Why Do Firms Hedge?
An Asymmetric Information Model

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In the 1980s, volatility in interest rates and currencies led corporate financial managers to consider and undertake hedging on a scale that was unprecedented. This awareness of hedging as a risk management tool was driven by a number of factors. There was (and still is) the pervasive belief that disasters like the savings and loan disaster of 1980s could have been avoided if the institutions were properly hedged against interest rate risk (this view was expressed in the Wall Street Journal on August 17, 1993). Similarly, numerous anecdotal examples of firms having large losses or going bankrupt due to their failure to hedge exchange rate movements exist. One famous example is Laker Airways, where Laker’s costs of borrowing (on airplanes) were in dollars while its revenues were split evenly between dollars and pounds (see Shapiro [1989, pp. 275–276] for more details on this example). Facilitating hedging, there has been an enormous increase in the liquidity of many derivative markets (whether traded or over the counter) that provide instruments for hedging corporate risk.1

Given this attention to hedging at a corporate level and the proliferation of complex hedging techniques (largely driven by academic research on option and futures pricing models), the paucity of academic research on the fundamental question of why and when firms should hedge is surprising. In fact, the traditional full information perfect capital markets model of the firm says very little about why firms hedge and implies that whether firms hedge or not is irrelevant as investors can undertake the necessary hedging activities by themselves. This view is expressed in an article by Culp and Miller [1995]; they state that “most value maximizing firms do not, in fact, hedge.”

While the full information perfect capital markets paradigm has little to say about why firms hedge, other important paradigms like the option-pricing paradigm imply that equityholders may want to undertake risky activities as the option value of equity is increased by such variance-increasing activities. Hence, the main ideas of corporate finance have little to say on why firms hedge and seem to indicate that (if anything) there are strong incentives against hedging.

Our article presents an explanation as to why firms hedge based on a theory of managerial responses to asymmetric information. The key insight is that managers who have superior abilities with respect to some risks or uncertainties will try to ensure that their superior abilities are quickly discovered by the market. To ensure this, they will try to hedge those risks that are not under their control and where they have no exceptional abilities. Thus, hedging reduces the noise in the learning process by “locking-in the alpha” from the superior ability. In contrast,
managers with inferior abilities have incentives to reduce the efficiency of the learning process. They would prefer that all managers undertake risky variance-increasing activities. However, given that superior managers undertake hedging activities, lower ability managers may or may not hedge. As we will show, their decision to hedge or not to hedge depends on how much lower their ability is relative to managers of superior ability.

This insight assumes that managers are mainly concerned about their managerial reputations. A strong argument can be made that managerial compensation is related to equity performance. Hedging activities are inherently risk reducing and reduce the value of the equity when there is preexisting debt or when there are government loan guarantees or FDIC insurance (as is the case for banks). If managers hold some fraction of equity and are also concerned about equity values, higher ability managers will not hedge unless the ability difference is substantial. When they do hedge, the equity option value forgone acts like a signaling cost in a traditional signaling model. The higher ability manager knows that his performance is going to be better. As a consequence, the probability of bankruptcy is much less, making the equity option less valuable at the margin. The lower ability manager has a greater probability of bankruptcy and hence has a more valuable equity option at the margin. Consequently, the equity option increases the incentives of the higher ability manager to hedge.

Our model thus provides a rationale for why firms hedge and when firms will undertake hedging activities. In particular, our model indicates that if costs of hedging like the reduction in the equity option value are present, the firm managers will undertake hedging activities only when they believe that they have superior abilities and that these superior abilities result in performance substantially higher than that of other managers in the industry of lesser ability so as to compensate for these costs of hedging. In addition, our model indicates a relationship between managerial compensation and hedging policy. The greater the fraction of equity that the manager owns, the higher the implicit cost of hedging, and hence, hedging occurs only when the manager has a substantial performance differential over managers in other related firms.

These results are consistent with a recent empirical study by Tufano [1995] on the gold mining industry. Tufano found that younger managers are more likely to hedge than older managers. Since there is likely to be greater uncertainty about the ability of younger managers, this is consistent with the implication of our model that greater dispersion of abilities leads to more separation, that is, higher ability young managers hedge, while lower ability young managers do not hedge. With a cost of hedging, the model delivers the conclusion that a high difference in abilities leads to a “separating” equilibrium while a low difference in abilities leads to a “pooling” equilibrium (where managers of higher and lower abilities do not hedge). Again, this is consistent with Tufano’s results.

A second testable implication of our model is as follows. When the costs of hedging are low, less separation occurs. In contrast, when the costs of hedging are high, more separation occurs. Therefore, the value of firms that hedge relative to firms that do not hedge is much higher when the cost of hedging is higher. This is an implication of the model that is potentially testable. This implication is consistent with Tufano’s result that managers with stock options do not hedge as such managers face high costs of hedging. However, a test of our model would also require that managers with stock options who hedge must be of substantially higher ability than managers without stock options who hedge (with the usual ceteris paribus assumption).

While hedging is not observable, subsequent to the event, one sometimes knows whether a firm hedges or not. This is because ex-post firms voluntarily disclose hedging activity in the notes to the balance sheet. If our story of hedging is correct, the profits (earnings) of firms in the quarter or year that they hedge should be higher and the volatility of cash flows should be lower than usual. Such a test differs from tests of signaling models where profits (earnings) are higher for firms that signal as no volatility implications are present. Because hedging reduces volatility, the volatility reduction is an additional implication. Some preliminary empirical work by DeGeorge, Moselle, and Zeckhauser [1995] using Compustat data is supportive of this implication of the hedging hypothesis.

Our explanation differs from the other previous explanations that have been provided. In an early article on this topic, Smith and Stulz [1985] proposed risk aversion and taxes as rationales for firm hedging. Campbell and Kracaw [1987] used the ideas in Holmstrom [1979] to propose hedging as a method for ameliorating agency
problems. Froot, Stein, and Scharfstein [1992] used the fact that outside capital is costly to raise as a rationale for hedging. In a article that is closer to our ideas, DeMarzo and Duffie [1993] presented a model with risk aversion and symmetric information where hedging is optimal. None of these articles presents what to us seems to be the fundamental rationale for hedging: the idea of “locking-in” performance (or alpha in an investment context). We believe that this is an important reason why hedging occurs and thus emphasize it in our model. None of the other articles in the literature consider what the costs of hedging are and how these costs for hedging are balanced against the incentives to hedge. The closest article in the literature is by Ljungqvist [1994], who shows in a model with asymmetric information that there is an incentive for firms with low output to speculate. This is somewhat similar to some of our results on the model without hedging costs.3

Our emphasis on hedging as a decision by managers to “lock-in” performance where they have an advantage and eliminate risks that are not under their control is important. Hedging is not elimination of all risk but the management of risk.4 Alternative explanations like bankruptcy costs or the higher external costs of funding would imply corner solutions where managers try to eliminate all risks; this distinction between risks where managers have an advantage and risks that are not controllable by the manager is less important. We do not believe that hedging involves the elimination of all risk but rather only risks where the manager or firm does not have any special advantage. In particular, our model implies that firms hedge their risks only when they are sufficiently different from other similar firms in their abilities and share a common risk that is not controllable by managers. Only under these circumstances is it worthwhile to pay the implicit cost of hedging, the reduced equity option value or the reduced FDIC insurance option value.

Finally, we emphasize that our model is not a signaling model as hedging is an unobservable activity. This unobservability of hedging makes it closer to the models of learning by Holmstrom [1982], Campbell and Marino [1994], Fudenberg and Tirole [1986], Narayanan [1985], Palfrey and Spatt [1985], Scharfstein and Stein [1990], and Stein [1989]. In these models, agents do not have private information but undertake unobservable actions that affect the ability of the market or other agents to learn about the agent’s underlying talent. Our model differs from these models in that the agent undertaking the unobservable action has some private information about his ability; this plays an important role in determining his optimal action. The other articles that we are aware of with this idea that agents have differing incentives to manage the measurement process are by Allen and Gale [1990], Prendergast and Stole [1996], and Zweibel [1995]. Finally, DeGeorge, Moselle, and Zeckhauser [1996] presented a similar idea.6

In Allen and Gale [1990], the incentives of agents to manipulate the learning process is used in the context of a model that tries to explain why contracts do not contain all contingencies. Prendergast and Stole [1996] found that managers exaggerate their own information to appear as fast learners when they are young but eventually become conservative and are unwilling to use new information. In Zweibel’s [1995] work on corporate conservatism, managers who are behind tend to make efficient decisions while those ahead of the game tend to be conservative in their decision making.

While all of these articles also model the idea that there are managerial incentives to distort the learning process, we believe that the unique contribution of our article is to identify the costs and benefits of hedging. In particular, while hedging improves the informativeness of the learning process (the benefit), it has the associated cost of giving up some of the equity option value. This benefit–cost trade-off is important in delivering the results that we obtain that hedging should be undertaken by higher ability managers only when the benefits (the greater informativeness of the learning process) exceed the costs (the equity option value reduction).

THE MODEL WITHOUT HEDGING COSTS

We consider a model where managers run a firm for two periods. In the first period, the manager can invest $1 in a project. The project yields a random amount $1 + \gamma - \epsilon \Delta \tau$ at the end of the second period, hence $\gamma - \epsilon \Delta \tau$ represents profits. The random payoff $\gamma$ is that part of profits is under the control of the manager and can take one of two values $\gamma_h$ or $\gamma_l$ where $\gamma_h > \gamma_l$. The probability of the payoff $\gamma$ yielding the high state ($\gamma_h$) is $p$, which depends on the ability of the manager.

In contrast, the random factor $\Delta \tau$ is not under the control of the manager. This random factor $\Delta \tau$ can take the values $\delta$ or $-\delta$ with equal probability. The realization of
the random factor is observable to the market. However, the firm’s exposure to this factor \( e \) is not known to the market and can take one of three values, \( h \), 0, or \(-h\). The ex-ante probability of zero exposure is \( s \) while the probability of exposures \( h \) or \(-h\) is 0.5(1 - \( s \)). We also assume that \( y_i - y = h \delta \). In this discrete space model, this assumption is needed to ensure that stochastic process due to the product of the exposure \( e \) conditional on the realization of the factor \( \Delta r \) is noise for the learning undertaken by the market.

There are a number of interpretations that one can give to this setup. One is that of a bank where the payoff \( y \) represents credit risks where different banks differ in their abilities, while the payoff \( \Delta r \) represents interest rate risks for which banks (or classes of banks) do not have any competitive advantage in prediction. While interest rate changes are observable, the exposure to interest rate risks, \( e \), is not known by the market. Often, we will use this interpretation to illustrate our ideas. Another interpretation is that of a multinational firm where the process \( y \) represents part of the profits that is related to the ability of managers, while the process \( \Delta r \) represents exchange rate risks, and \( e \) is the unknown exposure to exchange rate risks.

Given the assumptions that we have made on the discrete state space, four possible states of the world can occur; we will label them \( z_1, z_2, z_3, \) and \( z_4 \). The probability of each state occurring, conditioning on the interest rate factor realization, depends on whether the manager chooses to hedge or not hedge the cash flow and is shown in Exhibit 1. In this exhibit, index \( i \) refers to the manager’s specific ability (we will discuss this further later) and the indexes \( NH \) and \( H \) represent the actions not hedging and hedging, respectively.

The probability distribution over the outcomes \( z_1 \) to \( z_4 \) is different when the manager hedges and when the manager does not hedge. When the manager does not hedge, the market cannot distinguish whether profits are high because the manager has good ability (skill) or because of luck (conditional on the factor \( \Delta r \), the exposure \( e \) was in the right direction). For example, with no hedging (in either interest rate factor realization) the market does not know whether the outcome \( z_2 = y_i \) has occurred or whether the outcome \( z = y_i + h \delta \) has occurred. This inability to decompose profits perfectly is crucial for hedging to be valuable. If the market were able to know the exposures to interest rate risk, there would no role for managerial hedging.

In the example we presented of the bank manager, this corresponds to an inability to distinguish whether the manager is skillful at credit analysis or whether the manager had a positive exposure to interest rate risk and the interest rate movement was favorable. We note that in our set up, the symmetry assumption ensures that the ex-post realization of the factor \( \Delta r \) is uninformative about the exposure \( e \) and the posterior probabilities of the outcomes \( z_1 \) to \( z_4 \) conditional on the realization \( \Delta r \) are just the prior probabilities. As a consequence, we suppress the dependence of the learning process on the realization of the factor \( \Delta r \).

Finally, when the manager does hedge, we assume he completely hedges the risk away by setting his exposure to zero. In the example we gave of the bank, this corresponds to making sure that the hedged bank is neutral to interest rate risk. In the example of the multinational firm, it corresponds to locking in revenues or costs so that they do not depend on the fluctuations in exchange rates. When the manager hedges, the outcomes \( z_1 \) and \( z_4 \) do not occur. The profit process is less noisy and more informative about the abilities of the agents (if both managers were not to hedge).

The exact probability with which the various states occur (given the manager decides to hedge or not hedge) depends on the ability of the manager. To capture this differential ability of managers, we assume that there are two types of managers—the high ability manager (manager 1) and the low ability manager (manager 2). The market’s prior probability that the manager is of higher ability is 0.5. We also assume that \( p_1 = 1 - p_2 \); this symmetry assumption ensures that none of our results are driven by asymmetric profit structures. We define \( d = p_1 - p_2 \). This is a measure of the difference in abilities and plays an important role.

### EXHIBIT 1
Probability of Each State Occurring

| State | NO HEDGING | Probability of Occurrence \( p(z_i | NH, t) \) | HEDGING | Probability of Occurrence \( p(z_i | H, t) \) |
|-------|------------|---------------------------------|----------|---------------------------------|
| \( z_1 \) | \( y_i + h \delta \) | 0.5(1 - \( s \)) \( p_1 \) | 0.5(1 - \( s \)) \( 1 - p_1 \) | 0.5(1 - \( s \)) \( p_1 \) |
| \( z_2 \) | \( y_i \) | \( p_2 \) | \( 1 - p_2 \) | \( 1 - p_2 \) |
| \( z_3 \) | \( y_i - h \delta \) | \( p_1 \) | \( 1 - p_1 \) | \( 0 \) |
in our results. Finally, $s > 1/3$, an assumption that is needed to ensure that when both firms do not hedge, the posterior probabilities are monotone in the outcome $z_j$. Thus $m = s - 0.5(1 - s)$ is always positive. Low $s$ values imply greater noise due to fluctuations in the exogenous noise process.

Given this basic structure, we specify the objective of the manager. We follow Holmstrom [1982] and Holmstrom and Ricart i Costa [1986] and assume that managers receive the expected value for that period up front as the wage payment. However, contracts are renegotiated at the end of each period. If the ability of the manager, $p_i$, were known, manager $i$ would receive as value $p_i y_b + (1 - p_i) y_i - r$ as wage payments. Setting $\beta = y_b - y_i$ and $\kappa = y_i - r$, under full information the manager receives a wage $p_i \beta + \kappa$.

However $p_i$ is not known, and hence the manager obtains $\hat{\beta}(0.5p_i + 0.5p_2) + \kappa = \beta(0.5) + \kappa$ (using the fact that $p_2 = 1 - p_j$) as the wage for the first period. The renegotiated wage for the second period is then given by:

$$B[P(1 | z_j)p_1 + P(2 | z_j)p_2] + \kappa. \quad (1)$$

Here, $P(1 | z_j)$ is the posterior probability of the manager having higher ability given that outcome $z_j$ is observed (a similar definition holds for $P(2 | z_j)$). We suppress the constants $\beta$ and $\kappa$—they are scaling variables and play no role in the results of this section. Since the current wage is fixed, the manager maximizes the expected value of his future expected wage in the next period. This implies that the manager cares only about his reputation and not about the equity value of the firm. In the next section, we relax this assumption by allowing managers to hold equity.

Hedging is an unobservable activity. Thus, the market cannot condition its posterior beliefs about the ability of the manager on whether a firm hedges or not; beliefs are functions only of observed profits. Given our specification of the manager’s objective, we define the Bayesian Nash equilibrium of the updating game as

(a) a market belief function $m^*$: $\{z_j, z_2, z_3, z_4\} \rightarrow [0,1]$, where $m^*(z_j)$ is the posterior probability, given the observation $z_j$, that the manager is of higher ability.

(b) a firm action function $a^*$: $\{1,2\} \rightarrow [0,1]$, where $a^*(i)$ is the probability with which the manager hedges (we are allowing for mixed strategies here).

We require that the pair \{$m^*(\cdot), a^*(\cdot)$\} satisfy the conditions that

1. given $m^*(\cdot)$, manager $i$ chooses

$$a^*(i) \in \arg \max_{a(i)} U(m^*(\cdot), a(i), i) \quad (2)$$

where

$$U(m^*(\cdot), a(i), i) = a(i) \left(\sum_{j=1}^4 p(z_j | H, i) m^*(z_j)(p_1 - p_2) + p_2\right)$$

$$+ (1 - a(i)) \left(\sum_{j=1}^4 p(z_j | NH, i) m^*(z_j)(p_1 - p_2) + p_2\right) \quad (3)$$

2. given $a^*(i)$, $i = 1, 2$, the market belief function $m^*(\cdot)$ satisfies

$$m^*(z_j) = \frac{p(z_j | a^*(1))}{p(z_j | a^*(1)) + p(z_j | a^*(2))} \quad (4)$$

for all $z_j$ observed in equilibrium. Here $m^*(z_j)$ is the equilibrium posterior probability that the manager is of higher ability given the profit level $z_j$. By definition,

$$p(z_j | a^*(i)) = a^*(i) p(z_j | H, i)$$

$$+ (1 - a^*(i)) p(z_j | NH, i). \quad (5)$$

The definition of $m^*(z_j)$ requires Bayes consistency over all profit levels observed in equilibrium but does not restrict market beliefs when a profit level is not observed. This occurs only when both firms hedge. In this case, profit levels $z_3$ and $z_4$ are not observed and there is no restriction on market beliefs conditional on observing these states.

Finally, we note that our definition of the strategies chosen by the manager allows for randomization. In the context of our discrete space model, randomization can be interpreted as partial hedging.

**Equilibrium in the Model without Hedging Costs**
A key idea behind our approach to hedging is the notion that managers of higher abilities will hedge so as to lock-in their profits and improve the informativeness of profits about their superior abilities. Therefore, we first investigate an intuitive equilibrium where the higher ability manager always hedges.

**Theorem 1** An equilibrium involving the higher ability manager hedging always exists. The lower ability manager’s decision depends on the difference in abilities between the managers, $d$, in the following way:

(i) For $0 < d < \delta^*_H$, $a^*(2) = 1$.
(ii) For $\delta^*_H < d < \delta^*_S$, $0 < a^*(2) < 1$.
(iii) For $\delta^*_S < d < 1$, $a^*(2) = 0$.

We refer to this equilibrium as a Type A equilibrium.

**Proof**: See Appendix.

We label the equilibrium in Theorem 1 as a Type A equilibrium. A Type A equilibrium occurs for all possible values of parameter $s$ that correspond to all possible variances for the noise process $r$. Thus, the Type A equilibrium requires no restrictions on the parameter space.

The intuition behind the Type A equilibrium is as follows. It is in the interest of the higher ability manager to hedge because hedging leads to a more informative learning process. However, when the difference in the abilities of the two managers is low, it pays for the lower ability manager to follow suit and also hedge. A low difference in abilities implies that the learning process is not informative even when both managers hedge (although more informative than when both managers do not hedge); by not hedging, the low ability manager runs the risk of the extreme states $z_i$ and $z_j$ occurring and revealing his ability. In contrast, when the difference in abilities is high, the learning process is very informative when both firms hedge; by not hedging, the lower ability manager has some chance of being mistaken as the higher ability manager when the realizations of the process $r$ are favorable.

While Type A equilibrium is most consistent with our insight and exists no matter what the parameter configuration is, we need to investigate other possible equilibria. We first show that no equilibrium exists where the lower ability manager hedges and the higher ability manager does not hedge or randomizes between hedging and not hedging.

**Theorem 2** The lower ability manager hedging and the higher ability manager not hedging or randomizing between hedging and not hedging can never constitute an equilibrium.

**Proof**: See Appendix.

The previous analysis considers the possibility of equilibrium where either the higher ability manager hedges or the lower ability manager hedges. We consider now equilibrium where the higher ability manager randomizes between hedging and not hedging. These are characterized in the following.

**Theorem 3** The other Bayesian Nash equilibria of the model are as follows:

**Type C equilibrium**: This equilibrium may occur only when $s < 0.5$ (high exogenous noise variance). The equilibrium strategies of the managers of different abilities is $a^*(1) = a^*(2) = 1/(2(1 - s))$.

**Type D equilibrium**: This equilibrium occurs when $s < 0.5$ (high exogenous noise variance). The equilibrium strategies of the managers of different abilities is $a^*(1) = 0$ and $0 < a^*(2) < 1$.

**Proof**: See Appendix.

A Type C equilibrium involves higher ability managers and lower ability managers randomizing between hedging and not hedging exactly to the same degree. This equilibrium is knife-edged as the values $a(1) = a(2)$ are chosen so that managers of any ability $p$ are indifferent between hedging and not hedging. Such equilibria may occur when the noise in the interest rate process is high (low $s$).

A Type D equilibrium leads to the less intuitive outcome that higher ability managers do not hedge while lower ability managers randomize. The intuition is as follows. When the parameter $s$ takes low values, fluctuations in the $r$ process add a lot of noise to the inference process. If the higher ability manager does not hedge, then beliefs when extreme outcomes occur place weight on the inference that the manager is good. If the lower ability manager mimics and also does not hedge, then extreme outcomes are informative: $z_i$ is more consistent with a higher ability manager, while $z_j$ is more consistent with a lower ability manager. Intermediate outcomes have little information as the noise.
in the \( r \) process is high. Thus, the lower ability manager has incentives not to hedge to achieve these intermediate outcomes where learning is low. Hence, he randomizes between hedging and not hedging.

The presence of this less intuitive equilibrium for some parameter values may seem odd at first glance. It is important to remember that in this model there are no costs associated with hedging or not hedging. As a consequence, the optimal strategies depend on what market beliefs exist about intermediate states and extreme states. In general, the intuition that hedging is valuable for the higher ability manager depends on the idea that it improves learning by the market. As a consequence, the market views extreme states as indicating a lower ability manager. In the less intuitive equilibrium, extreme states are viewed as more informative than intermediate states because the noise in the interest rate process is high and the difference in abilities is low. The multiplicity of equilibria in models with no signaling costs is a well-known fact (see Crawford and Sobel [1982] and the related literature on “cheap talk”). We will argue next that there are costs associated with hedging and that these costs are lower for managers of higher ability. As we will see, this cost differential is sufficient to eliminate this counterintuitive equilibrium.

In this section, we have demonstrated the existence of an intuitive equilibrium where the high ability manager hedges while the low ability manager does not hedge. This equilibrium exists without any parameter restrictions and embeds the idea that it was in the interest of the high ability manager to hedge so as to ensure a profit process that is more informative. The low ability manager in this equilibrium may or may not hedge; this decision depended on the difference in their abilities. If the difference in ability was large enough, the lower ability manager will not hedge (to lower the informativeness of the profit process) in the hope that luck will turn in his favor.

THE MODEL WITH MANAGERS WHO HOLD EQUITY

In demonstrating the results in the previous section, we have ignored the possibility that hedging may have costs associated with it. The existence of one important cost follows from the insight of the option pricing model. If the firm is being run in the interests of equityholders, the incentives of the equityholders are to undertake activities that increase variance, not reduce variance. If the firm undertakes hedging activities, it is reducing the exposure of the firm to certain risks and hence the variance of profits. In the example of the bank that we considered, such variance-reducing activities lower the value of the FDIC insurance option. Consequently, hedging has an implicit cost associated with it from the perspective of the equityholders. We explore the implications of the costs associated with hedging in this section.

It is reasonable to assume that managers will be sensitive to the effects of their decision making on equityholders and that the wages they receive may include an equity component (this would induce some alignment of their objectives with that of the equityholders). Hence, we assume that in addition to caring about the effects of their decisions on managerial reputation, managers hold a fraction \( \alpha \) of the equity of the firm.

In addition to allowing the manager to hold a fraction of the firm’s equity, we allow for external claimants such as debtholders or for insurance such as FDIC insurance. The effect of either of these two possibilities is similar, and we choose to use FDIC insurance in our analysis.

The Model with Hedging Costs

Institutions such as savings and loans and banks have access to deposit insurance whose value is reduced by hedging. Thus hedging has an associated cost. However, a key insight that is akin to that in signaling models is the following. This implicit cost of hedging is different for managers of different abilities. Firms where managers have higher ability have a lower probability of going bankrupt. Hence the marginal cost of undertaking hedging activities, which is the reduced value of the FDIC insurance (or the reduced wealth transfer from debtholders), is lower for a firm that has higher ability.

There are two conclusions from this intuition that are important. First, hedging will not occur unless the difference in abilities is large enough because there is an associated cost to hedging. However, when hedging occurs, the marginal cost differential results in the higher ability manager hedging and the lower ability manager not hedging. When we analyze the model extension in
the following, we will see that these two intuitive ideas are substantiated.

To operationalize FDIC insurance, we assume that $z_4 < 0$ while $z_2 > z_3 > 0$. The bank goes bankrupt in state $z_4$. In this case, the FDIC pays $-z_4$ to the depositors. Thus, the expected value of firm $i$ when it hedges and when it does not hedge is now given by

$$V(H,i) = p_i z_2 + (1 - p_i)z_3$$
$$V(NH,i) = 0.5(1 - s)p_i z_1 + [p_i m + 0.5(1 - s)]z_2 + [(1 - p_i)m + 0.5(1 - s)]z_3$$
$$V(NH,i) - V(H,i) = -0.5(1 - s)(1 - p_i)z_3 > 0 \quad (6)$$

When the firm (bank) makes a loss, the FDIC steps in and pays depositors. Hence, the expected value while not hedging is always higher than that while hedging. More importantly, this difference in expected values is higher for the lower ability manager as the FDIC option is more valuable to the lower ability manager.

The definition of a Bayesian Nash equilibrium is the same as before except that the manager’s objective, given the optimal market belief function $m^*(\cdot)$, is now (compare with Equation (3)):

$$U(m^*(\cdot),a(i),i) = a(i) \left( \sum_{j=1}^{4} p(z_j | H,i)m^*(z_j)(p_i - p_2) + p_2 + \frac{\alpha}{\beta} V(H,i) \right)$$
$$+ (1 - a(i)) \left( \sum_{j=1}^{4} p(z_j | NH,i)m^*(z_j)(p_i - p_2) \right. \left. + p_2 + \frac{\alpha}{\beta} V(NH,i) \right). \quad (7)$$

Here, $\beta$ is not $y_h - y_l$ but $\eta(y_h - y_l)$ where $\eta$ is the fraction of output that the manager receives in the future. This formulation is similar to that of Prendergast and Stole [1996] in that their study also uses an objective function in which managers care for both profits and end of period reputations. The fraction $\alpha$ represents the extent to which the manager cares for the current value while the fraction $\beta$ represents the extent to which the manager cares for the future value.

We first investigate the existence of the Type A equilibrium (the intuitive equilibrium) where the higher ability manager hedges. As we have discussed, the presence of cost of hedging should deter existence of the Type A equilibrium when the difference in abilities is low. On the other hand, when Type A equilibrium does exist, lower ability managers should be less willing to hedge relative to the case where managers only care for their reputations because of the cost differential between the two kinds of managers. This intuition turns out to be validated.

**Theorem 4** Intuitive Type A equilibria involve $a^*(1) = 1$ and exist for $d > \delta(s)$; $\delta(s)$ is always well defined. Thus, Type A equilibria always exist for high enough differences in managerial abilities. The behavior of the lower ability manager is as follows:

1. $\delta(s) < d < \delta_l^0$ and $d > \delta_h^0$, the lower ability manager does not hedge, $a^*(2) = 1$.
2. $\delta_l^0 < d < \delta_l^0$ and $\delta_h^0 < d < \delta_h^0$, the lower ability manager randomizes, $0 < a^*(2) < 1$.
3. $\delta_l^0 < d < \delta_h^0$ the lower ability manager hedges, $a^*(2) = 1$.

As hedging costs increase from 0, the region $(\delta_l^1, \delta_h^1)$ first vanishes, leaving only the region $(\delta_l^0, \delta_h^0)$ where randomization occurs. For large enough hedging costs, the region $(\delta_l^0, \delta_h^0)$ vanishes and the lower ability manager does not hedge, $a^*(2) = 0$. Finally, the region where $a^*(2) = 1$ (hedging) when managers hold equity is a strict subset of the region where $a^*(2) = 1$ (hedging) when managers only care for their reputations (in Theorem 1).

**Proof:** See Appendix.

Exhibit 2 provides the exact regions characterized in Theorem 4 for the values $s = 0.4$ (high exogenous noise variance) and $s = 0.75$ (low exogenous noise variance).

Type A equilibria do not exist for low differences in ability due to the positive costs of hedging. A sufficiently large difference in ability is required to make hedging a viable strategy for the higher ability manager. Depending on the cost of hedging, they may or may not involve a region where the lower ability manager hedges. However, when there is a region where the low ability manager hedges, the equilibrium takes the following form—for intermediate differences in ability, the lower ability manager does not hedge. As the difference in
ability increases, he randomizes and eventually hedges. For sufficiently high differences in ability, the manager randomizes again and eventually does not hedge.

The intuition for such behavior is as follows. When differences in ability are small and the higher ability manager finds it optimal to hedge, the lower ability manager does not because his cost is higher. As the difference in ability increases, so do the incentives to mimic the higher ability manager, provided the learning process is not too informative. This leads to an intermediate region where the lower ability manager hedges. As the difference in ability becomes large, however, the learning process when both managers hedge is very informative. In addition, the lower ability manager faces an increasingly higher hedging cost as the probability of going bankrupt increases. This leads him to prefer not to hedge.

We next turn to regions where the differences in ability are very low. In such regions, the Type A equilibrium does not exist. Intuitively, both managers will not hedge. We thus investigate the Type B equilibrium where the higher ability manager does not hedge. This equilibrium is characterized in Theorem 5.

**Theorem 5** Type B equilibrium is an equilibrium where \( a^*_2(2) = 0 \) and \( 0 \leq a^*_1(1) < 1 \). Thus, the lower ability manager does not hedge and the higher ability manager does not hedge or randomizes between hedging and not hedging (note that we do not include the case where the higher ability manager hedges as this is a Type A equilibrium). Type B equilibria have two subcases:

(i) Equilibria with \( a^*_1(1) = 0 \). These exist for \( d \leq \gamma(s) \) where \( \gamma(s) > \delta(s) \). \( \gamma(s) \) is determined by the behavior of the higher ability manager when \( s > 0.5 \) and by the behavior of the lower ability manager when \( s < 0.5 \).

(ii) Equilibria with \( 0 < a^*_1(1) < 1 \). These equilibrium exists for a region where \( \delta(s) < d < \pi(s) \) where \( \pi(s) \leq \gamma(s) \).

**Proof:** See Appendix.

Type B equilibria contain the important equilibrium (Type B(i)) where both firms do not hedge. Since \( \gamma(s) > \delta(s) \), we have constructively proved that equilibria we believe are intuitive always exist. In fact at \( \delta(s) \), both the “pooling” equilibrium (both firms not hedging) and the “separating” equilibrium (higher ability firm hedges, lower ability firm does not) are both equilibria. Exhibit 3 provides the exact regions where Type A and Type B(i) equilibria exist for the values \( s = 0.4 \) (high exogenous noise variance) and \( s = 0.75 \) (low exogenous noise variance).

To complete the characterization of the model, we need to document what other equilibria exist, in addition to the previously discussed equilibria.

**Theorem 6** In addition to the equilibria of Theorems 4 and 5, the only other Bayesian Nash equilibrium of the model is as follows.
**Theorem 5:** Regions where Type A and Type B(i) exist for the Values $s = 0.4$ and $s = 0.75$

---

**A**

Equilibrium where Both Managers Do Not Hedge (Type B(i)) and

Equilibrium where the Higher Ability Manager Hedges (Type A)

---

**B**

Equilibrium where Both Managers Do Not Hedge (Type B(i)) and

Equilibrium where the Higher Ability Manager Hedges (Type A)

---

**Type D equilibrium.** This equilibrium involves $a^*(1) = 0$ and $0 < a^*(2) < 1$. It cannot occur for $s > 0.5$. It may occur for $s < 0.5$ depending on the parameters $s$ and $d$. If

$$
\alpha(d) = \frac{\alpha}{d} > \frac{d^3}{1 + d} \quad \text{or}
$$

$$
(\alpha - \alpha(d)) \frac{|y_r - h\delta|}{\alpha(d)\eta} > 1
$$

then Type D equilibria do not exist.

**Proof:** See Appendix.

Theorem 6 characterizes the other equilibria in this model. The Type D equilibrium is the less intuitive equilibrium we have discussed. As discussed in the previous section, this equilibrium requires high noise in the process $r$ (low $s$). As previously mentioned, we think that the Type D equilibrium is less reasonable when there is a cost of hedging as the marginal cost of hedging is lower for the higher ability manager. The bound we present eliminates this equilibrium; essentially, this condition requires that the manager cares sufficiently for the equity value of the firm relative to his future career reputation.

The bound given in Theorem 6 depends on the difference in managerial abilities. When the difference in managerial abilities is $1/3$, the value of the bound is $1/36$. When $d = 1$, the bound has its highest value of $1/2$.

In particular, suppose $|y_r - \delta|$ is $1/2$ (high state profits are twice the absolute value of low state losses assuming the worst realization of the exogenous shock). Also, let $\eta = 0.1$ (the manager cares for $10\%$ of future output). Then, the bound when $d = 1/3$ yields $\alpha > (1/9)\eta$ or $\alpha > 0.0111$. The bound when $d = 1$ yields $\alpha > 1.5\eta$ or $\alpha > 0.15$. Hence in the worst possible case ($d = 1$), one needs the weight on current equity to be $1.5$ times the weight on the share of future output. If the conditions of Theorem 6 are met, the cost differential between managers of differing abilities is such that behavior involving the lower ability manager hedging is implausible and does not occur.

We now discuss the empirical implications of our model. While hedging is not observable, subsequent to the decision, one sometimes knows whether a firm’s hedges or not. This is because ex-post firms voluntarily disclose hedging activity in the notes to the balance sheet. If our story of hedging is correct, the profits (earnings) of firms in the quarter or year that they hedge should be higher and the volatility of cash flows should be lower than usual. Such a test differs from tests of signaling models where profits (earnings) are higher for firms that signal; no volatility implications are present. Because hedging reduces volatility, the volatility reduction is an additional implication. Some preliminary empirical work by DeGeorge, Moselle, and

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**Exhibit 3**

**Theorem 5:** Regions where Type A and Type B(i) exist for the Values $s = 0.4$ and $s = 0.75$
Zeckhauser [1995] using Compustat data is supportive of this implication of the hedging hypothesis.

A second empirical implication of our model is as follows. When the difference in abilities is high and there are costs to hedging, we are more likely to find hedging. In contrast, when the difference in abilities is low, the equilibrium that occurs involves no hedging by both kinds of managers. This implication is borne out in a recent empirical study by Tufano [1995] on the gold mining industry. First, he found that younger managers are more likely to hedge than older managers. Since there is likely to be greater (lesser) uncertainty about the ability of younger (older) managers, a higher (lower) difference in abilities is consistent with younger (older) managers. Hence, Tufano’s work provides much support for our approach to hedging.13

A third empirical implication of our model is as follows. When the costs of hedging are low, less separation through hedging occurs. In contrast, when the costs of hedging are high, more separation through hedging occurs. Therefore, the value of firms that hedge relative to firms that do not hedge is much higher when the cost of hedging is higher. This is another implication of the model that is potentially testable. Tufano’s finding that the managers with stock options do not hedge is consistent with our idea that higher costs of hedging lead to less overall hedging. However, a test of our approach would require that if managers with stock options hedge, their firm value must be much higher than that of managers without stock options who hedge.14

CONCLUSIONS

We have presented a model wherein managers use hedging as an indirect vehicle to communicate their abilities. While hedging itself is not observable, the firm’s decision to hedge or not to hedge and the market’s beliefs about this decision affects the inferences that the market makes from firm profits. In a model where managers care only for their reputations, we show the existence of an equilibrium where the higher ability manager hedges while the lower ability manager hedges when the ability difference is low but does not hedge when the ability difference is high. When differences in ability are low, the learning process when both managers hedge is not very informative (although more informative than when both managers do not hedge), while not hedging allows the lower ability manager to be discovered when extreme states occur. Thus, the lower ability manager prefers to hedge. When the difference in abilities is high, the learning process when both managers hedge is very informative and the lower ability manager prefers not to hedge because there is a chance that low interest rate realization leads to high profits.

We next allow managers to hold equity in their firms. If FDIC insurance exists (as in the case of banks) or the firm has pre-existing debt, hedging is costly. However, it is more costly for the lower ability manager because his probability of going bankrupt is higher. In such a case, not hedging is the equilibrium when the ability difference is low.

For a sufficiently high difference in abilities, there is an equilibrium where the higher ability manager hedges. In this equilibrium, the lower ability manager behaves as follows—for lower ability differences he does not hedge as his cost of hedging is higher. As the ability difference increases, there is a region where he hedges. This occurs when reputation effects become more important and the difference in ability is still not too high. Finally, for large enough differences in ability he does not hedge because the learning process when both managers hedge is very informative and his probability of going bankrupt is increasing. Relative to the model when managers care only for their reputations, more separation occurs in the case where they own equity.

Our results indicate that hedging occurs when higher ability managers are substantially different from lower ability managers or the costs of hedging are low. This substantiates the casual belief that hedging locks up higher profit opportunities in the same way that an arbitrageur locks up positive alpha arbitrage opportunities.
Appendix

Proof of Theorem 1: Given that \( a(1) = 1 \) and that \( 0 \leq a(2) \leq 1 \), the market beliefs are

\[
\begin{array}{c|c}
\text{Profit} & \text{Market beliefs } m(\cdot) \\
\hline
z_1 & 0 \\
z_2 & \frac{p_1 + a(2)p_2}{1 - (1 - a(2))(1 - s)0.5(3p_2 - 1)} \\
z_3 & \frac{p_2 + a(2)p_1}{1 - (1 - a(2))(1 - s)0.5(3p_1 - 1)} \\
z_4 & 0 \\
\end{array}
\]  
(A-1)

The posterior beliefs are those consistent with Bayes rule except when \( a(1) = 1 \). In that case, both firms hedge and thus \( z_1 \) and \( z_4 \) are off the equilibrium path. We assume the off-equilibrium posterior for profit levels \( z_2 \) and \( z_3 \) is 0.

For the higher ability manager to find it optimal to hedge, we must have,

\[
U(m^*(\cdot), H, 1) > U(m^*(\cdot), NH, 1) \\
\iff p_1m(z_2) + p_2m(z_3) > \left[ p_1m + 0.5(1 - s) \right] m(z_2) + \left[ p_2m + 0.5(1 - s) \right] m(z_3) \\
= 0.5(1 - s)(3p_1 - 1)m(z_2) + 0.5(1 - s)(3p_2 - 1)m(z_3) > 0. 
(A-2)
\]

Note that \( m(z_2) > m(z_3) \). Thus,

\[
0.5(1 - s)(3p_1 - 1)m(z_2) + 0.5(1 - s)(3p_2 - 1)m(z_3) \\
> 0.5(1 - s)(3p_1 - 1)m(z_2) + 0.5(1 - s)(3p_2 - 1)m(z_3) \\
= 0.5(1 - s)m(z_2) > 0 
(A-3)
\]

Thus, under these market beliefs, the higher ability manager’s best strategy is to hedge.

Next, consider the lower ability manager. If the lower ability manager randomizes between hedging and not hedging, then

\[
U(m^*(\cdot), H, 2) = U(m^*(\cdot), L, 2) \\
\iff p_1m(z_2) + p_2m(z_3) = \left[ p_1m + 0.5(1 - s) \right] m(z_2) + \left[ p_2m + 0.5(1 - s) \right] m(z_3) \\
= 0.5(1 - s)(3p_2 - 1)m(z_2) + 0.5(1 - s)(3p_1 - 1)m(z_3) \\
\leq (3p_2 - 1)p_2[1 - (1 - a(2))(1 - s)0.5(3p_2 - 1)] \\
= -(2 - 3p_2)p_2[1 - (1 - a(2))(1 - s)(3p_1 - 1)] \\
\iff (1 - a(2)) = \frac{(3p_2 - 1)(1 - p_1) + (2 - 3p_2)p_1}{(3p_1 - 1)(2 - 3p_2)0.5(1 - s)}. 
(A-4)
\]

The right-hand side of Expression (A-4) is monotone in \( p_1 \). Setting the right-hand side of the last equation in (A-4) equal to 0 yields

\[
3p_1 - 1 - 3p_2 + p_1 + 2p_2 - 3p_1^2 \\
= (6p_1 - 2 - 9p_2^2 + 3p_1)0.5(1 - s) \\
\iff 6p_1 - 6p_1^2 - 1 = (9p_1 - 9p_2^2 - 2)0.5(1 - s) \\
\iff p_1p_2 = \frac{2s}{9s + 3}. 
(A-5)
\]

This yields the solution \( \zeta^*_S = 0.5 + 0.5(1/\sqrt{3}) \). Setting the right-hand side of the last equation in (A-5) to 1 yields

\[
3p_1 - 1 - 3p_2^2 + p_1 + 2p_2 - 3p_1p_2 \\
= (6p_1 - 2 - 9p_2^2 + 3p_1)0.5(1 - s) \\
\iff 6p_1 - 6p_1^2 - 1 = (9p_1 - 9p_2^2 - 2)0.5(1 - s) \\
\iff p_1p_2 = \frac{2s}{9s + 3}. 
(A-6)
\]

This defines \( \zeta^*_S(s) = 0.5 + 0.5\sqrt{\frac{s + 3}{9s + 3}} \) which is decreasing in \( s \). Randomized strategies are viable in the region \( p_1 \in (\zeta^*_S, \zeta^*_S(s)) \). Since the difference in abilities \( d \) in \( h \), there is equivalently a region \((\delta^*_S, \delta^*_S(s)) \) such that for \( d \in (\delta^*_S, \delta^*_S(s)) \), randomization is optimal for the lower ability manager. When \( d < \delta^*_S \), the best strategy for the lower ability manager is to hedge, and when \( d > \delta^*_S(s) \), the best response is to not hedge.

Proof of Theorem 2: We show that no equilibrium where the low ability manager hedges while the high ability manager randomizes can exist. In this case, the posterior beliefs are

\[
\begin{array}{c|c}
\text{Profit} & \text{Market beliefs } m(\cdot) \\
\hline
z_1 & 1 \\
z_2 & \frac{a(1)p_1 + (1 - a(1))[p_1m + 0.5(1 - s)]}{a(1)p_1 + (1 - a(1))[p_1m + 0.5(1 - s)] + p_2} \\
z_3 & \frac{a(1)p_2 + (1 - a(1))[p_2m + 0.5(1 - s)]}{a(1)p_2 + (1 - a(1))[p_2m + 0.5(1 - s)] + p_1} \\
z_4 & 1 \\
\end{array}
\]  
(A-7)

The lower ability manager hedges. Thus,
The proof is constructive and the lower ability manager does not hedge.

Type A equilibria require \( (A-10) \)

\[
\begin{align*}
\text{We first look at the case where } a(2) = 1; \text{ the lower ability manager hedges. We thus need that} \\
(2p_1 - 1)(6p_1 - 6p_1^2 - 1) > -\frac{\alpha}{\beta}zp_i p_i \\
\end{align*}
\]

where \( p_2 = 1 - p_1 \) is used. Note that the left-hand side is zero at \( p_1 = 0.5 \), while the right-hand side is strictly positive. Also, the left-hand side has derivative \( 4(9p_1p_2 - 2) \) and is a concave function with a maximum at \( p_1 = 2/3 \). The right-hand side is a linear function in \( p_1 \). Hence, there are exactly two intersections or none. There exists a region \( p_1 \in (\zeta_1^l, \zeta_1^r) \) (or \( d \in (\delta_1^l, \delta_1^r) \)), possibly empty in which the lower ability manager finds it optimal to hedge.

Now consider the case where the lower ability manager does not hedge; \( a(2) = 0 \). The inequality for the lower ability manager simplifies to

\[
\begin{align*}
\left(2 - \frac{1}{p_1}\right)[6p_1p_2 - 1 - 0.5(1-s)(9p_1p_2 - 2)] \\
< -\frac{\alpha}{\beta}z_1[1 - 0.5(1-s)] + 0.25(1-s)^2(9p_1p_2 - 2). \\
\end{align*}
\]

The second derivative of the left-hand side of Equation (A-12) is

\[
\begin{align*}
-\frac{2}{p_1^2}[6p_1p_2 - 1 - 0.5(1-s)(9p_1p_2 - 2)] \\
+ \left[6 - \frac{9}{2}(1-s)\right](1 - 2p_1) \\
+ \left[6 - \frac{9}{2}(1-s)\right]\frac{1}{p_1^2}(1 - 2p_1) - 2\left(2 - \frac{1}{p_1}\right) < 0. \\
\end{align*}
\]

Thus the left-hand side is concave in \( p_1 \). The right-hand side has derivative \( -\frac{\alpha}{\beta}z_1 \times 0.25(1-s)^29(1-2p_1) \) and thus is a decreasing concave function. At \( p_1 = 0.5 \) and \( p_1 = 1 \), the left-hand side is negative (or zero) while the right-hand side is always positive; thus, if an intersection occurs at least two intersections exist.

Equation (A-12) with equality is a cubic equation and has at most three real roots; we need to ensure that this third root is not between \( p_1 = 0.5 \) and \( p_1 = 1 \). But at \( p_1 = 0 \), the left-hand side is \( s \) and the right-hand side is \( 0 \). Thus a root exists between \( p_1 = 0 \) and \( p_1 = 0.5 \). Call the two intersection points between 0.5 and \( \zeta_2^l \) and \( \zeta_2^l, \zeta_2^r < \zeta_3^u \). Next, while considering randomized strategies, we show that \( \zeta_2^r < \zeta_2^l < \zeta_3^u < \zeta_3^l \). In the region outside \( (\zeta_2^l, \zeta_2^r) \) (the corresponding region for \( d \) is \( (\delta_2^l, \delta_2^r) \)), the lower ability manager does not hedge.
In the region within \( (\zeta_1, \zeta_2) \) (the corresponding region for \( d \) is \( (\delta_1, \delta_2) \)), the lower ability manager hedges. In the intermediate region, he randomizes.

Randomization is an optimal strategy if \( U(m(\cdot), H, 2) = U(m(\cdot), NH, 2) \), or

\[
2 \frac{1}{P_1} \left[ 6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2) \right] = -\frac{\alpha}{\beta} z_s \left[ 1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^20.25(1 - s)^2(9p_1p_2 - 2) \right]. \tag{A-14}
\]

The left-hand side is \( 1 - (1 - a(2))(1 - s) > 0 \) at \( p_1 = 0 \) while the right-hand side is 0. There is at least one root between 0 and 0.5. At \( p_1 = 0.5 \) the left-hand side is 0, while the right-hand side is a positive number. Finally, for \( p_1 = 1 \), the left-hand side is \( -1 - (1 - a(2))(1 - s) < 0 \), while the right-hand side is still positive. Thus, there are at two intersections or no intersections at all between 0.5 and 1; more than two cannot occur as there are at most three roots to the equation and one root lies between 0 and 0.5. Call these intersection points \( \zeta_1^{(a)} \) and \( \zeta_2^{(a)} \).

The left-hand side of Equation (A-14) is concave in \( p_1 \); while the right-hand side is decreasing and concave in \( p_1 \). Suppose \( a(2') > a(2) \). Then, for \( p_1 < 2/3 \), \( LHS(a(2'), p_1) > LHS(a(2), p_1) \) and for \( p_1 > 2/3 \), \( LHS(a(2'), p_1) < LHS(a(2), p_1) \). Also \( RHS(a(2'), p_1) > RHS(a(2), p_1) \).

Suppose \( a(2') > a(2) \) and \( \zeta_1^{(a)} < \zeta_2^{(a)} \). Because \( \zeta_1^{(a)} < 2/3 \), the assumption that \( \zeta_2^{(a)} < 2/3 \) is justified by the very same argument. To show this claim, note that at \( p_1 = \zeta_1^{(a)} \),

\[
LHS(a(2'), p_1) = RHS(a(2'), p_1) \equiv (p_1 - p_2) [6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2)]
= -\frac{\alpha}{\beta} p_1z_s \left[ 1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^20.25(1 - s)^2(9p_1p_2 - 2) \right]. \tag{A-15}
\]

Then at \( p_1 = \zeta_1^{(a)} \),

\[
LHS(a(2), p_1) < RHS(a(2), p_1) \equiv (p_1 - p_2) [6p_1p_2 - 1 - (1 - a(2))0.5(1 - s)(9p_1p_2 - 2)]
< -\frac{\alpha}{\beta} p_1z_s \left[ 1 - (1 - a(2))0.5(1 - s) + (1 - a(2))^20.25(1 - s)^2(9p_1p_2 - 2) \right]
\equiv (1 - 3p_1p_2) > \left( 6p_1p_2 - 1 \right)0.5(1 - s)(9p_1p_2 - 2)(2 - a(2') - a(2)) -0.25(1 - s)^2(9p_1p_2 - 2)^2(1 - a(2))(1 - a(2')) \tag{A-16}
\]

where \( LHS(a(2'), p_1) = RHS(a(2'), p_1) \) has been used. The left-hand side of the last equation in (A-16) is lowest when \( p_1 = 0.5 \), where its value is 0.25. The right-hand side of the last equation in (A-16) is highest at 0.5, provided \( p_1 < 2/3 \). At \( p_1 = 0.5 \), the right-hand side of last equation in (A-16) has value 0.125 (1 - s), which is less than 0.25. Thus, a root of the equation exists between 0.5 and \( \zeta_2^{(a)} \). Hence \( \zeta_1^{(a)} < \zeta_2^{(a)} \).

To prove that \( \zeta_1^{(a)} > \zeta_2^{(a)} \), consider first the case where \( \zeta_1^{(a)} > 2/3 \). In this case, \( LHS(a(2), p_1) > LHS(a(2'), p_1) > RHS(a(2'), p_1) \). Hence \( \zeta_1^{(a)} > \zeta_2^{(a)} \). On the other hand, if \( \zeta_1^{(a)} < 2/3 \), an argument similar to that in the prior paragraph works and at \( p_1 = \zeta_1^{(a)} \), \( LHS(a(2), p_1) > RHS(a(2), p_1) \). Hence, a second root exists at some \( p_1 \) value greater than \( \zeta_1^{(a)} \).

This proves the proposition.

Finally, it is easy to show that as \( -\alpha/\beta z_s \) increases, the region \( (\zeta_1, \zeta_2) \) disappears, leaving the region \( (\zeta_1', \zeta_2') \) within which randomization occurs. Eventually, this region also disappears, and for all \( p_1 \) values, the lower ability manager finds it optimal not to hedge.

We now turn to the higher ability manager. He hedges as long as

\[
U(m(\cdot), H, 1) > U(m(\cdot), NH, 1)
\equiv (p_1 - p_2) \left[ 0.5(1 - s)(3p_1 - 1)m(z_s) + 0.5(1 - s)(3p_2 - 1)m(z_s) \right] > -\frac{\alpha}{\beta} z_s p_10.5(1 - s) \tag{A-17}
\]

After much simplification, this inequality reduces to

\[
\frac{p_1 - p_2}{p_2} \left[ 2 - 6p_1p_2 - (1 - a(2))0.5(1 - s)(4 - 15p_1p_2) \right] > -\frac{\alpha}{\beta} z_s \left[ 1 - (1 - a(2))0.5(1 - s) + 0.25(1 - a(2))^2(1 - s)^2(9p_1p_2 - 2) \right] \tag{A-18}
\]

The left-hand side of Equation (A-18) is strictly increasing in \( p_1 \), while the right-hand side of the same equation is strictly decreasing in \( p_1 \). Also the left-hand side of Equation (A-18) is zero at \( p_1 = 0.5 \), while the right-hand side of the same equation is positive. At \( p_1 = 1 \), the left-hand side of Equation in (A-18) is \( \infty \), while the right-hand side of
the same equation is a finite number. Thus, \( \zeta(a(2)) \) exists such that for \( p_1 > \zeta(a(2)) \), hedging is optimal.

Next, we show that \( \zeta(a(2)) < \zeta^{(2)} \). To prove this, note that we need to compare Equation (A-14) with Equation (A-18). The right-hand side of Equation (A-18) is less than in Equation (A-14) as \( p_2 < p_1 \). The left-hand side of Equation (A-18) is greater than the left-hand side of Equation (A-14), provided that

\[
2 - 6p_1p_2 - (1 - a(2))(4 - 15p_1p_2) > 0
\]

\[
\iff 1 - 4p_1p_2 - (1 - a(2))(1 - s)(1 - 4p_1p_2) > 0
\]

Because this inequality holds, \( \zeta(a(2)) < \zeta^{(2)} \). Define \( \zeta(s) = \zeta(a(2)) \). Then, \( \zeta(s) < \zeta^{(2)} \). The equilibrium constructed here then holds for \( p_1 > \zeta(s) \) (similarly we have \( d > \delta(s) \)). In this region, given the lower ability manager’s optimal action, the higher ability manager finds it optimal to hedge.

**Proof of Theorem 5:** Type B equilibria involve the lower ability manager not hedging \( \{U(m(\cdot), H, 2) < U(m(\cdot), NH, 2)\} \) and the higher ability manager not hedging or randomizing \( \{U(m(\cdot), H, 1) < U(m(\cdot), NH, 1) \) or \( U(m(\cdot), H, 1) = U(m(\cdot), NH, 2)\} \).

Define the functions

\[
F_1(a(1)) = \frac{a(1)(1 - s)(3p_1 - 1) + p_1m + 0.5(1 - s)}{a(1)(1 - s)(3p_1 - 1) + m + 1 - s} - \frac{(1 - a(1))p_1}{1 - a(1)p_1} - \frac{p_2(1 - a(1)p_2)}{1 - a(1)p_2}
\]

\[
F_2(a(1)) = \frac{a(1)(1 - s)(3p_1 - 1) + p_1m + 0.5(1 - s)}{a(1)(1 - s)(3p_1 - 1) + m + 1 - s} - \frac{(1 - a(1))p_1}{1 - a(1)p_1} - \frac{p_2(1 - a(1)p_2)}{1 - a(1)p_2}
\]

\[
(p_1 - p_2)F_2(a(1)) < -\frac{\alpha}{\beta}p_1z_1.
\] \hspace{1cm} (A-21)

The inequality for the higher ability manager

\[
(p_1 - p_2)F_1(a(1)) = -\frac{\alpha}{\beta}p_2z_4
\] \hspace{1cm} (A-22)

if he randomizes, or

\[
(p_1 - p_2)F_1(0) < -\frac{\alpha}{\beta}p_2z_4
\] \hspace{1cm} (A-23)

if he does not hedge. In the Internet appendix, in part (b) of the proof of Theorem 3, we show that when \( s > 0.5 \), \( F_1(0) > 0 \) and \( F_2(0) < 0 \). Since \( 0 < -\frac{\alpha}{\beta}p_2z_4 < -\frac{\alpha}{\beta}p_2z_4 \), an equilibrium where both managers do not hedge exists when

\[
(p_1 - p_2)F_1(0) < -\frac{\alpha}{\beta}p_2z_4
\]

\[
\iff (p_1 - p_2)
\]

\[
\left\lbracket \frac{(3p_1 - 1) p_1m + 0.51 - s}{m + 1 - s} + (3p_2 - 1) p_2m + 0.5(1 - s)}{m + 1 - s} - p_1^2 - p_2^2 \right\lbracket
\]

\[
< -\frac{\alpha}{\beta}p_2z_4
\]

\[
\iff (p_1 - p_2)(2s - 1)(2 - (p_1^2 + p_2^2) - 1)
\]

\[
< -\frac{\alpha}{\beta}p_2z_4(m + 1 - s).
\] \hspace{1cm} (A-24)

The left-hand side of the last equation in (A-24) is increasing in \( p_1 \), while the right-hand side is decreasing. Also, \( LHS(0.5) = 0 < RHS(0.5) \), while \( LHS(1) > 0 = RHS(1) \). Thus, a unique solution \( \omega(s) \) exists such that for all \( p_1 < \omega(s) \) (similarly, \( d < \gamma(s) \), Equation (A-24) holds.

To prove that \( \omega(s) > \zeta(s) \) (or \( \gamma(s) > \delta(s) \)), argue as follows. At \( \zeta(s) \) when the market believes that the higher ability manager hedges and the lower ability manager does not, \( U(m(\cdot), H, 1) = U(m(\cdot), NH, 1) \) and \( U(m(\cdot), H, 2) < U(m(\cdot), NH, 2) \). But this is just
\[(p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta} p_2 z_4\]
\[(p_1 - p_2)F_1(1) < -\frac{\alpha}{\beta} p_1 z_4.\]  
(A-25)

Since \(dF_2(\sigma(1))/d\alpha(1) > 0\),
\[(p_1 - p_2)F_2(0) < (p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta} p_2 z_4\]  
(A-26)

and
\[(p_1 - p_2)F_2(0) < 0 < -\frac{\alpha}{\beta} p_1 z_4.\]  
(A-27)

Thus, both managers not hedging is not an equilibrium and \(\omega(\sigma) > \zeta(\sigma)\) (or \(\gamma(\sigma) > \delta(\sigma)\)).

When \(s < 0.5\), \(F_2(0) < 0\) and \(F_1(0) > 0\). Now an equilibrium with both managers not hedging exists as long as
\[(p_1 - p_2)F_2(0) < -\frac{\alpha}{\beta} p_1 z_4\]
\[(p_1 - p_2)(2s - 1)(4p_1 p_2 - 1) < -\frac{\alpha}{\beta} p_1 z_4(m + 1 - s)\]  
(A-28)

The left-hand side is a convex increasing function. The right-hand side is linear function with slope \(-\frac{\alpha}{\beta} z_4(m + 1 - s)\). Because \(LHS(0.5) = 0 < RHS(0.5)\), one intersection or none exists. Thus \(\omega(\sigma)\) (and \(\gamma(\sigma)\)) is well defined.

To prove that \(\omega(\sigma) > \zeta(\sigma)\) (and \(\gamma(\sigma) > \delta(\sigma)\)), argue as follows. At \(\zeta(\sigma)\),
\[(p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta} p_2 z_4\]
\[(p_1 - p_2)F_1(1) < -\frac{\alpha}{\beta} p_1 z_4.\]  
(A-29)

Now \(F_1(1) > F_2(1)\). If we show that \(F_2(0) < F_1(1)\), we are done, as then
\[(p_1 - p_2)F_2(0) < (p_1 - p_2)F_1(1) = -\frac{\alpha}{\beta} p_2 z_4\]
\[(p_1 - p_2)F_1(0) < 0 < -\frac{\alpha}{\beta} p_1 z_4.\]  
(A-30)

When \(p_1 < 2/3\), \(dF_2(\alpha(1))/d\alpha(1) > 0\). Then, \(F_2(0) < F_1(1) < F_1(1)\) follows. When \(p_1 > 2/3\), the inequality \(F_2(0) < F_1(1)\) is simplified as follows:
\[
\left(1 - 2s\right) \left(1 - 2p_1 p_2\right) \frac{m + 1 - s}{m + 1 - s} \times \left[(m + 1 - s)^2 + 0.5(1 - s)(m + 1 - s)\right] + 0.25(1 - s)^2(9p_1 p_2 - 1)
\]
\[
< (3p_1 - 1)
\]
\[
\times \left[0.5(1 - s)(3p_1 - 1) + p_2 m + 0.5(1 - s)\right] + m + 1 - s
\]
\[
+ (3p_2 - 1)
\]
\[
\times \left[0.5(1 - s)(3p_2 - 1) + p_2 m + 0.5(1 - s)\right] + m + 1 - s
\]
\[
\Rightarrow (1 - 2s)(1 - 4p_1 p_2)
\]
\[
\frac{m + 1 - s}{m + 1 - s} \times \left[(m + 1 - s)^2 + 0.5(1 - s)(m + 1 - s)\right] + 0.25(1 - s)^2(9p_1 p_2 - 1)
\]
\[
< (m + 1 - s)^2 - 0.5(1 - s)(2 - 9p_1 p_2)
\]
\[
+ (m + 1 - s)2m(1 - 3p_2 p_2)
\]
\[
+ (m + 1 - s)0.5(1 - s)
\]  
(A-31)

On the left-hand side, the term associated with \(9p_1 p_2 - 2\) is negative as \(p_1 > 2/3\). On the right-hand side, \(2 - 9p_1 p_2\) and \((m + 0.5 - s)(1 - s)\) are positive. Thus, it suffices that
\[
(1 - 2s)(1 - 4p_1 p_2)\left[(m + 1 - s) + 0.5(1 - s)\right]
\]
\[
< (m + 1 - s)2m(1 - 3p_2 p_2) + (m + 1 - s)0.5(1 - s)
\]
\[
\Rightarrow (1 - 2s)(1 - 4p_1 p_2) < (m + 1 - s)2m(1 - 3p_2 p_2)
\]
\[
+ (m + 1 - s)0.5(1 - s)
\]  
(A-32)

where \(m + 1 - s + 0.5(1 - s) = 1\) is used. This last inequality is true as
\[
1 - 2s < (m + 1 - s)2m + (m + 1 - s)0.5(1 - s)
\]
\[
\Rightarrow 1 - 2s < 0.5(1 + s)\left[\frac{s}{2} - \frac{1 - s}{4} + \frac{1 - s}{2}\right]
\]
\[
\Rightarrow 1 - 2s < 0.5(1 + s)\frac{2}{4} + 1 - s
\]
\[
\Rightarrow 3 < 9s.
\]  
(A-33)

Since \(s > 1/3\), this is true. Hence, \(\omega(\sigma) > \zeta(\sigma)\) (and \(\gamma(\sigma) > \delta(\sigma)\)).

We now turn to equilibria where the high ability manager randomizes and the low ability manager does not hedge.
When $1/3 < s < 0.5$, we know that $F_i(0) < 0$ and that $dF_i(a(1))/da(1) > 0$. Hence, there is a solution to the problem $(p_1 - p_2)F_i(1) = A_{min}p_2$. Thus, a solution to the equation $(p_1 - p_2)F_i(a(1)) = Ap_2$ exists in the range $(0, A_{max})$. However, at $A = 0$, $(p_1 - p_2)F_i(0) = 0$ and no solution exists. At $A = 1$, $(p_1 - p_2)F_i(0) = (p_1 - p_2)F_i(1) = Ap_2 < Ap_1$, and a solution exists. Let $G(A) = (p_1 - p_2)F_i(x(A)) - Ap_1$, where $x(A)$ solves $(p_1 - p_2)F_i(x) = Ap_2$. Then,

$$
\frac{dG}{dA} = (p_1 - p_2)\frac{dF_i}{dx} \frac{dx}{dA} - p_1
$$

$$
= (p_1 - p_2)\frac{dF_i}{dx} \left( p_2 \frac{dF_i}{dx} - p_1 \right)
$$

$$
= p_2 \frac{dF_i}{dx} - p_1
$$

$$
< 0 \quad (A-34)
$$

using the fact that $dF_i(x)/dx > dF_i(x)/dx$. Hence, there is an unique region $(A_{min}, A_{max})$ where a solution exists. From this, given $A$, we can show that there is a unique region $\delta(s) < d < \pi(s)$ where $\pi(s) = \gamma(s)$ (the bound $\delta(s)$ comes from noting that the equality $(p_1 - p_2)F_i(1) = Ap_2$ defines $\delta(s)$).

When $0.5 < s < 1$, the proof is a little different. At $A = 0$, we know that $F_i(0) < 0$. Hence, the behavior of equilibrium depends on the higher ability agent. Since $F_i(0) > 0$, there is an $A_{min}$ such that $(p_1 - p_2)F_i(0) = A_{min}p_2$. By the same argument as before, there is also an $A_{max}$ such that $(p_1 - p_2)F_i(1) = A_{max}p_2$.

We claim that equilibrium exists in the region $(A_{min}, A_{max})$. At $A_{min}$, we know that $(p_1 - p_2)F_i(0) < 0 < A_{min}p_1$. Hence, equilibrium exists. The fact that $dF_i(x)/dx > dF_i(x)/dx$ then implies that $(p_1 - p_2)F_i(x(A)) < Ap_1$ where $x = a(1)$ is the randomized strategy for the higher ability given $A$. From this we can deduce that given $A$, there is a unique region $\delta(s) < d < \pi(s)$ where $\pi(s) = \gamma(s)$. Again, the bound $\delta(s)$ comes from noting that the equality $(p_1 - p_2)F_i(1) = Ap_2$ defines $\delta(s)$.

**Proof of Theorem 6**: The theorem consists of three parts and is shown in the Internet appendix.

ENDNOTES

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1For example, in late 2015, open interest in Eurodollar futures totalled 11.3 million contracts and the S&P 500 futures, 5-year, and 10-year Treasury note futures each had open interest of more than 2 million contracts.

2Tufano [1995] also found that the greater the equity holdings of the manager, the more likely one will observe hedging. It is difficult to use this observation directly as what matters in our model is the cost of hedging and not the fraction of equity held. A proxy for the cost of hedging must simultaneously account for the fraction of equity held and the value of the equity option or FDIC insurance that hedging destroys.

3In Ljungqvist’s setup, absent speculation, there is perfect discrimination between the good and bad types of managers. In our setup, this is not true as output is not perfectly revealing about the ability of the manager.

4It is interesting to note that Culp and Miller [1995] stated that “absent superior information, value-maximizing firms may not only avoid hedging, but may well shun the underlying activity itself.” See Breeden [1984, 1989] for earlier discussions of risk management.

5Ljungqvist [1994] also allowed for manipulation of the measurement process. Matthews and Mirman [1983] are also closer to our model with costs of hedging. The model with costs of hedging can be viewed a noisy signaling as is the case of the model of Matthews and Mirman [1983].

6DeGeorge, Moselle, and Zeckhauser [1996] assumed that firms with higher means have lower costs of variance reduction. In contrast, we make no such assumptions. Additionally, we model a cost for hedging that we believe to be of importance and one that yields the differential costs of hedging for firms with different mean cash flows.

7In this discrete state model, it is difficult to model partial hedging. Since we consider mixed strategies, one could view mixing between hedging and not hedging as partial hedging.

8We note that the wage does not depend on the realization of the interest rate factor, $\Delta r$.

9Again, we note that the updated wage does not depend on the realization of the interest rate factor, $\Delta r$.

10There is also an equilibrium that we do not discuss as it only exists for $s = 0.5$. In this case, we can have an equilibrium where $a(1) = a(2) = 0$.

11The signaling models of Ross [1977], Bhattacharya [1979], Harris and Raviv [1985], and Brennan and Copeland [1987] have a related formulation of the managerial objective function.
With positive costs of hedging, the Type C equilibrium that involve randomization by managers of both higher and lower abilities does not exist.

We also note that Tufano [1995] found no support for the costly bankruptcy approach to hedging in his study. However, Geczy, Minton, and Schrand [1997] looked at a large cross-section of firms and found evidence more consistent with the transactions cost of raising capital and presence of growth opportunities hypothesis put forward by Froot, Scharfstein, and Stein [1993].

Tufano [1995] also found that the greater the equity holdings of the manager, the more likely one will observe hedging. It is difficult to use this observation directly as what matters in our model is the cost of hedging and not the fraction of equity held. A proxy for the cost of hedging must simultaneously account for the fraction of equity held and the value of the equity option or FDIC insurance that hedging destroys.

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