Financing Insurance

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Research Agenda on Collateralized Finance

- Bulk of (external/debt) **financing collateralized in practice**
  - Households: mortgages, car loans, etc.
  - Firms: tangible assets primary determinant of leverage
  - Intermediaries: real estate finance (commercial and residential)
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- Tractable dynamic model of collateralized financing

- Key friction: **limited enforcement**
  - Enforcement of repayment by borrower limited to tangible assets
  - Novel assumption: **no exclusion**
  - Implication: **collateral constraints**
    - Promises are not credible unless collateralized
  - Implementation: complete markets in one-period Arrow securities
    - Tractable!
Dynamic Collateralized Finance – Implications

(1) **Risk management** – Rampini/Viswanathan (2010, 2013)
- Involves state contingent promises and needs collateral
- Opportunity cost: forgone investment
- Severely constrained firms do not hedge
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   - Determinant: fraction tangible assets required for production
   - Leasing has repossession advantage and permits greater borrowing
   - Severely constrained firms lease
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(3) **Durability** – Rampini (2019)
   - Durable assets have larger financing needs and are harder to finance
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- Durable assets have larger financing needs and are harder to finance

(4) **Financial intermediation** – Rampini/Viswanathan (2019)
- Intermediaries with collateralization advantage require capital
- Rich implications for economic dynamics
Financing Insurance – Synopsis

- **Intertemporal aspect to insurance**
  - Insurance premia paid up front; insurance is state-contingent savings
  - Smoothing over time and insurance across states linked
  - Obstacle as important as moral hazard and adverse selection?

Under stationary distribution, insurance is incomplete with probability 1; absent with positive probability globally increasing in net worth and income.

Theory of insurance firms

Insurers' balance sheets
- Assets: collateralized loans
- Liabilities: diversified portfolio of insurance claims

Consequence of intertemporal aspect to insurance
- Collateral scarcity... lowers equilibrium interest rate... reducing insurance & increasing inequality
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  - *precautionary* (increases when income gets riskier)
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Stylized Facts on Household Risk Management

- **Insurance by U.S. households**
  - **Cross section** – Brown/Finkelstein (2007)
    - Health/long-term care coverage increase in income, wealth, age
  - **Within-household variation** – Fang/Kung (2012)
    - “[I]ndividuals who experience negative income shocks are more likely to lapse all [life insurance] coverage.”
  - Consumption data – Blundell/Pistaferri/Preston (2008)
    - “full insurance of transitory shocks except among poor households”

- **Insurance by farmers in developing economies**
  - Consumption data – Townsend (1994)
    - “[s]ome evidence that the landless are less well insured”

- **Rainfall insurance**
  - Giné et al. (2008), Cole et al. (2013)
    - Participation increases in wealth; decreases in borrowing constraints
  - Most frequently stated reason for not purchasing insurance:
    - “insufficient funds to buy insurance”

- **Crop insurance**
  - Casaburi/Willis (2018)
    - Take-up of pay at harvest (not up-front) crop insurance much higher

- **Corporate risk management**
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- Corporate risk management
Dynamic Model of Insurance

- Discrete time, infinite horizon

Households

- Preferences: $E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$ where $\beta \in (0, 1)$, $u(c)$ strictly increasing, strictly concave, continuously differentiable, $\lim_{c \to 0} u_c(c) = \infty$, $\lim_{c \to \infty} u_c(c) = 0$

- Income $y(s)$ Markov chain on state space $s \in S$ with transition matrix $\Pi(s, s') > 0$ and $\forall s, s_+, s_+ > s, y(s_+) > y(s)$

- Income net of non-discretionary spending for health, accidents, ...

- Notation: $y' \equiv y(s')$, $\underline{s} = \min\{s : s \in S\}$, $\overline{s} = \max\{s : s \in S\}$, etc.
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Lenders (partial equilibrium/exogenous $R$ for now)

- Risk neutral, discount at $R^{-1} \in (\beta, 1)$, deep pockets, abundant collateral
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Lenders (partial equilibrium/exogenous \( R \) for now)

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Limited enforcement – default without exclusion

- Optimal dynamic contract can be implemented with complete markets in one-period ahead Arrow securities subject to short sale constraints (special case of collateral constraints)

- Related: R/V (2010, 2013)
Household’s Dynamic Insurance Problem

- Recursive formulation

- Given $s$ and $w$, household solves

  \[ v(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w', s')|s] \]  

  subject to budget constraints for current and next period, $\forall s' \in S$

  \[ w \geq c + E[R^{-1} h'|s] \]  

  \[ y' + h' \geq w' \]  

  and short sale constraints, $\forall s' \in S$

  \[ h' \geq 0 \]

- Arrow securities $h'$ for each state $s'$ (and associated net worth $w'$)

- Endogenous state variable: net worth $w$ (cum current income)
Characterization of Dynamic Insurance Problem

- Well-behaved dynamic program
  - Return function concave; constraint set convex
  - Operator defined by (1) to (4) satisfies Blackwell’s sufficient conditions
  - Solution: ∃! value function $v$; strictly increasing; strictly concave
Characterization of Dynamic Insurance Problem

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- First order conditions
  - Denote multipliers on (2) and (3) by \( \mu \) and \( \beta \Pi(s, s') \mu' \) and on (4) by \( \beta \Pi(s, s') \lambda' \)
  - Ignore non-negativity constraints on consumption (not binding)

\[
\begin{align*}
\mu &= u_c(c) \\
\mu' &= v_w(w', s') \\
\mu &= \beta R \mu' + \beta R \lambda'
\end{align*}
\]

- Envelope condition: \( v_w(w, s) = \mu \)
  - Value function continuously differentiable
Insurance is Increasing (Prop. 1)

- **Richer households insure more states**
  - (i) Set of states that household insures \( S_h \equiv \{ s' \in S : h(s') > 0 \} \) is increasing in net worth \( w \) given current state \( s \), \( \forall s \in S \)

- **Richer households better insured/spend more on insurance**
  - (ii) For \( w_+ > w \) and denoting net worth next period associated with \( w_+ (w) \) by \( w_+ (w') \), we have
    - \( w_+ \geq w \) and \( c_+ \geq c' \), \( \forall s' \in S \), i.e., \( w_+ \) and \( c_+ \) statewise dominate and hence FOSD \( w' \) and \( c' \), respectively; moreover, \( h_+ \geq h' \), \( \forall s' \in S \), and \( E[h'_+ | s] \geq E[h' | s] \)
    - consumption across insured states constant, i.e., \( c' = c_h \), \( \forall s' \in S_h \), and \( c_h \) is strictly increasing in \( w \)
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- Richer households better insured/spend more on insurance
  - (ii) For $w_+ > w$ and denoting net worth next period associated with $w_+ (w)$ by $w'_+ (w')$, we have
    - $w'_+ \geq w'$ and $c'_+ \geq c'$, $\forall s' \in S$, i.e., $w'_+$ and $c'_+$ statewise dominate and hence FOSD $w'$ and $c'$, respectively; moreover, $h'_+ \geq h'$, $\forall s' \in S$, and $E[h'_+|s] \geq E[h'|s]$
    - consumption across insured states constant, i.e., $c' = c_h$, $\forall s' \in S_h$, and $c_h$ is strictly increasing in $w$

- Remarks:
  - This does not say which states are insured
  - All statements are conditional on state $s$
Income Processes with Positive Persistence

- **Stochastically monotone Markov chain**

- Consider Markov chains which exhibit a notion of positive persistence

**Definition 1 (Monotone Markov chain).** A Markov chain $\Pi(s, s')$ is **stochastically monotone**, if it displays first order stochastic dominance (FOSD) if $\forall s, s_+, \hat{s}', s_+ > s, \sum_{s' \leq \hat{s}'} \Pi(s_+, s') \leq \sum_{s' \leq \hat{s}'} \Pi(s, s')$
Stochastically monotone Markov chain

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$$\sum_{s' \leq s'} \Pi(s_+, s') \leq \sum_{s' \leq s'} \Pi(s, s')$$

Remarks:

- Distribution of states next period conditional on current state $s_+$
- FOSD distribution conditional on current state $s$ for all $s_+ > s$

IID is special case: $\Pi(s, s') = \Pi(s'), \forall s \in S$, is stochastically monotone
Insurance with Stochastic Monotonicity (Prop. 2)

- Assume that $\Pi(s, s')$ is stochastically monotone

- **Key property**
  - (i) Marginal value of net worth $v_w(w, s)$ is decreasing in state $s$
Insurance with Stochastic Monotonicity (Prop. 2)

- Assume that $\Pi(s, s')$ is stochastically monotone

- **Key property**
  - (i) Marginal value of net worth $v_w(w, s)$ is decreasing in state $s$

- **Insurance is globally increasing**
  - (ii) Household insure a lower interval of states, $S_h = \{s', \ldots, s'_h\}$ given $w$ and $s$; net worth next period $w'$, insurance $h'$, set of insured states $S_h$, and insured consumption $c_h$ are all monotone increasing in $w$ and $s$
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- **Intuition**
  - Higher current income means FOSD shift in income next period ⇒ lower marginal value of current net worth
  - If property is satisfied, households insure lower income realizations more
Insurance with Stochastic Monotonicity (Cont’d)

- Proof of key property - Prop. 2

- Define operator $T$ as

$$Tv(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^2} u(c) + \beta E[v(w', s')|s]$$

subject to equations (2) through (4)

- Sketch: Show that if $v$ has property that $\forall s, s_+, s_+ > s$, $v_w(w, s_+) \leq v_w(w, s)$, then $Tv$ (and fixed point) inherit property
“Richer Households are Better Insured” (Prop. 2, Part (iii))

- Assume income process independent: $\Pi(s, s') = \pi(s'), \forall s, s' \in S$

- Richer households insure more states/higher net worth
  - Net worth in insured states $w(s') = w_h, \forall s' \in S_h$, and $w_h$ is increasing in $w$
Assume income process \textbf{independent}: \( \Pi(s, s') = \pi(s'), \forall s, s' \in S \)

Richer households insure more states/higher net worth

- Net worth in insured states \( w(s') = w_h, \forall s' \in S_h \), and \( w_h \) is increasing in \( w \)

\textbf{Richer households lower variance of net worth/consumption}

- Variance of net worth \( w' \) and consumption \( c' \) next period is decreasing in current net worth \( w \)
Insurance is Incomplete (Prop. 3)

- Assume income process is stochastically monotone

- “Poor households cannot afford insurance”
  - (i) At net worth $w = y$ in state $s$, household does not insure at all, that is, $\lambda' > 0$, $\forall s' \in S$, and $S_h = \emptyset$
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- High income households are not completely insured
  - (ii) At net worth $w = \bar{y}$, household does not insure highest state next period, that is, $\lambda(s') > 0$ and $S_h \subsetneq S$, $\forall s \in S$
Basic Financing Insurance Trade-off

- Poor shift net worth to present not across states next period

\[ v_w(w(s), s') = v_w(w(\bar{s}'), \bar{s}') \]

Current consumption

\[ \Pi(s, \bar{s}') \]

\[ \Pi(s, s') \]

\[ v_w(w(\bar{s}'), \bar{s}') \neq v_w(w(s'), s') \]

No insurance
Assume that $\Pi(s, s')$ is stochastically monotone.

Existence and uniqueness

(i) There exists a unique stationary distribution of net worth.

Support of net worth distribution

(ii) Support of stationary distribution is subset of $[\underline{w}, \underline{w}_{bnd}]$ where $\underline{w} = y$ and $\underline{w}_{bnd} \geq \overline{y}$ with equality if $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$. 

Insurance increasing, incomplete, absent with positive prob.

(iii) Under stationary distribution, insurance is increasing, incomplete with probability 1, and completely absent with strictly positive prob.
Insurance in Long Run – Stationary Distribution (Prop. 4)

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- Existence and uniqueness
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A Theory of the Insurance Function?

- Behavior of insurance similar to savings
  - **Insurance is “state-contingent saving”**
  - Buffer stock savings with state-contingent buffer stocks

- Friedman (1957), *A Theory of the Consumption Function* (page 39)
  - “These regressions show **savings to be negative at low measured income levels, and to be a successively larger fraction of income, the higher the measured income**. If low measured income is identified with ‘poor’ and high measured income with ‘rich,’ it follows that the ‘poor’ are getting poorer and the ‘rich’ are getting richer. The identification of low measured income with ‘poor’ and high measured income with ‘rich’ is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof.”
Parameters: $y' \in \{0.8, 1.2\}, p = 0.5$, CRRA with $\gamma = 2$
Precautionary Nature of Insurance (Prop. 5)

- **Definition:** Behavior is **precautionary** if it increases when risk increases (MPS on $y'$)

- Assume income process independent: $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$

Remarkably:
Risk aversion sufficient
No assumptions on prudence ($u_{ccc}(c)$) required
Contrast: Classic precautionary savings result in models with incomplete markets (Bewley (1977), Aiyagari (1994), Leland (1968))
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  - $\tilde{\pi}(s')$ is a mean-preserving spread (MPS) of $\pi(s')$
  - Then $\tilde{E}[\tilde{h}'] \geq E[h']$
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(Novel) Global Monotonicity – Why?

- **Household finance**
  - Positive persistence of income further lowers marginal value of net worth when current income realization is high
  - Yields *globally increasing insurance* theorem
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- **Corporate finance**
  - Positive persistence in cash flows due to productivity shocks
    - Positive persistence productivity shocks implies conditional expected productivity higher
    - **Investment opportunities** raise marginal value of net worth when current productivity is high
  - Effects go in opposite direction; no global monotonicity result
Household Finance with Durable Goods

- Preferences: \( u(c) + g(k) \) where \( k \) is durable good (e.g., housing)

- Durable goods
  - ... as collateral for state-contingent debt \( b' \)

  \[
  \theta k(1 - \delta) \geq Rb'
  \]

  where \( \delta \) is depreciation rate and \( \theta \in [0, 1) \) is collateralizability
  - ... imply additional financing needs
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- **Equivalent insurance formulation**
  - Fully lever durables: set \( \hat{b}' = R^{-1}\theta k(1 - \delta) \) and pay down

\[
\varphi \equiv 1 - R^{-1}\theta(1 - \delta)
\]

  - Insure with Arrow securities \( h' \) subject to short sale constraints

\[
h' \equiv \theta k(1 - \delta) - Rb'
\]
Household’s Problem with Durable Goods

- Given $s$ and $w$, household solves

$$v(w, s) \equiv \max_{c,k,h',w' \in \mathbb{R}_+^2 \times \mathbb{R}^2_s} u(c) + \beta g(k) + \beta E[v(w', s')|s]$$  \hspace{1cm} (5)

subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + \varphi k + E[R^{-1} h'|s]$$  \hspace{1cm} (6)

$$y' + (1 - \theta)k(1 - \delta) + h' \geq w'$$  \hspace{1cm} (7)

and short sale constraints, $\forall s' \in S$

$$h' \geq 0$$  \hspace{1cm} (4)
Household’s Problem with Durable Goods

Given \( s \) and \( w \), household solves

\[
v(w, s) \equiv \max_{c, k, h', w' \in \mathbb{R}^2_+ \times \mathbb{R}^2} u(c) + \beta g(k) + \beta E[v(w', s')|s]
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subject to budget constraints for current and next period, \( \forall s' \in S \)

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w \geq c + \varphi k + E[R^{-1}h'|s]
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\[
h' \geq 0
\] (4)

Remarks

- Net worth \( w \) is cum income and durable goods net of borrowing
- Investment Euler equation for durable goods

\[
1 = \beta g_k(k) \frac{1}{\varphi} + E \left[ \beta \mu' \frac{(1 - \theta)(1 - \delta)}{\varphi} \right] s
\]
Insurance with Durable Goods: Properties (Prop. 6)

- Properties generalize

Net worth next period $w'$ strictly increases in $w$, given $s$.

Increasing insurance with stochastic monotonicity

Household never insures the highest state next period.

Marginal value of net worth $v_w(w, s)$ is decreasing in $s$.

Poor households do not participate in insurance markets.

For sufficiently low net worth, the household is constrained against all states next period.

Net worth next period $w' \geq y' + (1 - \theta)k(1 - \delta) > y'$.
Insurance with Durable Goods: Properties (Prop. 6)

- **Properties generalize**

- **Monotonicity**
  - Net worth next period $w'$ strictly increases in $w$, given $s$
  - Insurance $h'$ does not necessarily increase in $w$

- **Incomplete insurance** (with $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$)
  - Household never insures the highest state next period

- **Increasing insurance with stochastic monotonicity**
  - Household insures lower interval of states
  - Marginal value of net worth $v_w(w, s)$ is decreasing in $s$

- **Poor households do not participate in insurance markets**
  - For sufficiently low net worth, household is constrained against all states next period
  - Net worth next period
    \[ w' \geq y' + (1 - \theta)k(1 - \delta) > y' \]
Insurance with Durable Goods: Example

- Financing needs for durables reduce insurance and increase net worth accumulation.

- Parameters: $y' \in \{0.8, 1.2\}$, $p = 0.5$, CRRA with $\gamma = 2$, $g = 2$, $\theta = 0.8$
Determine $R$ in equilibrium to clear market for collateralized claims.
Equilibrium and Effect of Collateral on Insurance

- Determine $R$ in equilibrium to clear market for collateralized claims

- **Theory of insurance firms**
  - Asset-and-liability structure result of intertemporal nature of insurance

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Financing Insurance
Determine $R$ in equilibrium to clear market for collateralized claims

**Theory of insurance firms**
- Asset-and-liability structure result of intertemporal nature of insurance

**Collateral scarcity**
- In equilibrium, $\beta R \leq 1$, with equality iff $\theta \geq \bar{\theta}$
- When $\theta < \bar{\theta}$, collateral is scarce and $\beta R < 1$ as previously assumed
Stationary Equilibrium

- **Definition (Stationary Equilibrium)** A stationary equilibrium is
  
  allocation \( x(z) \equiv \{c(z), k(z), h'(z), w'(z)\} \) for each household given
  
  \( z \equiv \{w, s\} \)
  
  interest rate \( R \)
  
  stationary distribution \( F(z) \)

  such that

  - \( x(z) \) solves each household’s problem in (5)-(7) and (4), given \( z \)
  
  - market for state-contingent promises clears

  \[
  \int_z E[b'(z)|s]dF(z) = 0 \tag{12}
  \]

  or equivalently, supply of collateralized claims equals demand for state-contingent claims \( h'(z) \)

  \[
  \int_z \theta k(z)(1 - \delta)dF(z) = \int_z E[h'(z)|s]dF(z). \tag{13}
  \]
Theory of Insurance Firms

- Interpretation of market clearing condition
  - Representative insurance company

- Insurers’ balance sheet

\[
\int z \theta k(z)(1 - \delta)dF(z) = \int z E[h'(z)|s]dF(z)
\]

Assets: mortgages
Liabilities: diversified insurance claims

- Assets: collateralized loans ("mortgages")
- Liabilities: diversified portfolio of insurance claims

Why do insurers/insurance sector have positive assets?

- Consequence of intertemporal nature of insurance
- Insurance premia are paid up-front
Aggregation and Resource Constraints

- Define $W \equiv \int_z w(z) dF(z)$, $C \equiv \int_z c(z) dF(z)$, $K \equiv \int_z k(z) dF(z)$, $H' \equiv \int_z h'(z) dF(z)$.

- Aggregate budget constraints (6) and (7) $\forall s' \in S$

$$W = C + \varrho K + E[R^{-1}H'] \quad (14)$$

$$E[y'] + (1 - \theta)K(1 - \delta) + E[H'] = W \quad (15)$$
Define $W ≡ \int_z w(z) dF(z)$, $C ≡ \int_z c(z) dF(z)$, $K ≡ \int_z k(z) dF(z)$, $H' ≡ \int_z h'(z) dF(z)$.

Aggregate budget constraints (6) and (7) $\forall s' \in S$

$$W = C + \varphi K + E[R^{-1} H'] \quad (14)$$

$$E[y'] + (1 - \theta)K(1 - \delta) + E[H'] = W \quad (15)$$

Using market clearing $\theta K(1 - \delta) = E[H']$, (14) and (15) imply

$$W = C + K = E[y'] + K(1 - \delta) \quad (16)$$

so aggregate wealth = consumption + capital stock or

$$E[y'] = C + \delta K$$

so aggregate wealth = output + depreciated capital stock
Full Insurance in Economy with Abundant Collateral

- Suppose $\beta R = 1$; then

\[ \mu = \mu' + \lambda' \geq \mu', \quad s' \in S \]

so $\mu = \mu' = \mu^*$; full insurance

- Let $R^* = \beta^{-1}$ and $r^* \equiv R^* - 1$; consumption $c^*$, net worth $w^*$, and durable goods $k^*$ constant and investment Euler equation

\[ r^* + \delta = \frac{g_k(k^*)}{u_c(c^*)} \tag{17} \]
Full Insurance in Economy with Abundant Collateral

■ Suppose $\beta R = 1$; then

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■ Let $R^* = \beta^{-1}$ and $r^* = R^* - 1$; consumption $c^*$, net worth $w^*$, and durable goods $k^*$ constant and investment Euler equation

$$r^* + \delta = \frac{g_k(k^*)}{u_c(c^*)} \quad (17)$$

■ Using $\phi^* = 1 - R^{*-1} \theta (1 - \delta)$, write (6) and (7) as

$$w^* = c^* + \phi^* k^* + E[R^{*-1} h'^*], \quad (18)$$

$$\gamma' + (1 - \theta) k^*(1 - \delta) + h'^* = w^*, \quad \forall s' \in S. \quad (19)$$
Feasible? – As long as $h^* \geq 0$, $\forall s' \in S$
Feasible? – As long as $h'^* \geq 0$, $\forall s' \in S$

Using (16) to substitute for $w^*$ in (19)

$$h'^* = \theta k^*(1 - \delta) - (y' - E[y']) \geq 0, \quad \forall s' \in S$$

that is, **sufficient pledgeability**

$$\theta \geq \bar{\theta} \equiv \frac{y(s') - E[y']}{{k^*}(1 - \delta)} \geq 0$$
Effect of Collateral Scarcity on Interest Rate and Insurance

- **Collateral scarcity**: $\theta < \bar{\theta}$

- Then $\beta R < 1$ as previously assumed
  - At $\beta R = 1$ excess demand for collateralized claims
Collateral scarcity: $\theta < \bar{\theta}$

Then $\beta R < 1$ as previously assumed

- At $\beta R = 1$ excess demand for collateralized claims

Incomplete insurance and previous characterization applies
Effect of Collateral Scarcity on Interest Rate and Insurance

- **Collateral scarcity:** $\theta < \bar{\theta}$

- Then $\beta R < 1$ as previously assumed
  - At $\beta R = 1$ excess demand for collateralized claims

- **Incomplete insurance and previous characterization applies**

- $R < 1$ is possible, that is, negative interest rates $r = R - 1$
Effect of Collateral Scarcity on Interest Rate and Insurance

- Effect on interest rate, insurance, durable stock, and welfare loss

![Graphs showing the effect of collateralizability on interest rate, aggregate insurance, aggregate durable goods, and welfare loss.](image_url)
Effect of Collateral Scarcity on Inequality

- When collateral is scarce, wealth distribution fans out

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Financing Insurance
Conclusion – Focus: Intertemporal Nature of Insurance

- **Collateral scarcity** implies $\beta R < 1$

- When collateral is scarce, **equilibrium insurance**
  - ... is incomplete, **globally increasing, and precautionary**
Collateral scarcity implies $\beta R < 1$

When collateral is scarce, equilibrium insurance is incomplete, globally increasing, and precautionary.

Financing aspect of insurance key to understanding:
- basic patterns in insurance and household risk management
- asset-and-liability structure of insurers
Conclusion – Focus: Intertemporal Nature of Insurance

- **Collateral scarcity** implies $\beta R < 1$

- When collateral is scarce, **equilibrium insurance**
  - ... *is incomplete, globally increasing, and precautionary*

- Financing aspect of insurance key to understanding
  - **basic patterns** in insurance and household risk management
  - **asset-and-liability structure of insurers**

- Friction as important as moral hazard and adverse selection?
Comparison to Bewley (1977) Economy

- **Precautionary savings** $h$ instead of insurance $h'$, $\forall s' \in S$

- Given $s$ and $w$, household solves

$$ V(w, s) \equiv \max_{c, h, w' \in \mathbb{R}_+ \times \mathbb{R}^{S+1}} u(c) + \beta E[V(w', s') | s] $$

subject to budget constraints for current and next period, $\forall s' \in S$

$$ w \geq c + R^{-1}h $$
$$ y' + h \geq w' $$

and short sale constraint

$$ h \geq 0 $$

- Same key property: $v_w(w, s)$ decreasing in $w$ and $s$ (same proof strategy)

- With $FOSD$, $h$ (weakly) increasing in $w$ given $s$, but (weakly) decreasing in $s$ given $w$

- “Insurance” in Bewley model not increasing in $s$ (Prop. D.1)
Precautionary savings vs. insurance

Parallels

Since $v_w(w, s)$ decreasing in $w$ and $s$, envelope condition implies consumption $c$ increasing in $w$ and $s$

Thus precautionary savings $h$ and insurance expenditure $E[R^{-1}h'|s]$ both increasing in $w$ (given $s$) and decreasing in $s$ (given $w$)

Key distinction

Insurance $h'$ increasing in $s$ (although insurance expenditure $E[R^{-1}h'|s]$ decreasing in $s$)

Insurance is increasing in $s$ ($w'$ increasing in $s$)

Precautionary savings $h$ decreasing in $s$ implies $w(s')$ decreasing in $s$

“Insurance” in Bewley model not increasing in $s$
Precautionary Savings in Bewley Model (Prop. D.2)

- **Key condition:** Convexity of marginal utility $u_c(c)$

- Assume income process independent: $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$

- **Marginal value of net worth**
  - If $u_c(c)$ is convex, then $v_w(w)$ is convex

- **Precautionary savings guaranteed only if $u_c(c)$ convex**
  - Suppose $\tilde{\pi}(s')$ MPS of $\pi(s')$
  - If $u_c(c)$ convex, then $\tilde{h} \geq h$

- New proof of classic result
  - Leland (1968), Sandmo (1970), Sibley (1975), and Kimball (1990)

- Contrast: Precautionary insurance requires only risk aversion
Parameters: \( y(s') \in \{0.8, 1, 1.2\} \); \( \pi(s') = \pi_{\sigma}, 1 - 2\pi_{\sigma}, \pi_{\sigma}; \) with \( \pi_{\sigma} = 0, 0.2, 0.5 \).
Durable Goods Price Risk Management

- Stochastic durable goods price $q(s)$
- Down payment $\varphi(s) \equiv q(s) - R^{-1} \theta E[q'|s](1 - \delta)$

**Household’s problem:** Given $s$ and $w$, household maximizes (5) subject to budget constraints for current and next period, $\forall s' \in S$

\[
\begin{align*}
    w & \geq c + \varphi(s)k + E[R^{-1}h'|s] \\
    y' + (1 - \theta)q'k(1 - \delta) + h' & \geq w'
\end{align*}
\]  

(24)  

(25)

and the short sale constraints, $\forall s' \in S$

\[
h' \geq 0
\]  

(4)

**Effect of durable goods price**

- Investment Euler equation for durable goods

\[
1 = \beta \frac{g_k(k)}{\mu} \frac{1}{\varphi(s)} + E \left[ \beta \frac{\mu'}{\mu} \frac{(1 - \theta)q'(1 - \delta)}{\varphi(s)} \right]_s
\]

- Price affects down payment $\varphi(s)$ and net worth $w'$ directly
Durable Goods Price Risk: Example

- Hedging durable goods prices ($q' \in \{0.95, 1.05\}$) (and income)

- Durable goods price affects cost of durables, net worth, and required down-payments
Increasing Durable Goods Price Risk Management

- **Full pledgeability of resale value \((\theta = 1)\)** (Prop. 8)

- With logarithmic preferences, household income insurance is increasing and household does not hedge price risk
  - Why? - Price risk separable; does not affect marginal utility of net worth

- With isoelastic preferences and \(\gamma < 1\), risk management is increasing
  - Intuition
    - Equivalent to economy with preference shocks
    - With \(\gamma < 1\), \(q(s)\) lowers marginal utility (similar: Campbell (1996))
Risk Management and Rent vs. Buy Decision

- **Renting:** Ease of repossession allows higher leverage
  - See Eisfeldt/Rampini (2009) and R/V (2013)
  - Pay rental fee in advance

\[
    u_l(s) \equiv rq(s) - (E[q'|s] - q(s)) + E[q'|s](\delta + m)
\]

where \( m \) is monitoring cost

- Financially constrained households rent

- **Renting affects incentives to hedge**
  - High (implicit) leverage induces hedging
  - Sign of hedging demand can flip (related: Sinai/Souleles (2005))
**Household’s problem:** Given $s$ and $w$, household maximizes (5) subject to budget constraints for current and next period, $\forall s' \in S$

$$w \geq c + \varphi(s)k_o + R^{-1}u_l(s)k_l + E[R^{-1}h'|s] \tag{14}$$

$$y' + (1 - \theta)q'k_o(1 - \delta) + h' \geq w' \tag{15}$$

non-negativity constraints on owned and rented durables,

$$k_o, k_l \geq 0 \tag{16}$$

and short sale constraints, $\forall s' \in S$

$$h' \geq 0 \tag{4}$$

with $k = k_o + k_l$
High leverage of renting induces risk management \((m = 0.02)\)

Renters hedge states with high house prices!