dyer (Businger and Yaglom, 1971; Dyer, 1974)
form $\phi_m(\zeta) = (1 - 16\zeta)^{-1/4}$ for $\zeta < 0$ (i.e. unstable conditions), which are the stability conditions explored here.

### 3.2.2 A spectral budget

If $\bar{\epsilon}$ is a conservative quantity across the turbulent energy cascade (i.e. total TKE production is balanced by TKE transfer across all wavenumbers within the inertial subrange and is further balanced by TKE removal through viscous dissipation at large wavenumbers), then a simplified spectral budget can be derived for any wavenumber $k$ given as (Hinze, 1959; Panchev, 1971):

$$\bar{\epsilon} = -\frac{d\bar{U}}{dz} \int_{k}^{\infty} F_{ww}(s) ds + \frac{g}{T} \int_{k}^{\infty} F_{wT}(s) ds$$

$$+ F(k) + 2\nu \int_{0}^{k} s^2 E_{TKE}(s) ds, \quad (3.4)$$

where the first, second, third and fourth terms on the RHS represent as before the mechanical and buoyant production of TKE in the range of $[k, \infty]$, the transfer of TKE in the range $[k, \infty]$, and the viscous dissipation in the range of $[0, k]$. Two conditions are imposed on $F(k)$ so as to ensure that this spectral budget recovers its time-averaged TKE counterpart in Eq.3.3 when spectrally integrated across all $k$. The first condition is that at $k = 0$, $F(0) = 0$, and

$$\bar{\epsilon} = -\frac{d\bar{U}}{dz} \int_{0}^{\infty} F_{ww}(s) ds + \frac{g}{T} \int_{0}^{\infty} F_{wT}(s) ds =$$

$$-\frac{d\bar{U}}{dz} \langle \overline{u'd'} \rangle + \frac{g}{T} \overline{w'T'}, \quad (3.5)$$

so that $\int_{0}^{\infty} F_{ww}(s) ds = \overline{u'd'}$ and $\int_{0}^{\infty} F_{wT}(s) ds = \overline{w'T'}$ maintain the balance between production (mechanical and buoyant) and $\bar{\epsilon}$. 33
The second condition is that as \( k \to \infty \), \( F(\infty) \to 0 \), and

\[
\bar{\epsilon} \approx 2\nu \int_{0}^{\infty} s^2 E_{\text{tke}}(s) \, ds,
\]

or \( \bar{\epsilon} \) is primarily explained by viscous contributions at very large \( k \). The transfer of turbulent kinetic energy \( F(k) \) across wavenumber \( k \) is related to the action of the triple moments and pressure-velocity interaction (i.e. \( \int_{0}^{\infty} F(k) \, dk = (1/2) \partial \left( w^2 (u'^2 + v'^2 + w'^2) \right)/\partial z + \rho^{-1} \partial (\overline{u'})^2/\partial z \)), the two transport terms in the TKE budget), requires closure. The Heisenberg model (Heisenberg, 1948) can be employed to achieve such a closure and is given by

\[
F(k) = \nu_t(k) \left[ \text{curl} \, \overline{u} \right]^2 \approx 2\nu_t(k) \int_{0}^{k} s^2 E_{\text{tke}}(s) \, ds,
\]

where \( \overline{u} \) is a ‘macro-scale’ component of the velocity, and \( \nu_t(k) \) is the so-called Heisenberg eddy viscosity (Heisenberg, 1948). It is produced by the motion of eddies with wavenumbers exceeding \( k \) and is given by

\[
\nu_t(k) = C_H \int_{k}^{\infty} \frac{E_{\text{tke}}(s)}{s^3} \, ds,
\]

where \( C_H \) is the Heisenberg constant to be discussed later. With these approximations and closure assumptions, the spectral budget for the TKE in the ASL reduces to

\[
\bar{\epsilon} = -\frac{d\bar{U}}{dz} \int_{k}^{\infty} F_{wu}(s) \, ds + 2 (\nu_t(k) + \nu) \int_{0}^{k} s^2 E_{\text{tke}}(s) \, ds
\]

\[
+ \frac{g}{T} \int_{k}^{\infty} F_{wT}(s) \, ds.
\]
3.2.3 The spectral budget at $k_a = 1/z$

While valid for all $k$, Eq. 3.9 is now evaluated at a specific $k_a = 1/z$ within the ASL. It is assumed that for $kz > 1$ (i.e. small scales), $E_{tke}(k)$ and $F_{wu}(k)$ are described by their conventional Kolmogorov (Kolmogorov, 1941) and Lumley’s (Lumley, 1967) scaling forms respectively, given as

$$E_{tke}(k) = C_o \varepsilon^{2/3} k^{-5/3},$$  \hspace{1cm} (3.10)$$

and

$$F_{wu}(k) = -\frac{d\bar{U}}{dz} C_{uw} \varepsilon^{1/3} k^{-7/3},$$ \hspace{1cm} (3.11)

where $C_o$ is the Kolmogorov constant and $C_{uw}$ is a similarity constant (Saddoughi and Veeravalli, 1994; Pope, 2000; Ishihara et al., 2002; Katul et al., 2013b). Using these spectral and co-spectral scaling expressions for $k > 1/z$,

$$\frac{d\bar{U}}{dz} \int_{k_a}^{\infty} F_{wu}(s) ds = -\frac{3}{4} \left( \frac{d\bar{U}}{dz} \right)^2 C_{uw} \varepsilon^{1/3} k_a^{-4/3},$$ \hspace{1cm} (3.12)

and

$$\nu_t(k_a) = C_H \int_{k_a}^{\infty} \frac{C_o \varepsilon^{2/3} s^{-5/3}}{s^3} ds = \frac{3C_H C_o^{1/2} \varepsilon^{1/3}}{4k_a^{4/3}}.$$ \hspace{1cm} (3.13)

With the Kolmogorov microscale defined as $\eta = (\nu^3/\varepsilon)^{1/4}$, it is interesting to note that the ratio of turbulent to molecular viscosity at $k = k_a = 1/z$ is given by

$$\frac{\nu_t(k_a)}{\nu} = \frac{3C_H C_o^{1/2}}{4(k_a \eta)^{4/3}}.$$ \hspace{1cm} (3.14)

Because $k_a \eta \ll 1$ in the ASL, $\nu \ll \nu_t(k_a)$, and $\nu + \nu_t \approx \nu_t$ in Eq. 3.9.

To evaluate the buoyancy production for $k_a \ll k < \infty$, the co-spectral scaling for $F_{wT}(k)$ is to be used and is given by (Kader and Yaglom, 1991; Kaimal and Finnigan,
\[F_{wT} = C'_{wT} \frac{dT}{dz} \epsilon^{1/3} k^{-7/3},\]  
\[(3.15)\]

where
\[C'_{wT} = \left(1 - \frac{3}{2} \frac{(4/3) C_T}{C_o} \frac{\zeta}{\phi_m(\zeta) - \zeta}\right) C_{wT},\]  
\[(3.16)\]

and where \(C_T = 0.8\) is known as the Kolmogorov-Obukhov-Corrsin constant (Corrsin, 1951), \(C_{wT}\) is a co-spectral similarity constant (Kaimal and Finnigan, 1994; Katul et al., 2013a, 2014), and \(T_s = -\overline{w'T'}/u_*\) is the ASL temperature scale. The derivation of Eq.\(3.15\) assumes that production and dissipation terms in the temperature variance budget are also in balance (Katul et al., 2013a). It is hereby noted that using the definition of \(L, T_s\) can be written as
\[T_s = \frac{u_*^2}{\kappa (g/T) L}.\]  
\[(3.17)\]

Also, MOST scaling for the mean temperature profile results in,
\[\frac{dT}{dz} = \frac{T_s}{\kappa z} \phi_T(\zeta),\]  
\[(3.18)\]

where \(\phi_T(\zeta)\) is the stability correction function for heat, which can be described by the Businger-Dyer equation as \(\phi_T(\zeta) = \phi_m(\zeta)^2\) in unstable conditions (Dyer, 1974). Justification beyond dimensional considerations for \(\phi_T(\zeta) = \phi_m(\zeta)^2\) are reviewed elsewhere (Li et al., 2012a) and are not repeated here.

Using the definition of \(F_{wT}(k)\), the buoyancy production contribution can be determined as
\[\frac{g}{T} \int_{k_a}^{\infty} F_{wT}(s) ds = \frac{3}{4} \frac{g}{T} \frac{T_s}{\kappa z} \phi_T(\zeta) C'_{wT} \epsilon^{1/3} k_a^{-4/3}.\]  
\[(3.19)\]
Again, in the ASL, \( \frac{dU}{dz} = \phi_m(\zeta) u_a/(\kappa z) \), \( \tau = (\phi_m(\zeta) - \zeta) u_a^2/(\kappa z) \), and using \( F_{wu}(k) \) from Eq. 3.11 and Eq. 3.19 along with Eq. 3.17, Eq. 3.9 simplifies to

\[
\int_0^{k_z} s^2 E_{tke}(s) ds = \frac{u_a^2}{z^2} C_s,
\]

where \( C_s(\zeta) \) varies with \( \zeta \) and is given by

\[
C_s = \frac{2}{3C_H C_o^{1/2}} \left( (\phi_m(\zeta) - \zeta)^{2/3} \kappa^{4/3} / \kappa^2 \right) - \left( \frac{3}{4} \right) C_{uw} \phi_m(\zeta)^2 / \kappa^2 - \left( \frac{3}{4} \right) \zeta C_{wT} \phi_T(\zeta) / \kappa^2 \). \quad (3.21)

Any formulation for \( E_{tke}(k) \) requires an analytical expression for \( C_s \). To estimate \( C_s \), standard values for the constants \( \kappa = 0.4 \), \( C_o = 0.55 \), \( C_{uw} = 0.15 \), and \( C_H = (8/9) C_o^{-3/2} \) as derived for isotropic conditions (Schumann, 1994) are used. The other constant \( C_{wT} \) is taken as (Kaimal and Finnigan, 1994) \( C_{wT} = 3 C_{uw} \). By this particular choice of constants for \( C_o \), \( C_H \), and \( C_{uw} \), the issue of projecting the three dimensional transport term to one dimensional wavenumber form has been bypassed (Ishihara et al., 2002). The variations of \( C_s \) with \( \zeta \) are shown in Figure 3.1 and the discussion on \( E_{tke}(k) \) is now based on how \( C_s \) varies with atmospheric stability. It is evident from Figure 3.1 that for \(|\zeta| < 0.1 \) (labeled as Zone I), \( C_s \) is approximately constant. Also, for \(|\zeta| > 0.5 \) (labeled as Zone II), \( C_s \) is well represented by an approximate power-law of the form \( C_s = p (\zeta)^q \). When this power-law form is fitted to the expression of \( C_s(\zeta) \) given by Eq. 3.21 and for \(|\zeta| > 0.5 \), an acceptable statistical fit is achieved when \( p = 1.6 \) and \( q = 0.6 \), with a coefficient of determination \( R^2 > 0.99 \). For these two zones describing \( C_s \) variations with \( \zeta \), it is possible to derive explicit analytical expressions for \( E_{tke}(k) \) and track the consequences of these expressions to \( \sigma_u/u_a \).
3.2.4 Formulation for Zone I

To solve for \( E_{tke}(k) \) in Eq. 3.20 for \( k_z \leq 1 \), a power-law solution with undetermined coefficients is first assumed so that \( E_{tke}(k) = a_1 k^{b_1} \). This assumed power-law solution is then inserted into Eq. 3.20 to yield

\[
a_1 z^{-3-b_1} = \frac{u_*^2}{z^2} C_s, \quad (3.22)
\]

where \( C_s \) is approximately a constant independent of \( z \) in Zone I. A plausible polynomial matching between the left- and right-hand side yields \(-3 - b_1 = -2\) or \( b_1 = -1\), and \( a_1 = 2C_s u_*^2 \). This analysis suggests that \( E_{tke}(k) = C'_{TKE} u_*^2 k^{-1} \), thereby recovering the \(-1\) power-law in the spectrum of TKE (Tchen, 1953; Panchev, 1971; Perry and Abell, 1977; Turan et al., 1987; Katul and Chu, 1998; Nikora, 1999; Katul et al., 2012), where \( C'_{TKE} = 2C_s \). Furthermore, as argued elsewhere (Banerjee and Katul, 2013), \( \sigma_u^2 \approx \sigma_v^2 + \sigma_w^2 \) and thus \( E_{tke}(k) \approx E_u(k) \) for \( k < k_a \) in the ASL.

To summarize,

\[
E_u(k) = \begin{cases} 
C'_{TKE} u_*^2 k^{-1}, & \text{if } k_z \leq 1 \\
C_o \epsilon^{2/3} k^{-5/3}, & \text{otherwise}
\end{cases} \quad (3.23)
\]

A further discussion about the \( k^{-1} \) power law is necessary at this point. Specifically, a \( k^{-1} \) scaling at low wavenumbers \((k)\) for the streamwise turbulent velocity spectrum \( (E_u(k)) \) has been prevalent in the turbulence literature since the early 1950s and across many experiments and simulations (Tchen, 1953, 1954; Klebanoff, 1954; Hinze, 1959; Pond et al., 1966; Bremhorst and Bullock, 1970; Panchev, 1971; Bremhorst and Walker, 1973; Perry and Abell, 1975, 1977; Korotkov, 1976; Bullock et al., 1978; Hunt and Joubert, 1979; Kader and Yaglom, 1984; Perry et al., 1986, 1987; Turan et al., 1987; Erm et al., 1987; Perry and Li, 1990; Erm and Joubert, 1991; Kader and Yaglom, 1991; Yaglom, 1994; Katul et al., 1996; Katul and Chu, 1998; Jimenez, 1999; Nikora, 1999; Katul et al., 2012; Calaf et al., 2013; Yang et al.,
2014) both for wall bounded flows and ASL turbulence. However, a few studies have not observed a clear $k^{-1}$ scaling or argued against its existence in the unstably stratified ASL (Kaimal, 1978; Antonia and Raupach, 1993; Morrison et al., 2002) and others have found the existence only under certain constraints (Nickels et al., 2005) including very high Reynolds number and $z/\delta < 0.02$. Here, the $k^{-1}$ spectrum is assumed to extend from $k \geq 1/H$ to $kz \leq 1$, where $H = \alpha \delta$, and $\alpha \delta$ is a measure of the largest size of eddies. Below $k = 1/H$ (i.e. at very large scales), the spectrum can be assumed to be flat, i.e., a constant value $(C_{TKE} u_*^2 (1/H)^{-1})$ determined from the continuity requirement at $k = 1/H$. This assumption leads to a value of $\alpha = 1$ thereby retaining some energy in the very-large scale motion and eliminates the need to have a parameter ($\alpha$) susceptible to fitting exercises. Hence, unless otherwise stated, $\alpha = 1$. Experimental datasets for near-neutral conditions appear to support this assumption, such as the $E_u(k)$ reported in Perry et al. (1986). It is important to note that this formulation for Zone I assumes $C_s$ to be constant and the three regime $-5/3$, -1 and flat spectrum holds. It is also to be noted that although the spectral solution here might not be the most accurate representation, given that the spectrum might have some curvature at the intersection of the different domains, and $E_u(k)$ has been assumed to be approximately equal to $E_{tke}(k)$, it allows analytical tractability and provides a physical basis to expand Townsend’s hypothesis to mildly unstably stratified flow as shown layer.

3.2.5 Formulation for Zone II

While evaluating Eq. 3.20, it was assumed that $C_s$ is approximately constant thereby explaining the $-1$ power-law in $E_u(k)$. However, beyond moderately unstable (e.g. $-\zeta > 0.5$), the assumptions that $E_{tke}(k) \approx E_u(k)$ and the existence of a $-1$ power-law for $kz < 1$ might be questionable given the dependence of $C_s$ on $\zeta$.

As a first step to address these issues, the form of $E_{tke}(k)$ at $kz < 1$ is still
assumed to be a power law as used to derive Eq. 3.22. Moreover, the simplified power-expression for $C_s = p (\zeta)^q$ is now used instead of a constant $C_s$. The modified form of Eq. 3.22 now becomes

$$\frac{a_1 z^{-3-b_1}}{3 + b_1} = \frac{u_x^2 p z^q}{z^2 L^q},$$

(3.24)

which upon simplification yields $b_1 = -1 - q = -1.6 \approx -5/3$ and $a_1 = (2-q)u_x^2 p/L^q$. This result is rather interesting because it supports the hypothesis that another extended $-5/3$ scaling law, instead of the $-1$ scaling law might be applicable at $kz < 1$ as indicated by the Kansas data (Kaimal and Finnigan, 1994) though this $-5/3$ scaling is not necessarily associated with an inertial subrange. In fact, two separate $-5/3$ regimes with a break in the vicinity of $kz = 1 - 4$ has already been reported for unstable conditions using previous long-term atmospheric surface layer measurements (Kader and Yaglom, 1991). For analytical tractability, these two $-5/3$ regimes are not distinguished and are assumed to be approximated by a single power-law with an exponent of $-5/3$. If the details of the transition from one $-5/3$ regime to another are known, then they can be incorporated.

Unfortunately, even with this simplification, the presence of a $-5/3$ alone at low $k$ is not entirely prognostic because (i) no clear estimate can be made about the low-wavenumber breakpoint till which the $-5/3$ scaling can be extended and assumed to be valid, (ii) the precise shape of the spectrum is lacking in the vicinity of $-\zeta = 0.5$ where the $-1$ power-law collapses and the $-5/3$ power-law forms. Also, these two issues become compounded for large $z$ as the velocity statistics become quasi-isotropic thereby violating the assumption that $E_{tke}(k) \approx E_u(k)$ at low $k$ in the spectral budget for $E_{tke}(k)$.

However, this near isotropic state may actually offer some guidance because $E_w(k)$ can still be idealized via two regimes, a near flat-portion at low $k$ and a $-5/3$ portion at larger $k$. It is argued here that the $E_u(k)$ spectrum may approach the $E_w(k)$
The modeled $C_s(\zeta)$ according to Eq. 3.21 and a fitted with a power law of the form $C_s = p(\zeta)^q$, where $p = 1.6$ and $q = 0.6$. It is found that the power law is a good fit for $C_s(\zeta)$ after $-\zeta = 0.5$ represented by a dotted line (labeled as Zone II). A constant $C_s$ appears to be reasonable for $-\zeta < 0.5$ (labelled as Zone I).

spectrum as the tendency towards isotropization is approached, which is likely to hold far from the boundary or at large $z$ (and for $-\zeta > 0.5$). If it is assumed that the breakpoint of the now-constant to $-5/3$ regime spectrum is at $1/(\gamma_1 z)$, where $\gamma_1$ is assumed to be greater than unity (eddy sizes are larger than $z$ at this break-point in $E_w(k)$), the $E_u$ spectrum can be described as

$$E_u(k) = \begin{cases} C_o \varepsilon^{2/3} (1/\gamma_1 z)^{-5/3}, & \text{if } k\gamma_1 z \leq 1 \\ C_o \varepsilon^{2/3} k^{-5/3}, & \text{otherwise.} \end{cases}$$  

(3.25)

while the form of the flat spectrum ($k\gamma_1 z \leq 1$) is obtained from continuity requirement at $k = 1/\gamma_1 z$.

To summarize, for instability conditions beyond moderate ($-\zeta > 0.5$) and at
distances sufficiently far from the boundary so that near-isotropic conditions are approached, the $E_u(k)$ shape is dramatically altered from its near-neutral to moderately unstable conditions, resulting in a different formulation for $\sigma_u/u_*$. This domain of formulation and the associated spectral shape is marked on Figure 3.1 as Zone II. The dynamics in the intermediate zone (named zone III) is unknown and beyond the scope of the present work. In this zone, the $E_u(k)$ spectrum evolves from the spectral shape of the three regime -5/3, -1, flat (in zone I) to the two regime -5/3, flat spectra (in zone II).

3.2.6 The longitudinal velocity variance

For Zone I, integrating $E_u(k)$ from Eq. 3.23 from zero to infinity, the variance is

$$\sigma_u^2 = \int_0^{1/H} C'_{TKE} u_*^2 (1/H)^{-1} dk$$

$$+ \int_{1/H}^{k_a} C'_{TKE} u_*^2 k^{-1} dk + \int_{k_a}^{\infty} C_o \bar{\epsilon}^{5/3} k^{-5/3} dk. \quad (3.26)$$

Substituting $\bar{\epsilon} = (\phi_m(\zeta) - \zeta) u_*^3/(\kappa z)$ and performing the integration,

$$\sigma_u^2 = C'_{TKE} u_*^2 + C'_{TKE} u_*^2 \ln \left( \frac{H}{z} \right)$$

$$+ \frac{3}{2} \frac{C_o u_*^2}{\kappa^{2/3}} (\phi_m(\zeta) - \zeta)^{2/3}. \quad (3.27)$$

Normalizing $\sigma_u^2$ by $u_*^2$, Eq. 3.27 recovers the Townsend’s attached eddy hypothesis form (Marusic et al., 2013)

$$\frac{\sigma_u^2}{u_*^2} = B_1(\zeta) - A_1(\zeta) \ln \left( \frac{z}{\delta} \right), \quad (3.28)$$

where

$$B_1 = \frac{3}{2} \frac{C_o}{\kappa^{2/3}} (\phi_m(\zeta) - \zeta)^{2/3} + C'_{TKE} \ln(\alpha) + C'_{TKE} \quad (3.29)$$
and

$$A_1 \approx C_{TKE}'$$  \hspace{1cm} (3.30)

are the intercept and slope, respectively. Recall that $C_{TKE}' = 2C_s$ is dependent on $\zeta$ via Eq. 3.21 but only weakly in Zone I.

In summary, the derivation here unfolds some (but not all) necessary conditions for which MOST might explain distortions in $\sigma_u^2/u_*^2$ around its near-neutral state as specified by Townsend’s hypothesis due to thermal stratification. If the base-line neutral state of $\sigma_u^2/u_*^2$ involves a dimensionless variable $z/\delta$ (instead of being a constant as in $\sigma_w/u_*'$), then under the restrictive conditions highlighted here, MOST may correct for deviations from this neutral base-line state via $A_1(\zeta)$ and $B_1(\zeta)$ due to finite $\zeta$. This derivation is associated with a $k^{-1}$ power-law scaling in $E_u(k)$ for $kz < 1$, which is not likely to hold for moderately unstable (i.e. $-\zeta > 0.5$) conditions and most definitely will not hold for approximate convective conditions (i.e. $-\zeta > 5$). Nonetheless, maintaining the $C_{TKE}' = 2C_s$ dependence on $\zeta$ (instead of a constant) as in Eq. 3.21 allows the initial discussion of $\sigma_u/u_*$ scaling to be anchored to established norms to boundary-layers in near-neutral conditions. Stated differently, the derivation here shows that if the shape of the spectrum of $u$ is unaltered by $\zeta$ (specified in Zone 1), and if variations in $\zeta$ are primarily attributed to $L$ instead of $z$, then a straight-forward extension of Townsend’s attached eddy hypothesis can explain variations in $\sigma_u^2/u_*^2$ via $A_1(\zeta)$ and $B_1(\zeta)$ due to finite $\zeta$. The adequacy and robustness of this extension to Townsend’s hypothesis will be further discussed. This derivation also makes clear that with increasing instability, if the shape of the $u$ spectrum changes from its Zone I state and if the $-\zeta$ increases are due to either $z$ or $L$, then a straight-forward extension of Townsend’s attached eddy hypothesis via $A_1(\zeta)$ and $B_1(\zeta)$ is likely to be insufficient as is the case for Zone II.

For Zone II and with the large $z$ assumption (but still in the ASL), and upon
integrating the spectrum in Eq. 3.25, the longitudinal velocity variance is given by

\[
\frac{\sigma_u^2}{u_s^2} = (1 + \frac{3}{2}) \frac{C_o}{\kappa^{2/3}} (\phi_m(\zeta) - \zeta) \frac{2}{3} \gamma_1 \frac{2}{3}.
\]  

(3.31)

Hence, if \( \gamma_1 \) is known, how \( \sigma_u/u_s \) varies with \( \zeta \) for unstable to convective conditions can be predicted. There are a number of assumptions that complicate the determination of \( \gamma_1 \). The first pertains to the presence of two \(-5/3\) power-laws for small and large \( k \) instead of a single power-law. The degree of separation in wavenumber space and the shape of the connecting spectrum between these two \(-5/3\) exponents can certainly impact the numerical value of \( \gamma_1 \). Moreover, there are large uncertainties in the shape of the spectrum for scales much larger than \( \gamma_1 z \), and a constant extension to \( k = 0 \) is over simplistic at best.

3.3 Results

3.3.1 Comparison with laboratory experiments and the near neutral limit (Zone I)

Before the effects of atmospheric (in)stability are discussed, the \( \sigma_u^2/u_s^2 \) scaling is first presented for the near-neutral case where \( \phi_m(0) = \phi_T(0) = 1 \). For Zone I, the shape of the predicted spectrum from the spectral budget can be found in Figure 1.

Also, for these near-neutral stability conditions, the general log-law (Marusic et al., 2013) \( \sigma_u^2/u_s^2 = B_1 - A_1 \ln(z/\delta) \) recovered from the equation 3.28 is observed across several laboratory experiments as discussed elsewhere (Banerjee and Katul, 2013).

The estimates for \( A_1 = 1.2 - 1.4 \) and \( B_1 = 1.6 - 2.3 \) reported in Marusic et al. (2013) are reasonably recovered when \( \alpha = 1 \). Using a flat spectrum at \( kH < 1 \), the constant \( B_1 \) can be estimated as \( B_{1a} = C_o/\kappa^{2/3} (1 + 3/2 + \ln(\alpha)) = 2.5 \), which appears to be commensurate with the upper limit set by Marusic et al. (2013). The parameter \( A_1 \) is estimated as \( A_1 \approx C_o/\kappa^{2/3} \), which with a \( C_o = 0.55 \) and \( \kappa = 0.4 \) result in \( A_1 \approx 1 \). This estimate is again commensurate with the lower limit provided by Marusic et al. (2013).
To begin unfolding the effects of atmospheric stability on $\sigma_u^2/u_*^2$, consider the case where $C_s$ predicted from equation 3.21 is inserted into equation 3.22 without any modification (i.e. variations in $\zeta$ are assumed to originate from variations in $L$ not $z$) and the spectrum for the formulation for Zone I holds. The outcome of this exercise shows how the two parameters associated with Townsend’s hypothesis for $\sigma_u^2/u_*^2$, namely the intercept $B_1$ and slope $A_1$, might be nonlinearly modified by $\zeta$ within the ASL (though as earlier noted, equation 3.22 does not correctly capture the spectral dynamics for Zones II and III). Both parameters appear approximately constant up to $-\zeta < 0.5$ (i.e. Zone I) and then are predicted to dramatically increase as more unstable conditions are approached. In the neutral limit, they approach the values $B_1 = 2.5$ and $A_1 = 1$ respectively as discussed earlier.

The derivation here also brings into focus why $\zeta$ is not a unique parameter explaining the variability in $\sigma_u/u_*$ as predicted by MOST for $\sigma_u/u_*$. As earlier mentioned, it has been known for some time now that $\delta/L$ is a more dominant parameter affecting $\sigma_u/u_*$ for ASL flows (Lumley and Panofsky, 1964; Panofsky et al., 1977). The spectral budget here predicts how the dimensionless groups $\zeta = z/L$ and $z/(\alpha \delta)$ describe $\sigma_u/u_*$, at least for the conditions when $\zeta = z/L$ variations are dominated by $L$ instead of $z$ and the onset of the $-1$ power-law in $E_u(k)$ for $kz < 1$ is not nullified by thermal stratification. Only for illustration purposes, Figure 3.2 shows computed $\sigma_u/u_*$ varying with both $z/\delta$ and $\zeta = z/L$ for $\alpha = 1.0$ using a straight-forward extension of Townsend’s hypothesis with $B_1$ and $A_1$ modified by $\zeta$. The individual scaling behavior of $\sigma_u/u_*$ with $\zeta$ and $\delta/L$ are also shown in the figure and the model is again found to produce a $1/3$ scaling behavior in the limits of large $|\delta/L|$ and $|z/L|$. It is to be noted that that $z$ is fixed at $5m$, $-\zeta$ is varied from $10^{-3}$ to $10$ and $\delta$ is varied from $10^2 m$ to $10^3 m$. In terms of asymptotic behavior in the neutral limit, the model converges to $\sigma_u/u_* = 2.3$ and the functional form by Panofsky et al. (1977) converges to $\sigma_u/u_* = 2.0$, which corroborates to the range 2-3 reported in the literature (Lum-
Figure 3.2: (a) The computed variation of $\sigma_u/u*$ with $z/\delta$ and $-\zeta = z/L$ at $\alpha = 1$ and $z = 5$ m shown as a surface plot. This formulation is based on the extrapolation of the formulation for Zone I (i.e. a straight-forward extrapolation of Townsend’s hypothesis with $B_1$ and $A_1$ modified by $\zeta$). (b) Variation of $\sigma_u/u*$ with $-\zeta = -z/L$. Similar scaling behavior obtained from the functional forms described in Panofsky et al. (1977) and Wilson (2008) are also shown. (c) Variation of $\sigma_u/u*$ with $-\delta/L$. Similar scaling behavior obtained from the functional forms described in Panofsky et al. (1977) and Wilson (2008) are also shown. The $(-\zeta)^{1/3}$ and $(-\delta/L)^{1/3}$ scaling in semi-log representation is depicted in both figures (b) and (c) for reference.

Note the increase in $\sigma_u/u*$ with increasing $\delta$ for a fixed $z$, which is also observed in Figure 3 of Banerjee and Katul (2013). Furthermore, for a fixed $\delta$, the increase in $\sigma_u/u*$ is also observed on the surface plot with increasing $|\zeta|$. Functional forms of the variation of $\sigma_u/u*$ empirically fitted to several experiments have been provided by Panofsky et al. (1977) as $\sigma_u/u* = [4 + 0.6(\delta/(L))^{2/3}]^{1/2}$. Using another expansive data set, Wilson (2008) provided a modification to Panofsky’s form by incorporating the $z$ and $\delta$ variation as $\sigma_u/u* = \left(\left[4 + 0.73(\delta/(L))^{2/3}\right]\left[1 - (z/\delta)^{0.25}\right]\right)^{1/2}$. These empirical
Figure 3.3: Comparisons between model estimates and the functional forms provided by Wilson (2008) and Panofsky et al. (1977) for two different heights (a) \( z = 5\) m and (b) \( z = 50\) m.

functions represent a large corpus of data on variations in \( \sigma_u/u_\ast \) and serve as a logical basis for evaluating the proposed formulation.

Because of the constraint \( z \ll \delta \) that is not too different from the limit in Nickels et al. (2005) to ensure the onset of a -1 power-law in the longitudinal velocity spectrum, the comparisons are shown for a reasonably small \( z = 5\) m (i.e. \( z/\delta < 0.02 \)) in panel (a) of Figure 3.3 and the comparisons appear surprisingly reasonable even for moderately unstable conditions \( -\zeta > 1 \) given the lack of any tunable parameter here. However, for a high (separated by an order of magnitude) value of \( z = 50\) m (i.e. \( z/\delta > 0.02 \)), the comparisons among the models remain strongly correlated but biased by a multiplier as shown in panel (b) of Figure 3.3. Recall that when deducing Eq. 3.22, variations in \( \zeta \) were momentarily assumed to originate from variations in \( L \) not \( z \). This simplification allowed the polynomial matching to recover
the -1 power-law in the spectra at the expense of ignoring large variations in $z$ when determining $C_s(\zeta)$. This deficiency is discussed later after comparisons with a recent field experiment, AHATS, is presented.

### 3.3.2 Comparisons with the AHATS field experiment

Assumptions made in deriving the spectral budget are now explored using ASL data collected as part of the Advection Horizontal Array Turbulence Study (AHATS) conducted near Kettleman City, CA, USA from June 25 to July 17, 2008 (Salesky et al., 2013). Here, data from the AHATS profile tower is considered where velocity and temperature measurements were collected using Campbell Scientific CSAT-3 triaxial sonic anemometers at heights of $z = 1.51, 3.30, 4.24, 5.53, 7.08,$ and $8.05$ m. Raw data were sampled at 60 Hz, then down-sampled to 20 Hz during preprocessing. The data were divided into blocks of 27.3 minutes, or $32768 = 2^{15}$ data points per block to permit the use of Fast Fourier Transforms in spectral density estimation. The coordinate system was aligned with the mean wind direction so that $U_2 = 0$ for each block as earlier noted. Only blocks of data with wind angles of $|\alpha_w| \leq 45^\circ$ were included in the analysis. Several quality control criteria were employed during the pre-processing. Blocks of data that exhibited more than a 30% deviation of $\phi_w = \sigma_w/\upsilon_*$ from the value predicted by MOST were discarded (Lee et al., 2004). To minimize the effects of nonstationarity on the calculated statistics, non-stationary ratios (Vickers and Mahrt, 1997) for the streamwise (RNu), cross-wind (RNv), and vector (RNS) velocity components were examined for each period. Blocks of data were excluded from the analysis if RNu, RNv, or RNS $\geq 0.5$. Blocks of data were also excluded if the measured $\upsilon_* \leq 0.1$ m s$^{-1}$ or measured $\rho c_p \sqrt{T^*} \leq 10$ W m$^{-2}$ (Högström, 1988). Finally, because MOST requires the assumption that the turbulent fluxes do not vary with $z$ within the ASL, periods where either momentum or heat flux varied more than 20% with $z$, as quantified by the coefficient of variation across $z$, were
The mean velocity and temperature gradients were calculated by fitting a second-order polynomial in \( \ln z \) to the calculated mean profiles, e.g. 

\[
U(z) = Af_m \ln z + Bf_m (\ln z)^2 + Uf_0,
\]

where \( Af_m, Bf_m, \) and \( Uf_0 \) are constants determined through linear regression fitting. The polynomial fit can then be differentiated to determine the mean gradient by:

\[
\frac{\partial U}{\partial z} = \frac{Af_m}{z} + \frac{2Bf_m \ln z}{z}.
\]

The mean turbulent kinetic energy dissipation and temperature variance dissipation rates were simultaneously estimated by linear regressions to the compensated second-order structure functions, e.g.

\[
r_1^{-2/3}D_{11}(r_1) = c_2 \epsilon^{2/3} = ar_1 + b
\]

for spatial lags \( r_1 \) in the range \( 0.2 \leq r_1 \leq 2.0 \) m (Salesky et al., 2013).

Here, \( r_1 \) is determined from Taylor’s frozen turbulence hypothesis (Taylor, 1938). In an analysis of deviations from MOST, Salesky and Chamecki (2012) found that best-fit curves to \( \phi_m(\zeta) \) and \( \phi_T(\zeta) \) from the AHATS data set were similar to the Businger-Dyer empirical form. The local balance assumptions for the TKE and temperature variance budgets are also examined here using the AHATS data. In Figure 3.4, the local imbalance (i.e. dissipation – production) of the TKE \( (\beta_E) \) and temperature variance \( (\beta_T) \) budgets are presented in panels (a) and (b), respectively. For unstable conditions, the imbalance of the TKE budget in panel (a) scatters around \( \beta_E = 0 \). In panel (b), the imbalance in the temperature variance budget \( \beta_T \) also scatters around zero for the majority of the data runs. However, data points with large values of \( \beta_T \) occurred for runs with \( \phi_T < 0 \), i.e. a counter-gradient heat flux. Discussions of the local imbalance of the TKE budget can also be found for other recent studies (Wilson, 2008; Salesky et al., 2013). Not withstanding these
issues, it can be surmised here that for unstable and near-neutral conditions, the TKE budget is close to a local balance \( \beta_E(\zeta) \approx 0 \) on average and does not exhibit any significant dependence on \( \zeta \) (Salesky et al., 2013). Spectra from the AHATS, shown in Figure 3.5, were calculated using the 60Hz data (the inertial range is not really present in the 20Hz data) using periods of approximately 54.8 minutes. The use of such long periods allowed an extension of the measured spectra to a small wavenumber range, which is needed to assess the slope in the \( k_z < 1 \) range. Each spectrum was first normalized by \( C_0 \bar{e}^{2/3} \) so as to collapse the inertial range and then spectra were averaged in bins of \( \zeta \) as indicated in the figure. Panels (a) and (b) do suggest a transition from \( k^{-1} \) to \( k^{-5/3} \) scaling for \( k_z < 1 \) with increasing instability for \( E_{TKE} \) and \( E_u \), respectively consistent with the predictions here for Zones I and II. To further investigate the spectral scaling, compensated spectra using \( k^{5/3} \) are also shown in Figure 3.5(c),(d) and using \( k^1 \) shown in Figure 3.5(e),(f).

The measured spectra in most runs from AHATS support the existence of a decade of \(-1\) power-law scaling in \( E_u(k) \) and \( E_v(k) \) but not \( E_w(k) \) at \( k_z < 1 \) for small \(-\zeta < 0.5\) (consistent with theoretical predictions for Zone I). Also, the AHATS
Figure 3.5: (a)-(b) $E_u(k)$ and $E_{TKE}$ spectra from AHATS experiment averaged over bins of $-\zeta$ illustrating the existence of a $-1$ power-law in the large scales for near neutral conditions transitioning to a $-5/3$ power law for strongly unstable conditions. Panels (c)-(f) show compensated versions of the spectra to emphasize the existence of the $-1$ and $-5/3$ scalings for $kz < 1$.
measured spectra suggest a collapse of a -1 power-law in $E_u(k)$ and $E_v(k)$ and the initiation of an extended -5/3 power-law beyond $kz < 1$ around an atmospheric stability limit $-\zeta > 0.5$ as predicted from the $C_s(\zeta)$ in Zone II) and shown in Figure 3.5. The finding that the $E_v(k)$ spectrum is similar to the $E_u(k)$ spectrum ($E_u(k) = E_v(k)$) further supports the assumption that $E_u(k) + E_v(k) \gg E_w(k)$ and hence $E_{tke} = (1/2)(E_u(k) + E_v(k) + E_w(k)) \approx E_u(k)$ at low k. In fact, the assumption of $\sigma_u^2 \approx \sigma_v^2 + \sigma_w^2$ and thus $E_{tke}(k) \approx E_u(k)$ for $k < k_a$ in the ASL can be also tested using the AHATS data. Figure 3.6 shows a one to one comparison between $\sigma_u^2$ and $\sigma_v^2 + \sigma_w^2$ computed from AHATS data and the assumption can be deemed to be fair.

As atmospheric instability increases beyond $-\zeta > 0.5$, two $k^{-5/3}$ regions can be identified in $E_{TKE}$ in Figure 3.5(c) (i.e. in pre-multiplied form) consistent with measurements reported elsewhere Kader and Yaglom (1991) for unstable ASL conditions. As noted earlier, these two separated $-5/3$ regimes are ‘lumped’ together into a single regime here for analytical tractability in Zone II and uncertainty associated with such simplifications are absorbed in $\gamma_1$.

To illustrate the role of $\delta$ on $\sigma_u/u_*$ from the AHATS, $\sigma_u/u_*$ was calculated for
the morning periods from 0800-1000 PDT (assumed to have smaller $\delta$) and the afternoon periods from 1400-1600 PDT (assumed to have larger $\delta$) using the straightforward extension of Townsend’s hypothesis with stability dependent $A_1$ and $B_1$. Because a direct measurement of $\delta$ was not available, the average boundary layer depth for morning and afternoon periods was estimated using a slab model described in Juang et al. (2007) and discussed in the appendix B. From the slab model, values of $\delta = 289 \pm 31$ m for the morning (0800-1000 PDT) period, and $\delta = 836 \pm 37$ m for the afternoon (1400-1600 PDT) were obtained. This estimates are consistent with a visual estimate for $\delta = 780 \pm 130$ m from the plots of daily soundings available from the NCAR project website.

For each of these distinct $\delta$ and at a given $z$, the $\sigma_u/u_*$ approximately collapse on a single curve as $\zeta$ varies. Consistent with earlier model calculations shown in Figure 3.2 (panel a), the afternoon $\sigma_u/u_*$ is found to be higher than its morning counterpart for the same $\zeta$, signifying the effects of $\delta$ at a fixed $z$. Noting that $\alpha$ was pre-set to unity in all calculations, the variations at both $z$ and $\delta$ are reasonably predicted by the model as shown in Figure 3.7. This agreement is also not due to any artificial self-correlation arising from $u_*$ impacting the dependent and independent variables jointly as measured and modeled dimensional $\sigma_u$ also agree (not shown). However, the model is found to be biased about 15% with a coefficient of determination exceeding 0.7. It is evident that modeled $\sigma_u/u_*$ over-predict the measurements for the smaller $\delta$ (or morning) values, at least when compared to the larger $\delta$ case. This bias may already be hinting that the modeled $E_u(k)$, with its predicted $-1$ power-law form (as derived for Zone I), is not correctly reproducing the measured $E_u(k)$ in Zone II as expected when straight-forward extension of Townsend’s hypothesis with stability dependent $A_1$ and $B_1$ is employed in this zone. Recall that the AHATS spectra also show a shift from $-1$ to $-5/3$ exponent in $E_u(k)$ with increasing instability.
Figure 3.7: Measured $\sigma_u/u_*$ from the AHATS experiment for two different heights ((a) and (b)) of the boundary layer-in the morning and at afternoon, for two different heights of measurement ($z$). The data shows variation with $-\zeta$, as well as $\delta$. Both variations appear to be predicted by the model depicted by solid and dash-dotted lines for two different $\delta$ values for morning ($\delta = 289$ m) and afternoon ($\delta = 836$ m).

3.3.3 Comparisons with literature data

Two different types of field data, compiled in Panofsky et al. (1977), are now discussed against model prediction in Figure 3.8 when using equations 3.30 with $C_s$ given by equation 3.21. For clarity, Figure 3.8 shows the comparisons between model and measurements for the tower and aircraft data reported in Panofsky et al. (1977). The tower data measured at different heights (ranging from 4 m to 32 m but precise heights not reported) are represented here by an average $z = 18$ m, and the model calculations assume $\alpha = 1$ (i.e. no tunable parameter). The tower data is reasonably predicted by the model for all stability (unstable) conditions. The aircraft data, however, is found to be over-predicted (again consistent with the bias noted for the
AHATS experiment for the smaller $\delta$). To be clear, a number of cautionary notes about the aircraft data must be highlighted in this comparison. The aircraft data were analyzed in Panofsky et al. (1977) by assuming $\sigma_u = \sigma_v$, the measurement heights were sufficiently large (> 100m) so that neglecting Coriolis effects or the flux transport terms in the TKE or, equally important, the temperature variance budgets as done here may be questionable. Furthermore, the assumption that $E_{tke}(k)$ is dominated by $E_u(k)$ clearly breaks down at these heights, where the degree of anisotropy is far weaker when compared to the tower measurements collected near the surface. Notwithstanding these issues, it is to be noted that when $\alpha = 1$ is replaced with $\alpha = 0.25$, the proposed model can reproduce the reported aircraft $\sigma_u/u*$ to high fidelity. However, such a small $\alpha$ (or any other empirical reduction to it with increasing $z$) indicates significant reductions in low-frequency energy that cannot be theoretically explained by the current solution to the spectral budget. That is, significant differences exist between the $\sigma_u/u*$ formulations for Zones I and II (or equations 3.28 and 3.31) reflecting fundamental differences in the scaling laws describing the shapes of the energy spectra for these two zones. This will be discussed further in subsequent sections.

Interestingly, a one-third scaling of $\sigma_u/u*$ with $\zeta$ for a fixed boundary layer height has been prevalent in some studies reporting a $\sigma_u/u* = 2.7(1 - 3\zeta)^{1/3}$ (Panofsky and Dutton, 1984; Hsieh and Katul, 1997). The spectral budget model is found to retrieve that 1/3 scaling for large $-\zeta$ reasonably as shown in Figure 3.9 though such 1/3 scaling is known to be contaminated by self-correlation given that $u*$ variations impact dependent and independent variables here. A comparison between measured and modeled $\sigma_u$ was repeated (not shown) and the agreement between them was commensurate with the AHATS agreements. The proposed scaling law and the model are also found to represent the data over grass and bare soil reasonably although the
grassland data is found to be more scattered due to the fact that it was collected in non-ideal ASL conditions (within a large forest clearing). To summarize, no unique dimensionless group describes variations in $\sigma_{u}/u_*$ for the unstable ASL, but there are few length scales (at minimum $z$, $\alpha \delta$, and $L$) that must be accommodated. Even in the near-neutral ASL, $\sigma_{u}/u_*$ varies with $z/\delta$ as earlier shown by Townsend, perhaps explaining why the scatter in field data is large for such near-neutral conditions. Also, in contrast to the scaling proposed by Panofsky et al. (1977), $\zeta$ remains an essential variable here needed to explain some of the variations in $\sigma_{u}/u_*$, at least for very small $z/\delta$ lending further support to the proposed amendment by Wilson (2008).

**Figure 3.8:** Comparisons between modeled and measured $\sigma_{u}/u_*$ using data from Panofsky et al. (1977) for the tower and aircraft platforms. This model is represented by a single $\alpha = 1$. 
Figure 3.9: Variation of $\sigma_u/u_*$ with $-\zeta = -z/L$ for $\alpha = 1.0$ and three different $z/\delta = 0.05, 0.01$ and $0.005$. The data over grass and soil from Hsieh and Katul (1997) (HK97) are superimposed on the figure also showing the $1/3$ power-law scaling relation proposed in Hsieh and Katul (1997) as $\sigma_u/u_* = 2.7(1-3\zeta)^{1/3}$.

3.3.4 Results pertaining to Zone II

It was assumed before that variation in $\zeta$ originated primarily from $L$ while evaluating Eq. 3.20. This assumption unexpectedly reproduced well $\sigma_u/u_*$ even when $-\zeta > 0.5$ provided $z$ was small (as expected from tower measurements). However, when comparing to aircraft measurements, this assumption results in the model being biased for large $z$ as shown in Figure 3.8 and Figure 3.3(b). The fate of the model for somewhat larger $z$ and $-\zeta$ but still for $z/\delta < 0.1$ (i.e. flow within the ASL) requires further examination that has already been discussed in section 3.2.5. The final form of $\sigma_u/u_*$ from this discussion (Zone II) can be found in Eq. 3.31. This formulation is for valid for $-\zeta > 0.5$ (Zone II) where $z$ is sufficiently large. The need
for a large \( z \) separate from the requirement of \(-\zeta > 0.5\) is to ensure near-isotropic conditions become prevalent (i.e. \( E_w(k) \) and \( E_u(k) \) as well as their wavenumber integrations are commensurate) away from the boundary but otherwise maintaining all the assumptions in the idealized ASL. This formulation is tested against the functional forms proposed by Panofsky et al. (1977) and Wilson (2008) and the aircraft data provided in Panofsky et al. (1977) and shown in Figure 3.10. It is found that a value of \( \gamma_1 = 2 \) is a reasonable estimate, i.e., the inertial behavior is found to continue up to a height commensurate with twice the observation height when \( z \) is large and \( \zeta > 0.5 \) (Zone II). Interestingly, this estimate appears to be consistent with variations in the spectral peaks of \( kE_w(k) \) reported for the Kansas experiments. For example, Kaimal and Finnigan (1994) report that the spectral peaks inferred from measured \( kE_w(k) \) when normalized by their near-neutral limit \((=f_{p_w}(\zeta))\) are given as \( f_{p_w}(\zeta) = (1 - 0.7|\zeta|)^{-1} \) when \(-\zeta < 1\), and \( f_{p_w}(\zeta) = 3.23 \) when \( 1 < -\zeta < 0.1\delta/L \). That is, as isotropic conditions are approached away from the boundary (i.e. \( E_w(k) \approx E_u(k) \)) which is expected in the large \( z \) limit, the Kansas data do suggest that \( 1 < \gamma_1 < 3.23 \) (as inferred from \( E_w(k) \)), consistent with the intermediate \( \gamma_1 = 2 \) found here. Recall that the \( E_u(k) \) spectrum assumed here (i.e. flat for \( k < (\gamma_1z)^{-1} \) and exhibits a \(-5/3\) scaling for \( k > (\gamma_1z)^{-1} \)) has its well-defined spectral peak at \( k = (\gamma_1z)^{-1} \) where \( kE_u(k) \) is maximum, consistent with its \( kE_w(k) \) counterpart when \( z \) is large and \(-\zeta > 0.5\). Also, it is to be noted that \( \gamma_1 \) is impacted by the assumption of a single \(-5/3\) exponent as given by equation 3.24, an assumption not supported by the AHATS spectra in Figure 3.5.

3.4 Conclusion

A spectral budget method has been discussed to explain the characteristics of the normalized streamwise turbulent intensity \((\sigma_u^2/u_*^2)\) under unstably stratified atmosphere. This budget showed why a straightforward extension of Monin-Obukhov similarity
Figure 3.10: (a) Comparisons between predictions from equation 3.31 and functional forms provided by Panofsky et al. (1977) and Wilson (2008) using $\gamma_1 = 2$. (b) One to one comparison with aircraft data provided by Panofsky et al. (1977).

theory (MOST) to $\sigma_u^2 / u^2$ is inadequate. Analytical solutions to this spectral budget were possible for two limiting conditions: 1) $z/\delta < 0.02$ and $|z/L = \zeta| < 0.5$ (labeled as Zone I), and 2) $0.02 \ll z/\delta < 0.1$ and $|\zeta| > 0.5$ (labeled as Zone II). The $z/\delta = 0.02$ limit was independently derived from laboratory studies (near-neutral conditions) and found reasonable here, and the $-\zeta = 0.5$ limit was inferred from Figure 3.1 while the spectral analysis of the AHATS data also indicated a similar trend. The first condition ensured the onset and maintenance of a -1 power-law in $E_u(k)$ at low $k$ for mildly unstable conditions. The second condition is associated with a deterioration of the -1 power law in $E_u(k)$, and its eventual replacement with a $-5/3$ scaling beyond $kz < 1$. The $\sigma_u$ comparisons have been checked for self correlation and it has been found out that any self or spurious correlation is not the cause
for the fair agreements.

The work here showed that $\sigma_u^2/u_*^2$ is found to conform to the logarithmic scaling anchored to Townsend’s attached eddy hypothesis for near-neutral conditions (or Zone I), while the coefficients of this log-law are found to be modified by MOST for mildly unstable conditions. The required low $z/\delta \ll 1$ condition to observing a -1 power-law is compatible with laboratory measurements by Nickels et al. (2005) for neutral boundary layer flows. That is, the Townsend’s attached eddy hypothesis, the height conditions promoting the -1 power-law ($z/\delta < 0.02$), and MOST expansion for non-neutral flows to mildly unstable in the ASL are all interconnected. Interestingly, the $\sigma_u^2/u_*^2$ derived for $z/\delta < 0.02$ and $|\zeta| < 0.5$ (Zone I) appears to be robust to variations in $|\zeta| < 0.5$ and agreement with measurements and other empirical models were surprisingly found to hold up to $-\zeta = 10$ for tower measurements where $z$ was small. In the case of higher $z$ but still $z/\delta < 0.1$ and a finite $-\zeta > 0.5$ (Zone II), the longitudinal spectrum loses its -1 scaling consistent with the laboratory findings in Nickels et al. (2005) and follows an approximate -5/3 scaling at low-wavenumbers consistent with Kader and Yaglom (1991) though this exponent is not connected to the inertial subrange. Using this asymptotic argument for $z/\delta < 0.1$ and a finite $-\zeta > 0.5$, a different model for $\sigma_u/u_*$ was derived and shown to agree with earlier models and aircraft data for those conditions. Naturally, ‘stitching’ these two limiting conditions via some ad-hoc function is possible (e.g. Zone III) though such stitching does not guarantee correct predictions of $E_u(k)$ at $kz < 1$. Progress on how to transition from one formulation to another for $\sigma_u/u_*$ can greatly benefit from Large Eddy Simulation runs where $z$, $L$, and $\delta$ are allowed to vary. Perhaps more broadly, the fact that $E_u(k)$ does not exhibit a single ‘canonical’ shape across all $z$, $L$, and $\delta$ may explain the lack of ‘non-universal’ form in $\sigma_u/u_*$ when all these length scales are simultaneously varied.
4.1 Introduction

The scaling laws of the root mean squared longitudinal turbulent velocity component \( \sigma_u \) normalized by the friction velocity \( u_* \) with distance \( z \) from a solid boundary has received renewed attention in meteorology and turbulence (Alfredsson et al., 2011; Smits and Marusic, 2013; Poggi et al., 2006; Cai et al., 2008; Hsieh and Katul, 2009; Hansen et al., 2012; McKeon, 2013; Stevens et al., 2014; Yang et al., 2014). While it has been known for some time that \( \sigma_u/u_* \) in the atmospheric surface layer (ASL) deviates from the expectations set by the Monin and Obukhov similarity theory (MOST) even for near-neutral conditions (Bradshaw, 1978; Kader and Yaglom, 1990; Kaimal, 1978; Kaimal and Finnigan, 1994; Charuchittipan and Wilson, 2009), MOST scaling for \( \sigma_u/u_* \) remains the ‘workhorse’ model for practical problems because alternative scaling laws that retain the simplicity of MOST remain lacking (Zilitinkevich and Calanca, 2000). The goal of this work is to offer a potential al-
ernative to MOST for the stable ASL for ‘idealized’ conditions. These idealized conditions include stationary planar homogeneous flows in the absence of subsidence and well-developed turbulence so as to ensure an extensive inertial subrange spectrally separating production from viscous dissipation scales.

A universal logarithmic scaling with $z$, originally proposed by Townsend (1976) and expressed as $\sigma_u^2/u_s^2 = B_1 - A_1\log(z/\delta)$ for neutral turbulent boundary layers, has been recently confirmed by different high Reynolds number laboratory experiments and large eddy simulations (Marusic et al., 2013; McKeon, 2013; Smits and Marusic, 2013; Stevens et al., 2014), where the atmospheric boundary layer (ABL) height $\delta$ is featured as a significant dynamical variable. However, a satisfactory phenomenological theory predicting how $\sigma_u^2/u_s^2$ is altered by thermal stratification in idealized stably stratified ASL flows while recovering its well established near-neutral limit remains elusive and frames the scope here. By phenomenological theory here, we mean a theory that relates a number of empirical observations on $\sigma_u^2/u_s^2$ in a way that is consistent with the fundamental equations but the theory itself is not directly derived from these fundamental equations. Hence, as a starting point for developing such a phenomenological theory, a number of observations and findings must be reviewed.

First, under near-neutral conditions and for an idealized ASL, the mean velocity ($U$) exhibits a logarithmic scaling with $z$ roughly for the same range of $z/\delta$ as $\sigma_u/u_s$. The logarithmic scaling for the mean velocity is expressed as $U^+ = \kappa^{-1}\log(z^+) + C_w$ where $\kappa$ is the von Kármán constant and $C_w$ is a surface roughness coefficient, $U^+ = U/u_s$ and $z^+ = u_s z/\nu$ are non-dimensional velocity and length from the wall, respectively. Second, in the presence of thermal stratification, the mean velocity is reasonably described by incorporation of a stability correction function for momentum $\phi_m(\zeta)$ that is only a function of the stability parameter $\zeta = z/L$ according to MOST, where $L$ is the Obukhov length and $\zeta < 0$ indicates unstable stratification.
and $\zeta > 0$ indicates stable stratification. Links between the energy spectrum of turbulence and $\phi_m(\zeta)$ (or similar measures) have received renewed interest in the last decade (Sukoriansky et al., 2006; Katul et al., 2011b; Li et al., 2012a; Salesky et al., 2013; Sukoriansky and Galperin, 2013; Katul et al., 2013a, 2014; Li et al., 2015a) but will not be elaborated on here. Third, several studies reported measurements of spectra, co-spectra of momentum and heat fluxes as well as the corresponding integral length scales, and their variations with the stability (Kaimal et al., 1972; Kaimal, 1973; Caughey, 1977; Högström, 1990; Launiainen, 1995; Canuto et al., 2008; Basu et al., 2014). This dependence of integral scales on $\zeta$ will be explicitly accommodated in the proposed spectral budget model. Fourth, a balance between the turbulent kinetic energy dissipation rate, mechanical production and buoyant destruction (Wyngaard and Coté, 1971; Katul et al., 2014) is often assumed though several datasets suggest the existence of a finite transport term that can vary with $\zeta$ as reviewed elsewhere (Salesky et al., 2013). Implications and biases due to a stability-dependent non-negligible transport term will be discussed later in the appendix C but not explicitly included at this stage. Lastly, it was noted by Wyngaard and Coté (1971), Nieuwstadt (1984), and Olesen et al. (1984) among others that all dimensionless parameters resulting from local combinations of flow variables reach a uniform value at sufficiently high $\zeta$ (but still maintaining a sufficiently developed turbulent state) thereby losing their $z$ dependence. This phenomenon is referred to as $z$-less stratification and its limit is generally assumed to be close to $\zeta \approx 1$ or 2 (Olesen et al., 1984). However, measurements of $\sigma_u/u_*$, compiled by Pahlow et al. (2001), suggest that a power-law scaling in $\zeta$ appears to be maintained even when $\zeta > 2$, which was used to argue against the $z$-less scaling for $\sigma_u$. However, the analysis of Pahlow et al. (2001) was problematic due to the presence of the so-called sub-meso motions. The revised analysis (Basu et al., 2006) of the same dataset (augmented with others) did not show such spurious behavior. The spectral model proposed in
this study will be used to partly clarify some (but not all) of the conditions promoting
z-less stratification.

The proposed framework builds on a previous spectral budget method developed
to explain the logarithmic scaling exhibited by $\sigma_u^2/u_*^2$ when $\zeta = 0$ (Banerjee and
Katul, 2013). A later study expanded the spectral budget method to describe $\sigma_u/u_*$
under unstable conditions (i.e., $\zeta < 0$) by the addition and appropriate modeling
of a buoyant production term in the spectral budget (Banerjee et al., 2014). It has
been found that $z$, $\delta$ and $L$ are all parameters required for describing the variation
of $\sigma_u/u_*$. Analytical results for $\sigma_u/u_*$ from the theory were also possible in two limit-
ing conditions identified as $z/\delta < 0.02$, $-\zeta < 0.5$ and $0.02 << z/\delta < 0.1$, $-\zeta > 0.5$
(Banerjee et al., 2014). While progress has been made towards understanding sev-
eral characteristics of $\sigma_u/u_*$ in the unstable boundary layer, there has been limited
progress for the stable atmospheric boundary layer (SABL), which is the main focus
here. This lack of progress is partly experimental because contamination from non-
turbulent phenomena such as gravity waves, drainage forces from change in terrain
slope, low level jets and meso-scale motions, and passage of clouds during nocturnal
conditions, which are known to impact $\sigma_u$, are ubiquitous in field experiments under
stable conditions (Pahlow et al., 2001; Cava et al., 2004). Thus, the SABL is char-
acterized by different levels of balance between mechanical production of turbulent
kinetic energy (TKE) and buoyant damping under different conditions ranging from
almost non-turbulent to well mixed conditions (Ohya et al., 1997). In low wavenum-
bers, the effects of gravity waves dominate close to the Brunt-Väisälä frequency (N)
and can influence the turbulent spectra and co-spectra of the velocity component
(Einaudi and Finnigan, 1981; Leyi and Panofsky, 1983; Finnigan et al., 1984; Hunt
et al., 1985; Rohr et al., 1988; Edwards and Mobbs, 1997; Mahrt et al., 2001; Cava
et al., 2004; Campos et al., 2009; Mahrt et al., 2012). Heat and pollutants can be
transferred vertically by waves and low-frequency turbulent motions and a vertical
diffusion length scale of $\sigma_w/N$ for heat and pollutant transfer has also been identified where $\sigma_w^2$ is the vertical velocity variance (Hunt et al., 1985). Weinstock (1981) reported a strong correlation between the turbulent kinetic energy dissipation rate and $\sigma_w$. The proposed spectral budget here will not consider such non-stationarities, the role of gravity waves (and hence $N$), and intermittent switching between turbulent and non-turbulent states. The turbulence in the SABL is assumed to be sufficiently developed so that a sufficiently large separation between the integral length scale and the viscous dissipation length scale (or the Kolmogorov microscale) always exists and Kolmogorov’s theory for the inertial subrange (Kolmogorov, 1941) approximates velocity and temperature spectra for all scales bounded between the integral and Kolmogorov scales. This is consistent with the observation by Huang and Bou-Zeid (2013) that this ‘continuously turbulent SABL’ is different from the ‘intermittently turbulent SABL’. The ‘continuously turbulent SABL’ is also described as ‘weakly stable’ and is the subject of the present work while the ‘intermittently turbulent SABL’ is often designated as ‘very stable’ boundary layer (Mahrt, 1998; Huang and Bou-Zeid, 2013). Approaches that analyzed similar problems where the spectrum of (vertical) velocity and temperature shift from waves at large scales (characterized by a -3 power-law) to inertial at smaller scales (characterized by a -5/3 power-law) are also not within the scope here and are reviewed elsewhere (Sukoriansky et al., 2006; Sukoriansky and Galperin, 2013). The proposed derivation also does not require or assume any link between the turbulent kinetic energy dissipation rate and $\sigma_w$.

4.2 Theory

4.2.1 Background and definitions

A coordinate system is defined with $x$, $y$, and $z$ forming the longitudinal, lateral, and vertical directions, respectively. Standard Reynolds decomposition notation is also employed so that the longitudinal $(u)$, lateral $(v)$, and vertical $(w)$ velocities
are decomposed into a stationary mean and fluctuating or turbulent quantities using
\[ u = U + u', \quad v = V + v', \quad \text{and} \quad w = W + w' \]
with \( V = 0 \) based on the choice of the coordinate system. Potential temperature and pressure are also decomposed into a mean \((T\text{ and }P)\) and turbulent fluctuations \((T'\text{ and }p')\), respectively. As earlier noted, it is assumed that the flow is characterized by high Reynolds and Peclet numbers (negligible molecular viscosity and diffusivity compared to their turbulent counterpart), stationary \((\partial(\cdot)/\partial t = 0)\), planar homogeneous in the mean \((\partial(\cdot)/\partial x = \partial(\cdot)/\partial y = 0)\) and without subsidence \((W = 0)\) or significant mean horizontal pressure gradient \((\partial P/\partial x = \partial P/\partial y = 0)\) as common to MOST. When these assumptions are incorporated into the equations for mean horizontal velocity and mean temperature, they result in \(du'/dz = 0\) and \(d\overline{u'T'}/dz = 0\), where \(\overline{u'u'}\) and \(\overline{u'T'}\) are the momentum and sensible heat fluxes, respectively. Beyond ignoring molecular diffusion and viscosity, the high Reynolds and Peclet numbers are assumed to be sufficiently large so as to further suppress formation of gravity waves or laminarization of the flow. A large spectral separation is also assumed between the integral length scale of the flow and the Kolmogorov microscale so that scales bounded by these two limits maintain an approximate locally homogeneous and isotropic state.

### 4.2.2 The Turbulent Kinetic Energy (TKE) budget

The TKE budget for such an idealized ASL can be written as

\[
\frac{\partial e}{\partial t} = 0 = -\overline{u'u'}\frac{dU}{dz} - \beta \overline{u'T'} - \frac{\partial}{\partial z} \left( \frac{1}{2} \overline{u'^2 + v'^2 + w'^2} \right) + \frac{1}{\rho} \overline{w'p'} - \overline{\epsilon}, \tag{4.1}
\]

where \(e = (1/2) (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)\) is the TKE, \(\sigma_u^2 = \overline{u'^2}\), \(\sigma_v^2 = \overline{v'^2}\), and \(\sigma_w^2 = \overline{w'^2}\), \(\beta = g/T\) is the buoyancy parameter, \(g\) is the gravitational acceleration, \(\rho\) is the mean air density, \(t\) denotes time and and \(\overline{\epsilon}\) is the mean TKE dissipation rate, \(T\) denotes potential temperature. The first, second, third terms on the right hand side (RHS) of Eq. 4.1 are the mechanical production and buoyant destruction of
TKE, and the transport of TKE by turbulence and pressure-velocity interactions, respectively. For several ASL applications, the production, buoyant and dissipation terms can exist in a near balance (Townsend, 1976; Pope, 2000; Wilson, 2008). However, some datasets demonstrated that the transport term cannot be neglected, especially in stable stratification and the imbalance in the TKE budget is suggested to vary with the stability parameter $\zeta = z/L$ (Salesky et al., 2013). A consequence of this imbalance on the $\sigma_u$ scaling will be explored separately in the appendix C but this imbalance is momentarily ignored. In case of a TKE balance, (i.e., negligible transport), MOST (Monin and Obukhov, 1954) yields \( \frac{dU}{dz} = \phi_m(\zeta) u_*/(\kappa z) \) and $\bar{\epsilon} = (\phi_m(\zeta) - (\zeta)) u_*^3/(\kappa z)$ where $L = -u_*^3/(\kappa \beta \overline{w'T'})$, and $\phi_m(\zeta)$ is the stability correction function for momentum as mentioned earlier. In case the transport terms are significant, $\bar{\epsilon}$ can be modified as $\bar{\epsilon} = (\beta_2(\zeta) + \phi_m(\zeta) - (\zeta)) u_*^3/(\kappa z)$ following Salesky et al. (2013), where $\beta_2(\zeta)$ is a function of $\zeta$ that may be determined from experimental datasets.

Other common quantities are now defined. The flux Richardson number ($R_f$) is given by the ratio of the buoyant destruction rate of TKE ($\beta \overline{w'T'}$) to the mechanical production rate of TKE ($-P_m$)

$$R_f = \frac{\beta \overline{w'T'}}{-P_m} \quad (4.2)$$

and the gradient Richardson number ($R_g$) is given as

$$R_g = \frac{\beta \Gamma}{S^2} \quad (4.3)$$

where $P_m = -S \overline{u'T'}$, $S = dU/dz = \phi_m u_*/(\kappa z)$ is the mean velocity gradient and $\Gamma = dT/dz = \phi_T T_*/(\kappa z)$ is the mean potential temperature gradient. $u_*$ is, as before, the friction velocity defined as $-u_*^2 = \overline{u'u'}$ and $T_*$ is defined as $T_* = u_*^2/(\kappa \beta L)$. 67
Using these definitions, the flux Richardson number can be related to $\zeta$ using

$$R_f = \frac{\beta \bar{w}' T'}{\bar{u}' w'} = \frac{\beta \bar{w}' T'}{-u_*^3 \phi_m(\zeta)/(Kz)} = \frac{\zeta}{\phi_m(\zeta)}. \quad (4.4)$$

Unless otherwise mentioned, $\phi_m(\zeta)$ can be described by the widely used empirical Businger-Dyer (Businger and Yaglom, 1971; Dyer, 1974) form: $\phi_m(\zeta) = (1 + 4.7\zeta)$ for $0 < \zeta < 2$, which is the range of stability conditions explored here. This is commensurate with the critical flux Richardson number ($R_f = 0.2$, same for the Gradient Richardson number ($R_g$) as well) for the applicability of local similarity theory since when $\zeta = z/L = 2$, $R_f = \zeta/\phi_m = 0.2$ (Grachev et al., 2013). The concomitant temperature stability correction function ($\phi_T$) is also assumed to be given by its MOST representation as $\phi_T = 0.74 + 4.7\zeta$. To a leading order, $\phi_m(\zeta) \approx \phi_T(\zeta)$ though for $R_g > 0.1$, this approximation no longer holds (Zilitinkevich et al., 2013; Katul et al., 2014). With this background, the spectral budget for TKE and its connection to $\sigma_u$ are presented.

4.3 A spectral budget formulation for TKE

At an arbitrary wavenumber $k$, the spectral budget can be expressed as (Hinze, 1959; Panchev, 1971; Banerjee et al., 2014)

$$\bar{\epsilon} = \frac{dU}{dz} \int_{k_a}^{\infty} F_{wu}(s) ds - \frac{g}{T} \int_{k_a}^{\infty} F_{wT}(s) ds + F(k) + 2\nu \int_0^{k_a} s^2 E_{tke}(s) ds, \quad (4.5)$$

where the first, second, third and fourth terms on the RHS represent the mechanical production and buoyant destruction of TKE in the range of $[k_a, \infty]$, the transfer of TKE in the range $[k_a, \infty]$, and the viscous dissipation in the range of $[0, k_a]$ respectively. Using the Heisenberg model, for all eddies between the wavenumbers 0 and $k_a$, the action of smaller eddies can be represented by an additional viscosity,
since the way in which these smaller eddies transfer momentum is similar to the action of ordinary friction. This additional viscosity must depend upon the intensity of the small eddies, i.e. on the part of the spectrum with large wavenumbers by means of a certain integral that was formulated based on dimensional arguments by Heisenberg (1948). Using this argument, the transfer term can be ‘closed’ as

$$ F(k) = \nu_t(k) |\text{curl } \bar{u}|^2 \approx 2\nu_t(k) \int_0^{k_a} s^2 E_{tke}(s) ds, \quad (4.6) $$

where $\nu_t(k)$ is the wavenumber dependent Heisenberg viscosity. With this simplification, the spectral budget Eq. 4.5 becomes (Banerjee et al., 2014)

$$ \bar{\epsilon} = \frac{dU}{dz} \int_{k_a}^{\infty} F_{wu}(s) ds + 2(\nu_t(k) + \nu) \int_0^{k_a} s^2 E_{tke}(s) ds - \frac{g}{T} \int_{k_a}^{\infty} F_{wT}(s) ds. \quad (4.7) $$

### 4.3.1 Modeling the mechanical production term

To link the spectral budget to MOST scaling, it is assumed that $k_a = 1/(\gamma(\zeta) z)$ within the ASL, where $\gamma(\zeta)$ varies with $\zeta$ and is specified later. $\gamma z$ represents a stability dependent break-point between the large scales and the inertial scales, which may be related to the integral length scale of the flow but is not necessarily identical to it. The reduction of $\gamma$ with stability from a neutral limit magnitude of unity implies that the extent of the inertial subrange reduces with increasing $\zeta$.

For a wavenumber $k$ that satisfies $k \gamma(\zeta) z > 1$, the traditional inertial subrange scaling applies (Kolmogorov, 1941) so that

$$ E_{tke}(k) = C_o \bar{\epsilon}^{2/3} k^{-5/3}, \quad (4.8) $$

where $C_o = (33/55)C_K$ is the Kolmogorov constant when $k$ is interpreted as a one-dimensional cut, and $C_K = 1.55$ is the Kolmogorov constant associated with three-dimensional wavenumbers. The $33/55$ originates from a 1-d interpretation (along
x) of the TKE spectrum that is locally isotropic given by 

\[ E_{tke}(k) = (1/2)(E_u(k) + E_v(k) + E_w(k)), \]

with \(33/55 = (18/55 + 24/55 + 24/55)/2 = (66/2)/55\) since the Kolmogorov spectral constants for the individual one-dimensional velocity spectra are \((18/55)C_K, (24/55)C_K\) and \((24/55)C_K\) respectively (Banerjee and Katul, 2013). Using the co-spectral budget formulation provided by Katul et al. (2014), the following form can be derived for \(k > k_a\)

\[ F_{wu}(k) = C_{uw} \bar{e}^{1/3} S k^{-7/3}, \]  

which is in agreement with the accepted ‘-7/3’ scaling for the co-spectra of momentum flux the inertial subrange (Lumley, 1967), where \(C_{uw} = (2C_o)/(5A_u)\), and \(A_u \approx 1.8\) is the Rotta constant. Hence, the production term in the spectral budget Eq. 4.7 can be simplified by substituting \(\bar{e}\) in 4.9 with \(\bar{e} = (\beta_2(\zeta) + \phi_m(\zeta) - \zeta) u^3_s/(\kappa z)\), as well as setting \(\beta_2(\zeta) = 0\) as:

\[
\frac{dU}{dz} \int_{k_a}^{\infty} F_{wu}(s) ds = \left( \frac{\phi_m u}{\kappa z} \right) \left( \frac{2 C_o \left( \phi_m - \zeta \right)^{1/3}}{5 A_u} \right) \left( \frac{\phi_m u}{\kappa z} \right) \left( \frac{3}{4} k_a^{-4/3} \right) = \frac{u^3 2 3 C_o \phi_m^2 \left( \phi_m - \zeta \right)^{1/3}}{\kappa^{7/3}} \gamma^{4/3} \quad (4.10)
\]

### 4.3.2 Modeling the buoyant destruction term

Using the co-spectral budget formulation for \(F_{wT}(k)\) provided in Katul et al. (2014), the following form can be derived for \(k > k_a\)

\[ F_{wT}(k) = C_{wT} \bar{e}^{1/3} \Gamma k^{-7/3}, \]  

where \(C_{wT} = (2C_o Q)/5A_T\), \(A_T \approx 1.8\) is the Rotta constant for heat assumed to be the same as its momentum counterpart (=\(A_u\)) here, and \(Q(\zeta)\) varies with \(\zeta\) as shown
later. It is to be noted that $C_{wT} \approx C_{uw} Q(\zeta)$. Hence, the buoyant destruction term in the spectral budget Eq. 4.7 can be simplified as

$$
\frac{g}{T} \int_{k_a}^{\infty} F_{wT}(s) ds = \left( \frac{g}{T} \right) \left( \frac{2 C_o (\phi_m - \zeta)^{1/3} u_*}{(\kappa z)^{1/3}} \right) \left( \frac{u_*^2}{\kappa^6 L} \right) \left( \frac{\phi_T}{\kappa z} \right) Q \left( \frac{3}{4} k_a^{-4/3} \right)
$$

$$
= \frac{u_*^3}{z} \frac{2 C_o \phi_T \zeta Q (\phi_m - \zeta)^{1/3}}{(1/4) A_T \kappa^{7/3}} \gamma^{4/3} \tag{4.12}
$$

The term $Q$ is a dimensionless quantity varying with $\zeta$ defined as

$$
Q = 1 + \frac{\beta C_T N_T}{(1 - C_{1T}) C_o \Gamma} \tag{4.13}
$$

where $C_T \approx 0.8$ is the Kolmogorov-Obukov-Corrsin constant for the temperature spectrum in the inertial subrange (along $x$), $C_{1T} = 3/5$ is a constant associated with the isotropization of the production term in the co-spectral budget of temperature flux and can be predicted from Rapid Distortion Theory (RDT) for isotropic turbulence, and $N_T$ is the thermal variance dissipation rate defined as

$$
N_T = \frac{u'T}{\beta} \Gamma P_m = -\frac{1}{\beta} R_T \Gamma P_m = -\frac{\zeta \Gamma S u_*^2}{\phi_m \beta} \tag{4.14}
$$

Hence $Q$ can be simplified as

$$
Q = 1 - \frac{\beta C_T}{(1 - C_{1T}) C_o \Gamma} \frac{\zeta \Gamma u_*^2 \phi_m}{(\phi_m - \zeta) \phi_m \beta \kappa z} = 1 - \frac{C_T \zeta}{(1 - C_{1T}) C_o (\phi_m - \zeta)}. \tag{4.15}
$$

It is interesting to note that $Q(\zeta) \approx C_{wT}/C_{uw} < 1$ is consistent with the findings from the Kansas experiment (Kaimal and Finnigan, 1994).

4.3.3 Spectral budget at $k_a = 1/(\gamma z)$

The spectral budget equation can now be used to find the spectral form of $E_{TKE}(k)$ (which is later linked to $\sigma_u^2$) in the range $0 - k_a$ using the modeled forms of the
mechanical production and buoyant destruction terms. It is necessary to recast the spectral budget equation (Eq. 4.7) in the following form

\[
\bar{\epsilon} = \frac{d\bar{U}}{dz} \int_{k_a}^{\infty} F_{wu}(s) ds + \frac{g}{k_a} \int_{k_a}^{\infty} F_{wT}(s) ds = 2(\nu_t(k)) \int_{0}^{k_a} s^2 E_{tke}(s) ds
\]  

(4.16)

where it is assumed that \( \nu_t \gg \nu \) i.e., the turbulent viscosity is much higher than the molecular viscosity. Using Eq. 4.10 and Eq. 4.12, Eq. 4.16 is rewritten as

\[
2(\nu_t(k)) \int_{0}^{k_a} s^2 E_{tke}(s) ds = \frac{u_m^3}{z} \left[ \frac{\phi_m - \zeta}{\kappa} - \frac{2}{5} \frac{C_o \phi_m^2 (\phi_m - \zeta)^{1/3}}{A_u} \frac{\gamma^{4/3}}{\kappa^{7/3}} \right] + \frac{2}{5} \frac{C_o \phi_T \zeta Q (\phi_m - \zeta)^{1/3}}{A_T} \frac{\gamma^{4/3}}{\kappa^{7/3}} \]  

(4.17)

Assuming \( E_{tke}(k) \) is inertial for \( k \) bounded by \([k_a, \infty]\) given by Kolmogorov’s theory (i.e. ignoring exponential cutoff due viscous effects at high \( k \)), \( \nu_t \) can be determined as (Banerjee et al., 2014)

\[
\nu_t(k_a) = \frac{3C_H C_o^{1/2} \bar{\epsilon}^{1/3}}{4 k_a^{4/3}},
\]  

(4.18)

where \( C_H \) is the Heisenberg constant defined as \( C_H = (8/9)C_o^{(-3/2)} \). Substituting the definition of \( \nu_t(k) \) from Eq. 4.18, Eq. 4.17 can be written as

\[
\int_{0}^{k_a} s^2 E_{tke}(s) ds = \frac{u_m^2}{z^2} \frac{2}{3} \frac{C_H C_o^{1/2}}{C_o^{1/2}} \left[ \frac{(\phi_m - \zeta)^{2/3}}{\kappa^{2/3} \gamma^{4/3}} \right] - \frac{2}{5} \frac{2 C_o \phi_m^2}{A_u} \frac{\gamma^{4/3}}{\kappa^{7/3}} + \frac{2}{5} \frac{C_o \phi_T \zeta Q}{A_T} \frac{\gamma^{4/3}}{\kappa^{7/3}},
\]  

(4.19)

which is consistent with Banerjee et al. (2014)

\[
\int_{0}^{k_a} s^2 E_{tke}(s) ds = \frac{u_m^2}{z^2} C_{st},
\]  

(4.20)

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where $C_{st}$ here is defined as

$$C_{st}(\zeta) = \frac{2}{3 C_H C_o^{1/2}} \left[ \frac{(\phi_m - \zeta)^{2/3}}{\kappa^{2/3} \gamma^{4/3}} - \frac{23 C_o \phi_m^2}{54 A_u \kappa^2} + \frac{23 C_o \phi_T \zeta Q}{54 A_T \kappa^2} \right].$$

(4.21)

It is to be noted that in the neutral limit, $\zeta = 0$, $\phi_m = 1$, $\phi_T \approx 1$ and $\gamma = 1$, thus recovering $C_{st}$ to be

$$C_{st-neutral} = \frac{2}{3 C_H C_o^{1/2}} \left[ \frac{1}{\kappa^{2/3}} - \frac{23 C_o}{54 A_u \kappa^2} \right].$$

(4.22)

With appropriate values of the constants ($C_H \approx 1$, $C_o = (33/55)1.55$, $\kappa = 0.4$, $A_u = 1.8$), yields a $C_{st-neutral} \approx 0.56$.

The function $\gamma(\zeta)$ is now discussed. As earlier noted, the integral length scale decreases with increasing $\zeta$ (Kaimal et al., 1972; Kaimal, 1973; Caughey, 1977; Högström, 1990; Launiainen, 1995; Canuto et al., 2008). While $\gamma z$ is not identical to the integral length scale as earlier noted, its dependence on $\zeta$ is assumed to follow a similar pattern. For this purpose, $\gamma(\zeta) \approx (1 + a_L \zeta)^{-1}$ so as to ensure a $\gamma(0) = 1$, where $a_L$ is a decay coefficient. Possible decay rates of $\gamma$ with increasing $\zeta$ are discussed and are shown in Figure 4.1.

The variations of the integral length scale of the longitudinal velocity normalized by $z$ are shown as black circles on Figure 4.1, which were computed by Salesky et al. (2013) using the Advection Horizontal Array Turbulence Study (AHATS) data collected in the ASL. A good fit to this data is $a_L = 3$. The dash-dot line refers to the integral length scale of the vertical velocity whose functional form has been computed from the Kansas experiment as $(0.55 + \zeta)^{-1}$ (Kaimal and Finnigan, 1994). It is to be noted that its neutral limit is greater than 1, which indicates a larger length scale at lower wavenumbers than $z$. However, when normalized by this neutral limit, it can also be used as a guide to how the break-point shifts with increasing stability. In addition, because the integral length scale of the vertical velocity is closely aligned
with this breakpoint, it may be better suited for approximating $\gamma z$ than the integral length scale of the longitudinal velocity. As evident from Figure 4.1, its variation is quite similar to the variation of the integral length scale of the longitudinal velocity. Interestingly, $\gamma(\zeta) \approx 1/\phi_m(\zeta)$ is also shown for reference and appears to be also similar to the one inferred from the integral length scale of the vertical velocity. For this reason, this aforementioned form is used for analytical tractability. Notwithstanding which precise approximation to use, it appears that a value of $a_L$ in the range 3 to 4.7 is reasonable.

### 4.3.4 Formulation for $C_{st}:$ Numerical fitting

To proceed with an analytical derivation, the integral expressed in Eq. 4.20 must be simplified. Figure 4.2 shows the variation of the function $C_{st}$ with $\zeta$. As observed, $C_{st}$ suggests the possible existence of a two-regime formulation somewhat similar to the result of Banerjee et al. (2014). In zone I, $C_{st}$ is roughly flat until $\zeta = 0.1$, assuming the neutral limit of $C_{st-neutral} = 0.56$ which is designated as $c = 0.56$. In zone II, $C_{st}$ can be described by an approximate power law of the form $C_{st} = a \zeta^b$, where $a$ and $b$ can be obtained by a numerical curve fitting as $a \approx 2.966$ and $b \approx 0.66$ with a coefficient of determination of 0.99. Although a smooth transition between the two regimes (zone I and zone II) can be possibly formulated by an interpolation scheme, it is assumed that $\zeta \approx 0.1$ is the location of this transition for all practical purposes. With this two-part formulation for $C_{st}$, the shape of the $E_{tke}$ spectrum can now be inferred for each regime in the limit $0 - k_a$.

### 4.3.5 Formulation for zone I

Since the form of $E_{tke}$ in the limit $0 - k_a$ is unknown, it may be assumed to be a power law, i.e.,

$$E_{tke}(k)_{0-k_a} = rk^l.$$  \hfill (4.23)
Substituting this power law expression in Eq. 4.20, the following identity can be derived

\[
\left( \frac{r}{3 + l} \gamma^{3-l} \right) z^{3-l} = (u_*^2 c) z^{-2}
\]  

(4.24)

Equating terms in the left hand side (LHS) and RHS of Eq. 4.24, one can obtain

\[
l = -1; r = 2u_*^2 c \gamma^2
\]  

(4.25)

while the parameter \( c = C_{st-neutral} = 0.56 \) is known a-priori as discussed earlier.

Thus to summarize, for zone I,

\[
E_{tke}(k) = \begin{cases} 
C_{tke}' u_*^2 k^{-1}, & \text{if } k \gamma z \leq 1 \\
C_0 \varepsilon^{2/3} k^{-5/3}, & \text{otherwise.}
\end{cases}
\]

(4.26)

where \( C_{tke}' = 2c \gamma^2 \). Thus the \( E_{tke} \) spectrum recovers the much discussed \( k^{-1} \) scaling as reviewed elsewhere (Banerjee et al., 2014). Furthermore, it can also be assumed that the \( k^{-1} \) scaling is valid until \( kH = 1 \) where \( H \) is a measure of the size of largest turbulent eddies in the SABL and the spectrum is flat at lower wavenumbers, i.e.,

\[
E_{tke} = C_{tke}' u_*^2 (1/H)^{-1} \text{ form continuity requirement at } kH = 1.
\]

4.3.6 Formulation for zone II

Following the same approach presented in section 4.3.5, \( E_{tke} \) is assumed to be a power law, i.e.,

\[
E_{tke}(k)_{0-k_a} = pk^q.
\]

(4.27)

Substituting this general power law expression in Eq. 4.20, the following identity can be derived

\[
\left( \frac{p}{3 + q} \gamma^{3-q} \right) z^{3-q} = \left( \frac{u_*^2 a}{L^b} \right) z^{b-2}
\]

(4.28)
Equating terms in the LHS and RHS of Eq. 4.28, one can obtain

\[ q = -1 - b; \quad p = \frac{u_*^2 a (2 - b)}{L^b} \gamma^{2-b}; \quad (4.29) \]

where \( a \) and \( b \) are known a priori from \( C_{st} \). It is interesting to note that \( q = -1 - b \approx -1.67 \approx -5/3 \), which indicates that the \( E_{tke} \) spectrum follows a scaling law analogous to its inertial value at higher \( \zeta \). Because of the similarity in the spectral exponents and to ensure spectral continuity, we simply extend the inertial subrange spectrum up to some scale \( k \gamma_1(\zeta) z \), where \( \gamma_1 \) is discussed later. However, it must be emphasized that the main mechanisms leading to the approximate -5/3 scaling here differ from those in the inertial subrange. However, for maintaining maximum simplicity here, the \( E_{tke} \) spectrum can be described as

\[ E_{tke}(k) = \begin{cases} 0, & \text{if } k \gamma_1 z \leq 1 \\ C_o \epsilon^{2/3} k^{-5/3}, & \text{otherwise.} \end{cases} \quad (4.30) \]

Now the determination of \( \gamma_1 \) is discussed. It is known that the turbulent (but not necessarily the total) kinetic energy in \( \sigma_u \) is quite suppressed and eddies are appreciably deformed by stratification at eddy sizes much larger than the Ozmidov length scale \( L_0 = (\bar{v}/N^3)^{1/2} \), where \( N = [(g/T)(dT/dz)]^{1/2} \). At these large scales, non-turbulent phenomena such as gravity waves become energetically prevalent but the turbulent contribution to the energy is small (or negligible). For the idealized ASL considered here where \( \zeta > 0.1 \), \( L_0/(\kappa z) \sim \left[ (\phi_m(\zeta) - \zeta) \gamma^{1/2}(\zeta \phi_T(\zeta))^{-3/4} \right] \). To a first order, the \( L_0/(\kappa z) \) declines at a rate commensurate with \( [\phi_m(\zeta)]^{-1} \) when setting \( \phi_T(\zeta) \sim \phi_m(\zeta) \sim \zeta \) and \( (\phi_m(\zeta) - \zeta) \sim \zeta \). An assumption of maximum simplicity again is that the turbulent component of the kinetic energy spectrum lacks significant energy at \( k \gamma_1(\zeta) z \leq 1 \) with \( \gamma_1(\zeta) \) declining with \( \zeta \) at a rate commensurate with \( \gamma(\zeta) \). However, instead of determining \( \gamma_1 \) from \( L_0 \), it will be determined so as to ensure continuity between zones I and II when deriving \( \sigma_u^2 \), as will be discussed later.
Another interesting observation is that the spectrum exhibits an apparent inertial signature in its exponent (=−5/3) at higher stability. Given that the spectrum is also found to follow −5/3 scaling at highly unstable condition (Banerjee et al., 2014), it can now be stated that the signature of buoyancy is an apparent ‘inertialization’ or transition to an approximate −5/3 scaling from a combination of −5/3 and −1 scalings dominating the $E_{tke}$ spectrum for near-neutral conditions.

4.3.7 Linking the longitudinal velocity variance to TKE

For neutral and unstable atmospheric surface layer, it was argued elsewhere (Basu et al., 2010; Banerjee and Katul, 2013; Banerjee et al., 2014) that $\sigma_u^2 \approx \sigma_v^2 + \sigma_w^2$ and thus $e = (1/2)(\sigma_u^2 + \sigma_v^2 + \sigma_w^2) \approx (1/2)(\sigma_u^2 + \sigma_v^2) \approx \sigma_u^2$. It is assumed that this condition is also valid for stable conditions, especially at high Reynolds and Peclet numbers when turbulence is well separated from gravity waves. To test this assumption, experimental data (described in detail in section 4.4) collected over two distinct vegetation canopies at the Duke forest over four years are used. Figure 4.3 demonstrates a one to one comparison between $\sigma_u^2$ and $e$ for two different seasons (i.e., summer and winter) and for two different vegetation canopies, a second growth oak-hickory hardwood (HW) forest and a Loblolly pine plantation. In all cases it can be assumed that $\sigma_u^2 \approx e$ irrespective of surface cover and stability parameters (the slopes are 0.80, 0.84, 0.70 and 0.76 for winter pine, summer pine, winter HW and summer HW, respectively). Thus the TKE spectrum can be integrated to obtain $\sigma_u^2$ for the purposes here.
4.3.8 The longitudinal velocity variance

Integrating the $E_{tke}(k)$ spectra from the formulation for zone I (Eq. 4.26) to obtain the longitudinal velocity variance results in

$$\sigma_u^2 = \int_0^{1/H} C_{TK} u_s^2 (1/H)^{-1} ds + \int_{1/H}^{k_a} C_{TK} u_s^2 s^{-1} ds + \int_{k_a}^{\infty} C_{o} \epsilon^{3/2} s^{-5/3} ds. \quad (4.31)$$

Assuming $H = \gamma \delta$ and substituting the values from Eq. 4.26, Eq. 4.31 becomes

$$\frac{\sigma_u^2}{u_s^2} = 2c\gamma^2 - 2c\gamma^2 \log(\frac{\tilde{z}}{\delta}) + \frac{3}{2} \frac{C_o}{\kappa^{2/3}} (\phi_m - \zeta)^{2/3} \gamma^{2/3}, \quad (4.32)$$

which can also be written in a form consistent with Townsend’s attached eddy hypothesis as

$$\frac{\sigma_u^2}{u_s^2} = B_1 - A_1 \log(\frac{\tilde{z}}{\delta}), \quad (4.33)$$

where

$$B_1 = 3 \frac{C_o}{2 \kappa^{2/3}} (\phi_m - \zeta)^{2/3} \gamma^{2/3} + 2c\gamma^2 \quad (4.34)$$

and

$$A_1 = 2c\gamma^2 \quad (4.35)$$

where $\gamma = \gamma(\zeta)$ and thus the Townsend parameters now vary (mildly) with stability for near neutral to slightly stable conditions.

If the second formulation for zone II is used, the $E_{tke}$ can be integrated to derive the longitudinal velocity variance as

$$\frac{\sigma_u^2}{u_s^2} = \frac{3}{2} \frac{C_o}{\kappa^{2/3}} (\phi_m - \zeta)^{2/3} \gamma^{2/3}. \quad (4.36)$$

It is also interesting to note that the boundary layer height is not present in the formulation for zone II, which indicates that at higher stability $\sigma_u$ is no longer
sensitive to the boundary layer height (as expected). The parameter $\gamma_1$ can be found out by imposing continuity condition on $\sigma_u^2 / u^2_w$ at $\zeta = 0.1$ (i.e., the transition between the two formulations). Equating Eq. 4.32 and Eq. 4.36 at $\zeta = 0.1$, $\gamma_1$ can be calculated to have a near neutral limit $\gamma_1 \approx 1$ and can be assumed to reduce with stability somewhat similar to $\gamma(\zeta)$. Again for simplicity, we choose $\gamma_1(\zeta) = 1 / \phi_m(\zeta)$ based on the scaling argument that the Ozmidov length scale $L_o$ decreases with increasing stability as $1 / \phi_m(\zeta)$. Unfortunately, this continuity constraint results in weak scale separation between all the key length scales here - $L_0$, $z$, $L$, and $\delta$ (at least when compared to zone I). Given the number of assumptions used to separate the two formulations and the associated uncertainties, predictions based on the formulation for zone I and zone II are separately compared to data for all $\zeta$ and biases are then analyzed as will be shown in section 4.5 and the appendix D. It is also to be noted that both formulations predict the correct neutral limit of $\sigma_u / u^*_w$ (between 2 and 3).

4.4 Datasets

To evaluate the model, four datasets are used. One dataset is a published laboratory experiment (Ohya et al., 1997), one is an experiment over a large lake, and two datasets are from long-term monitoring initiatives above a pine and a hardwood forest. The forest experiments are separated into summer and winter seasons to reflect large and small leaf area index (LAI) values.

The laboratory experiment was conducted by Ohya et al. (1997) in a wind tunnel where stably stratified flows were generated by cooling the aluminum floor to $3^\circ C$ and maintaining the ambient air at $50^\circ C$. The experimental runs covered a range of $u^*_w$ from 0.013 to 0.13 m s$^{-1}$ varying over an order of magnitude. The boundary layer height was measured and a range $0.1 < z / \delta < 0.5$ was reported in their results. Further details on the experiments and datasets can be found in Ohya et al. (1997).

The lake data include 20Hz eddy covariance measurements of three-dimensional
velocity and air temperature at four different heights (1.65, 2.30, 2.95, and 3.60 m) above an extensive lake surface. Details about the lake data and quality control measures can be found elsewhere (Vercauteren et al., 2008; Huwald et al., 2009; Li and Bou-Zeid, 2011; Li et al., 2012b). In particular, the averaging interval is 30 minutes and data with fluxes measured at the four heights differing by more than 10% are excluded so as to maintain the constant flux assumption required by the derivation here and MOST scaling for mean velocity and temperature (Li et al., 2015b). The stability range covered in the lake measurements are mildly stable conditions ($\zeta = 0.001 - 0.25$). As a result, although the boundary layer height is not measured in the experiments, they are expected to be not very low (on the order of hundreds of meters). Figure 4.4 shows the variations of $u_*$ with $\zeta$ in the lake data. It is clear that the $u_*$ decreases with increasing stability, which is expected. Interestingly, the decrease of $u_*$ is captured by $\gamma(\zeta)$ with $a_L = 3$ (the black line) and $a_L = 4.7$ (the red line), as shown in Figure 4.4. Note that $u_{*,\text{neu}}$ indicates the neutral $u_* = 0.21 \text{ m s}^{-1}$.

The forest experiments are conducted over two different but adjacent vegetation canopies: a Loblolly Pine plantation and a 100 year-old second growth oak hickory Hardwood forest. The data include 10 Hz eddy covariance measurements of three dimensional velocity and air temperature and the details of the experiments can be found elsewhere (Juang et al., 2008; Katul et al., 2012). The data span over four years and are divided into summer and winter season. Since the experiments were conducted over the canopy, the $\zeta = z/L$ is modified as $\zeta = (z - d)/L$, where $d$ is the zero-plane displacement height. This $d$ is set to be zero in winter for the hardwood canopy and set to be $(2/3)h_c$ otherwise for both forests, where $h_c$ is the canopy height ($h_c = 33\text{ m}$ for the Hardwood forest and $h_c = 15\text{ m}$ for the Pine forest). The height of measurement ($z$) is 39.5m for the Hardwood forest and 15.5m for the Pine forest. The Pine forest had a LAI varying from 2.65-4.56 $m^2 m^{-2}$ and the Hardwood
forest had a maximum LAI about 6 $m^2 m^{-2}$. The stability covered by this dataset ranges from mildly stable to highly stable conditions ($\zeta = 0.0001 - 10$). $u_*$ also varies with stability with modal values of 0.06 m s$^{-1}$ in the summer and 0.35 m s$^{-1}$ in the winter for both Hardwood and Pine canopies. The two forests were also collocated so they were subjected to similar meteorological conditions. There are 11449 and 14977 30-minute runs for the pine data in the summer and winter, respectively. For the Hardwood, there are 13049 and 15045 30-minute runs in the summer and winter, respectively. To assess the applicability of MOST in the canopy sub layer (CSL), which is roughly 2-3 times of canopy height, some of the stored high frequency data above the pine forest is used. The second order structure function $D_{uu}$ for $u$ is calculated independently, which is regressed with $Cv\tau^{2/3} r^{2/3}$ where $Cv \approx 2$ (Stull, 1988) and $r$ is the lag in meters in the longitudinal direction (computed by multiplying the sampling time interval with the mean velocity using Taylor’s frozen turbulence hypothesis). Within the inertial subrange, $D_{uu} = Cv\tau^{2/3} r^{2/3}$. It is found that the range where this is true is approximately between 0.3m and 3m. From this regression, the dissipation rate $\tau$ is computed. The mechanical $(-\overline{uw'U'/dz} = u_*^3/(\kappa z)\phi_m)$ and buoyant production terms ($(g/T)\overline{w'T'}$) in the TKE budget are then calculated using the dataset assuming MOST is valid in the CSL. If the independently computed production and dissipation are in balance, it suggests that MOST may be used in the canopy sublayer and that the TKE budget is, to reasonable approximation, represented by a balance between production and dissipation terms. Figure 4.5 shows a one to one comparison between production and dissipation terms for the high frequency Pine data collected over twelve different days for near neutral and more stable conditions shown in black and red symbols, respectively. Overall, the terms are reasonably balanced suggesting that MOST may be applied to relevant mean flow quantities in the CSL.

The boundary layer height was not measured in these experiments, either. How-
ever, since the dataset covers highly stable conditions, $\delta$ can be expected to drop significantly. On the other hand, $\delta$ was assumed not to appreciably exceed the average neutral ABL height of about 1000 m. This limit was determined from separate measurements in neutral conditions by Salesky et al. (2013). To have a realistic measure of $\delta$ at stable conditions, Zilitinkevich’s model (Zilitinkevich, 1972) is invoked. It is estimated that $\delta \approx \lambda \sqrt{(R/\mu)}$ where $R \approx 10$ is a constant (Zilitinkevich, 1972), $\lambda = \kappa u_*/f$, $\mu = \lambda / L$ and $f = 10^{-4}$ rad s$^{-1}$ is the Coriolis parameter. Using this approximation, it is estimated that $\delta \approx 50$ m under highly stable conditions. As a more improved estimation over the model of Zilitinkevich (1972), another model by Arya (1981) is used and is given by $\delta = au_*/f$ where $a \approx 0.3$. However, this modification is not found to impact the results significantly. Thus these four datasets provide a wide range of conditions in terms of $u_*$, stability and roughness (smooth: wind tunnel and lake experiments; very rough: forest experiments) to assess the proposed model.

4.5 Results and Comparisons

4.5.1 Evaluation of the model for $\sigma_u$

Although two regimes have been identified for the formulation of $\sigma_u$, we first answer the following question: to what extent can the formulation for zone I explain the experiments? The performance of the formulation for zone II will be explored later. Recall that the formulation for zone I recovers Townsend’s form and explains how the Townsend parameters evolve with $\zeta$, which is why exploring this formulation for the entire dataset is of interest. Figure 4.6 shows the comparison with the wind tunnel experiment conducted by Ohya et al. (1997), where $\sigma_u$ is plotted against $Ri_g$. It is to be noted that the gradient Richardson number $Ri_g$ can be rewritten in terms of the stability parameter $\zeta = z/L$ as $Ri_g = \zeta (\phi_T(\zeta)/\phi_m(\zeta))^2$ (Kaimal and Finnigan, 1994). Three different $z/\delta$ ratios, 0.1, 0.25 and 0.5, are used in order to cover the
data and the comparisons are satisfactory. To cover the range of $u_*$ (0.013-0.13) reported in Ohya et al. (1997), $u_*$ is made dynamic by multiplying $u_{*,neu}$ by $\gamma(\zeta)$ guided by Figure 4.4.

For the lake data, the range of $\zeta$ covered by the lake data spans from 0.001 to 0.025, as shown in Figure 4.4, and hence the boundary layer height is estimated not to be very small under such mildly-stable conditions (recall that the boundary layer height is not measured). Figure 4.7 shows the comparison between modeled and measured $\sigma_u$ for the lake data when $\delta$ is calculated based on the model by Arya (1981) as discussed in section 4.4. As can be seen, the calculated boundary layer height $\delta$ increases with increasing $z/L$ from about 1000 m (under near-neutral conditions) to about 100 m under mildly-stable conditions. Interestingly, the calculated boundary layer height from Arya’s model over the lake does not change significantly when $z/L > 0.1$. However, it is noted that using a variable $\delta$ is not appreciably better when compared to calculations with a constant $\delta = 500$ m and $\delta = 1000$ m.

Figure 4.8 shows the comparison with field data collected over the two forests. In Figure 4.8, all panels show the comparison between measured and modeled (our model and MOST) $\sigma_u$ in terms of their variations with $\zeta$. Ensemble averages of measured and modeled $\sigma_u$ are shown and their variability are represented by black and red error-bars, respectively. As expected, $\sigma_u$ (in dimensional form) is found to reduce with stability, since the effects of mechanical turbulence is diminished by the effects of buoyancy. Panels (a) (Summer, Hardwood), (d) (Winter, Hardwood), (g) (Summer, Pine) and (j) (Winter, Pine) show the aforementioned comparisons with a preset boundary layer height $\delta = 50$ m since the data were mostly collected at night time when the boundary layer height is reduced to great extent. For $\delta = 50$ m, the comparisons are found to be satisfactory given that the ensemble averages almost follow each other. There is some unavoidable scatter in the data when they are observed at the low stability regime. On the other hand, at the stability regime
beyond $\zeta = 2$, the modeled $\sigma_u$ drops below the measured level. This indicates that there might be some residual turbulence in the data not captured by the model when zone I formulation alone is used. However, instrument signal-to-noise ratio is also lower under very stable conditions and cannot be discarded. Panels (b), (e), (h) and (k) in the middle column show these comparisons with a $\delta = 1000$ m, which is about the height of the ABL under mildly stable to near-neutral conditions (Stull, 1988). With $\delta = 1000$ m, the model overestimates $\sigma_u$ at low stabilities. Panels (c),(f),(i) and (l) show the comparisons between measured and modeled $\sigma_u$ from MOST, which is given as $\sigma_u = 2.5u_*$ (Stull, 1988). As shown, MOST predictions often overestimate the data. Figure 4.9 demonstrates this feature better in which all panels show one to one comparisons between the modeled (x-axis) (our model and MOST) and measured (y-axis) $\sigma_u$. As described before, the comparisons with $\delta = 50$ m (Panels a to d) are satisfactory but our model overestimates the data with a $\delta = 1000$ m (Panels e to f). MOST predictions, as before, overestimate the data. One important point to note for Figures 4.7 and 4.8 is that the measured friction velocity $u_*$ for each data point is used to drive the model since $u_*$ varies appreciably.

Since the forest dataset spans over such a wide range of stability, the comparisons with the formulation for zone II are also shown in Figure 4.10. Panels (a) (Summer, Hardwood), (c) (Winter, Hardwood), (e) (Summer, Pine) and (g) (Winter, Pine) show one to one comparisons between the modeled and measured $\sigma_u$. Panels (b) (Summer, Hardwood), (d) (Winter, Hardwood), (f) (Summer, Pine) and (h) (Winter, Pine) show variations of $\sigma_u$ with $\zeta$. Both one to one comparisons and variations with $\zeta$ show that $\sigma_u$ is also reasonably captured by the formulation for zone II. However, it is again stressed that the boundary layer height is not present in the formulation for zone II, which is different from the formulation for zone I. The errors and biases in the predictions using both formulations for the complete range of stability is discussed in the appendix D. The main finding is that both model formulations are
not particularly biased when comparisons are conducted separately for $\zeta < 0.1$ and $\zeta > 0.1$, presumably due to the fact that $\gamma_1$ value is derived by matching the two formulations at $\zeta = 0.1$. Hence, in the zone-II formulation, $C_{st} \approx 0.6$ for $\zeta < 0.1$ instead of an asymptote to 0 (because of a zero additive parameter). Also, the fact that $(z-d)/\delta$ becomes on the order of 0.5 means that the $B_1$ term becomes dominant (instead of $A_1$) for $\zeta > 0.1$ in the zone-I formulation. A consequence of this outcome is that the effects of $\delta$ also become muted in the zone-I formulation.

4.5.2 Scaling variables

An important point to be stressed in Figure 4.8 is that measured and modeled dimensionless quantity $\sigma_u/u_*$ is not compared. Rather, $\sigma_u$ is compared directly to avoid the known problem of self-correlation (Hicks, 1978, 1981; Bruin, 1982; Pahlow et al., 2001; Andreas and Hicks, 2002; Hartogensis et al., 2005; Cava et al., 2008; Banerjee et al., 2014). The reason for such self correlation is the common presence of $u_*$ in both variables - the ordinate $\sigma_u/u_*$ and the the abscissa $\zeta = z/L$, where $u_*^3$ dictates the Obukhov length $L$.

Some previous studies (Pahlow et al., 2001; Juang et al., 2008) reported an increase of $\sigma_u/u_*$ with $\zeta$ and this increase exhibits an approximate 1/3 scaling, which is indicative of possible artificial self-correlation. The forest experiments used here also show a similar 1/3 scaling behavior when $\sigma_u/u_*$ is plotted against $\zeta$, as illustrated in Figure 4.11. Note again that $\sigma_u$ itself reduces with increasing stability as shown in Figure 4.8. Hence, to separate out the effect of stability on $\sigma_u$, the variation of $\sigma_u$ with $\zeta$ from the model is shown for three different but pre-fixed $u_*$ (i.e. no artificial self-correlation) in panel (a) of Figure 4.12. Panel (b) of Figure 4.12 shows the variation of non-dimensional $\sigma_u/u_*$ with $\zeta$ and since the $u_*$ is fixed (i.e. only the sensible heat flux varies), the curves show a self-similar behavior and decrease with increasing stability. Note the boundary layer height is also fixed in Figure 4.12
though in reality δ will vary with increasing stability.

4.5.3 Predicting z-less stratification

In Figure 4.12 and also in Figure 4.8, it can be observed that σ_u becomes independent of ζ after ζ ≈ 2 indicating that σ_u is no longer dependent on z. This observation indicates the possible onset of z-less stratification and is commensurate with the same limit ζ ≈ 2 reported by other studies such as Olesen et al. (1984) and Kaimal and Finnigan (1994). Figure 4.13 shows the variation of σ_u with ζ for the second formulation (zone II) under high stability conditions. While the range of variation of σ_u is quite small, it also reaches z-less stratification reasonably at ζ ≈ 2 (the model results are extended to ζ = 10 only for illustrating the near-leveling off when no other constraint is employed to suppress turbulence). As a result, the spectral budget model here predicts the z-less scaling for σ_u under high stability conditions. The ‘1/3’ scaling of σ_u/u* reported in some previous studies (Pahlow et al., 2001) may be the outcome of self-correlation and cannot be used to argue against the existence of the z-less stratification in isolation of other metrics.

4.6 Conclusion

A spectral budget framework has been developed to describe the longitudinal turbulent intensity (σ_u) under stable atmospheric stratification. In presence of stable stratification, although turbulence is generated by mechanical processes, buoyancy effects counter mechanical effects and consequently reduce the turbulent intensity. Thus, in the spectral budget framework, the mechanical production term and the buoyant destruction term compete with each other. Two stability regimes are identified and separated at ζ = z/L ≈ 0.1. In the first regime (ζ ≤ 0.1) the resulting normalized longitudinal velocity variance is found to be consistent with the log-law proposed by Townsend although the Townsend’s coefficients are found to vary with
stability parameters. The second regime ($\zeta > 0.1$) results in a truncated spectrum exhibiting a -5/3 scaling even at low wavenumbers. Interestingly, the second regime formulation does not contain the boundary layer height. Thus it can be concluded that at high stabilities, $\sigma_u$ is not sensitive to the boundary layer height.

The model outcomes using the formulation for regime I are compared to data from a controlled lab experiment, a field campaign over a smooth lake surface and two extensive datasets collected over tall vegetation canopies. The formulation for regime II are also compared with the field data over canopies. It is also found that $\sigma_u$ decreases with increasing stability as expected. However, some previous studies had predicted an increase of $\sigma_u/u_s$ with increasing stability parameter $\zeta = z/L$. This may be partly explained by the artificial self-correlation as a consequence of choosing common scaling variables in abscissa and ordinate. Hence, to separate out the effect of increasing stability on turbulent intensity, $\sigma_u$ is deemed to be a more robust variable compared to $\sigma_u/u_s$. With this choice, the same data that show increasing trend of $\sigma_u/u_s$ with $z/L$ display a reduction of $\sigma_u$ with increasing $z/L$, which is satisfactorily predicted by the model with zone I formulation. The modeled $\sigma_u$ with a pre-fixed $u_s$ flattens as $z/L$ approaches 0 and recovers the correct neutral limit. The modeled $\sigma_u$ also flattens and becomes asymptotic after $z/L = 2$. This result may signify that the model supports the existence of $z$-less stratification for a pre-set $u_s$ and also correctly captures the limit $z/L = 2$ as the onset of $z$-less stratification. The second formulation as a limiting scenario for high stability regimes predicts an upper limit for the onset of $z$-less stratification after $\zeta \approx 2$. From an operational perspective, if the boundary layer height is not measured precisely, it can be calculated from a simple model and the predicted $\sigma_u$ would not suffer much, because of the low sensitivity to $\delta$ (the order of magnitude is important, not the precise value). The proposed framework here brings together seemingly unrelated theories of turbulence such as Kolmogorov’s hypothesis, Heisenberg’s eddy viscosity,
Townsend’s hypothesis and MOST under a common framework. It also provides a physical basis for illustrating links between a number of well established constants.
Figure 4.1: Variation of $\gamma(\zeta)$ with the stability parameter $\zeta = z/L$. Black circles show the normalized integral length scale for vertical turbulent velocity from AHATS data. Dashed line indicates a fit to AHATS data as $\gamma = 1/(1+3\zeta)$. Dash and dot line represents the normalized integral length scale for vertical turbulent velocity from Kansas experiment. Black line indicates $\gamma(\zeta) = 1/\phi_m(\zeta)$, where $\phi_m$ is the stability correction function for momentum determined from the Kansas experiment.
Figure 4.2: Variation of $C_{st}$ with the stability parameter $\zeta = z/L$. Zone I and zone II are marked and the transition between the two zones is at $\zeta \approx 0.1$. $C_{st}$ is nearly constant in zone I but follows a power law in zone II.
Figure 4.3: Comparison between the turbulent kinetic energy \((e)\) and \(\sigma_{u}^2\) for Pine and Hardwood canopies for two different seasons- Summer and Winter. The slopes are 0.80 (Winter, Pine), 0.70 (Winter, HW), 0.76 (Summer, HW) and 0.84 (Summer, Pine)
Lake Data

\[ u^* = \frac{u^*_{neu}}{1+4.7\zeta} \]

\[ u^* = \frac{u^*_{neu}}{1+3\zeta} \]

Figure 4.4: Variations of \( u_* \) with \( \zeta = z/L \) in the lake data. \( u^*_{neu} \) is the \( u_* \) under neutral conditions (\( = 0.21 \text{ ms}^{-1} \)).
Figure 4.5: Comparisons between production and dissipation terms in the TKE budget from Pine data collected in the Duke Forest over twelve days. Only stable stratifications are shown, near neutral conditions are shown in red and more stable conditions are shown in black symbols.
Figure 4.6: Variations of $\sigma_u$ with the gradient Richardson number $Ri_g$ from the lab experiment and from the model.
Figure 4.7: Comparison between measured and modeled $\sigma_u$ for the lake experiment with $\delta$ computed from Arya’s model ($R^2 = 0.61$). Left panel shows the variation of the calculated boundary layer height.
Figure 4.8: Variations of $\sigma_u$ with $\zeta = z/L$ from the forest experiments and from the model using the formulation for zone I (left and middle panels) and MOST (right panels). The comparisons are shown for two different canopies (Pine and Hard Wood) and two different seasons (Summer and Winter) as indicated by the titles of the panels.
Figure 4.9: Comparisons between measured and modeled $\sigma_u$ using the formulation for zone I (left and middle panels) and MOST (right panels) for the forest experiment. The comparisons are shown for two different canopies (Pine and Hard Wood) and two different seasons (Summer and Winter) as indicated by the titles of the panels.
Figure 4.10: (a), (c), (e), (g): Comparisons between measured and modeled $\sigma_u$ using the formulation for zone II for the forest experiments. (b), (d), (f), (h): Variations of $\sigma_u$ with $\zeta$ from the forest experiments and from the model using the formulation for zone II.
Figure 4.11: Variations of $\sigma_u/u_*$ with $\zeta$ from the forest experiments. Red line shows the $\zeta^{1/3}$ scaling as an indication of self-correlation.
Figure 4.12: Variations of $\sigma_u$ (a) and $\sigma_u/u_*$ (b) with $\zeta$ for three different but preset $u_*$ from the model using the formulation for zone I. The flattened lines at $\zeta = 2$ indicate $z$-less stratification.
Figure 4.13: Variations of $\sigma_u$ with $\zeta$ from the model using the formulation for zone II.
Mean flow near edges and within cavities situated inside dense canopies

5.1 Introduction

Flows within and near forest gaps and cavities are becoming increasingly environmental and ecological in scope. Re-colonization of disturbed areas, seed and pollen dispersal, control of pests by vegetated barriers, and risk analysis of gene flow are but sample applications where estimates of the mean flow field, at minimum, is necessary (Nathan et al., 2002, 2011b,a). Any spatial heterogeneity in the stretch of the canopy along the mean wind direction ($x$), such as the presence of an edge, a clearing, or a gap generates a mean pressure gradient $dP/dx$. At some intermediate height above the gap or edge, the mechanical production of turbulent kinetic energy tends to be small as these production terms scale with the inverse of height from the boundary ($1/z$). Hence, the key terms in the mean longitudinal momentum budget at large $z$ generally reduce to the mean pressure gradient and the two advective terms. We call this simplified state a ‘turbulently inviscid flow’ or simply ‘inviscid flow’. However, in the vicinity of the canopy, gradients in turbulent stresses become significant along
with $dP/dx$. Within the canopy volume but far from any edge, the drag force also becomes a leading term in the mean momentum balance in addition to the turbulent stress gradients and possibly $dP/dx$. We refer to the interplay between the pressure gradient, the turbulent stresses, and the drag force as the planar-homogeneous ‘turbulent canopy flow’. To what degree the inviscid flow and the turbulent canopy flow, being asymptotic limiting states to the general problem of flow inside forest gaps, dictate the bulk flow properties near gaps and clearing, frames the compass of this work. This question is explored using a combination of a streamfunction-vorticity model and recent flume experiments. The ‘turbulent canopy solution’ is first tested across a wide range of canopy types in the absence of a non-zero $dP/dx$. Next, flow modification near inhomogeneous forest edges is tested by comparing the model runs against published wind-tunnel experiments and large-eddy simulation (LES) studies. After this, the mean flow inside cavities with different boundary configurations is explored using new flume experiments. Both ‘inviscid flow’ and ‘turbulent canopy solution’ are implemented for gaps and the zones ‘ruled’ by the aforesaid flow regimes are analyzed.

The fundamental dynamics of the mean flow near forest edges is often categorized into two types: entry and exit. The entry problem involves modelling the transition of an equilibrated flow above an extensive clearing into an extensive tall uniform forested canopy, while the exit problem considers the flow from such a forest into the clearing. In the entry problem, the upwind flow senses the forest patch by means of a mean pressure field, leading to the generation of a spatially varying $dP/dx$. The velocity field responds to the variations in $dP/dx$ rapidly by means of advection. The flow decelerates on the windward side of the edge and later adjusts itself due to the canopy drag to some adjustment length scale (Belcher et al., 2003), comparable to the ‘impact zone’ length scale. Further downstream, the flow homogenizes, with the generation of an internal boundary layer (IBL), which is defined as the flow over a
different surface, as it forms within an existing boundary layer (Stull, 1988; Garratt, 1992). Very far from the forest edge, the flow statistics re-equilibrate with the forest canopy and only vary in the vertical direction. For an exit flow from the forest, as the flow departs the forest, it accelerates with occasional recirculation in the case of a denser upstream canopy resembling back-facing step (BFS) flow (Cassiani et al., 2008; Detto et al., 2008). Fig. 5.1 depicts the different zones in the flow with the corresponding internal length scales following Belcher et al. (2003). The impact region, adjustment region, canopy interior, canopy shear layer, roughness change region, exit region and the far wake can be identified in this schematic.

These two configurations are commonly explored using Reynolds-averaged models that employ first-order closure schemes (Li et al., 1990; Peltola, 1996), $E - \varepsilon$ (Liu et al., 1996) or $k - \varepsilon$ (Katul et al., 2004; Frank and Ruck, 2008; Dalpe and Masson, 2009) to solve for the profiles of the mean velocity and Reynolds stresses inside and above the canopy. These models are routinely compared to wind-tunnel observations and/or field experiments (Raynor, 1971; Irvine et al., 1997). Over the last two decades, a number of LES studies have resolved some of the key energetic scales of turbulence inside and around gaps and edges in canopies. These LES studies offer a detailed view of the recirculation regions, re-attachment regions, and their sensitivity to leaf area index (LAI) (Yang et al., 2006; Cassiani et al., 2008; Dupont and Brunet, 2008; Schlegel et al., 2012). In fact, a classical field study already depicted a recirculation region in a forest clearing by means of smoke drift (Bergen, 1975). Recent experimental studies also reported flow adjustment dynamics in the presence of fixed and porous obstructions (Rominger and Nepf, 2011). A few LES studies described in detail the development of coherent structures generated due to the canopy transitions (Fesquet et al., 2009; Gavrilov et al., 2010, 2011; Huang et al., 2011) across edges and gaps using fully turbulent theory for the canopy and the space above the canopy as well. Exit flow from a forest or rough-to-smooth
transition has been shown experimentally and via LES to be analogous to a BFS problem (Detto et al., 2008; Cassiani et al., 2008). Another study (Belcher et al., 2012) puts forward a comprehensive discussion regarding the influence of complex terrain and forest edges on the mean and turbulent flow statistics, indicating their relevance to the international FLUXNET program for measuring scalar fluxes across regional climates and biomes (Baldocchi et al., 2001). The present work banks on all these results in exploring the problem at hand using a minimalist first-order closure model of turbulence (Holland, 1989; Li et al., 1990; Peltola, 1996).

5.2 Theory

5.2.1 General Considerations

The mean continuity and mean momentum conservation equations for a high Reynolds number, incompressible flow in the absence of Coriolis and buoyancy effects within and above rigid canopies are represented as (Li et al., 1990)

\[
\frac{\partial U_i}{\partial x_i} = 0, \quad (5.1)
\]

and

\[
\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial R_{ij}}{\partial x_j} - F d_i. \quad (5.2)
\]

where \(U_i\) are the mean velocity components with \(U_1 = U, U_2 = V,\) and \(U_3 = W\) along directions \(x_i,\) where \(x_1 = x, x_2 = y,\) and \(x_3 = z\) are the longitudinal, lateral, and vertical directions, respectively. The coordinate system is aligned so that \(U_2 = 0.\)

The term \(U_i U_j\) represents the mean advective acceleration terms, \(\rho\) denotes the mean fluid density, \(P\) denotes the mean departure from hydrostatic pressure and \(R_{ij}\) denotes the Reynolds stress tensor that encodes the effects of turbulence on the mean flow field, and \(F d_i\) is the drag force in direction \(x_i\) induced by the presence of
the canopy elements and parametrized as

\[ F_{d_i} = C_d a |U| U_i = |U| U_i/L_c, \]  \hspace{1cm} (5.3)

where \( C_d \) is a dimensionless local foliage drag coefficient that varies between 0.1 and 0.3 for several terrestrial vegetation canopies (Katul et al., 2004), \( a \) is leaf area density that can vary appreciably with \( z \) depending on the distribution of foliage within the canopy, \(|U| = \sqrt{U^2 + W^2}\) is the mean wind speed and \( L_c \) is the adjustment length scale.

Upon employing first-order closure principles, the Reynolds stress tensor \( R_{i,j} \) can be written as

\[ R_{ij} = K_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \]  \hspace{1cm} (5.4)

where \( K_t \) is the turbulent eddy viscosity formulated using a mixing-length model as

\[ K_t = l_m^2 \sqrt{\left( \frac{\partial U_i}{\partial x_j} \right)^2 + \left( \frac{\partial U_j}{\partial x_i} \right)^2}, \]  \hspace{1cm} (5.5)

and where the \( l_m \) is the canonical mixing-length. Following the argument in Wilson and Shaw (1977), \( l_m \) is given as

\[ l_m = \min \left( k_v z, \alpha h, 2\beta^3 L_c; \ z/h < 1 \right) \text{ or } k_v (z - d) \]  \hspace{1cm} (5.6)

for \( z/h > 1 \) where \( k_v = 0.4 \) is the Von Karman constant, \( \beta \) is a constant related to the attenuation coefficient of the mean velocity varying between 0.1 and 0.3 depending on the canopy density (Campbell and Norman, 2012; Poggi et al., 2004), \( h \) is canopy height and \( d \) is the zero-plane displacement for momentum calculated as the centroid of the drag force (Thom, 1971; Jackson, 1981; Poggi et al., 2004) in the vertical direction using

\[ d = \frac{\int_0^h F_{d_1}(z) z \, dz}{\int_0^h F_{d_1}(z) \, dz}. \]  \hspace{1cm} (5.7)
The premise is that the eddies increase at a rate proportional to $k_v z$ from the surface, until they attain a constant size proportional to $\alpha h$, where $\alpha$ is a constant that takes the value of $k_v (z - d)/h$ to maintain continuity in $l_m$ at the canopy-air interface (but not smoothness). Where the leaf area density $a(z)$ is sufficiently high, eddy sizes are limited by the high concentration of foliage and accounted for by the expression $2\beta^3 L_c$. That is, where $a(z)$ is sufficiently high, the mixing-length is governed by this term, otherwise the constant mixing-length expression $\alpha h$ dominates. Thus using the minimum of the three expressions in Eq. 5.6 can be justified. Taking gradient with respect to $x_i$ to Eq.5.2 along with $\partial U_i / \partial x_i = 0$, and rearranging the terms,

$$\frac{1}{\rho} \frac{\partial^2 P}{\partial x_i x_i} = -\frac{\partial}{\partial x_i} \left( \frac{\partial U_i U_j}{\partial x_j} \right) - \frac{\partial F d_i}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \frac{\partial R_{ij}}{\partial x_j} \right).$$  \tag{5.8}$$

Eq. 5.8 has now the conventional form of a Poisson’s equation for $P$, given as

$$\nabla^2 p = -f$$  \tag{5.9}$$

where $p = P/\rho$ and $f$ is the negative of the right-hand side of the Eq. 5.8.

5.2.2 The Mean Streamfunction-Vorticity Representation

Because flows inside cavities, edges, and gaps may be characterized by mean recirculation zones, a vorticity-streamfunction formulation (Pozrikidis, 2009) that accommodates the formation of such zones is preferred and can be derived as follows. The $U$ and $W$ mean momentum equations are written separately as

$$\frac{\partial U}{\partial t} + \frac{\partial U U}{\partial x} + \frac{\partial U W}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial R_{11}}{\partial x} + \frac{\partial R_{13}}{\partial z} - F d_1,$$  \tag{5.10}$$

and

$$\frac{\partial W}{\partial t} + \frac{\partial U W}{\partial x} + \frac{\partial W W}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{\partial R_{13}}{\partial x} + \frac{\partial R_{33}}{\partial z} - F d_3.$$  \tag{5.11}$$
where the subscripts 1 and 3 are interchangeably used with the subscripts \( x \) and \( z \).

Eq. 5.10 and Eq. 5.11 are differentiated with respect to \( z \) and \( x \), respectively and subtracted to obtain the time rate of change of the mean vorticity

\[
\frac{\partial \omega}{\partial t} = T_1 + T_2 + T_3
\]  

(5.12)

where \( T_1, T_2, \) and \( T_3 \) include the effects of the advective terms, turbulence, and drag force on the mean vorticity and are given by

\[
T_1 = \left( \frac{\partial^2 U U}{\partial x \partial z} + \frac{\partial^2 U W}{\partial z \partial z} - \frac{\partial^2 U W}{\partial x \partial x} - \frac{\partial^2 W W}{\partial x \partial z} \right),
\]  

(5.13)

\[
T_2 = - \left( \frac{\partial^2 R_{11}}{\partial x \partial z} + \frac{\partial^2 R_{13}}{\partial z \partial z} - \frac{\partial^2 R_{13}}{\partial x \partial x} - \frac{\partial^2 R_{33}}{\partial x \partial z} \right),
\]  

(5.14)

\[
T_3 = \left( \frac{\partial F d_1}{\partial z} - \frac{\partial F d_3}{\partial x} \right).
\]  

(5.15)

The left-hand side of Eq. 5.12 is the time rate of change of the mean vorticity, defined as

\[
\omega = \left( \frac{\partial W}{\partial x} - \frac{\partial U}{\partial z} \right).
\]  

(5.16)

The mean velocities are related to the mean vorticity through a conventional mean streamfunction. This streamfunction \( \psi \) is defined such that

\[
U = \frac{\partial \psi}{\partial z},
\]  

(5.17)

and

\[
W = - \frac{\partial \psi}{\partial x}.
\]  

(5.18)

The mean streamfunction and the mean vorticity are also linked by a Poisson’s equation

\[
\nabla^2 \psi = -\omega.
\]  

(5.19)
The algorithm implemented to solve this equation is briefly highlighted in the subsequent section.

5.2.3 Turbulently Inviscid Scheme

The turbulently inviscid scheme can be described as the condition where the effects of the turbulent Reynolds stress is neglected. The flow is thus bereft of turbulent diffusivity, which is an analogy to the molecular viscosity in the first-order closure principle and hence is called turbulently inviscid. The term \(B\overline{R}_{ij}\overline{B}_x\) is dropped from Eq. 5.2 and therefore its contributions are omitted in the subsequent analysis. It is to be noted that the gradient of the stress is neglected, not the stress itself and another significant observation is that the governing momentum equation under the inviscid condition is first-order non-linear while its fully turbulent counterpart is second-order non-linear. Hence the turbulent and the inviscid schemes are significantly different from each other.

5.2.4 Numerical Implementation and Boundary Conditions

A finite difference formulation on an uniform grid is used to solve Eq. 5.19; the algorithm is outlined in the Appendix E. For solving the Poisson equation (5.19), four boundary conditions must be specified. The flow is assumed to be equilibrated with the underlying surface at the entrance and exit locations resulting in the upstream and downstream boundaries to be horizontal and \(\partial \psi / \partial x = 0\). At the top boundary, set far from the clearing or cavity, the streamlines are assumed to be parallel to each other and correspond to the outer-layer velocity (\(U_{out}\)), assuming \(U_{out}\) is planar uniform. Hence, a Dirichlet boundary condition of \(\psi = (1/2) \int U_{out} \Delta z\) is imposed. At the bottom boundary,

\[
\frac{\partial \psi}{\partial z} = \frac{U_{st}}{k\nu} \ln \left( \frac{\Delta z}{z_0} \right)
\]

(5.20)
where $u_{*g}$ is the friction velocity near the ground (or forest floor), $\Delta z$ is the vertical grid spacing ($> z_0$) and $z_0$ is the roughness length of the surface (not the vegetation). The formulation in Eq. 5.20 is consistent with the mixing-length in Eq. 5.6.

The pressure Poisson’s equation, Eq. 5.9 is solved with the following boundary conditions: $\partial P/\partial x = 0$ at the bottom and $P = 0$ at the other three sides. The vorticity formulation has a number of advantages. First and foremost, it is not implicit. The structure of Eq. 5.12 allows it to be solved in a conventional, time marching method, as the vorticity only appears on the right-hand side. Moreover, continuity is automatically satisfied in a streamfunction formulation. The need to update the pressure and the velocity simultaneously is also eliminated in such a scheme. The disadvantages are that such a scheme requires specification of boundary condition on the streamfunction instead of the velocity and hence is not straightforward.

5.3 Results and Discussions

Prior to addressing the main problem, the model is first evaluated against published uniform canopy experiments and LES presented in the literature. In the first sub-section, results from previous studies on horizontally homogeneous canopies are reported and compared against model runs to assess the model performance when only the Reynolds stresses and the drag force dominate the mean longitudinal momentum balance. This comparison is intended to provide an independent evaluation of the modeled $l_m$ across a wide range of leaf area density shapes and canopy heights when the flow is allowed to attain a stationary and planar homogeneous state. Next, flow modifications introduced by dense forest edge(s) are explored via comparisons with previous LES studies, field, and wind-tunnel experiments. In this case, apart from the Reynolds stress and the drag force, the pressure gradient and the advective terms are the key terms in the momentum budget equation (Eq. 5.2) close to the edge and in the canopy. In the last sub-section, the proposed formulation is
employed for the more complex scenario where a combination of porous and solid configurations are used to describe the boundaries of a cavity intended to mimic a forest-gap in a flume. The extent to which the inviscid solution and the full solution agree with laser Doppler anemometry is discussed for porous-porous, solid-porous, porous-solid, and solid-solid cavity boundaries.

5.3.1 Horizontally Homogeneous Canopy

The case of a horizontally homogeneous canopy has been extensively discussed in the context of turbulent closure schemes (or mixing-length specification) (Shaw and Schumann, 1992; Katul et al., 2004; Wang, 2012). The model runs are presented against field experiments for seven different dense canopies ranging from a short corn canopy to a tall mixed hardwood forest (Fig. 5.2). The vertical inhomogeneity is large due to the large variability in $a(z)$. The mean horizontal velocity component ($U$) and the Reynolds stress ($R_{13}$) profiles are compared against field measurements reported in Katul et al. (2004) and the other original sources such as Amiro (1990) for aspen, spruce and Jack pine, Wilson et al. (1982) for corn, Katul and Chu (1998) for Loblolly pine, Leuning et al. (2000) for rice, Meyers and Baldocchi (1991) for hardwood and Kelliher et al. (1998) for Scots pine.

The key observations from these comparisons are as follows:

1. The modeled mean velocity within the canopy agrees with field measurements in many cases (see Table 5.1). The largest disagreements were for the hardwood (HW) canopy cases, where a topography-induced mean pressure gradient (Lee et al., 1994) (not included in the model calculations here) may have been responsible for generating a secondary maximum. Another reason for this mismatch may be attributed to the severe vertical heterogeneity in $a(z)$, which is mostly concentrated in the top one third of the canopy. It should also be noted that several of the disagreements with the field measurements are also evident
for the rice (RI) canopy, though these measurements have anomalous noise due to a sequential sampling employed. The comparisons with the stress profiles and the mean velocity profiles are shown in Table 5.1.

2. As a whole, it can be stated that the first-order closure model with such a mixing-length used to resolve the turbulent features inside the canopy is acceptable for modelling the mean flow. It is to be noted that the present work is not intended to improve the existing canopy turbulence models or their canonical mixing-length, but to establish confidence in the model being used in resolving some of the key aspects of turbulence inside canopies.

The results for a horizontally homogeneous canopy have also been compared to a LES study (Shaw and Schumann, 1992) and the model runs display similar quality of agreement to these simulations (Fig. 5.3). It is to be observed that for

Table 5.1: Regression statistics for the one to one comparisons between experimental data (abscissa) and modeled (ordinate) $U$ and $R_{13}$ for horizontally homogeneous canopy. The $R^2$ represents the coefficient of determination and $RMSE$ indicates the root mean square error in the regression. Slopes and Intercepts for the linear regression are also reported.

| Variable | Species | $R^2$ | Slope | Intercept | $RMSE$
|----------|---------|-------|--------|-----------|--------
| $U$  | AS      | 0.86  | 0.81   | 0.00      | 0.06   |
| $U$  | CO      | 0.99  | 0.93   | -0.01     | 0.01   |
| $U$  | HW      | 0.01  | 0.01   | 0.18      | 0.02   |
| $U$  | PI      | 0.94  | 1.26   | -0.01     | 0.05   |
| $U$  | RI      | 0.59  | 0.58   | -0.01     | 0.05   |
| $U$  | SP      | 0.94  | 0.87   | 0.01      | 0.05   |
| $U$  | SPI     | 0.76  | 0.61   | 0.13      | 0.04   |
| $U$  | JPI     | 0.97  | 0.49   | 0.17      | 0.03   |
| $R_{13}$  | AS      | 0.90  | 1.04   | 0.04      | 0.09   |
| $R_{13}$  | CO      | 0.98  | 1.23   | -0.01     | 0.05   |
| $R_{13}$  | HW      | 0.70  | 0.86   | -0.00     | 0.14   |
| $R_{13}$  | PI      | 0.89  | 0.89   | -0.02     | 0.12   |
| $R_{13}$  | RI      | 0.72  | 0.84   | 0.07      | 0.13   |
| $R_{13}$  | SP      | 0.99  | 0.75   | -0.03     | 0.04   |
| $R_{13}$  | SPI     | 0.92  | 0.92   | 0.06      | 0.08   |
| $R_{13}$  | JPI     | 0.99  | 0.66   | -0.08     | 0.02   |
a homogeneous canopy, vertical profiles of velocity or stress are self-similar at any section. For both LAI cases ($LAI = 5$ and 2), the modeled velocity follows LES except in the lower layers of the canopy and close to the ground, while for the stress, the deviations are more significant, particularly above the canopy. However, they follow the same average trend. These results reiterate that the ‘turbulent canopy solution’ is a plausible approximation.

5.3.2 Presence of a Forest Edge

Spatial gradients are generated when a forest edge is introduced into an otherwise planar homogeneous flow field. When the flow enters the forest, the pressure increases at the upstream to the entry point and then is reduced as the flow progresses into the forest. The streamlines become distorted at the edge and a recirculation zone with an observable length scale is generated immediately past the edge. The strength of this recirculation is dependent on the drag of the canopy. Higher $C_d$ amounts to stronger recirculation (Dalpe and Masson, 2009). The proposed model formulation is now applied to the edge-problem described in Yang et al. (2006) and the model runs are compared to the wind-tunnel experiments reported by them (Figs. 5.4 and 5.5).

While other experiments have also been reported on the development of turbulence across forest edges, e.g., Morse et al. (2002), it is the comparison between both LES and wind-tunnel data that makes this dataset ideal for our study.

The structure of the domain in a forest entry problem is shown in Fig. 5.4. The canopy is preceded by a grassland with uniform $a(z)$ that represents a field situation; the canopy $a(z)$ is obtained by digitizing the wind-tunnel data presented in Figure 4 of Yang et al. (2006). The recirculation zone is clearly visible after the edge and the streamlines become parallel at a height of about $3h$, commensurate with the vertical extent of the canopy sublayer (Raupach, 1981). The distortion of the streamlines upstream of the edge is notable and appears to be a feature in both the model runs.
presented here and the data presented in Yang et al. (2006). In the bottom row of Fig. 5.4, the same model runs are presented with the top boundary much higher above the canopy to show the dependence on the upper boundary condition. The upper boundary condition forces zero gradient in the streamlines and the conclusion is that the streamlines become approximately parallel some $3h$ to $5h$ above the ground. However, the mean features of flow distortion around the edge remain the same. The building up and decaying of pressure is shown using a surface plot in Fig. 5.4. It is to be noted that in the absence of any pressure data, the modeled pressure field is normalized by the maximum pressure to illustrate its relative spatial distribution with respect to the streamlines. The measured and modeled wind profiles (Fig. 5.5) at the specified sections show acceptable agreement with the LES model runs. The impact region upstream of the forest edge, the adjustment region immediately downstream of the forest edge and the roughness change region downstream and just above the canopy are all visible in Fig. 5.4. Different zones of flow distortion in a canopy transition according to Belcher et al. (2003) are identified on Fig. 5.4. The impact region, adjustment region, canopy interior flow and roughness change region can be visually identified in Fig. 5.4. The length scale $L_c$, a measure of the efficiency of the canopy to remove momentum (Belcher et al., 2003), can be estimated as $L_c/h = (C_d LAI)^{-1}$ (Belcher et al., 2003). In particular, the co-location of zones of concentrated vorticity as delineated by the streamlines and the span of the adjustment region determined from $L_c$ ($x/h \approx 2.5$, where $C_d = 0.2$ and $LAI = 2$) is noted in Fig. 5.4. Moreover, the relaxation of the pressure buildup near the forest edge shown in Fig. 5.3 occurs over a distance into the forest comparable to $x/h \approx 7.5$, which is also comparable to $x/L_c \approx 3$ noted in Belcher et al. (2003). It is worth mentioning that the pressure field in Fig. 5.4 is qualitatively similar in terms of the spatial distribution to the entry problem calculations in Li et al. (1990).

A comparison of the modeled pressure has been made with the reported surface
pressure measurements in Nieveen et al. (2001) for smooth-to-rough (entry problem) transitions. The forest edge is located at $x/h = 0$. For the entry problem (Fig. 5.6) negative $x/h$ indicates the grassland before the forest and positive $x/h$ denotes the region inside the forest and past the edge. The edge has a tapered structure as evident from the figure to provide a smoothed and more realistic transition in the entry problem. The field measurements reported in Nieveen et al. (2001) are ground pressure. Pressure at three different levels have been presented in Fig. 5.6. It is observed that the modeled ground pressure is shifted relative to the measurement by a distance of 0.5 $h$. The phase relations between longitudinal pressure profiles are also to be noticed. The peak of the pressure pulse has been observed to be at 0.6 $h$ inside the canopy for the entry problem for both the model run and the measurement. The measurements do not report the decline of pressure immediately past the edge as depicted in the model run. This decrease, however has been reported by other studies as Wilson and Flesch (1999) and Li et al. (1990). Some of the uncertainties in this comparison can be attributed to lack of knowledge on field conditions (e.g. outer velocity, drag coefficient, leaf area density and distribution near the edge). Also, the model indicates vertical gradients in the horizontal pressure distribution (and their phase relations to the surface across various levels) as is evident from Fig. 5.6).

5.3.3 Presence of the Gap: Comparison with LES

Moving from a forest entry into a canopy gap configuration, the mean streamfunction-vorticity formulation has been employed to the problem scenario described in Schlegel et al. (2012) and the model runs have been compared to the LES results and field data reported in their study. The $a(z)$ described in that study has been digitized and used in the model. The position of the gap is shown in Fig. 5.7. The gap in between the canopies is assumed to be a grassland. The length and height of the domain are normalized by the canopy height. The mean streamlines are now used to describe
the flow dynamics in the gap. As the flow exits the forest, a weak recirculation zone appears before the edge. The streamlines illustrate a concentration of vortices within the lower canopy at the entry. The size of these vortices are commensurate to predictions from Belcher et al. (2003). The streamlines become almost parallel at a height of about $3h$ as before. The different zones of action in a canopy transition are identified in Fig. 5.7. Adjustment region, canopy interior, roughness change region, exit region and another adjustment region before the gap are identified. The velocity profiles from LES and field data at four qualitatively differently positions are digitized from Schlegel et al. (2012) and they are found to compare well with the computed profiles (Fig. 5.8), except close to the forest floor. Disagreement between model and data begins at about $0.75h$. Some of these differences are due to lower boundary conditions introduced by the streamfunction formulation and failure of K-theory in this zone.

5.3.4 Presence of Gap: Comparison with lab experiments

Finally, extremes in gap density configuration for the same gap size and bulk flow rate are explored via model calculations suppressing (i.e. inviscid) and activating the turbulent viscosity. The model is run for four different scenarios corresponding to flume measurements, namely porous-porous (P-P), porous-solid (P-S), solid-porous (S-P) and solid-solid (S-S) configurations, with the flow exiting the first and ‘facing’ the second. These configurations are intended to amplify or relax pressure perturbations at the two gap interfaces. To retain the same canopy formulation, the solid configuration is modeled using a very high (hundred times the porous case) $a(z)$. This assumption is fair because a very high $a(z)$ implies a near zero velocity within the mattress (but not pressure). The experiments have been performed in a large recirculating flume described elsewhere (Fontan et al., 2013). Porous mattresses have been used to simulate the effects of canopy on the flow while the solid mattress was
constructed using a stainless steel sheet covering the entire porous mattress. The porous mattresses were composed of open-cell polyurethane foam characterized by a homogeneous and isotropic structure with high permeability. This high permeability allows for the development of a flow resembling a perturbed mixing layer near the porous medium characterizing many dense canopy flows as described in Manes et al. (2011). Laser Doppler Anemometry (LDA) was used to acquire the two velocity components at various positions within and above the gap. Dye Laser visualization (DLV) runs were used to decide on an optimum gap size. Very small gap size themselves dictate the vortical structure and in a very large gap size, the flow equilibrates after the drop resembling an independent exit, then entry configuration. It has been found that a gap size of $3h$ is ‘rich in gap dynamics’, that is the configuration ‘allows a separated shear layer to be initiated and developed but with a minimum re-attachment zone.’ Effectively, the flow is not allowed to equilibrate between re-attachment and the adjustment again. Due to the measurement techniques and materials used, velocity measurements inside the mattress are not possible. To address the primary question here, measured and modeled $U$, $W$ and $R_{13}$ are compared. For the model runs, the ‘full model’ and the ‘inviscid flow model’ outside the mattress domain are also compared. It is recalled that the inviscid model runs are carried out by suppressing the turbulent terms in the two mean momentum budget equations (Eq. 5.2). Before presenting the mean velocity comparisons, the effects of the gap interfaces on the pressure perturbation are first discussed.

The computed pressure fields (Fig. 5.9) for all four configurations demonstrate the sensitivity of the flow to the gap porosity. The pressure features inside the canopy are retained in Fig. 5.9 for clarity of presentation. The results show:

1. In the case of porous-porous (P-P) configuration, there is a zone of decreased pressure past the exit edge and a zone of increased pressure just before the
entrance edge, followed by another zone of decreased pressure. The pressure perturbation intensities are comparable and the pressure field appears quasi-symmetrical about a vertical axis positioned in the centre of the gap.

2. In the case of porous-solid (P-S) configuration, there is a strong zone of increased pressure just before the entrance edge, followed by a zone of decreased pressure. The symmetry noted for P-P is now broken. The origin of this asymmetry is due to the fact that the solid interface (or the very high leaf area density approximation to it) is far more effective at blocking the flow than the porous interface.

3. In the case of solid-porous (S-P) configuration, there is a zone of strongly decreased pressure past the exit edge and a zone of mildly increasing pressure just before the entrance edge, followed by a zone marked by mild decrease in pressure. The symmetry noted in P-P is again broken here.

4. In the case of solid-solid (S-S) configuration, there is a zone of decreased pressure past the exit edge and a zone of increased pressure just before the entrance edge, followed by another zone of decreased pressure. The intensities are comparable, but stronger and far more spatially localized than the P-P configuration. Also, the symmetry is restored.

5. The inviscid flow field shows similar trends as the full model that includes the Reynolds stresses, again suggesting that the large-scale pressure perturbations in gaps may be far more influenced by the bulk flow gradients than by the gradients in Reynolds stresses.

How do these pressure perturbations impact the spatial distribution of the two velocity components is discussed next. Only two (P-P and S-P) of the four flow
configurations are presented in Fig. 5.10 and Fig. 5.11 as they illustrate all possible configurational heterogeneity. The results for the P-S and S-S cases were not published previously, though the setup and Reynolds numbers used are the same as those provided in Fontan et al. (2013). As before, \( z/h = 1 \) denotes the top of the canopy and the the bulk flow is from left to right. The positions of the canopy or solid region appear as white or blocked regions. It is observed that the computed and measured \( U \) reasonably agree. The spatial \( W \) fields are moderately comparable but the \( R_{13} \) fields are poorly reproduced. The \( W \) increases past the exit-edge and above the entrance-edge, while it decreases above the exit-edge. Inside the gap, the agreement between measured and modeled \( W \) is better than outside. The mean flow field produced by including the turbulent component is more spatially diffused than the inviscid field (as expected). There is a zone of decrease in \( R_{13} \) between the two edges. However, this zone is displayed upward for the computed field, compared to the experimental data. This may be caused by a sharply increasing mixing-length over and past the canopy, which may be unrealistic for such a configuration. The entire turbulent stress is set to be zero for the inviscid solution across the entire domain because of the zero stress at the water surface. For the porous-porous configuration, the intensities of the important zones are comparable to each other on the two sides. However, in Fig. 5.11, the flow transverses past a solid configuration and encounters a porous configuration (the solid-porous case). Hence the zones marked by increase and decrease of \( W \) are positionwise similar to the porous-porous case, although characterized by higher intensity. Like before, the solution generated by including the turbulent component is more diffused when compared to its inviscid counterpart. It is important to notice that like pressure, \( W \) is a good indicator of sensitivity of the flow field to spatial heterogeneity as it is explicitly generated from spatial variability in \( U \).

To emphasize the recirculation regions, streamlines for all the four configurations
are presented in Fig. 5.12. These plots depict a definite recirculation zone in each case with an observable length scale. The flow appears to be more distorted around the solid edges. It is to be noted that the inviscid solution can produce the bulk movement of these vectors and offers a preview of the blue-print or skeleton of the recirculation region within the gap. One to one comparisons for two (Porous-Porous or P-P and Solid-Porous or S-P) of the four configurations are provided for both $U$ and $W$ in Table 5.2 and Table 5.3. Table 5.2 reports the statistics for the entire problem domain and Table 5.3 repeats for the gap domain only. It is observed that for all the configurations, the agreement between measured and modeled $U$ is good, and average to poor for $W$ though regions of positive and negative $W$ are reasonably delineated by both model calculations. All these observations reinforce the premise that in case of a complex flow adjustment problem, the spatial patterns in the bulk flow dynamics may be captured by a ‘turbulently inviscid’ flow. It is well established now that inside the gap, the pressure gradient $dP/dx$ governs the flow dynamics. The ‘inviscid flow solution’ can capture the main heterogeneity of the pressure and can propagate it to the mean velocity via the advective terms. This is substantiated by the correct location of the recirculation region in Fig. 5.10 and

Table 5.2: Regression statistics for the one to one comparisons between experimental data (abscissa) and modeled (ordinate) $U$ and $W$ for two configurations for the complete domain. The $R^2$ represents the coefficient of determination and $RMSE$ indicates the root mean square error in the regression.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Configuration</th>
<th>Solution Type</th>
<th>$R^2$</th>
<th>Slope</th>
<th>Intercept</th>
<th>$RMSE$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>P-P</td>
<td>Turbulent</td>
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<td>0.07</td>
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<td>P-P</td>
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<td>0.88</td>
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<td>0.10</td>
</tr>
<tr>
<td>$U$</td>
<td>S-P</td>
<td>Turbulent</td>
<td>0.93</td>
<td>0.82</td>
<td>0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>$U$</td>
<td>S-P</td>
<td>Inviscid</td>
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<td>0.82</td>
<td>-0.02</td>
<td>0.13</td>
</tr>
<tr>
<td>$W$</td>
<td>P-P</td>
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<td>0.21</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>$W$</td>
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<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$W$</td>
<td>S-P</td>
<td>Turbulent</td>
<td>0.20</td>
<td>0.35</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$W$</td>
<td>S-P</td>
<td>Inviscid</td>
<td>0.22</td>
<td>0.22</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Fig. 5.11. The recirculation is occurring in a zone below the zone of maximum shear stress at $z/h = 1$. This may be the reason why the inviscid scheme and the turbulent scheme involving K-theory agree in this location.

5.4 Conclusion

The extent to which turbulent and turbulently inviscid solutions to the mean momentum balance explain the mean flow across forest edges and cavities inside dense forested canopies has been explored. A mean streamfunction-vorticity formulation has been proposed and used to address this question. The advantages of this approach are, (i) guaranteeing conservation of fluid mass, and (ii) capturing recirculation patterns without requiring interactive solutions to the mean velocity and the Poisson equation for pressure. This vorticity formulation has been implemented for a planar homogeneous canopy where the vertical variability in leaf area density has been accounted for. It has been shown that the first-order closure model performance is similar to conventional $k - \epsilon$ models inside a canopy lending confidence in the parameterization of the canonical mixing-length. Next, the forest edge problem has been explored and the turbulent solution has been found to describe the bulk spatial patterns of the mean flow near the edge. The proposed formulation has been found to predict the signatures of the different length scales observed in a canopy transition as

<table>
<thead>
<tr>
<th>Variable</th>
<th>Configuration</th>
<th>Solution Type</th>
<th>$R^2$</th>
<th>Slope</th>
<th>Intercept</th>
<th>$RMSE$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.85</td>
<td>0.06</td>
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</tr>
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</tr>
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<td>0.74</td>
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<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>$W$</td>
<td>P-P</td>
<td>Inviscid</td>
<td>0.52</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$W$</td>
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<td>0.39</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$W$</td>
<td>S-P</td>
<td>Inviscid</td>
<td>0.25</td>
<td>0.25</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 5.3: Same as Table 5.1, but only for the cavity domain.
proposed by analytical approaches (Belcher et al., 2003) and as documented by field experiments and LES. Finally, the ‘clearing inside canopy’ or the so-called ‘cavity’ problem has been solved for the inviscid and turbulent solutions. It has been found that the inviscid solution can describe the bulk flow dynamics in some intermediate zone inside the cavity. These observations are consistent with the proposition put forward in Belcher et al. (2003, 2012), arguing that the shear stresses are of a less in order of magnitude than the so-called inertial or advective terms for much of the flow and even more in the impact region upstream of the edge or in the gap. This implies that the mean flow within the cavity is governed by the horizontal advection rather than the vertical turbulent transport terms. The inviscid solution cannot capture the large vertical heterogeneity in the mean velocity above the canopy, and the more detailed turbulent features within the gap. However, these features can be reasonably captured via first-order closure representations in the turbulent solution. Focusing on the gap region only, both solutions are comparable in terms of agreement with the measurements. Given the ability of this model to capture the pressure variations and the mean advective acceleration terms, it is sufficient for exploring the distributions of scalars and roughness-induced flow adjustments on complex topography.
Figure 5.1: Schematic showing different zones of adjustment to the canopy transition according to Belcher et al. (2003). (i) Impact region; (ii) adjustment region; (iii) canopy interior; (iv) canopy shear layer; (v) roughness-change region; (vi) exit region; (vii) far wake.

The quantities $h$, $L$, $L_C$, $l_s$ denote canopy height, length of canopy stretch, adjustment length scale and height of canopy shear layer respectively.
Figure 5.2: Leaf area density ($m^2/m^3$), normalized mean horizontal velocity $U/U_{max}$ and normalized Reynolds stress $R_{13}/R_{13,max}$ profiles for seven different vegetation canopies. The canopy types are aspen (AS), corn (CO), hardwood (HW), Loblolly pine (PI), rice (RI), spruce (SP), Scots pine (SPI) and Jack pine (JPI). The lines represent model runs and the markers represent the field measurements cited in Katul et al. (2004). Vertical height $z$ is normalized by the canopy height $h$. Maximum value is chosen as normalization reference to provide a uniform basis for comparison among the three parameters. This basis is maintained throughout this study.
FIGURE 5.3: $LAD$, normalized horizontal mean velocity and normalized Reynolds stress for two different leaf area indices ($LAI$) as indicated in the figure. Normalization basis are same as before. Blue lines indicate model runs and black solid dots indicate LES runs from Shaw and Schumann (1992) digitized by us.
Figure 5.4: Computed streamlines and normalized pressure for the forest edge configuration scenario described in Yang et al. (2006). The colormap on the streamline plot shows the position of the canopy in terms of the LAD. The bottom row presents the same scenario with a much higher top boundary to show the dependence of the problem to the upper boundary condition which forces the streamlines to be parallel.
Figure 5.5: Normalized velocity profiles at different positions for the forest entry problem described by the wind-tunnel data (Fig. 4) of Yang et al. (2006). The normalization and legend conventions are the same as before.
Figure 5.6: Normalized surface pressure across a forest edge in an entry problem (top panel) described in Nieveen et al. (2001). The black markers correspond to field measurements presented in Nieveen et al. (2001) and the blue lines depict the computed pressure from the full model. Normalization has been achieved by maximum positive value to provide common basis of comparison. The location and tapered structure of the edge has been qualitatively sketched by the black lines. The solid, dashed and dotted lines indicates ground pressure, pressure at half the canopy height and pressure at the canopy height respectively, used to illustrate the magnitude of the vertical pressure variation.
Figure 5.7: Computed streamlines and pressure for the problem scenario described in Schlegel et al. (2012). The different zones according to Belcher et al. (2003) are visually identified. An adjustment zone before the second edge is identified as (viii).
Figure 5.8: Normalized velocity profiles at different positions for the problem of flow exiting the forest described in (Fig. 9, HOM case) Schlegel et al. (2012). The red lines show the LES data and the black dots represent field data. Normalization and legend conventions are same as before.
Figure 5.9: Normalized pressure fields qualitatively describe the asymmetries originating from different flow configurations for turbulent (top row) and turbulently inviscid flow (bottom row) and for all four configurations.
Figure 5.10: Spatial variation of mean horizontal and vertical velocity ($U$ and $W$) and Reynolds stress ($R_{13}$) for the porous-porous (P-P) configuration. Turbulent and inviscid solutions are presented in the first two columns. The third column presents the LDA measurements. The $U$ and $W$ are normalized by the maximum $U$ velocity, i.e., the outer layer velocity. This is a logical basis for normalization as the streamlines become parallel at the top of the domain as evident from Fig. 5.6. $R_{13}$ has been normalized by its maximum value, to provide a common basis of comparison for the experimental and computed data.
Figure 5.11: Spatial variation of mean horizontal and vertical velocity ($U$ and $W$) and Reynolds stress ($R_{13}$) for the solid-porous (S-P) configuration. Turbulent and inviscid solutions are presented in the first two columns. The third column presents the LDA measurements.
Figure 5.12: Streamlines for all four configurations. The arrows indicate the direction and magnitude of flow velocity at different positions for different flow configurations. First column presents the full solution that includes the turbulent stresses, the second column presents the inviscid solution and the third column presents the measurements.
Flume experiments on wind induced flow in static water bodies in the presence of protruding vegetation

6.1 Introduction

The monetary value and multiple ecosystem services provided by static water bodies such as wetlands and marshes are rarely disputed (Begon and Harper, 1986; Bennett et al., 2002; Costanza et al., 1998; Fonseca and Fisher, 1986; Fonseca and Cahalan, 1992; Gambi et al., 1990; Gleason et al., 1979; Jordanova and James, 2003; Nepf, 1999, 2012a; Pergent-Martini et al., 2006; Prigent et al., 2001; Reusch and Chapman, 1995; Ward et al., 1984); however, characterization of the flow field, needed in all such ecosystem valuation, remains the subject of active research. Operational models for water flow in wetlands commonly assume the flow to be analogous to a wide and shallow open channel described by the so-called Saint-Venant equations that are then mathematically closed for the energy losses using a Manning-type formula with an associated friction factor as recently reviewed elsewhere (Katul et al., 2011a). Because the flow depth in wetlands is shallow, wind effects can be sufficiently large
so as to induce flow even in the absence of any gravitational gradient. These wind effects on the flow have traditionally been lumped into changes in the friction factor, with little theoretical or experimental underpinning, which is the main motivation for this work. By no means this is a unique criticism to such an operational framework. Another common criticism is the lack of explicit inclusion of the effects of vegetation on both - bulk and turbulent flow quantities needed for the purposes here. Such vegetation characterization on the bulk flow has often been directed to drag or flow resistance estimation for unidirectional flow but in the absence of wind (Fairbanks and Diplas, 1998; Kubrak et al., 2008; Lee et al., 2004; Montakhab et al., 2012; Nepf, 1999; Nepf and Koch, 1999; Nepf and Vivoni, 2000; Nepf, 2012a; Righetti, 2008; Sukhodolov and Sukhodolova, 2009; Sun and Shiono, 2009; Tanino and Nepf, 2008; Wilson et al., 2003). A number of studies have also been concerned with detailed description of turbulent processes needed in modeling movement of particulate matter inside aquatic vegetation, characterization of dispersion, and lateral diffusion (Huang et al., 2008; Lightbody and Nepf, 2006; Murphy et al., 2007; Nepf et al., 1997b,a; Serra et al., 2004). In unidirectional flow through a vegetation canopy, the shear layer on top of a canopy generates canopy scale turbulence that is pushed down to the canopy displacement length (Nepf, 2012a; Poggi et al., 2004). At the bottom of the canopy, turbulence is generated by stem scale wakes (Nepf, 2012a; Poggi et al., 2004). In dense canopies, the intensity of turbulence is reduced by sheltering (Raupach and Thom, 1981; Finnigan, 2000), which plays a positive role in sediment retention and prevention of bed erosion (Baptist, 2003; Jordanova and James, 2003; Jordanova et al., 2006; Liu and Shen, 2008; Neumeier and Ciavola, 2004; Schmid et al., 2005; Sharpe and James, 2006; Shucksmith et al., 2011; Stephan et al., 2005; Zong and Nepf, 2011). However, all these experiments did not consider the problem of wind-induced flow within emergent aquatic vegetation, the main compass of this work.
Any wetland or channel featuring aquatic vegetation is naturally subjected to wind flow and wind-generated waves that can influence the flow-field inside the water body, which is further complicated by the presence of emergent vegetation also subjected to the wind and consequent oscillation depending upon their flexibility. In the absence of vegetation, the problem of wind blowing over a water surface is not particularly new and has a long history (Charnock, 1955; Csanady, 2004). Wind flowing over a static water body such as a lake or reservoir (as in the original work of Charnock) is the main source of mechanical energy for turbulent mixing inside the water body. The wind flowing over the water surface causes a drift current in the direction it blows thus perturbing the water surface, which is called wind set-up (Hellström, 1941; Tsanis, 1989). This local pressure gradient generated due to the ‘tilt of the surface’ (Tsanis, 1989) creates a reversed flow at the bottom of the water body, as well as ensuring mass continuity in a vertical plane. Here, the difference of the physical process governing the propagation of gravity waves and the wind set-up should be noted. Gravity wave is a self sustained process initiated out of an initial perturbation before the wave is damped. Whereas the wind set-up is continuously sustained by air motion, which injects energy into the water and applies shear stress on the surface. Studies like Baines and Knapp (1965); Cheung and Street (1988); Goossens et al. (1982); Komori et al. (1993); Kranenburg (1987); Langmuir et al. (1938); Longo et al. (2012); Sanjou et al. (2010); Sanjou and Nezu (2011); Tsanis and Leutheusser (1988); Tsanis (1989); Wu (1975) have reported experimental investigation of wind induced water currents, focusing on both the surface motion and the counter-current flow but without vegetation. Some studies have discussed a simple analytical model of wind set-up by constructing one, two and three dimensional models and engineering models for wind induced counter-current flow but without any vegetation (Bukatov and Zavyalov, 2004; Belcher et al., 1994; Cioffi et al., 2005; Harris et al., 1996; Hunter and Hearn, 1987; Jin and Kranenburg, 1993; Koçyigit
and Koçyigit, 2004; Meyer, 2011; Rodi, 1980; Wu and Tsanis, 1995; Yang, 2001; Yang et al., 2008). The role of waves in vegetated system has been considered, but in these cases, the entire wave was imposed on the vegetation primarily to mimic tidal systems so as to study wave attenuation, equivalent bed roughness and friction factor inside aquatic vegetation canopy under wave forcing (Bradley and Houser, 2009; Fonseca and Cahalan, 1992; Hansen and Reidenbach, 2011; Infantes et al., 2012; Kobayashi et al., 1993; Manca et al., 2012; Mendez and Losada, 2004; Riffe et al., 2011; Thomas and Cornelisen, 2003). Others have also discussed the nature of the flow field inside a flexible aquatic vegetation under the action of wave forcing by means of laboratory experiments (Coops and Van der Velde, 1996; Gambi et al., 1990; Luhar et al., 2010; Lowe et al., 2005a; Reidenbach et al., 2007; Tanino et al., 2005; Tanino and Nepf, 2008) and by modeling (Li and Yan, 2007; Li and Zhang, 2010; Mullarney and Henderson, 2010). Flapping motion of the vegetation, a generic feature of many aquatic vegetation under oscillatory forcing like waves, also appears to enhance nutrient uptake (Koch and Gust, 1999; Huang et al., 2011; Lowe et al., 2005b; Nepf, 2012b; Thomas and Cornelisen, 2003). On similar lines, a few studies have addressed the characterization of turbulent structures and detection of sweep-ejection cycles and traveling vortex induced synchronous progressive waving action on aquatic flexible vegetation called ‘Monami’ (Ghisalberti and Nepf, 2002, 2006, 2009; Nepf and Ghisalberti, 2008; Okamoto and Nezu, 2013; Oldham and Sturman, 2001).

It is evident from this literature survey that progress has been made in understanding (i) the dynamics of wind-shear-water interaction without vegetation, and (ii) the flow dynamics in presence of flexible vegetation under wave forcing. Yet, all these previous studies in the second category have dealt with wave forcing generated by wave-makers, i.e., the whole water mass has been subjected to a wave forcing. Under this condition, some studies like Luhar et al. (2010), Lowe et al. (2005a) and
Lowe et al. (2005b) have used linear wave theory to interpret their results— for example the decomposition of the instantaneous flow-field into phase averaged, coherent and turbulent components. Other studies examining counter-current flow without vegetation like Tsanis and Leutheusser (1988) and Tsanis (1989) have analyzed their results without any regard to linear wave theory and employed parabolic mixing length models to close their turbulent stresses.

The present work related to wind induced flow in a water body falls in the middle of these two aforementioned approaches. The presence of oscillating flexible vegetation increases the complexity of the problem. No previous reference of this problem has been found in the literature where the emergent vegetation is subjected to a dynamic wind loading, while the wind applies a shear on the water simultaneously subjecting the whole system to a wave-turbulent interaction. Hence, the first goal of the present work is to describe the onset and magnitude of wind-induced water flow in a standing water body in the presence of emergent vegetation with varying density and rigidity. To build a theoretical framework assisting future model development, a second goal is to delineate under what circumstances the wave and turbulence dominated regimes are separable so as to allow standard turbulence theory and standard linear wave theory to be applied at those decoupled regimes.

To address these goals and issues experimentally, Particle Imaging Velocimetry (PIV) experiments have been conducted in the laboratory to explore the characteristics of turbulence induced by wind shear on a static water body systematically for different water heights ($h$) and mean wind speeds ($U_a$) for each of the following scenarios: no vegetation, rigid sparse vegetation, rigid dense vegetation, flexible sparse vegetation and flexible dense vegetation. Analysis of the experimental data facilitates the understanding of the effects of $h$, $U_a$, vegetation flexibility and vegetation density, all of which are external conditions needed in describing the flow-dynamics within the water body. Another important aspect of the present attempt is that no
instance of PIV experiments of such a type involving flexible moving canopy and wind on water have been found in the literature although PIV experiments with rigid vegetation and moving water flow have been conducted in the past as reviewed elsewhere (Nezu and Sanjou, 2008). It is demonstrated in the present work that such a PIV experiment is possible indeed with proper handling and choice of materials and methods.

6.2 Experiment

The PIV experiments were conducted in the Fluid Mechanics workshop at the Institute of Hydroscience and Engineering (IIHR) - University of Iowa. The dimensions of the flume can be found in panel (a) of figure 6.1. The wind was generated by a fan (with three preset wind speed settings) mounted above the flume. For the experimental runs with the canopy, nylon cable ties (4 mm wide, 1 mm thick) 'planted' on a test bed were used as a model vegetation. Two different vegetation densities, \( \lambda_d = 0.39 \) for sparse and \( \lambda_d = 1.04 \) for dense, were used. Full tie (or canopy) height \( h_c = 27.3 \) cm was used for flexible vegetation, while the same ties and same vegetation density cut to \( h_c = 7 \) cm represent the rigid vegetation (but \( h < h_c \) in all experiments). The \( \lambda_d \) was calculated as \( \lambda_d = n \frac{w_t h}{s_v} \) following Nezu and Sanjou (2008), where \( n \) is number of cable ties, \( w_t \) is the width of each tie, and \( s_v \) is the test bed ‘vegetated’ area. The \( h \) was used here in the estimation of \( \lambda_d \) instead of \( h_c \) because the vegetation was emergent for all runs as earlier noted. The selection of an optimal \( \lambda_d \) is not trivial given that it is an optimization between maintaining realistic vegetation densities, as well as maintaining sufficient open area to allow particle imaging that can be challenging where the vegetation is flexible and oscillating. For the purpose of imaging, water was seeded with neutrally buoyant particles (spherical hollow glass spheres 110P8, Potters Industries) and illuminated with a laser light sheet passed through a thin (\( \approx 1.5 \) mm) slit carved on the test bed. A high fre-
frequency camera (Motion Xtra NX4-S1- Integrated Design Tools, Inc.) was used for imaging, and a sampling frequency of 30 Hz has been used. The PIV data analysis was conducted using the open source software PIVLab (Thielicke and Stamhuis, 2010). Images for the experimental set-up and the test bed are shown in panels (b) and (c) of figure 6.1. The $U_a$ was measured using a standard hand held anemometer approximately at $z_a = 5$ cm above the water surface. The wave height was measured (although not synchronized with every experimental run) with a wave gauge (Kenek water level measuring system, servo type, SW-10.1). Wave frequencies ($f_w$) were obtained by Fast Fourier Transforming (FFT) the time evolution of the water surface and choosing the dominant peak frequency. The measurements were conducted midway of the test bed so that the waves are fully developed and representative of the wave condition. The wavelength ($\lambda_w$) was measured from visual analysis of the images of the water surface and one mean value was used. The wave velocity ($U_w$) was determined by using $U_w = f_w \lambda_w$. The wave amplitude ($A_w$) was measured from the average wave height collected in the water surface elevation data. The quantity $A_w/\lambda_w$, called wave steepness factor, was then computed. The wind and wave conditions are listed in the table 6.1:

<table>
<thead>
<tr>
<th>$U_a$ (m/s$^{-1}$)</th>
<th>$f_w$ (Hz)</th>
<th>$\lambda_w$ (m)</th>
<th>$U_w$ (m/s$^{-1}$)</th>
<th>$A_w$ (m)</th>
<th>$U_w/U_a$</th>
<th>$A_w/\lambda_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.83</td>
<td>4.0</td>
<td>0.076</td>
<td>0.30</td>
<td>0.00100</td>
<td>0.108</td>
<td>0.013</td>
</tr>
<tr>
<td>3.33</td>
<td>4.2</td>
<td>0.076</td>
<td>0.32</td>
<td>0.00129</td>
<td>0.096</td>
<td>0.016</td>
</tr>
<tr>
<td>3.37</td>
<td>4.4</td>
<td>0.076</td>
<td>0.34</td>
<td>0.00134</td>
<td>0.100</td>
<td>0.018</td>
</tr>
</tbody>
</table>

The wave steepness factor does not exceed 0.05 in nature (Yang, 2001) and the experimental condition here is well below that limit. Also, the $U_a$ range does not vary appreciably, though all of the three selected $U_a$ are sufficiently large to generate turbulence in the water system. As shown later, these differences in $U_a$ also generate
different stresses at the water surface due to the presence of emergent vegetation. Three $h$ values were used in the experiment and are designated as $H1$, $H2$ and $H3$ respectively, and for each $h$, three $U_a$ (and thus wave) conditions designated as $W1$, $W2$, and $W3$ were explored. Hence, different combinations of $h$ and $U_a$ are designated as $HiWj$, where $i = 1, 2, 3$ and $j = 1, 2, 3$. For each of the 9 $HiWj$ combinations, 5 vegetation configurations were studied as earlier noted: no vegetation, sparse rigid, sparse flexible, dense rigid, and dense flexible. The water height characteristics are listed in the table 6.2. The classification of water depth used here matches the experimental condition in Yang (2001). The average wavelength is calculated based on linear wave theory according to the formula (Yang, 2001)

$$\lambda_w = \frac{gT_w^2}{2\pi \tanh \left( \frac{2\pi h}{\lambda_w} \right)},$$

(6.1)

where $T_w = 1/f_w$ is the wave period and $g$ is the gravitational acceleration. This computation ($= 0.091m$) overestimates the visually calculated wavelength of 0.076m by about 18%.

Table 6.2: Water height conditions explored showing the measured water depth $h$ and the relative water depth $h/\lambda_w$ along with the designation used.

<table>
<thead>
<tr>
<th>$h$ (m)</th>
<th>$h/\lambda_w$</th>
<th>Depth designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>1.64</td>
<td>Deep</td>
</tr>
<tr>
<td>0.070</td>
<td>0.91</td>
<td>Intermediate</td>
</tr>
<tr>
<td>0.025</td>
<td>0.32</td>
<td>Shallow</td>
</tr>
</tbody>
</table>

Given the small domain, lateral boundaries of the flume, and wind speed generation method, it is instructive to assess whether the experiments here are representative of large water bodies. To do so, $U_a$ in the absence of vegetation was used in combination with Charnock’s equation (Charnock, 1955) to estimate the momentum roughness height ($z_o$) and subsequent $A_w$ as well as the momentum flux at the air-water interface. Charnock’s equation, valid for large fetch in the absence of waves
generated from distant storms (or swells) and for stationary and planar homogeneous air flow well above the viscous or wave-induced roughness sublayer, is given as

\[ U(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_o} \right); \quad z_o = 0.011 \frac{u_*^2}{g}, \tag{6.2} \]

where \( \kappa = 0.4 \) is the Von Kármán constant and \( A_w = \alpha_w u_*^2 / g \), with \( \alpha_w \) being a proportionality constant depending on wavelength and steepness of waves being generated (Csanady, 2004). With \( z_a = 0.05 \) m and measured \( U_a \), the estimated air-side \( u_* = (0.15, 0.18, 0.19) \) m s\(^{-1}\), resulting in \( A_w = (0.9, 1.3, 1.4) \times 10^{-3} \) m, reasonably close to the measured \( A_w = (1.0, 1.29, 1.34) \times 10^{-3} \) m when \( \alpha_w = 0.4 \). Adjusting by air and water densities, these air-side \( u_* \) translate to water-side \( u_* = (0.18, 0.22, 0.23) \times 10^{-3} \) m s\(^{-1}\) at the air-water interface. The horizontally averaged PIV measurements of \( u_* \) near the water-air interface varied from 0.4 to 2.5 \( \times 10^{-3} \) m s\(^{-1}\) across all points near the surface suggesting that the Charnock’s equation for the air-side is a lower-limit on such momentum transfer. This underestimation by Charnock’s equation is partly due to the proximity of the \( U_a \) measurements near the surface and partly due to the fact that the Reynolds stress is not constant near the water surface. To illustrate the first point, estimated \( A_w \) and \( U_a \) measured at \( z_a = 0.05 \) m are used to compute the Keulegan number (wave height to viscous length scale) that varied from 10 to 15 in all cases, which is quite low for the application of Charnock’s equation. Ideally, Charnock’s equation applies in situations where the Keulegan number exceeds 100. Hence, the use of the ‘rough-formulation’ for inferring \( u_* \) from \( U_a(z) \) via Charnock’s equation for \( z_o \) is likely to be underestimated. Also, the fact that the stress is increasing in magnitude away from the interface (as discussed and shown later) suggests that the PIV measurements away from the interface may over estimate inter-facial values. Nevertheless, these calculations do suggest that the flume experiments ‘mimic’ key aspects of wind dynamics and inter-facial momentum transfer over large water
bodies in nature despite the primitive wind generation mechanism and lateral edges of the flume.

6.3 Data Analysis

6.3.1 General considerations

Processing of PIV images produces velocity time series (both for the horizontal $u$ and vertical $w$ components) at every grid point that populate the region of observation (imaged by the high frequency camera, approximately 15 cm x 15 cm) except for those points obstructed for imaging by some moving element or the locations of the air region and test bed. These regions are blocked for the aid of image processing so that only the seeded fluid part of the image is analyzed. The grid size in the aforesaid observation window is 63 x 63 grid points consistent for all experimental runs. An instantaneous velocity field is depicted in figure 6.2 for the purpose of visualization. Rolling motions and reverse flow can be observed - providing a snapshot of the rich dynamics under consideration. Summary statistics for any flow variable are constructed by first time averaging at a point then horizontally averaging time-averaged quantities across the camera window field to produce profiles treated as space-time averaged quantities as common in canopy flow studies (Poggi et al., 2004; Raupach and Shaw, 1982). That is, instantaneous $u$ and $w$ at a point are first decomposed into a time-averaged and fluctuating components given as

$$u = \bar{u} + u', \quad w = \bar{w} + w', \quad (6.3)$$

where over-bar denotes a time-averaged quantity and primes denote fluctuating quantity from time averages. The local stress $\bar{u}'\bar{u}'$ and the longitudinal ($\sigma_u = \sqrt{\bar{u}^2}$) and vertical ($\sigma_w = \sqrt{\bar{w}^2}$) intensities can be computed at every grid point as well. Again, horizontal averaging is then applied over all time-averaged points to yield profiles of space-time averaged quantities (Poggi et al., 2004). These space-time averaged
profiles are presented as a function of normalized height $z/h$, where $z$ is the distance from the flume bed. That is, $z/h = 0$ is the channel bottom and $z/h \approx 1$ is the location of the mean air-water interface. For a given canopy configuration and $U_a$, these space-time averaged profiles are presented as a function of $z/h$ for the three $h$ scenarios to emphasize the (significant) effects of water depth on these space-time averaged flow statistics.

Because the flow field consists of simultaneous wave and turbulence effects, interpretation of the experimental data is subject to different choices. For example, Luhar et al. (2010) suggested a decomposition of the velocity data from a study of superimposed tidal flow and turbulence into a mean component (obtained by binning the data into different phases based on upward zero crossings and ensemble averaging the bins), a coherent wave component and turbulent fluctuations. However, since no prior knowledge is available about the nature of the flow field for the present problem from the literature, a logical choice would be examining the spectra and co-spectra of the co-existing wave and turbulence in the time series, which constitutes much of the material for this section. Indication of a near -5/3 power-law in the spectra can be treated as signature of fine-scaled turbulence presumed to be locally homogeneous and isotropic, while a flat spectrum would indicate unavoidable white noise. Any excitation close to the dominant wave frequency could indicate prevalence of wave dominance. Given that the noise content of PIV data can be high in such experimental configuration, identifying the noise component and removing it from the time series prior to any averaging (time and planar) becomes necessary.

In the spectral analysis here, $S_x(f)$ refers to the one-sided spectrum at frequency $f$ (in cycles per unit time or Hz) of an arbitrary zero-mean stationary stochastic process $x(t)$ with variance $\sigma_x^2 = \mathbb{E}[x^2] = \int_0^\infty S_x(f)df$. All spectra are first calculated at a grid point using Welch’s averaged modified periodogram method with 4 sections and 50% overlap, and with each section windowed with a Hamming shape and the
computed periodograms for each window are then averaged. These spectra are then horizontally averaged for each \( f \) to produce an ensemble \( S_x(f, z) \). A similar procedure was used for all co-spectral calculations \( S_{uw}(f) \), where \( \overline{u'w'} = \int_{0}^{\infty} S_{uw}(f) \, df \).

For certain illustrations, the space-time averages - referred to as ensemble \( S_u(f, z) \), \( S_w(f, z) \), and \( S_{uw}(f, z) \) are compared at three typical \( z/h \) labeled as top \( (z/h = 0.9) \), middle \( (z/h = 0.5) \) and bottom \( (z/h = 0.1) \), respectively.

Spectra and co-spectra for the no vegetation scenario (as reference case) and the flexible dense scenario are shown in figure 6.3 and figure 6.4, respectively. Panels (a), (d) and (g) present \( S_u(f, z) \), panels (b), (e) and (h) present \( S_w(f, z) \), and panels (c), (f) and (i) present \( S_{uw}(f, z) \) at the three \( z/h \) previously mentioned (top, middle, and bottom), respectively. In every panel, the spectra and co-spectra are plotted for all three \( h \) cases - H1 (deep), H2 (intermediate) and H3 (shallow) to emphasize the role of \( h \) at a given \( z/h \). Only the highest \( U_a \) (i.e. W3) is used for all these cases for the purpose of demonstration (and these cases are labeled as H1W3, H2W3, H3W3 in the figure legends). The -5/3 power-law is also presented in the panels featuring the \( S_u(f, z) \) and \( S_w(f, z) \) spectra to identify signatures of inertial subrange turbulence (if any). The following observations can be made from figure 6.3 and figure 6.4:

- For the no vegetation scenario (in figure 6.3), the shallowest spectra (i.e. those associated with H3W3) do not follow the -5/3 power-law at any \( z/h \), although this data appear to be contaminated by a large noise component. The deep \( h \) case (i.e. H1W3) is also ‘noisy’ for large \( z/h \), but shows the signatures of persistent inertial turbulent structures with decreasing \( z/h \) - following the -5/3 power-law faithfully at the middle and bottom \( z/h \). The behavior of the intermediate \( h \) case (i.e. H2W3) falls between the deep and shallow \( h \) scenarios. This intermediate \( h \) case also displays the existence of an inertial sub-range in the middle and bottom \( z/h \) levels. Possible explanation for this behavior is that
the flow near the air-water interface is subjected to extensive disturbances that prevent any inertial subrange to be resolved. Also, the temporal sampling (i.e. 30 Hz) is too coarse to delineate a possible restricted inertial subrange followed by viscous dissipation range at high frequencies. This is analogous to spectra of turbulence near solid boundaries where the velocity spectra commonly lack extensive inertial subrange scales in the roughness (or buffer) regions. For the deep flow cases (i.e. H1W3), after turbulence is generated at the water-air interface, eddies get the allowance (higher $h$) to populate the full range of scales analogous to many shear flows when $h$ becomes sufficiently large. In sum, the spectra appear to share some analogies with other shear flows (e.g. an inverted boundary layer) in terms of expectations as to the onset of an inertial subrange, where the generation mechanism occurs at the air-water surface, and with sufficient distance from the air-water surface (i.e. decreasing $z/h$), an inertial subrange forms provided $h$ is sufficiently large. There exists a peak in the $S_w(f)$ close to 4 Hz, which is the dominant wave frequency, indicating the effects of the waves to be significant but only for the deepest $h$ (i.e. H1W3) thereby illustrating the role of $h$ at a given $z/h$. Interestingly, the wave effects are captured most significantly at the middle $z/h$. The fact that the vertical velocity spectra also capture the same wave effects demonstrates the existence of vertical wavy structures. A plausible explanation might be that the rolling motions (or orbitals) from the passage of the traveling waves move downwards from the air-water interface before being damped by the presence of turbulence close to the bottom. For the intermediate and shallow $h$ cases, the orbitals might also be appreciably distorted by the presence of strong turbulence as previously discussed. The fact that the wave amplitude appears to be most visible in the middle layers (of H1W3) differs appreciably from experiments conducted by wave-makers, where the wave generation frequency is significant.
at all depths.

- For the flexible dense scenario (in figure 6.4), all three $h$ cases of deep, intermediate and shallow closely follow the -5/3 power law behavior- indicating the presence of fine-scaled turbulence characterized by approximate inertial subrange scaling at all three $z/h$. This pattern is displayed by both $S_u(f, z)$ and $S_w(f, z)$. This observation is significant, indicating that the presence of flexible vegetation introduces new length scales by means of vegetative drag that then leads to fine-scaled turbulence. There also exists a peak in the spectra close to the 4 Hz wave frequency. However, contrary to the no vegetation scenario, the wave peaks are much more ‘diffused’ around the 4 Hz frequency and they can be observed at all $z/h$ and for all three $h$ cases - deep, intermediate and shallow. The impact of surface waviness is much enhanced due to a weakened turbulence by vegetative drag. Whether or not new frequency peaks are introduced due to the waviness of the vegetation can be another interesting issue.

- The co-spectra appear noisy in all cases for both no vegetation and flexible dense scenario (despite planar averaging). However, there is an indication of co-spectral activity (positive and negative) close to 4 Hz for the flexible dense scenarios, though the activity seems to average out in the range of 2-5 Hz (meaning no significant net momentum transfer due to the wave component).

As observed from this spectral analysis of the instantaneous velocity time series, wave and turbulence appear to be intertwined in a complex fashion even when the spectra are horizontally averaged. The noise content of the data appear high (and contaminates the co-spectra significantly). Hence, it becomes necessary to reduce the spectral signatures of the noise first. After dis-infecting the individual series from noise, it might be possible to separate out the wave and turbulence and discuss their
bulk properties since there is a signature of a higher spectral energy density close to the dominant wave frequency.

6.3.2 Separating signal from noise in Fourier domain

To proceed with the exercise of separating the $u$ and $w$ time series into ‘signal’ and ‘noise’ - a Lorenz curve type analysis is performed. This curve was first proposed by the economist M.O. Lorenz in 1905 to quantify whether the imbalance in wealth distribution based on taxable income is increasing or decreasing in time (Lorenz, 1905). Since its inception, this curve has been extended to measure imbalances in energy distribution of any time series and in an arbitrary domain (e.g. time, Fourier, or wavelet) as discussed elsewhere (Katul and Vidakovic, 1998; Lorenz, 1905; Vidakovic, 2009). The basis of this approach is that the highly energetic events are clustered into few frequencies and once the frequency modes are sorted on their energetic basis, the high energy modes can be collected and translated back into the time domain to obtain the ‘cleaned’ signal. The rest of the frequencies corresponding to the lower energetic modes can be translated back to time domain to re-construct the noise time series thereby allowing a determination of the signal-to-noise ratio (SNR).

To proceed with this approach, a ‘cutoff’ energy threshold needs to be established above which the energy would correspond to ‘signal’ and below which the energy would correspond to ‘noise’. The theoretical basis for this threshold, which is based on the maximum curvature in the Lorenz curve, is discussed in appendix F. Another important assumption in this approach is that the noise modifies the spectral energy content of a series but not the phase-angle (i.e. surrogate to time location of coherent energetic events). Constructing a Lorenz curve on the Fourier coefficients aids in establishing this threshold as has been discussed elsewhere (Katul and Vidakovic, 1996, 1998; Vidakovic, 2009). Further details about this filtering method can be found in the appendix F. Only sample results of the outcome of the analysis
are shown in figure 6.5, where the original $u$ time series, the Lorenz-cleaned and the noise series are displayed. The ‘signal’ and the ‘noise’, when summed reconstruct the original series (by definition) and is confirmed here by the zero residual between the reconstructed series and the original noise-infected time series.

To lend confidence in the aforesaid analysis, planar averaged (over all realizations at a particular height) spectral energy densities of the ‘signal’ and ‘noise’ should be constructed. The noise, if un-correlated or ‘white’, exhibits a flat spectrum that is most evident at high frequencies. Figure 6.6 shows spectra for the signal and the noise for the flexible dense scenario for all three different $h$ cases- deep (H1), intermediate (H2) and shallow (H3). Panels (a) - (b), (e) - (f) and (i) - (j) show the spectral energy density for signal and noise respectively for the $u$ at three different $z/h$ levels - top, middle and bottom. Panels (c) - (d), (g) - (h) and (k) - (l) show the spectral energy density for signal and noise respectively for the vertical velocity for the same experimental configuration and locations. As observed, the spectra for noise are commonly flat at all levels of the flow for both the $u$ (panels (b), (f) and (j)) and $w$ components (panels (d), (h) and (l)). This imparts some confidence to the filtering performed on the original time series given that the spectra of the noise were not previously assumed. On the other hand, signatures of $-5/3$ scaling behavior can be observed in the ‘cleaned’ signal both for the $u$ (panels (a), (e) and (i)) and $w$ (panels (c), (g) and (k)) spectra at all $z/h$ considered. With this separation between signal and noise, the SNR for different experimental runs can be estimated based on the lorenz-cleaned signal and noise variances. Panel (a) of figure 6.7 shows signal to noise ratios by performing one to one comparisons between the variance of the horizontal velocity signal ($\sigma_s^2$) versus the variance of noise ($\sigma_n^2$) for all the experimental runs. As evident from this figure, the SNR is about $3 - 4$, which is moderate but expected for PIV data in such a setting. The coefficient of determination ($R^2$) between the signal and the noise components for all the experimental runs was also conducted
suggesting that signal and noise are reasonably uncorrelated or weakly correlated. Interestingly, the $R^2$ values between the signal and noise are least for the deepest flow (H1) cases (0-0.15), increasing slightly with the shallower cases, 0-0.18 for the intermediate cases (H2) and 0-0.30 for the most shallow (H3) cases.

6.3.3 *Separating wave from turbulence in de-noised signals*

The waves, now visible in the de-noised series as peaks around the dominant wave frequency of 4 Hz in figure 6.6, have been determined from separate measurements. Instead of appearing as a spike at 4 Hz, the peak in the de-noised series is partially diffused towards lower frequencies. Hence, a frequency bandwidth of 2 - 5 Hz can be selected and it can be assumed that any activity (or energy) within this bandwidth is wave induced. The rest of the series contains turbulent effects. Any other effect from the movement of vegetation might be included via another frequency window. If it is outside a clear window width, it might show up in the turbulent spectra because by then the wave energy has already been transferred to turbulent kinetic energy. To achieve this filtering, spectral energy of a series is first computed from the Fourier components and the part of the series contained in the specified frequency window. The spectral energy between the extremities of the frequency window is then linearly interpolated. Any peak above the interpolated energy is considered part of the wave component. The rest of the series is considered turbulent. Translating back the wave and turbulent components to time domain yields the now separated turbulent and wave series. The filtering approach is explained in appendix G. Figure 6.8 shows ensemble averaged wave and turbulent spectra computed from the wave and turbulent series after they are separated from each other using the frequency filtering approach for the flexible dense scenario. Every panel display the spectral energy density for three depth cases, deep (H1) in black, intermediate (H2) in blue and shallow (H3) in red. Panels (a) and (b) show the wave and turbulence spectra
respectively for the horizontal velocity and panels (c) and (d) show the wave and turbulence spectra for the vertical velocity for the top level of the flow depth for each cases. Panels (e) and (f) show the wave and turbulence spectra respectively for the horizontal velocity and panels (g) and (h) show the wave and turbulence spectra for the vertical velocity for the intermediate level of the flow depth for each cases. Similarly, panels (i) and (j) show the wave and turbulence spectra respectively for the horizontal velocity and panels (k) and (l) show the wave and turbulence spectra for the vertical velocity for the bottom level of the flow depth for each cases. Figure 6.9 represents the wave and turbulent spectra but for the no vegetation scenario. Similar figures for the flexible sparse scenario, rigid dense and rigid sparse scenarios were derived but are not shown for brevity. Key observations from these figures for the wave and turbulence are as follows: For the no vegetation scenario, the spectrum of wave always display a peak around 4 Hz as earlier noted. The peak is not a sharp one but is rather diffused towards lower frequencies. The wave spectra follows some scaling above 5 Hz (the top extremity of the frequency filtering window), but not below 2 Hz (the low end of the filtering window). This indicates that the wave component does not influence high frequency motion associated with fine-scaled turbulent eddies. However, the larger eddies comparable to the wave orbitals in size distort the wave motion and diffuse the wave energy towards lower frequency. Any other low frequency motion below 2 Hz display a noise-like flat spectrum. This phenomenon is true for all three $z/h$, for all three $h$ cases and for both $u$ and $w$. Similar behavior is exhibited by the flexible dense scenario. Hence, the fate of the wave motion due to interaction between wave and turbulence appears similar irrespective of presence of vegetation. However, they might vary in the energy content of the dominant wave modes.

The behavior of the turbulent motion varies from scenario to scenario as expected. In the no vegetation scenario, the turbulence close to the air-water interface
is governed by detached eddies. This is indicated by a flat spectrum at the lower frequencies and a spectrum close to a -1 power law (a signature of detached eddies in classical boundary layers) at the higher frequencies for all three \( h \) cases (Banerjee and Katul, 2013; Kader and Yaglom, 1991; Katul et al., 1996; Katul and Chu, 1998; Nikora, 1999; Perry and Chong, 1982). However, for \( z/h < 0.5 \), the deep and intermediate \( h \) cases (H1 and H2) become inertial, displaying a -5/3 scaling. The shallow \( h \) case (H3) does not display such an inertial behavior. This can possibly be explained by the fact that for the shallow \( h \) cases, turbulent eddies get distorted by the wave orbitals throughout all \( z/h \) - whereas for the deep and intermediate \( h \) cases, the wave orbitals do not directly distort the turbulent eddies for the deeper layers \( z/h < 0.5 \). Also, the behavior of the \( w \) spectrum is similar to the \( u \) spectrum. For the flexible dense vegetation, the turbulence is much more structured - meaning here that power-laws are evident over a broad range of time scales in the spectral domain. Even close to the surface, all three different \( h \) cases display -5/3 power law spectra apart from the middle and bottom \( z/h \). This is due to the possibility that the vegetative drag produces fine-scaled turbulence following wake-production. There exist diffused peaks at low frequencies superimposed on the turbulent spectra. These peaks arise from the oscillation of the flexible vegetation by wind. For the flexible sparse vegetation, the turbulent spectra is less organized than the flexible dense vegetation. The spectra close to the surface do not display a -5/3 scaling, but rather one close to a -1 scaling for all three height cases. Close to the middle and bottom \( z/h \), the spectra follows -5/3 scaling closely. It is also important to note that the shallow \( h \) case (i.e. H3) appears more structured than its counterpart for the no vegetation scenario. This indicates the importance of vegetative drag in the generation of fine-scaled turbulence component. For the rigid dense vegetation, the wave spectra contains more energy for the shallow \( h \) cases. The intermediate and bottom \( z/h \) display -5/3 scaling- again indicating of the positive effect of vegetative drag on
generation of fine-scaled turbulence. However, there are no additional peaks on the turbulent spectra- reflecting the effect of rigid vegetation- with almost no oscillation with the wind. The rigid sparse vegetation scenario does not differ much from the rigid dense scenario, but shows more structure (i.e. spectral power-laws) than the no vegetation counterpart.

6.4 Results and Discussion

Following the analysis of the high frequency $u$ and $w$ time series, their de-noising, and their separation into wave and turbulence, the two study objectives can now be addressed using all the 9 $HiWj$ combinations for each of the 5 vegetation configurations. In particular, the effects of the 5 vegetation configurations on the space-time averaged mean flow, turbulence and wave energetics, momentum transport, and isotropy are presented and discussed. As the $u$ and $w$ series have already been de-noised using the Lorenz filtering approach previously discussed, and the signal component has been decomposed into wave and turbulence using spectral filtering analysis, standard Reynolds averaging can be attempted. Moreover, wave and turbulent components of intensity and stress can be separately computed. Analyzing the original signal is retained in the context of mean velocities as the series become automatically de-meaned during extraction of wave and turbulent components.

6.4.1 Mean horizontal velocity

Figure 6.10 displays spatially averaged (horizontally across every level) profiles of the the mean horizontal velocity ($\overline{U}$) obtained with time-averaged raw $u$. Every panel presents the horizontal-averaged $u$ for all five scenarios - namely no vegetation, rigid sparse, rigid dense, flexible sparse and flexible dense. Panels (a), (b) and (c) present the deep (H1), intermediate (H2) and shallow (H3) $h$ cases respectively for the slowest $U_a$ (W1). Panels (d), (e) and (f) present the deep (H1), intermediate (H2)
and shallow (H3) \( h \) cases respectively for the intermediate \( U_a \) case (W2). Similarly, panels (g), (h) and (i) present the deep (H1), intermediate (H2) and shallow (H3) \( h \) cases respectively for the fastest \( U_a \) case (W3). As before, the normalized height \( z/h \) is used in all data representation. Key observations from figure 6.10 are presented based on the \( h \) scenario as follows:

- For deep \( h \) flow (H1 cases) with no vegetation, the flow direction is windward at the top (\( z/h \approx 1 \)) and reversed near the bottom of channel, the inflection point being about \( z/h = 0.5 \) for all \( U_a \). With rigid sparse, rigid dense and flexible sparse vegetation, similar profiles as the no vegetation case are observed at all \( U_a \). With flexible dense vegetation and with increasing \( U_a \), the reverse flow diminishes and at the highest \( U_a \), the whole flow becomes windward in direction.

- For intermediate flow (H2 cases), the flow dynamics become richer. With no vegetation, the flow remains windward at the top and reverse at the bottom, the inflection point about \( z/h = 0.5 \) at all \( U_a \) as the H1 cases. With rigid sparse vegetation and with increasing \( U_a \), the reverse flow diminishes and at highest \( U_a \), the whole flow becomes windward as in the H1 cases. With rigid dense vegetation, the flow is strongly forward at all \( U_a \). With flexible sparse vegetation, oscillatory and almost stationary flow is observed in the upper layers (\( z/h > 0.7 \)) and uniform forward flow is noticed at the bottom. With flexible dense vegetation, the whole flow is forward at all \( U_a \) as before.

- For shallow flow (H3 cases), the whole flow is windward or forward at all \( U_a \) in all scenarios.

As observed, the presence of reversed flow is an important feature in wind shear induced turbulent flow. The existence of counter-current flow in the bottom layers has
been observed by other studies as earlier discussed in the Introduction section (Tsanis and Leutheusser, 1988; Tsanis, 1989). However, the effect of $U_a$, $h$ and vegetative drag or flexibility renders the dynamics more interesting. Irrespective of the presence of vegetation, as the flow becomes shallow (i.e. H3), the flow appears forward in the direction of $U_a$. In deep flow (i.e. H1), a high drag and flexible vegetation resists the counter-current movement at the bottom regions and makes the whole flow forward. At intermediate flow depth (i.e. H2), a similar effect of vegetation density is present as in the deep case, with the difference that the whole flow can become forward with a higher $U_a$. Figure 6.11 summarizes the key observations regarding the mean flow for all the experiments in the form of a binary phase diagram. Panel (a) summarizes the results for the no vegetation scenario, panel (b) for rigid sparse, panel (c) for rigid dense, panel (d) for flexible sparse and panel (e) for flexible dense scenario. A value of 1 in this presentation indicates fully forward flow and a value of 0 indicates existence of a counter-current flow at some location. The transition from one type of flow to another is represented by a number in between 0 and 1. The switching from one type of flow to another is thus visible at all 9 different combinations of $h$ and $U_a$. It is interesting to note that the regions and extent of switching are different in every vegetation configuration, thus delineating the effect of vegetation flexibility and density in a clearer fashion on the onset of reversed flow.

6.4.2 Momentum fluxes

The effects of turbulence and wave on momentum flux are computed from the turbulent and the wave time-series, separately. For reference, the momentum flux obtained from the raw PIV measurements are also shown and discussed. Figure 6.12 presents the horizontally averaged turbulent stress ($\overline{u'w'}$) profiles using the original series (i.e. includes wave and turbulence). As is the case for the mean flow analysis, every panel presents the stress for all five scenarios - no vegetation, rigid sparse, rigid dense,
flexible sparse and flexible dense. Panels (a), (b) and (c) present the deep (H1), intermediate (H2) and shallow (H3) $h$ cases respectively for the slowest $U_a$ case (W1). Panels (d), (e) and (f) present the deep (H1), intermediate (H2) and shallow (H3) $h$ cases respectively for the intermediate $U_a$ case (W2). Similarly, panels (g), (h) and (i) present the deep (H1), intermediate (H2) and shallow (H3) $h$ cases respectively for the fastest $U_a$ (W3). Figure 6.13 and figure 6.14 represent the stresses computed separately from the turbulent and wave series. As observed from figure 6.12, the total stress (wave and turbulence) is small at the surface ($z/h = 1$), increasing gradually with decreasing $z/h$, but then diminishing again near the bottom ($z/h = 0$). The shape of these non-monotonic stress profiles provide a window to the mechanism of momentum transfer inside the water body. Momentum is transferred downward from the surface while being attenuated inside the water body, resulting in a small bottom stress. In the presence of vegetation, this attenuation is rather severe, resulting in the fact the the Reynolds stress in the no vegetation scenario is significantly larger than the other scenarios with vegetation. In some cases, there exists positive values of total stress close to the bed. The figures clearly suggest that turbulent stress is larger than the wave stress by an order of magnitude. Also, the wave stress is generally larger at the surface, diminishing steadily inside the water body with decreasing $z/h$, while the turbulent stress is largest in the middle of the flow ($z/h = 0.5$). This implies that the wave component of momentum is injected at the surface originating from the impinging effect of the wave orbitals. The turbulent component of the momentum, on the other hand, is maximum inside the water body, probably generated from the Kelvin-Helmholtz instability that produces rolling motions inside the water body roughly collocated with the inflection points in the mean velocity profile. Wakes generated in the inter-vegetation spaces contribute to momentum transfer; however, the measurements being two dimensional, these structures cannot be resolved in this experiment and require three dimensional stereo-PIV measurements.
6.4.3 Turbulent and wave intensity

Apart from the momentum transfer, the origin and transfer mechanism of the turbulent and wave energies are now discussed. Since the wave and turbulent series are already separated, it is possible to compare their respective intensities (called $\sigma_{u,T}$ for turbulent and $\sigma_{u,W}$ for wave, respectively). Figures 6.15 and 6.16 present horizontally averaged profiles of $\sigma_{u,T}$ and $\sigma_{u,W}$ in the same fashion as figure 6.10 for all three $h$ and for all three $U_a$ cases. Figure 6.17 shows the relative importance of $\sigma_{u,W}$ with respect to $\sigma_{u,T}$ across all $z/h$ and for each case in the same arrangement.

Key observations from figures 6.15, 6.16 and 6.17 are as follows:

- For most of the cases, $\sigma_{u,T}$ is maximum at $z/h = 1$ and it reduces almost monotonically with decreasing $z/h$.
- For the deep $h$ case (H1), $\sigma_{u,T}$ is maximum for the rigid sparse scenario, followed by the rigid dense scenario, the no vegetation scenario, the flexible sparse scenario and the flexible dense scenario, respectively. It is clear that addition of flexibility and density increases sheltering. For the intermediate $h$ case (H2), the no vegetation case registers maximum $\sigma_{u,T}$. For the shallowest $h$ case (H3), this pattern is less clear but the flexible dense scenario always corresponds to the minimum $\sigma_{u,T}$, i.e., maximum sheltering.
- The wave intensity ($\sigma_{u,W}$) shows consistent profiles for all the scenarios suggestive of a possible similarity solution to its behavior in terms of $z/h$. However, the wave intensity is generally less than the turbulent intensity. Another important distinction is that for the deepest $h$ case (H1), the wave intensity is maximum slightly below the water surface, and it tends to decrease towards the air water interface ($z/h = 1$).
- The turbulent intensity is generally greater than the wave intensity (about
twice), except in the highest $U_a$ (i.e. W3) and for the deep $h$ (H1) cases. For the deep (H1) and intermediate (H2) $h$ cases, the turbulent intensity is much higher comparative to the wave intensity close to the bottom and not so much close to the top. On the other hand, for the shallowest case (H3), the relative importance of the turbulent and wave intensities are roughly consistent across $z/h$. This indicates that for the deep and intermediate $h$ cases, the turbulence effects penetrate much more in $z/h$ than the concomitant wave effects. However, for the shallow case, both the turbulence and wave effects penetrate the entire $z/h$ domain although the turbulent intensities remain higher than their wave counterpart.

6.4.4 Isotropy

The discussion on turbulent energetics invites the followup question regarding the isotropic (or lack of) nature in such a complex flow. To examine isotropic behavior in bulk flow statistics, Figure 6.18 shows a one to one comparison between the root-mean-squared (rms) component of the horizontal ($\sigma_u$) and vertical velocity ($\sigma_w$) horizontally averaged series. Panel (a) uses the raw velocity directly obtained from PIV analysis in the comparisons. Panel (b) uses the series after filtering out the noise, and panel (c) uses the separated turbulent from wave series and panel (d) uses the separated wave series. In all cases, the majority of the experimental runs show a near one to one behavior - indicating quasi-isotropic turbulent characteristics. This finding is relevant to operational models of turbulent dispersion noted in the introduction section, which require computation of turbulent diffusivity across different directions. For example, the horizontal and vertical turbulent diffusivity may be formulated as $K_x(z) \sim \sigma_u(z)^2 \tau(z)$ and $K_z(z) \sim \sigma_w(z)^2 \tau(z)$, where $\tau(z)$ is a relaxation time scale of the flow determined from the turbulent kinetic energy and its mean dissipation rate. Quasi-isotropic turbulent rms implies that $K_x(z) = K_z(z)$, which is a practical
simplification when modeling turbulence in such a complex configuration.

6.5 Summary and Conclusions

The wind induced wave-turbulence interaction in the presence of rigid and flexible vegetation for different water heights (deep, intermediate and shallow) and different wind speeds (slow, medium and fast, although not varying widely) under five different scenarios-no vegetation, rigid sparse vegetation, rigid dense vegetation, flexible sparse vegetation and flexible dense vegetation was explored. Particle Imaging Velocimetry (PIV) experiments have been conducted and spectral analysis have been performed on the measured velocity time series to separate wave and turbulence effects on energy and momentum transfer under the aforementioned conditions. A Lorenz curve analysis has been performed on the raw velocity data to identify and filter out the noise. Surface wave characteristics are measured separately to identify the dominant wave frequency where it exists. The de-noised velocity component series are then analyzed in the frequency domain to identify wave activities around the dominant wave frequency to separate out the wave and turbulent components of the signal. The no-vegetation scenario is found to be analogous to an inverted boundary layer flow where turbulence is generated in the air-water interface region and an inertial sub-range can be found deeper into the flow provided the flow depth is sufficiently high. On the other hand, the wave effects originate from the impinging motion of the rolling orbitals that move downward, resulting in higher activity close to the air-water interface before getting damped by the turbulent effects deeper inside the flow. Presence of flexible dense vegetation enhances the generation of fine scaled inertial turbulence by introducing new length scales via vegetative drag. The oscillation of the vegetation also diffuses the wave activity beyond the dominant wave frequency range. Another important feature of the coupled wind-wave-turbulent flow problem is the existence of counter-current mean flow in case of large flow depth.
However, the flow can switch to a fully-forward one when reducing flow depth, or with increasing vegetative drag or flexibility under higher wind speeds. After a critical shallow flow depth, the mean flow is always in the direction of the wind flow. In terms of momentum transfer, significant difference is found to exist between the wave component of momentum flux and the turbulent component of momentum flux. The wave momentum flux is injected from the surface and attenuated monotonically deeper inside the flow, while its turbulent counterpart is maximum at the middle of the flow depth and highly attenuated close to the bed in the presence of vegetation. In terms of intensity, the wave intensity is generally less than the turbulent intensity although the turbulent intensity penetrates deeper than wave intensity inside the water for deeper flows. In shallow flows, both components coexist throughout the depth. It is also important to note that the vertical velocity components tend to exhibit similar energetics and momentum transfer mechanisms, indicating a quasi-isotropic nature. Future efforts will attempt capturing such a complex interaction by constructing numerical models of increasing detail and further experimentation involving turbulence measurements in air and across the water flow.
Figure 6.1: (a) Schematic diagram of the flume used for the experimental runs. (b) Experimental set up, water seeded and illuminated by laser. (c) Test bed with model vegetation.
Figure 6.2: Instantaneous snap-shot of the two dimensional velocity field for the flexible sparse scenario, deepest water level (H1), and slowest wind speed (W1) termed as H1W1. The arrow size reflects the instantaneous velocity magnitude at the 64 x 64 grid points within a single image.
Figure 6.3: Spectra and co-spectra for the no vegetation scenario as a function of frequency $f$ computed from the raw time series. Panels (a), (d) and (g) represent the horizontally averaged spectral energy density for $u$ ($S_u(f)$) and for three different $z/h$ - top, middle, and bottom, respectively. Similarly, panels (b), (e) and (h) represent the spectral energy density for $w$ ($S_w(f)$) at the same three $z/h$ and panels (c), (f) and (i) represent the co-spectra ($S_{uw}(f)$) for the same three depths $z/h$. In every panel, the spectra and co-spectra are plotted for all three $h$ cases - H1 (deep - in black), H2 (intermediate - in blue) and H3 (shallow - in red) for the highest $U_a$ (i.e. W3). The $-5/3$ power-law is also plotted in a black dotted line in panels representing $S_u(f)$ and $S_w(f)$ spectra to locate signatures of inertial subrange turbulence.
Figure 6.4: Same as Figure 6.3 but for the flexible dense vegetation scenario.
Figure 6.5: Sample time series for the flexible dense scenario for the deepest water-highest wind case (H1W3) demonstrating the decomposition of a velocity into 'signal' and 'noise' using spectral analysis and Lorenz-curve filtering. The series in blue color represents the original velocity time series (cm s$^{-1}$). The series in red color represents the signal (the high energy component) and the series in black color represents the noise. The signal and the noise, when summed reconstruct the original series, indicated by a zero residual from the raw time series and is plotted in green color. Also it is noted that the coefficient of determination between the signal and the noise is almost zero- indicating they are completely uncorrelated.
Figure 6.6: Horizontally averaged spectra for the signal (\(S_{u,s}\)) and the noise (\(S_{u,n}\)) for the flexible dense scenario for all three different \(h\) cases- deep (H1), intermediate (H2) and shallow (H3) cases. Panels (a) - (b), (e) - (f) and (i) - (j) show the spectral energy density for signal and noise respectively for \(u\) at three \(z/h\) as before - top, middle, and bottom. Panels (c) - (d), (g) - (h) and (k) - (l) show the spectral energy density for the signal and noise respectively for \(w\) at the same three \(z/h\). As observed, the spectra for noise is almost flat (shown by cyan dashed line) at all levels of the flow for both \(u\) (panels (b), (f) and (j)) and \(w\) (panels (d), (h) and (l)).
Figure 6.7: Comparison between the variance of $u$ for the signal ($\sigma_{u,s}$) and noise ($\sigma_{u,n}$) components for all the experimental runs. The one-to-one line is also shown for reference.
Figure 6.8: Horizontally averaged wave $S_{x,W}$ and turbulent $S_{x,T}$ spectra computed from the wave and turbulent time series after they are separated from each other using the frequency filtering approach for the flexible dense scenario. Every panel display the spectral energy density for three $h$ cases, deep (H1) in black, intermediate (H2) in blue, and shallow (H3) in red. Panels (a) and (b) show the wave and turbulence spectra respectively for $x = u$ and panels (c) and (d) show the wave and turbulence spectra for $x = w$ for the three $h$ cases and the top $z/h$. Panels (e) and (f) show the wave and turbulence spectra respectively for $x = u$ and panels (g) and (h) show the wave and turbulence spectra for $x = w$ for the intermediate $z/h$. Similarly, panels (i) and (j) show the wave and turbulence spectra respectively for $x = u$ and panels (k) and (l) show the wave and turbulence spectra for $w$ for the bottom $z/h$ for all three $h$ cases.
Figure 6.9: Same as Figure 6.8 but for the no vegetation scenario.
Figure 6.10: Horizontally averaged mean horizontal velocity, denoted by $U(z)$, as a function of normalized height ($z/h$) using the raw PIV series. Every panel presents $U(z)$ for all five scenarios - namely no vegetation (black + symbols, NV), rigid sparse (red dashed line, RS), rigid dense (blue dash and dotted line, RD), flexible sparse (thick green dots, FS) and flexible dense (black line, FD). Panels (a), (b) and (c) present the deep (H1), intermediate (H2) and shallow (H3) cases respectively for the slowest $U_a$ case (W1). Panels (d), (e) and (f) present the deep (H1), intermediate (H2) and shallow (H3) cases respectively for the intermediate $U_a$ case (W2). Similarly, panels (g), (h) and (i) present the deep (H1), intermediate (H2) and shallow (H3) cases respectively for the fastest $U_a$ case (W3).
Figure 6.11: Key observations regarding $U(z)$ for all the experiments in the form of a binary phase diagram. Panel (a) summarizes the results for the no vegetation (NV) scenario, panel (b) for rigid sparse (RS), panel (c) for rigid dense (RD), panel (d) for flexible sparse (FS) and panel (e) for flexible dense (FD) scenario. A value of 1 in the colormap indicates fully forward flow in the direction of $U_a$ and a value of 0 indicates existence of a counter-current flow. The transition from one type to flow to another is represented by a number in between 0 and 1.
Figure 6.12: Same as Figure 6.10 but for the total stress \((uw')\) computed from the raw PIV series (i.e. including noise, wave, and turbulence).
Figure 6.13: Same as Figure 6.10 but for the turbulent Reynolds stress \( \langle u'w' \rangle \) computed from the separated turbulent series.
Figure 6.14: Same as Figure 6.10 but for the wave stress ($\overline{uw}$) computed from the separated wave series. Note here that the range of stress values are not identical across panels.
Figure 6.15: Same as Figure 6.10 but for the streamwise turbulent intensity $\sigma_{u,T}$. 
Figure 6.16: Same as Figure 6.10 but for the stream-wise wave intensity $\sigma_{u,W}$. 
Figure 6.17: Same as Figure 6.10 but $\sigma_{u,W}/\sigma_{u,T}$. 
Figure 6.18: Comparison between the rms components of $u \ (= \sigma_u)$ and $w \ (= \sigma_w)$. Moreover, panel (a) uses the raw velocity obtained from PIV analysis, panel (b) for the series after filtering out the noise $\sigma_{u,s}$ and $\sigma_{w,s}$, panel (c) uses the separated turbulent series $\sigma_{u,T}$ and $\sigma_{w,T}$ and panel (d) uses the separated wave series $\sigma_{u,W}$ and $\sigma_{w,W}$. The one-to-one line is also shown for reference.
This dissertation has presented a systematic study of turbulent flow in natural environments in presence of boundary conditions of increasing order of complexity by means of analytical, numerical and experimental methods. Chapter 1 has presented the general idea and motivation behind this work. A number of different fields such as hydrology, environmental sciences, atmospheric sciences, ecology, mechanical engineering etc. require the knowledge of turbulent flow near an interface, either a solid wall, or a porous media like vegetation, or water bodies. Studies involving land/biosphere-atmosphere interaction thus need a thorough understanding of the physics of turbulent flow near the earth surface, or forested ecosystems or over water bodies, which are often quite complicated to account for due to presence of heterogeneities such as thermal heating/cooling, presence of edges and gaps or complex topography in vegetation or presence of aquatic vegetation in wetlands. Simple dimensional arguments and similarity theories fail in presence of such complex boundary conditions. On the other hand, extensive computational approaches such as Direct Numerical Simulations (DNS) or Large Eddy Simulations (LES) might be used in these cases effectively but are computationally very expensive. The reason
for this is that for high Reynolds number problems encountered in the atmospheric dynamics literature, there exists a large scale separation between the atmospheric boundary layer height and the Kolmogorov microscale. So it is beyond the capacity of our existing computational resources to resolve these problems in three dimension and with a useful high resolution, i.e., small time steps and spatial grid sizes. Thus there exists a scope for an intermediate approach where phenomenological models can be used to connect the bulk macro states of the flow with the micro-states of the turbulent flow, thus uncovering the physics of the turbulence in a more intuitive manner as well as proving physical justifications for the occurrence of different parameters that are observed in experimental studies. This argument is central to the theme of this dissertation.

In chapter 2, a phenomenological model based on a spectral budget approach has been developed to describe the quasi universal logarithmic scaling displayed by the longitudinal turbulent velocity variance in numerous high Reynolds number experiments and also previously predicted by Townsend’s attached eddy hypothesis. This bulk property has been connected to the statistical distribution of turbulent kinetic energy (TKE) across a multitude of scales, i.e. the spectra, which represents the micro-state of turbulence. Chapter 3 and chapter 4 has extended this framework to explain the variation of the longitudinal velocity variance under thermal stratification (unstable and stable respectively) which cannot be explained by the traditional Monin Obukhov Similarity Theory (MOST). Future works in this area will attempt to analyze the effect of short-circuiting of the TKE spectra inside vegetation canopies on bulk turbulent statistics.

Chapter 5 has attempted to answer the question of how important it is to resolve the turbulence explicitly to capture the mean flow in presence of a more complex boundary condition such as a vegetation canopy with edges and gaps by means of a simple numerical investigation. It has been found out that although standard turbu-
lent closure models such as K-theory (gradient diffusion parameterization) are known to fail inside vegetation canopies, just the interplay between the advective terms, pressure gradients and drag forces are sufficient to resolve the bulk flow structures such as recirculation patterns across edges and gaps. While these results are valuable for model-data integration and calibration for flux measurements, the framework is planned to be used to model aerosol deposition over canopies containing edges and gaps over a complex topography.

Other biosphere-atmosphere interaction problems such as wind shear induced flow in static water bodies and associated gas exchange are extremely important for quantifying greenhouse gas emission potentials of wetlands and other water resource management problems. However the knowledge of the physics of turbulence in such a problem with the full complexity such as the presence of air-water interface and the presence of flexible protruding vegetations is rare in the literature although it is very crucial. Thus controlled experiments are necessary to uncover both the macro and micro-states of the turbulence that exist in the water body with a complex interaction with wind shear induced waves. Chapter 6 presents the experiments along with data analysis that attempts to separate wave and turbulence statistics and study the effect of environmental controls such as water height, wind speed, vegetation density and flexibility on the flow statistics and energy spectra. The states of the wave-turbulence interaction and bulk flow nature are found to be sensitive to these controls. Future work will attempt to study the nature of coherent structures in such a flow such as sweep-ejection cycles and also attempt numerical modeling of the problem.

Overall, a few general important issues can be learnt from this dissertation that can be used for various practical applications. Different types of land-atmosphere interaction models and climate models require some prescription of the turbulence without resolving it in a fine detail. The results presented in this dissertation can advance the way turbulence is prescribed-by linking the bulk statistics to the fine scale
structure of turbulence. For example, different climate models and LES models tradi-
tionally use similarity theories to parameterize the longitudinal velocity variance. This prescription can be made much more accurate by including more physics in terms of the boundary layer height and the stability parameters as demonstrated by the dissertation, which cannot otherwise be predicted by similarity theories. These improved predictions can be used in a plethora of applications which necessitate any prescription of turbulence such as dispersion modeling, foot-print estimation, weather predictions etc. In more complex scenarios, the dissertation provides guidelines on the degree of which the turbulence needs to be resolved. For example, dispersion and air pollution studies involving a major roughness transition such as street canions, forest edges etc. can still capture the first order effects of turbulence without explicitly prescribing it as demonstrated by the dissertation. For other operational aspects like wetland and reservoir management, the experiments discussed in the dissertation gives a general picture for the first time about the state of turbulence and mean flow- which can directly be used for sediment and nutrient transport models. These studies can also be used for wetland health monitoring and quantifying greenhouse gas emission potential of different water bodies relevant for climate studies.
Appendix A

Alternative estimate of Townsend Coefficients

An alternate estimate of $C'_{TKE}$ is now discussed. To evaluate this estimate based on $C_b$, it is necessary to provide numerical values for $C''_K$, $C_{uw}$ and $C_H$ in the context of a certain interpretation of $k$. For three-dimensional isotropic turbulence, it can be shown that $C_H = (8/9)C_o^{-3/2}$ as discussed elsewhere (Schumann, 1994). Interestingly, this isotropic estimate of $C_H$ can also be recovered from the spectral budget here when production and viscous terms are absent so that the energy transfer $F(k_a)$ at any arbitrary $k_a$ is entirely balanced by $\bar{\epsilon}$ resulting in

$$\bar{\epsilon} = \left[ \nu_t(k_a) \right]^2 \left[ \int_0^{k_a} 2(C_o(\bar{\epsilon})^{2/3}p^{-5/3}dp \right] = \left[ \frac{3C_H C_o^{1/2} \bar{\epsilon}^{-1/3}}{4k_a^{4/3}} \right] \left[ \frac{3}{4} k_a^{4/3} 2C_o(\bar{\epsilon})^{2/3} \right]. \quad (A.1)$$

Noting that $\bar{\epsilon}$ and $k_a$ cancel out in Eq. A.1, and upon re-arranging, the $C_H = (8/9)C_o^{-3/2}$ is recovered even when interpreting $k$ as a one-dimensional cut in the longitudinal direction. With this formulation for $C_H$, $C'_{TKE} = 2C_b = (C_o)(3/2)(\kappa^{4/3} - (3/4)C_{uw})/\kappa^2$. Using $\kappa = 0.4$, $C_{uw} = 0.15$ as discussed before in the derivation of $F_{wu}(k)$, and $C_o = 1.55 \times [(18 + 24 + 24)/2]/55 = 0.93$ result in $C_{TKE} = 1.5$. Estimating $C'_{TKE}$ requires a further discussion about the proportionality constant $\beta$ because
\( C_{TKE} = \beta C_{TKE} \). In the inertial subrange and at \( kz > 1 \), \( \beta = (18/55)C_K/(33/55)C_K \approx (18/33) \approx 0.5 \). However, in the integrated spectrum, there are contributions from all wavenumbers and \( \beta \approx 1.0 \) based on \( \bar{e} = (1/2)(\sigma_u^2 + \sigma_v^2 + \sigma_w^2) \) with \( \sigma_w^2 + \sigma_v^2 \approx \sigma_u^2 \). It may be surmised that \( \beta \) varies between 0.5 and 1.0. With \( \beta = 1.0 \), \( C'_{TKE} = 1.5 \), thereby estimating \( B_1 = C'_{TKE}(1 + \ln(\alpha)) + (3/2)C''_K/\kappa^{2/3} = 3.0 \) and \( A_1 = C''_{TKE} = 1.5 \), which are higher than the numbers reported by Marusic et al. (2013). With a \( \beta \) of 0.8 (between 0.5 and 1.0), it results in \( B_1 = 2.6 \) and \( A_1 = 1.1 \), close to the range provided by Marusic et al. (2013) \( (B_1 = 1.56 - 2.3 \) and \( A_1 = 1.21 - 1.33) \).
Appendix B

Discussion on the details of the slab model used to estimate boundary layer depth in AHATS

The average boundary layer depth for morning and afternoon periods was estimated using a slab model described in Juang et al. (2007). Briefly,

\[
\frac{d\delta}{dt} = \frac{(\overline{w' T'_{ps}} - \overline{w' T'_{p\delta}})}{\gamma \delta},
\]

where \(\overline{w' T'_{ps}}\) and \(\overline{w' T'_{p\delta}}\) represent the turbulent sensible heat fluxes at the surface and at the top of \(\delta\), \(\gamma\) is the local lapse rate of the mean slab potential temperature \(T_p\) just above \(\delta\). Invoking the standard assumption that \(\overline{w' T'_{p\delta}} = -\beta \overline{w' T'_{ps}}\) (Tennekes, 1973), setting \(\beta = 0.3\) (Kim and Entekhabi, 1998) and \(\gamma = 11.6 \times 10^{-3} \text{K m}^{-1}\) (Juang et al., 2007), the temporal variations of \(\delta\) can be predicted from the time series of measured sensible heat flux collected near the surface after imposing an initial condition on \(\delta\) at some time \(t_0\). This initial condition \(\delta(t_0)\) for the slab model is given by the nocturnal equilibrium formulation in Zilitinkevich (1972), i.e.

\[
\delta(t_0) = 0.4 \left( \frac{\langle u_* \rangle}{f} \langle |L| \rangle \right)^{1/2},
\]
where the angle brackets represent that nighttime average of a quantity and $f \approx 10^{-4}\text{s}^{-1}$ is the Coriolis parameter. Time series of the average values of $u_*$, $\overline{T_p}$, and $\overline{u^'T_p^'}$ for each 27.3 minute block of data from AHATS were used as input for this slab model. Data from 12 separate days of the AHATS experiment that were free from bad or missing daytime blocks in the variables of interest were used. The predicted time evolution of $\delta$ from the slab model for each day, together with the ensemble mean can be found in Figure B.1. The morning and afternoon periods are denoted with dashed vertical lines.
Appendix C

Effect of imbalance in the TKE budget

The first order effect of a stability dependent imbalance in the TKE Budget is now explored. As discussed before, in stable conditions, there might be an imbalance between the production and dissipation terms indicating that the transport terms might be significant. In that case, the spectral budget should also contain terms arising out of Fourier transformation of those terms but it becomes overwhelmingly complicated. Instead, a simple first order correction accounting for the imbalance is sought that can be perhaps encapsulated by the imbalance term $\beta_2(\zeta)$ in $\tau$. The stability variation of this term from the AHATS campaign is compiled by Salesky et al. (2013) and the data is fitted with a power law for the present work ($\beta_2 = 1.33\zeta^{0.42}$; see inset in figure C.1). It is to be noted that the neutral limit of this term is zero and thus the formulation I is still valid. The only modification is in the Townsend coefficient $B_1$.

$$B_1 = \frac{3}{2} \frac{C_o}{\kappa^{2/3}} (\beta_2(\zeta) + \phi_m - \zeta) 2^{3/3} \gamma^{2/3} + 2c\gamma^2.$$  \hspace{1cm} (C.1)

Figure C.1 shows the effect of this term. The top panel shows $\sigma_u$ computed from the first formulation using a $\beta_2(\zeta)$ that is set to zero (original formulation-solid line)
Figure C.1: First order effects of a stability dependent imbalance in the TKE Budget. Top panel shows $\sigma_u$ computed using the formulation for zone I with a $\beta_2(\zeta)$ set to be zero (solid line) and a power law function of $\zeta$ (dashed line). $\beta_2$ is computed by fitting a power law to the AHATS data compiled by Salesky et al. (2013) as shown in the inset. The bottom panel shows the variation of the associated Townsend parameter $B_1$ with stability for the two approaches.

and is set to be a power law function of $\zeta$ (dashed line). The bottom panel shows the variation of the associated Townsend parameter $B_1$ with stability for the two approaches. As can be observed, the formulation with the correction is slightly higher compared to the first formulation, but the difference is not highly significant.
Appendix D

Biases in the comparison to forest data

The biases arising from using both formulation I and formulation II for the entire range of \( z/L \) are now explored. Errors are calculated as the absolute value of the difference between the modeled and measured \( \sigma_u \). Probability density functions (pdf) of errors are shown in Figure D.1. Black lines indicate the pdf of errors in the comparison with the formulation for zone I. Dashed black lines indicate the pdf of errors in the comparison with the formulation for zone II. Dotted red lines indicate the pdf of errors in comparison with MOST. The top row indicates the domain of formulation I (\( z/L < 0.1 \)) and the bottom row indicates the domain for the formulation II (\( z/L > 0.1 \)). As observed, most errors are concentrated on the lower side for our formulation. While for MOST, errors are more concentrated on the middle ranges, specially for lower stabilities. Note that for the forest data with \( z/L > 0.1 \), MOST (\( \sigma_u = 2.5u_a \)) performs well and can be compared to our models. This finding perhaps proves the point that boundary layer height is not so significant at such stable conditions. Also, the stability dependence of \( \sigma_u \) is weak. Both of those results are consistent with the formulations for zone II.
Figure D.1: Probability density functions (pdf) of errors in the comparison between measured and modeled $\sigma_u$ with our formulations and MOST for the forest experiments. Black lines indicate the pdf of errors in the comparison with the formulation for zone I. Dashed black lines indicate the pdf of errors in the comparison with the formulation for zone II. Dotted red lines indicate the pdf of errors in comparison with MOST. The top row indicates the regime of formulation I ($z/L < 0.1$) and the bottom row indicates the regime for the formulation II ($z/L > 0.1$).
For solving the systems of equation described in sub-section 5.2.1, the following algorithm has been used.

1. Initiate $U$ and $W$ fields. For the $U$ field, an exponential profile is used inside the canopy and a logarithmic profile is used above the canopy. $W$ is set to zero uniformly.

2. Initiate the vorticity $\omega$ field using the assumed $U$ and $W$ fields using Eq. 5.16,

3. Find the vorticity at the next timestep using Eq. 5.12.

4. Using the new vorticity, solve the Poisson equation (Eq. 5.19) with boundary conditions to obtain the streamfunction $\psi$ at the new timestep.

5. Compute $U$ and $W$ from $\psi$ using Eq. 5.17 and Eq. 5.18.

6. Repeat the steps with the new $U$ and $W$ until the differences between successive iterations in vorticity falls below a pre-set tolerance value (usually $5\times10^{-3}$).

7. Using the converged vorticity, determine the final updated $\psi$, $U$ and $W$. 

8. Solve the pressure Poisson equation (5.9) to determine $p(x, z)$ using boundary conditions defined afterwards.

The numerical details are listed in Table E.1 where $h$, $x$, $z$, $dx$, $dz$ indicate canopy height, horizontal and vertical domain sizes, horizontal and vertical grid spacings, respectively. The grid parameters are normalized by canopy height $h$. $C_d$ and $LAI$ indicate canopy drag coefficient and the canopy leaf area index respectively.

Table E.1: Numerical details of the problems solved.

<table>
<thead>
<tr>
<th>Problem Reference</th>
<th>$x/h$</th>
<th>$z/h$</th>
<th>$dx/h$</th>
<th>$dz/h$</th>
<th>$C_d$</th>
<th>$LAI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform canopy (Shaw and Schumann, 1992)</td>
<td>9.6</td>
<td>3</td>
<td>0.21</td>
<td>0.07</td>
<td>0.15</td>
<td>2-5</td>
</tr>
<tr>
<td>Presence of edge (Yang et al., 2006)</td>
<td>30</td>
<td>6</td>
<td>0.67</td>
<td>0.11</td>
<td>0.20</td>
<td>2</td>
</tr>
<tr>
<td>Presence of edge (Nieveen et al., 2001)</td>
<td>10</td>
<td>2</td>
<td>0.2</td>
<td>0.04</td>
<td>0.15</td>
<td>2</td>
</tr>
<tr>
<td>Presence of gap (Schlegel et al., 2012)</td>
<td>8.4</td>
<td>3.5</td>
<td>0.1</td>
<td>0.08</td>
<td>0.15</td>
<td>7.1</td>
</tr>
<tr>
<td>Presence of gap (Fontan et al., 2013)</td>
<td>5</td>
<td>2.8</td>
<td>0.13</td>
<td>0.06</td>
<td>0.30</td>
<td>3.5</td>
</tr>
</tbody>
</table>
Appendix F

Lorenz curve method

Details on the Lorenz curve filtering method is briefly reviewed. As discussed in section 6.3.2, it is necessary to find the inflection of the Lorenz curve. The following approach can be used to construct the Lorenz curve in the Fourier domain, to identify the inflection point and to separate the signal and noise in the frequency domain for a sample time series $u(t)$. The Fast Fourier Transform (FFT) of a velocity time series (say $u(t)$) is first computed (labeled as $F_{u(t)}$) and the energy content of $F_{u(t)}$, labeled $E_F$, is determined from the squared Fourier amplitudes (i.e. $E_F = \text{Real}(F_{u(t)})^2 + \text{Imaginary}(F_{u(t)})^2$). These $E_F$ are sorted in ascending order from minimum to maximum energetic coefficient and their corresponding indices are tracked. The Lorenz curve is constructed by summing sequentially the coefficients and their associated $E_F$ normalized so that when all the Fourier coefficients are accounted for - the abscissa is unity, and this final point includes all the energy (or variance) in the original time so that - the ordinate is unity. Hence, when all the Fourier coefficients are eliminated, the energy content is zero and when all the Fourier coefficients are present, the entire energy or variance of the original series
is accounted. By sequentially removing the most energetic coefficient and repeating the process until the least energetic coefficient is removed (on the abscissa) and plotting the concomitant drop in energy (on the ordinate), the Lorenz curve is obtained. The Lorenz curve is a convex curve describing the cumulative energy (ordinate, [0-1]) contained in the smallest energy components (abscissa, [0-1]). A sample Lorenz curve is shown in Figure F.1 for the $u(t)$ for the flexible dense scenario for H1W3 (deepest flow, fastest wind) configuration. The convexity of the curve relative to the diagonal or one-to-one line (corresponding to a perfectly balanced energy distribution) is directly proportional to the energy disbalance shown in Figure F.1. Hence, based on the convexity of the Lorenz curve, a global thresholding criterion can be formulated whose premise is as follows: an energy cutoff can be selected such that the proportion at which the gain (in parsimony) by hard thresholding (or setting to zero its energy content) an additional $E_F$ coefficient is smaller than the loss in energy. This point on the Lorenz curve corresponds to the maximum convexity shown in Figure F.1. The normalization of the abscissa and ordinate of the Lorenz curve is needed so that fraction of coefficients and energy losses are measured on a scale of 0-1 and are equally weighted. After the coefficients associated with the ‘signal’ (i.e. the more energetic ones) and the noise ‘the least energetic ones’ are delineated in terms of their position on the Lorenz curve, the Inverse Fast Fourier Transform (IFFT) of the thresholded and non-thresholded coefficients can be computed to separately construct the noise and signal time series, respectively, assuming phase-angles are not altered. This is the outcome of the series in Figure 6.5.
Figure F.1: Lorenz curve (blue line) constructed following the procedure described in appendix F. The abscissa indicates the fraction of Fourier coefficients ($In$) associated with the original time series (unity indicates all Fourier coefficients are present) and the ordinate indicates the cumulative energy or squared Fourier amplitudes (unity indicates all the variance in the original series is explained when all Fourier coefficients are included). The dashed lines indicate the point computed from the maximum inflection point coinciding with the slope of the one-to-one line.
Appendix G

Frequency Filtering to separate wave and turbulence

After the signal is isolated in the frequency domain ($F_S$), the following algorithm is added to separate the wave and turbulence from the signal. A frequency window (2-5 Hz) is chosen, the extremities of which are labeled as $f_l$ (low frequency bound) and $f_h$ (high frequency bound). The amplitude ($A_S$) (obtained by taking square root of the sum of the squares of the real and imaginary parts) and phase information ($\phi$) of the signal within the frequency range are determined. Fit a straight line between the amplitudes at the extremities between $f_l$ and $f_h$. The interpolated straight line constitutes the amplitude of the turbulence signal ($A_t$). Compute the amplitude of the wave from $A_w^2 = A_S^2 - A_t^2$. Determine the Fourier coefficients of the wave by $F_w = A_w[\cos(\phi) + i\sin(\phi)]$ and turbulence by $F_t = A_t[\cos(\phi) + i\sin(\phi)]$. Compute IFFT on $F_w$ and $F_t$ to reconstruct the wave and turbulent time series.
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