Public Trust, The Law, and Financial Investment *

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Abstract
How does trust evolve in markets? What is the optimal level of government regulation and how does this intervention affect trust and economic growth? How do professional fees affect trust formation? In a two-stage theoretical model, we analyze the trust that evolves in markets, given the value of social capital, the level of government regulation, and the potential for economic growth. We show that when the value of social capital is high, government regulation and trustfulness are substitutes. In this case, government intervention may actually cause lower aggregate investment and decreased economic growth. In contrast, when the value of social capital is low, regulation and trustfulness may be complements. We analyze the optimal level of regulation in the market, given the conditions in the economy, and show that the absence of government intervention (a Coasian plan) is suboptimal in a culture in which social capital is not highly valued and when the potential for economic growth is low. We finally evaluate the effects of fees on the trust that forms in various cultures (high vs. low value to social capital) and compare our results with the implications of classic agency theory. Overall, our theoretical analysis in this paper is consistent with the empirical literature on the subject and we highlight novel predictions that are generated by our model.

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1 Introduction

It is well documented that public trust is positively correlated with economic growth (Putnam 1993; LaPorta, Lopez-de-Silanes, Shleifer, and Vishny 1997; Knack and Keefer 1997; Zak and Knack 2001) and with participation in the stock market (Guiso, Sapienza, and Zingales 2007a). These empirical findings raise several fundamental questions that we explore in this paper: How does trust form in markets? How does law and regulation affect the level of trust in the market? Are the law and trust always complements, or can they sometimes be substitutes? How can governments optimally affect the trust level that evolves in markets in order to maximize economic growth? How do professional fees affect the trust that forms in the market?

Existing empirical evidence offers contrasting answers to these questions. For example, La Porta et al. (1998, 2006) document substantial cross-sectional variation in the legal protection that investors receive in different countries, and posit that there exists a positive correlation between government regulation and market growth. Glaeser, Johnson, and Shleifer (2001) also argue for this positive relationship and use the differences between markets in Poland and the Czech Republic as a motivating example. In contrast, Knack and Keefer (1997) find that Scandinavian countries have substantial growth, despite the fact that their laws provide significantly less investor protection compared to common-law countries (LaPorta, Lopez-de-Silanes, Shleifer, and Vishny 1998). Likewise, Allen, Qian, and Qian (2005) study the emerging Chinese market and show that substantial growth of the private sector has occurred, despite the absence of a strict legal system. They assert that business culture and social norms play a large role in the productivity in China. Further, Allen, Chakrabarti, De, Qian, and Qian (2006) find that despite having a legal system with low investor protection in India, remarkably high growth has occurred due to a reliance on “informal and extra-legal mechanisms”. Based on all of these observations, the natural questions that arise are under what conditions is government intervention optimal (in the form of laws) and when is a Coasian approach more effective?

In order to address these questions, we develop a two-period theoretical model in which investors entrust their wealth to a continuum of heterogeneous agents and rely on the agents to honor their

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1 According to LaPorta et al. (1998), Scandinavian countries have the highest trust scores on World Value Surveys. Knack and Keefer (1997) show that the growth and high investment rate (GDP-scaled) in Scandinavian countries can be directly attributed to the high level of trust in these countries. For example, see Figures II and III in Knack and Keefer (1997).

2 A Coasian plan refers to a regime in which government regulation is minimized because market participants organize (or contract) to achieve efficient outcomes. See “Coase Versus The Coasians” for a good summary of this debate (Glaeser, Johnson, and Shleifer 2001).
fiduciary duty. Within a rational expectations framework, we analyze how public trust, aggregate investment, and economic growth change based on the legal environment and the social networks that are present in the market.

Before describing our model and results, three unique aspects of our notion of trust are worth highlighting. First, the ability of clients to rely on others (develop trust) in our model is calculative and arises from two sources: the law and culture. Calculative trust, as defined by Williamson (1993), means that investors rationally compute their trust level based on their subjective beliefs about the gambles they face. In making this calculation, they take into account two primary sources of trust. Trust that arises from the law evolves because investors can rely on the government to make sure that agents honor their fiduciary duty to clients. Trust that arises from culture evolves because investors can rely on a certain amount of professionalism or the social networks that have been established in the population. That is, in the latter type of trust, agents honor the fiduciary duty due to a social norm, not a formal law. In some circumstances, these two sources of trust may be complements, but in others they may be substitutes (Williamson 1993, Yamagishi and Yamagishi 1994).

Second, our concept of public trust differs from the previous notions of private trust and relationship building. The latter develop because participants interact repeatedly, often in a dynamic setting with an infinite horizon (e.g. Abreu 1988; Abdulkadiroglu and Bagwell 2005). The Folk Theorem is usually invoked, and because participants are allowed to punish each other for deviations from cooperation, this stabilizes the relationships that develop, but at the same time renders trust less important. Indeed, trust is more valuable when participants do not have a built-in governance mechanism (such as a punishment scheme) to protect their interests (Fukuyama 1995 and Zak and Knack 2001). This is the case when participants interact infrequently and/or the horizon is temporary (finite). Public trust becomes crucial for growth to occur, which is what we wish to model. Therefore, in our model, clients and agents interact over a finite horizon (two periods) and trust evolves as a public good due to both incentives and social norms, without the need for repeated interaction between the agents and clients.

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3This approach is consistent with Williamson (1993), Yamagishi and Yamagishi (1994), and Fukuyama (1995). Yamagishi and Yamagishi (1994) refer to these two types of trust as deterrent and benevolent trust.

4As such, the model that we pose is fully rational as all of the clients have consistent beliefs about the markets they face. Guiso, Sapienza, and Zingales (2007a) also adopt a calculative form of trust. In their model, investors rationally calculate their willingness to participate in the stock market.

5As we will discuss in the paper, the model could be generalized to include more periods. But what is critical is that the interaction should occur during a finite number of periods, so that trust plays a role in the relationship between the clients and agents.
Third, trust is only important when the contract between the parties is incomplete. That is, if state contingent contracts can be written and upheld by law, which protect the clients in all states of the world, then trust is a superfluous consideration. As Williamson (1993) points out, the ability to write such contracts renders trust unimportant to the relationship. In essence, when the contract is complete, investors can rely on the contract, rather than trust their investment agent. As such, even though state-contingent bonuses are common to many transactions, we restrict the contract space within the model to be necessarily incomplete, to then evaluate the role that trust has in the market.⁶

The model proceeds as follows. At the beginning of the game, a continuum of heterogeneous agents decide whether to pay a private cost to become trustworthy (good types) and act in their client’s best interest. Those who do not (opportunistic types) act in their own best interest and ignore their client’s well-being. We consider this cost to be linked to both the value that an agent derives from their social capital and the social pressures that result from the networks in which they participate. For example, if an agent has access to a well-developed social network that they can rely on, then they have a low cost of providing full service to the potential clients that they face. Additionally, this type of agent will also experience stronger social pressures to honor their obligations and will experience more “social disutility” when they fail to do so. In contrast, agents with poorly developed networks will not be able to honor their duty to their client with such ease and do not suffer a high utility penalty when they ignore their responsibilities to others.

The distribution of these costs (distribution of agents) characterizes the business culture of any population and defines the agents’ tendency to become trustworthy, given the incentives that they are given and the regulations they face. In equilibrium, the fraction of agents who become good-types represents the amount of public trust that exists in the market. Since clients are rational and have consistent beliefs, they properly calculate the level of public trust available in the market, even though they do not observe each agent’s individual choice. In each period, clients decide how much to invest with particular agents given the overall level of public trust and the protection offered by the government. Outcomes from the first period investment are publicly observable and therefore, the amount invested in the second period also depends on an agent’s outcome from the first period. In both periods, agents who are trustworthy maximize the outcome of the stochastic investment opportunity they face, whereas opportunistic agents do only what is required by law.

⁶As will become obvious, the model that we pose could be generalized to include contracts which have incentives. As long as they remain incomplete and the agents have some discretion, the results that we generate would not change qualitatively.
Based on the social culture that exists (i.e. the value of social capital), two different types of equilibria generically arise. In cultures where social capital is important (e.g. concave distribution functions), the public trust that develops is increasing in the potential productivity of the economy, and is decreasing in the amount of governmental regulation that is imposed (Type I Equilibrium).

That is, less public trust will form in these societies when laws governing the market are more strict. The intuition for this finding is that tough laws make it less rewarding for the marginal agent to reveal that they are trustworthy (through a public outcome). In fact, we show that strict laws may even displace public trust from the market altogether and in some cases more government intervention may actually lead to less aggregate investment and lower economic growth.

In contrast, in societies where social capital is less valuable, an additional class of equilibria can arise. In this case, government involvement increases public trust and aggregate investment in the market (Type II Equilibrium). That is, a more stringent legal system and the formation of public trust are complements. Interestingly, in these types of cultures, a higher potential productivity may lead to less aggregate investment in the market and lower economic growth. The intuition for this is that a higher productivity leads to more opportunism and therefore, clients are less willing to invest. Opportunities for growth may be lost because of higher incentives for opportunism.

Of course, the role of the government should be optimally determined based on the social culture in place and the tendency for public trust to develop. From the results already mentioned, we show that government regulation is less likely and may even be value-destroying when social capital is important in a society. In contrast, with a Type II equilibrium, regulation can be responsible for catalyzing both public trust in the market and economic growth. Most interestingly, we show that a Coasian plan is never optimal when the potential for productivity in the economy is low. That is, while the optimal level of government involvement may vary based on culture, the government will always expend resources to protect investors when potential for growth is low. There is always a role for some investor protection. This is an important finding as it sheds light on the previously mentioned debate over what type of law is optimal.

Finally, we consider the effect that professional fees have on the trust that forms in markets. We show that in a Type I equilibrium, trust is increasing in fees (incentives) as long as fees are relatively low. Once fees rise sufficiently high, however, trust begins to decrease as fees rise further. The reason that effort provision (i.e. becoming trustworthy) decreases after a threshold is because trust is a public good and as such, the benefit to becoming trustworthy is common among all agents. Once agents receive fees that are too high, it becomes harder for the marginal agent to
distinguish themselves when they are working harder for their client. Hence, trust formation does not necessarily rise as incentives increase. Throughout the analysis we compare our results with the predictions of standard agency theory.

There are two caveats that are important to address. First, for most of this paper, the social structure and the value to social capital is viewed as a primitive. Based on the distribution of costs of becoming trustworthy (the value to social capital), we analyze how much public trust evolves and the effect of government regulation on its formation. Thus, we accept Fukuyama’s view that social structure and culture have substantial inertia and that “durable social institutions cannot be legislated into existence the way a government can create a central bank or an army.” Indeed, Guiso, Sapienza, and Zingales (2007b) document empirically the extreme persistence of culture. There also exists previous work that has focused on the formation of social capital, primarily through the development of social norms and social networks; however, it is not our intention in this paper to model how business cultures primarily form, but to generate an analysis of how public trust evolves in relation to the laws that are set and how this affects economic growth. Further, our goal is to describe how the public (clients) benefits from the social networks that exist, even though they are not a part of these “private” relationships. In light of this, though, we do discuss the effect that the government has on social culture in Section 4 of the paper.

The second caveat is that the model in this paper could also be viewed as a model of hidden quality and might apply to economic settings besides financial investment. We feel that this is indeed true, since trust is also important when the quality of a good or service is hidden from a consumer. For example, consider a patient in a hospital who needs to trust a surgeon, or a consumer of a new pharmaceutical. While some protection is available by law, at least some of the components of quality are hidden, and trust is required to stabilize the trading relationship. Therefore, while we apply our model of trust to investments and economic growth, where it indeed adds considerable surplus, the model could be applied to other economic situations in which trust is important to reach Pareto superior outcomes.

The rest of the paper is organized as follows. In section 2, we set up our benchmark model and introduce our notions of public trust, the law, and social culture. Section 3 derives and characterizes the various equilibria of the game. Section 4 studies the role of the government in the market. Section 5 studies the effects of fees on trust formation. Section 6 concludes. The appendix contains all of the proofs.

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7See, for example, Kandori (1992); Greif (1994); Glaeser, Laiibson, and Sacerdote (2002); Bloch, Genicot, and Ray (2005); Mobius and Szeidl (2006); Robinson and Stuart (2006)
Each agent $j$ chooses whether to pay $d_j$ ($\tau$ realized) to become good types. The fraction who become trustworthy is given by $\tau$. Clients invest $p_1$ and receive zero if the investment fails or receive one if the investment succeeds. Good types maximize the potential for success, whereas opportunistic types only do what is required by law. At the end of $t = 1$, success ($S$) or failure ($F$) is publicly observed for each agent. At $t = 2$, investors invest $p_S$ if an agent succeeded last period or $p_F$ if the agent failed. Opportunistic types again only do what is required by law, whereas good types maximize the potential of the investment. Finally, $S$ or $F$ is realized and the game ends.

### 2 Market For Trust

Consider a two-stage model (Figure 1) in which a continuum of risk-neutral agents sell an investment opportunity to another continuum of risk-neutral clients in each period. The agent may have a different role depending on the specific investment type, but in all cases, they have a fiduciary duty to act in their client’s best interest. That is, the agent has a responsibility to use the capital in the best possible way to maximize the chances that the investment is successful. For $t \in \{1, 2\}$, define $p_t$ as the price that the client pays for the investment and $\phi$ as the fraction of $p_t$ that the agent keeps as a fee.\(^8\) In the market, $p_t$ is determined competitively, and we assume that the measure of clients is larger than that of the agents, so that when a transaction takes place, the client purchases the investment for its full expected value.

At the beginning of period one ($t = 1$), each agent $j$ chooses whether to pay a cost $d_j$ to become trustworthy and act in the best interest of their client (i.e. become a “good” ($G$) type). By becoming trustworthy, good types honor their client’s fiduciary duty and maximize the chances that the client receives a high payoff from the investment. If an agent chooses not to pay $d_j$, they only do what is required by law for their clients. The cost $d_j$ represents a durable investment (sunk cost) by some of the agents to protect their client’s interests. We restrict the actions of non-trustworthy agents by not allowing them to make such an investment at the beginning of $t = 2$. This, however, is without loss of generality in the two-period game, since it would never be rational for these agents

\(^8\)We treat the fee $\phi$ as exogenous and independent of the potential for production. In Section 5, however, we analyze the effect that changes in $\phi$ have on the trust that forms in the market and consider that $\phi$ affects the potential for production.
to pay $d_j$ at $t = 2$.\footnote{This will become clear when we analyze the optimal actions of the players in Section 3. If we would generalize the model to be $n < \infty$ periods in duration, it would never be optimal for agents to newly invest in this technology at the beginning of period $n$.}

Agents in the market are heterogeneous with respect to the cost $d_j$. Some agents have access to better social networks and are more efficient in providing full service to their clients. Given their relationships, they find it easier to rely on other market participants and offer better opportunities to outsiders. Additionally, agents who have more developed networks feel greater pressure to honor their responsibilities, which results in a higher social disutility if they disregard their duties to others. Since the outcomes from the investments are publicly observable both by clients and members of the social network, failure is associated with social disutility.\footnote{For example, see Kandori (1992).} Therefore, agents who are more “socially entrenched” (with a low $d_j$) are more likely to become trustworthy, given the incentives they face.\footnote{An alternative interpretation of this cost might be that agents who are more socially entrenched experience higher moral disutility from ignoring the interests of their clients. This type of moral disutility for shirking has been modeled previously by Noe and Robello (1994).} The opposite is true for an agent with a high cost $d_j$. In this case, they do not have access to the same channels and do not experience the same degree of disutility when they disregard their duties to others. Therefore, they are less likely to become trustworthy.\footnote{In an alternative specification of the model, the cost $d_j$ could be calculated as $d_j = c_j - s_j$, where $c_j$ is the cost of implementing systems to protect the interests of clients and $s_j$ is the disutility incurred if the agent shirks. For tractability, we prefer to characterize our agents with $d_j$, while keeping in mind both sources of each agent’s costs.}

Consider, for example, that each agent represents an investment broker who may either prepare to invest money on behalf of their client or not. Preparation requires effort and time as research is often involved. Access to social networks or connections allows some brokers to obtain information about potential investments in an easier fashion. Additionally, since the performance of each broker is publicly observable to members of their network, some brokers have greater incentives (pressure) to maintain a reputation in good standing.

There are other potential interpretations of the costs $d_j$, especially when each agent represents an entire organization, such as an entrepreneur or a CEO. Then, $d_j$ might also represent the cost of solving agency issues within the firm. As in Carlin and Gervais (2007), if employees are drawn from a highly ethical population, then the firm maximizes value by offering fixed wage employment contracts and avoiding the costs of risk-sharing.\footnote{See also Sliwka (2007).} If employees are prone to shirking or stealing because social norms are lax, then maximizing value requires costly incentives, which would then be parameterized by a high cost $d_j$. 

Let $F(d)$ be the distribution of costs in a population such that $d \in [0, 1]$, $F(0) = 0$, and $F(\cdot)$ is strictly increasing and twice continuously differentiable over the entire support. By assuming that $F(0) = 0$, we exclude the possibility that a fraction of agents are dependable no matter what incentives are present in the market. As such, each distribution $F(\cdot)$ characterizes the culture of a particular society and the tendency of people to honor their responsibility and be trustworthy. In the context of our model, $F(\cdot)$ measures the ease with which agents in a particular population can invest to help and/or protect their clients. For example, if

$$F_1(d) \geq F_2(d)$$

for all $d \in [0, 1]$, then we can call population 1 more trustworthy than population 2.

The shape (curvature) of $F(\cdot)$ is also important in characterizing a population and will play a key role in the types of equilibria that arise in the model. For example, if $F(\cdot)$ is concave, then the majority of agents in the population have relatively low costs of being socially responsible. Alternatively, if $F(\cdot)$ is convex, then there exists a significant mass of agents who have higher costs of becoming trustworthy.\(^{14}\) We will see in Section 3 that the specific characteristics of $F(\cdot)$ drive the type of behavior that is observed in equilibrium. Further, we will see in Section 4 that the characteristics of $F(\cdot)$ also dictate the optimal amount of regulation that a government should impose in the market.

Let $\tau$ denote the proportion of agents that pay the cost $d$. While $\tau$ is not observable by investors, it is correctly inferred in the rational expectations equilibrium that we derive. In this sense, the clients know exactly the fraction of agents who will take their fiduciary responsibility seriously, but for an individual agent, $\tau$ measures how much the client can trust them with their capital. As we will see, when there is more trust in the market (higher $\tau$), the expected productivity of the economy is higher, which is reflected by a larger $p_t$.

The outcome from the investment may be high (success, $S$) or low (failure, $F$). The client derives more utility $u_S$ from a successful investment, and for clarity we fix $u_S = 1$ and $u_F = 0$. The probability that a success or failure takes place is based on the type of agent that the client employs. Good types in the market (fraction $\tau$) succeed with probability $q \in [0, 1)$ and opportunistic types succeed with probability $\epsilon q$ where $\epsilon \in (0, 1)$.\(^{15}\) As such, we consider $q$ to be linked to the potential

\(^{14}\)In the analysis that follows, we also consider intermediate cases, in which the distributions have convex and concave regions.

\(^{15}\)We exclude $\epsilon = 0$ and $q = 1$ because when $\epsilon = 0$ and $q = 1$ both hold, agents can be perfectly screened based on their first period outcome. This occurs because when $q = 1$ good types never fail and when $\epsilon = 0$ opportunistic
growth in the economy. Also, we interpret $\epsilon$ as the degree to which the legal system governs the agent. An investment with a low level of $\epsilon$ is one in which the government requires less disclosure. With low $\epsilon$, the agent has more discretion to violate their fiduciary duty to their client. With a high level of $\epsilon$, the client is better protected by the law. In Section 4 we consider the optimal choice of $\epsilon$ for the government, given that implementation of the law is costly (that is, they face a cost $k_g(\epsilon)$, which we will specify later). Also, throughout what follows, we evaluate the effects of $q$ and $\epsilon$ on the trust $\tau$ that evolves and on economic growth.

The role of the government in setting the law is to delineate what all agents (good or opportunistic) must do to protect their clients. The associated cost to the agents is given by $k_a(\epsilon)$, which may be viewed as the cost of meeting government requirements (for example, processing certain paperwork or following the Sarbanes-Oxley Act). This cost $k_a(\epsilon)$ is the same for all agents and is independent of the decision each agent makes about whether to pay $d_j$. Also, we assume that $k_g(\epsilon)$ is the cost paid by the government to implement and fully enforce the law (i.e. make sure that agents indeed perform these minimum requirements). As such, no agent (good or opportunistic) would refrain from performing these tasks, as they would be detected for sure.

The clients make their investment up-front in each period $t \in \{1, 2\}$. Since clients cannot observe the agent’s type ($G$ or $O$) ex ante, the parameter $\tau$ measures the prior belief of each client about the agent with whom they have a relationship. As already mentioned, in equilibrium this belief equals the actual realized value of public trust. Once the first investment ($p_1$) is made with an agent, however, a success or failure is observed publicly. Agents who succeed in the first period are labeled with an $S$ and agents who fail are labeled with an $F$. Given the prior belief of the clients and the outcome from period one, the clients update their beliefs using Bayes’ law and form the posterior beliefs $Pr(G|S)$ and $Pr(G|F)$. They then use these beliefs to calculate the values for $p_S$ and $p_F$ that they are willing to invest with agents of each type at the beginning of period two. Once the agents are given $p_2 \in \{p_S, p_F\}$, opportunistic agents again ignore their duty to their client, while good types invest optimally. Once a final success or failure is realized, the clients are paid (if they recognize a payoff), and the game ends. The timing of the game is summarized in Figure 1.

It is important to note that we have assumed that each agent’s decision to pay $d_j$ is not publicly observable and cannot be credibly signaled to potential clients. This captures an important aspect of trust since clients in our model are considered “outsiders” to the production of successful types never succeed. This creates discontinuities in the agents’ payoff functions, which unnecessarily complicates the analysis.
investments. That is, when clients interact with an agent, they can neither observe the commitment that the agent has made to their well-being, nor the agent’s access to resources like social networks. If the client were an “insider” and could observe these attributes, then complete information would indeed make trust less important. Trust, however, becomes valuable when the client is an outsider and relies on the agent to protect their interests.

It is equally important to point out that we have restricted the contract space in this game in order to highlight the importance of trust in the market. Specifically, the bargaining power of the clients is low and they pay agents a fee that is independent of the future state of the world. Therefore, clients are not able to offer state-contingent bonuses to induce an effort provision by the agent. With such contracts, the client would be better able to protect themselves and would not have to rely as much on trust. The ability to write contracts that are protective to an investor makes trust less important to the relationship (Williamson (1993)). Trust becomes more valuable when contracts are incomplete and agents have discretion, which is what we wish to capture in this model. Therefore, while the model could be generalized to include contracts which have incentives (but would remain incomplete), the results would not change qualitatively as long as agents have some discretion and the clients are forced to calculate how much that they could trust them.

3 Endogenous Public Trust

We solve the game by backward induction and start by analyzing the optimal actions of the clients in period two.

3.1 Second Period Behavior

At the beginning of the second period, the clients calculate their expected return given the conditional probabilities \( \Pr(G|S) \) and \( \Pr(G|F) \) and invest based on the outcomes from period one. Using Bayes’ rule, the conditional probabilities are

\[
\Pr(G|S) = \frac{q\tau}{q\tau + \epsilon q(1 - \tau)} = \frac{\tau}{\tau + \epsilon(1 - \tau)}
\]

and

\[
\Pr(G|F) = \frac{(1 - q)\tau}{(1 - q)\tau + (1 - \epsilon q)(1 - \tau)}
\]
The investments are then calculated as

\[
p_S = q \Pr(G|S) + \epsilon q \Pr(O|S) \\
= q \Pr(G|S) + \epsilon q [1 - \Pr(G|S)] \\
= (1 - \epsilon) q \Pr(G|S) + \epsilon q
\]

and

\[
p_F = q \Pr(G|F) + \epsilon q \Pr(O|F) \\
= q \Pr(G|F) + \epsilon q [1 - \Pr(G|F)] \\
= (1 - \epsilon) q \Pr(G|F) + \epsilon q
\]

In what follows, we denote

\[
\Delta p \equiv p_S - p_F \\
= (1 - \epsilon) q \left[ \frac{\tau}{\tau + \epsilon (1 - \tau)} - \frac{(1 - q) \tau}{(1 - q) \tau + (1 - \epsilon q)(1 - \tau)} \right]
\]

as the investment difference between agents who experienced the two different outcomes. Notice that because \((1 - q) \epsilon < (1 - \epsilon q)\), the investment difference is always positive, unless \(\tau = 0\) or \(\tau = 1\), in which case \(\Delta p = 0\). Since agents receive a fraction \(\phi\) of the monies invested, \(\Delta p\) measures how much the clients reward (penalize) agents who had a success (failure) in period one. As we will see, the measure \(\Delta p\) plays a major role in the agents’ incentives to become a good type at the beginning of the game. The following proposition describes how \(\Delta p\) is affected by changing \(q, \epsilon, \) and \(\tau\), and will turn out to be useful later when we calculate the amount of trust that forms endogenously in the market.

**Proposition 1.** *(Properties of \(\Delta p\))*

(i) The investment difference \(\Delta p\) increases in \(q\) and decreases in \(\epsilon\).

(ii) If \(\epsilon = 1\) or \(q = 0\), \(\Delta p = 0\). For \(\epsilon \neq 1\) and \(q \neq 0\)

(a) The investment difference \(\Delta p\) is twice continuously differentiable, and strictly concave in \(\tau\) for \(\tau \in [0, 1]\).
Figure 2: The investment differential $\Delta p$ plotted as a function of trust $\tau$. In the first panel, the probability of success is $q = 0.5$ and $\epsilon$ varies between $\epsilon = 0.05$ (solid line), $\epsilon = 0.10$ (dashed line), and $\epsilon = 0.15$ (dashed-dotted line). In the second panel, $\epsilon = 0.10$ and the probability $q$ varies between $q = 0.4$ (solid line), $q = 0.5$ (dashed line), and $q = 0.6$ (dashed-dotted line).

(b) There exists $\bar{\tau}$ such that

$$\frac{\partial \Delta p}{\partial \tau} = \begin{cases} > 0 & \text{if } \tau < \bar{\tau} \\ < 0 & \text{if } \tau > \bar{\tau}, \end{cases}$$

where $\bar{\tau} \equiv \left[1 + \sqrt{\frac{1-q}{\epsilon (1-q)}}\right]^{-1}$.

The intuition of Proposition 1 can be appreciated by inspecting Figure 2. As the potential for productivity in the market increases ($q$ increases), the difference in relative investments widens. This occurs because clients gain more when an agent honors their responsibility to maximize their investment. A higher $q$ also means that the opportunity cost of shirking is higher, so clients increase the investment difference to provide incentives for agents to do the right thing. In contrast, as the level of $\epsilon$ increases, the investment difference decreases. This occurs because as the amount of discretion that agents have decreases, the amount of relative investment incentives that are required also decreases.

The relationship between trust ($\tau$) and the investment differential ($\Delta p$) is a bit trickier. When there is no trust ($\tau = 0$), the outcome in period one does not reveal any new information about the agents. Therefore, $\Delta p = 0$ when $\tau = 0$. For the same reason, when all agents are trustworthy ($\tau = 1$), $\Delta p$ is also zero. For trust levels $\tau \in (0, \bar{\tau})$, $\Delta p$ rises as trust increases. This occurs because
as \( \tau \) rises, the outcomes from the first period are more informative about the agents’ type. However, once the threshold \( \tau \) is reached, as \( \tau \) increases further, the outcomes in the first period become less informative and the optimal amount of \( \Delta p \) decreases. As such, in both panels of Figure 2, the investment differential \( \Delta p \) is a hump-shaped function of the trust \( \tau \). Note that \( \tau \in (0, 1) \) and is completely determined by \( q \) and \( \epsilon \). It is monotonically increasing in \( q \) and quadratic in \( \epsilon \).

### 3.2 First Period Behavior

Once the agents have made their choices about paying \( d_j \) and the level of public trust \( \tau \) is realized, clients rationally make their first period investments, which may be calculated as

\[
p_1 = \tau q + (1 - \tau)\epsilon q.
\]  

(2)

Interestingly, it is easy to show that the expected aggregate investment in each period is the same, that is,

\[
p_1 = \tau p_S + (1 - \tau)p_F.
\]  

(3)

More importantly, \( p_1 \) is a measure of the expected growth of the economy. That is, since \( p_1 \) measures the full expected value of the investment, the larger \( p_1 \) is, the higher the expected growth that the economy will experience as a result of the opportunity. Analyzing (2), \( p_1 \in [\epsilon q, q] \) and \( p_1 \) increases with \( \tau \). That is, as more public trust forms (\( \tau \) rises), the investment becomes more valuable, indicating higher economic growth. The link between trust formation and economic growth is entirely consistent with the findings of both Knack and Keefer (1997) and Zak and Knack (2001).

As we will see shortly, however, the effects of \( q \) and \( \epsilon \) on economic growth are ambiguous because they affect \( p_t \) directly and also through \( \tau \). Depending on the importance of social mores and the culture that exists (specifically on \( F(\cdot) \)), \( q \) and \( \epsilon \) may either increase or decrease economic growth.\(^{16}\)

We now determine the level of public trust that forms in the market, based on the agents’ decisions at the beginning of the game. Consider the initial decision faced by agents, namely whether to become trustworthy. The expected utility from the two choices are

\[
E[u_G] = \phi[p_1 + q p_S + (1 - q)p_F] - d - k_a(\epsilon)
\]

and

\[
E[u_O] = \phi[p_1 + \epsilon q p_S + (1 - \epsilon q)p_F] - k_a(\epsilon)
\]

(4)

\(^{16}\)Of course, if implementing the law were costless, setting \( \epsilon = 1 \) would maximize growth. In Section 4, we consider the case in which implementation is costly and analyze the optimal role of the government.
where $u_G$ is the utility of the good type and $u_O$ is the corresponding utility for an opportunistic type. A particular agent chooses to pay $d$ if

$$E[u_G] \geq E[u_O]$$

$$\phi[p_1 + q_pS + (1 - q)p_F] - d - k_a(\epsilon) \geq \phi[p_1 + \epsilon q_pS + (1 - \epsilon q)p_F] - k_a(\epsilon)$$

$$d \leq \phi(1 - \epsilon)q\Delta p.$$ 

As such, in any equilibrium of this game, the fraction of trustworthy agents, denoted $\tau^*$, is implicitly defined by

$$\tau^* = F(\phi(1 - \epsilon)q\Delta p(\tau^*)).$$ 

(5)

For convenience, we define the function $h(\tau, v)$ such that

$$h(\tau, v) = \tau - F(\phi(1 - \epsilon)q\Delta p(\tau)),$$

where $v = (q, \epsilon, \phi)$ is a particular vector in the parameter space $V \equiv [0, 1] \times (0, 1] \times [0, 1]$. It follows that in any equilibrium, $h(\tau^*, v) = 0$. Also, since $F$ and $\Delta p$ are twice continuously differentiable, $h \in C^2$ as well.

Proving existence of an equilibrium in this game is trivial since $\tau^* = 0$ (autarky) is always an equilibrium. When $\tau = 0$, $\Delta p = 0$ for all parameter values of $\phi$, $q$, and $\epsilon$, so that no agent wishes to deviate and become trustworthy. Therefore, a Prisoner’s Dilemma is always a potential outcome of the game, which is not surprising.

Our purpose here, though, is to characterize non-degenerate (non-autarkic) equilibria of the game in which trust evolves. Thus, we take the standard approach of Debreu (1970) and Mas-Colell (1985) and focus on “regular” positive trust equilibria for the rest of the paper \(^{17}\). We therefore make the following definition.

**Definition 1.** A regular equilibrium is a trust level $\tau^* > 0$ such that $h(\tau^*, v) = 0$ and $\frac{\partial h}{\partial \tau^*} \neq 0$. A regular equilibrium can be of two types:

1) A Type I equilibrium is a regular equilibrium $\tau_1^*$ at which $\frac{\partial h}{\partial \tau_1} > 0$.

2) A Type II equilibrium is a regular equilibrium $\tau_2^*$ at which $\frac{\partial h}{\partial \tau_2} < 0$.

\(^{17}\)The motivation for genericity analysis and focusing on “regular” equilibria in Debreu (1970) and Mas-Colell (1985) is to characterize general equilibria in exchange economies. Indeed, there are pathologic situations in which the excess demand function $z(p)$ might lead to an infinite number of equilibria, preventing comparative statics exercises. By limiting the focus to regular equilibria and proving that such pathologic cases are non-generic, local uniqueness and differentiability of the equilibria is guaranteed, thereby allowing for comparative statics to be generated.
It is indeed possible that other “pathologic” equilibria may arise in the game, especially when we do not restrict the curvature of $F$. For example, it is possible that $\frac{\partial h}{\partial \tau} = 0$ in which case it may be impossible to derive comparative statics because an infinite number of equilibria may exist. In what follows, though, we show that such pathologic equilibria are indeed non-generic, and only occur on a subset of the parameter space with zero measure. This allows us to restrict our attention to regular equilibria and derive meaningful comparative statics because all of the equilibria are locally unique and are amenable to the use of the Implicit Function Theorem \textsuperscript{18}.

Let $E \subset V$ denote the set of parameter values for which at least one positive trust equilibrium arises. We call $E$ the existence set, and will show in Propositions 3 and 4 that it is non-empty. At this point, however, we assume that $E$ is indeed non-empty to show that almost all of its elements give rise to regular equilibria (except a subset of measure zero).

**Proposition 2.** Let $T_v^*$ denote the set of positive trust equilibria that arises for a given $v \in E$. Then, except for a set of $v \in E$ of Lebesgue measure zero, $\frac{\partial h}{\partial \tau} \neq 0$ for all $\tau^* \in T_v^*$ and $T_v^*$ contains a finite number of elements.

The proof of Proposition 2 relies on a result from Mas-Colell (1985), which depends on Sard’s theorem. The result implies that except for a set of parameters having zero measure in the general parameter space, if a positive trust equilibrium exists, it will be regular and therefore its value will be differentiable in terms of the other market parameters. Proving such genericity implies a certain persistency of the types of trust equilibria that we focus on, and therefore motivates our choice to focus on differentiable equilibria, so that we can carry out comparative statics using the implicit function theorem.

Proposition 2 also implies that regular positive trust equilibria are locally unique and must either be of the Type I or Type II variant. As we will show, whether either of these two variants arises depends on the distribution function $F(\cdot)$ that is considered. When social capital is relatively valuable in the population ($F$ strictly concave), a Type I is the only regular positive-trust equilibrium that can arise. When social capital is less valuable in the population, we show that a Type II equilibrium may emerge as well. The different behavior of $h(\tau, v)$ around any regular equilibrium will imply that changes in $q$, $\epsilon$, and $\phi$ will affect trust formation and economic growth differently in

\textsuperscript{18}Note that this restriction is not necessary when we consider that $F(\cdot)$ is concave. In this case, any positive-trust equilibrium will be regular. The restriction will have more bite when we consider allowing $F$ to have both convex and concave regions as pathologic equilibria may arise in this case. However, restricting attention to generic equilibria allows us to generate meaningful comparisons of the types of equilibria that may arise.
these equilibria. As we also show, these properties will also affect the optimal level of government intervention in the market.

We begin first by analyzing economies in which the value to social capital is higher (i.e. \( F(\cdot) \) concave).

**Proposition 3.** *(High Value Social Capital: Type I Equilibria)* The equilibrium fraction of trustworthy agents is implicitly defined by (5). Suppose that \( F''(y) \leq 0 \) for all \( y \). Then, there exists an \( \bar{\epsilon} \) such that if \( \epsilon \geq \bar{\epsilon} \) the unique equilibrium involves \( \tau^* = 0 \), while if \( \epsilon < \bar{\epsilon} \), then there exists one, and only one, other equilibrium, in which \( \tau^*_1 > 0 \).

For any positive equilibrium public trust level, \( \tau^*_1 \) decreases in \( \epsilon \) and increases in \( q \). The maximum level of government intervention \( \tau \) increases in both \( q \) and \( F'(0) \). Finally, the aggregate amount invested in each period increases in \( q \), but decreases in \( \epsilon \) as long as

\[
\frac{d\tau^*_1}{de} < -\frac{1 - \tau^*_1}{1 - \epsilon}.
\]  

An example of a Type I equilibrium is given in Figure 3. According to Proposition 3, increasing the potential for economic productivity \( q \) leads to more public trust in the market. Additionally, as \( q \) increases, the ability for the market to sustain trust increases. For example, the amount of possible government intervention \( \tau \) that does not extinguish public trust rises as \( q \) increases. Importantly,
as the potential for productivity increases, the level of aggregate investment also increases. By Proposition 3, public trust increases with $q \left( \frac{dτ^∗}{dq} > 0 \right)$. According to (2), this implies that $\frac{dp}{dq} > 0$. Therefore, when social capital has value in a culture, as long as $\epsilon < 1$, a higher potential for productivity will actually lead to higher expected growth.

Proposition 3 also implies that public trust and government enforcement systems are substitutes in economies where social capital is valuable. As the government limits the potential loss from opportunism (higher $\epsilon$), the value of becoming trustworthy decreases, which results in a lower overall trust level. When $\epsilon \geq \bar{\epsilon}$, there is no public trust at all in equilibrium. As mentioned before, the cutoff point $\bar{\epsilon}$ in turn depends on the potential for productivity in the economy $q$ and on the distribution $F(\cdot)$. As $q$ rises, the benefit from becoming trustworthy increases, and it takes higher levels of government intervention to eliminate trust. Similarly, since $F''(\cdot) \leq 0$, as $F'(0)$ increases, more mass is shifted to lower costs of becoming “good”, and hence there is an increase in equilibrium public trust, ceteris paribus.

It remains ambiguous how economic growth is affected by $\epsilon$ in this setting. Certainly, given $q$, setting $\epsilon = 1$ maximizes growth, since all agents are forced by law to provide the maximum service to their clients. However, when implementing a maximally stringent legal system ($\epsilon = 1$) is prohibitively costly, it is valuable to consider the effect of $\epsilon$ on growth when $\epsilon < 1$. Indeed, there may exist values of $\epsilon < 1$ for which increasing $\epsilon$ actually decreases growth. Consider the marginal effect of increasing government intervention

$$\frac{dp_1}{d\epsilon} = q\frac{dτ^*_1}{d\epsilon} - \epsilon q\frac{dτ^*_1}{d\epsilon} + (1 - τ^*_1)q = (1 - \epsilon)q\frac{dτ^*_1}{d\epsilon} + (1 - τ^*_1)q$$

Since $τ^*_1$ decreases with $\epsilon$, growth will decrease in $\epsilon$ when

$$\frac{dτ^*_1}{d\epsilon} < -\frac{1 - τ^*_1}{1 - \epsilon}$$

This implies that if the elasticity of $1 - τ^*_1$ with respect to $1 - \epsilon$ is sufficiently high (less than $-1$), government intervention leads to lower aggregate investment by clients and lower economic growth. Intuitively, increasing $\epsilon$ then has two effects: it reduces the loss caused by opportunistic types, and it reduces the equilibrium level of public trust. More agents shirk, but the maximum loss from shirking is lower. Which effect dominates determines the overall effect on growth. As such, $\epsilon$ will have a negative effect on the economy when the equilibrium level of public trust is very responsive.
Figure 4: High Trust Equilibrium. Public trust $\tau_1^*$ and economic growth $p_1$ are plotted as a function of $\epsilon$. The distribution $F(\cdot)$ is uniform over $[0, 1]$ and $q = 0.5$. Both public trust and growth decrease monotonically as $\epsilon$ rises. Public trust is extinguished once $\epsilon$ reaches $\tau = 0.16$.

to changes in $\epsilon$.

Consider the example in Figure 4, in which public trust $\tau_1^*$ (dotted-line) and growth $p_1$ (solid-line) are plotted as a function of $\epsilon$. The distribution $F(\cdot)$ is uniform over $[0, 1]$ and $q = 0.5$. As is evident, both public trust and growth decrease monotonically as $\epsilon$ rises. Public trust is completely extinguished once $\epsilon$ reaches $\tau = 0.16$.

Now, we consider economies in which the value of social capital is lower. The following proposition characterizes the existence of Type I and Type II equilibria and shows that Type I and Type II equilibria are affected differently by changes in government intervention and the potential for growth.

**Proposition 4.** *(Low Value Social Capital: Type I and Type II Equilibria)* The equilibrium fraction of trustworthy agents is again implicitly defined by (5). Suppose that $F'(0) = 0$. Then, there exists a positive-measure set $E \subset V$ such that for all $v \in E$, at least two positive trust equilibria exist. The set $E$ consists of points with sufficiently high $q$ and $\phi$, and sufficiently low $\epsilon$. Further, there exists a set $N \subset V$, such that for all $v \in N$, no positive-trust equilibrium exists. The sets $E$ and $N$ comprise all of the points in $V$, except for a two-dimensional manifold of zero measure.

For almost all parameter values $v$ in the existence set $E$ (i.e. except for a subset of Lebesgue measure zero), the set of equilibria that arises is finite, and it consists exclusively of regular equi-
Figure 5: Low-Trust Equilibrium. The function \( F(\phi(1-\epsilon)q\Delta p(\tau)) \) is plotted as a function of \( \tau \). Two fixed points occur at \( \tau^*_1 \) and \( \tau^*_2 \). The other parameters are \( q = 0.82, \epsilon = 0.04, \phi = 0.2 \) and \( F \) is Beta(4,18).

Further, the equilibrium with the lowest positive trust level is a Type II equilibrium, and the equilibrium with the highest trust level is a Type I equilibrium.

In any Type II equilibrium (\( \tau^*_2 \)), trust is increasing in \( \epsilon \) and decreasing in \( q \), whereas in any Type I equilibrium (\( \tau^*_1 \)), trust is decreasing in \( \epsilon \) and increasing in \( q \). In any Type II equilibrium, aggregate investment \( p_t \) is increasing in \( \epsilon \), but decreasing in \( q \) if

\[
\frac{d\tau^*_2}{dq} < -\left[ \frac{\tau^*_2}{q} + \frac{\epsilon}{(1-\epsilon)q} \right].
\]

In any Type I equilibrium, the aggregate amount invested in each period increases in \( q \), but decreases in \( \epsilon \) as long as

\[
\frac{d\tau^*_1}{d\epsilon} < -\frac{1-\tau^*_1}{1-\epsilon}.
\]

Figure 5 depicts an example of the equilibria that arise when the value to social capital is low. Three equilibria exist in this case. As in Proposition 3, \( \tau^* = 0 \) is an equilibrium. Likewise, the fixed point \( \tau^*_1 > 0 \) has the same properties as the equilibria in Proposition 3. Note in Figure 5 that \( h(\tau, v) \) is indeed increasing at \( \tau^*_1 \). The third equilibrium \( \tau^*_2 \) has different characteristics. Since \( h(\tau, v) \) is decreasing at \( \tau^*_2 \), public trust is decreasing in \( q \) and increasing in \( \epsilon \), which has several important implications. A comparison between Type I and Type II equilibria is summarized in
Table 1: Comparison between Type I and Type II equilibria.

<table>
<thead>
<tr>
<th>Social Capital</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of $q$ on trust</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>Effect of $\epsilon$ on trust</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>Effect of $q$ on growth</td>
<td>$+$</td>
<td>$-/+$</td>
</tr>
<tr>
<td>Effect of $\epsilon$ on growth</td>
<td>$-/+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Table 1.

The fact that public trust decreases in a Type II equilibrium as the economy has a higher potential productivity (higher $q$) is intriguing. Indeed, in some markets as the opportunity for growth increases, the tendency for agents to ignore their fiduciary responsibility also increases. This type of behavior has been documented in several emerging markets (Zak and Knack 2001). The importance of this finding is that this may lead to lower aggregate investment and lower realized growth. If the condition in (7) holds, that is, if public trust decreases quickly as productivity increases, then the opportunity to produce may actually lead to lower economic growth\(^{19}\).

Proposition 4 also implies that public trust and government enforcement systems can be complements in markets where social capital has lower value. Further, increasing $\epsilon$ may have a positive effect on economic growth. Under the conditions in Proposition 4, $\frac{\partial p_t}{\partial \epsilon} > 0$. This means that in certain circumstances as the government requires more disclosure and limits the discretion of agents, clients are more apt to trust the market and make growth possible.

As in Proposition 3, though, too much government intervention can eliminate the formation of public trust altogether. However, in contrast, when social capital is low, the level of potential productivity must also be sufficiently high for public trust to form. In the model, this means that the existence set $E$ contains points around $\varphi \equiv (1,0,1)$. Intuitively, this means that clients either require social capital to be present or for there to be a reasonable return from proper investment. For example, Figure 6 depicts the sets of values of $\epsilon$ and $q$ that generate regular equilibria (Type I and Type II) in Proposition 4 for several members of the Beta family of distributions and a particular value of $\phi$. For the $\epsilon$-$q$ pairs above each curve, public trust is feasible, whereas below each curve public trust is impossible. Further, for any given value of $\epsilon$, there exists a minimum productivity potential $\bar{q}$ such that trust will only exist as long as $q \geq \bar{q}$. By inspection, the threshold

\(^{19}\)Note that (6) and (8) are the same. Also, the expression in (7) is qualitatively similar to that in (6) in that it represents a condition about the elasticity of equilibrium trust with respect to increases in the potential for productivity.
Figure 6: Values of $\epsilon$ and $q$ above the curve support two positive-trust equilibria.

$q$ is an increasing function of $\epsilon$, which means that as government intervention increases, a higher level of $q$ is required for public trust to be possible. We will consider the effect of $\phi$ on trust formation in Section 5.

The existence and characterization of these two types of equilibria motivate an analysis of the optimal government intervention in the market, which is the topic of the next section.

4 Coase Versus the Coasians Revisited

Until now, we have assumed that the level of government intervention $\epsilon$ is given exogenously. In this section, we analyze the government’s optimal choice of $\epsilon$, given the social culture $F(\cdot)$ that exists in the population and the potential for growth $q$ in the economy. We primarily focus on two aspects of this decision. First, we determine when a government should intervene through regulation versus when it should allow markets to function with minimal interference (a Coasian plan). Second, we derive comparative statics to compare the level of regulation that should arise in various economic settings. Throughout the following discussion, we relate our findings to previous empirical observations that have been documented in the literature.

We assume that regulation is costly for any government to implement. Specifically, we define
$k_g(\epsilon)$ as the cost that the government incurs when they implement a level of regulation $\epsilon$. Further, we define $c(\cdot)$ as the social cost of regulation $\epsilon$, which encompasses both the costs for the government and the costs for the agents (i.e. $k_g(\epsilon)$ and $k_a(\epsilon)$). For convenience, we restrict $c(\cdot)$ to be twice continuously differentiable, with $c(\epsilon) = 0$ for $\epsilon \leq \delta$, $c'(\epsilon) > 0$ for $\epsilon > \delta$, and $c'({\delta}) = 0$. The government’s problem is to choose an optimal $\epsilon$ to minimize the deadweight loss due to opportunism in the market plus the cost of implementing regulation. As we will show below, limiting the loss to opportunism is equivalent to maximizing economic growth in the market. Allowing the government to implement a $\delta$-level of enforcement without any cost represents a free-option for the government. The question that we will address then is whether the government exercises this free option and whether the government will further expend resources to choose $\epsilon^* > \delta$.\footnote{The magnitude of $\delta$ is small and can be reduced making this assumption as weak as one wants.}

The loss $L$ due to opportunism, given the setup in Section 2 may be expressed as

$$L = (1 - \epsilon)(1 - \tau^*)q.$$ 

Therefore, the government solves

$$\min_\epsilon L + c(\epsilon)$$

subject to

$$\tau^* = F(\phi(1 - \epsilon)q\Delta p(\tau^*)).$$

The following proposition outlines when it is optimal for the government to intervene versus implementing a Coasian plan.

**Proposition 5. (Coasian Economics Versus Government Intervention)**

(i) In any Type II equilibrium, $\epsilon^* > \delta$, that is, the government exercises its free option and chooses a strictly higher level of enforcement.

(ii) For any Type I equilibrium, there exists $\bar{q} > 0$ such that if $q < \bar{q}$, $\epsilon^* > \delta$. Otherwise, the government foregoes its free option to minimize the level of enforcement in the market.

According to Proposition 5, if the value to social capital is relatively low in a culture, and the market is in a Type II equilibrium, the government exercises its free option and further enhances investor protection. Further, if the value to social capital is relatively high, but the potential for growth in the economy is relatively low, the level of government regulation also exceeds $\delta$. This...
finding implies that Coasian plans are likely to be suboptimal when the potential for growth is low and/or the social culture is such that social capital is not highly valued. This is consistent with the comparison Glaeser, Johnson, and Shleifer (2001) make empirically between Poland and the Czech Republic. These two markets are assumedly fairly similar, and indeed government intervention has been shown to be value-enhancing.

It should be pointed out, however, that Proposition 5 does not assert that a Coasian plan (i.e. foregoing the free enforcement option) is never optimal. In contrast, it implies that a Coasian plan to let markets solve their own inefficiencies can only be optimal when the culture of the population values social capital and the potential growth in the economy is sufficiently high. This makes intuitive sense as these conditions naturally make a market ripe to develop without social planning. If people value their social stock within a business culture and there is a large potential for growth, these are the characteristics that would predict that a market would settle its own problems. This finding is consistent with empirical observations in Scandinavian countries (Knack and Keefer 1997), China (Allen, Qian, and Qian 2005), and India (Allen, Chakrabarti, De, Qian, and Qian 2006).

It is interesting to note that minimizing the deadweight loss to opportunism \( L \) is isomorphic to maximizing the level of aggregate investment and expected economic growth in the market. The loss to opportunism can be calculated as \( L = q - p_t \), so that minimizing \( L \) by choosing \( \epsilon \) is equivalent to maximizing \( p_t \). Therefore, the objective function in (9) could be re-written as

\[
\max_{\epsilon} p_t - c(\epsilon)
\]

subject to

\[
\tau^* = F(\phi(1 - \epsilon)q\Delta p(\tau^*)).
\]

In economic terms, since the level of aggregate investment \( p_t \) is a measure of both expected economic growth and the calculative trust in the market, minimizing the loss to opportunism is equivalent to maximizing overall trust that arises from both cultural and legal sources.

Of course, Proposition 5 only defines when a government must optimally intervene. The following proposition characterizes the relative amounts of government regulation that should exist, given the equilibria that arise.

**Proposition 6. (Comparative Statics on Optimal Regulation)**

(i) Consider an economy in which two regular trust equilibria arise such that \( \tau_1^* > \tau_2^* \). Then, the
optimal level of government intervention is higher in the Type II (low-trust) equilibrium than it is in the Type I (high-trust) equilibrium.

(ii) Consider two different economies that exhibit the same equilibrium level of public trust, but such that economy 1 is in a Type I equilibrium and economy 2 is in a Type II equilibrium. Then, the optimal level of government intervention in economy 1 is lower than that in economy 2.

The results in Proposition 6 imply that, within a population, if we were to compare a high trust Type I equilibrium versus a low trust Type II equilibrium (say, $\tau_1^* > \tau_2^*$), then we would expect more regulation to be present in the low-$\tau^*$ market. Likewise, when comparing two populations with the same amount of public trust $\tau^*$, when one values social capital highly and the other values it less, we should expect more government regulation in the latter market.

Proposition 6 is, therefore, consistent with the findings of both Glaeser, Johnson, and Shleifer (2001) and with Knack and Keefer (1997). That is, while Eastern European countries benefit from more government intervention, less regulation is required in Scandinavian countries since the value to social capital is higher. Therefore, it is not surprising given our model that these empirical findings coexist. In fact, with the insights we have drawn from our analysis, these two empirical observations are entirely consistent with each other.

It is important to point out that we do not entertain the possibility that the government can affect $F(\cdot)$ directly\(^{21}\). As pointed out by Fukuyama (1995), cultural “habits” have significant inertia, and may persist for long periods of time even after economic conditions have drastically changed. Clearly, however, governments are sometimes successful in improving social culture $F(\cdot)$, especially in the long-term. Consider the campaign by Bogotá mayor Antanas Mockus to build citizenship through teaching people to use symbols to reward and punish each other’s behavior. In one campaign people were given a plastic card with a “thumbs-up” on one side and a “thumbs-down” on the other. The cardholder would carry the card and use it to give other citizens feedback about their behavior. While the campaign was not an overwhelming success, it did cause people in Bogotá to improve their behavior towards each other, and did cause people in the city to view Bogotá more positively.

Another example is the famous inaugural words of President John F. Kennedy: “Ask not what your country can do for you, ask what you can do for your country.” This request to the people of the

\(^{21}\)Also, implicit in our analysis is that the cost $d_j$ is just a transfer to other members of the social network and does not represent a dead-weight loss. Therefore, $F(d)$ does not enter into the government’s optimization problem.
United States has become famous because it was instrumental in motivating a country to become more productive. In our model, these words would have the effect of changing the tendency for people to honor their responsibilities to each other and would change the distribution $F(\cdot)$. While we acknowledge the ability of leadership to alter $F(\cdot)$, we leave modeling this effect for future research.

5 Fees and Trust

So far in the paper, we have considered that the fees that clients pay to the agents are exogenously fixed and do not affect the potential for productivity. In this section, we analyze the effects that fees have on the trust that evolves in the market. The results that we derive differ depending on what type of equilibrium (Type I or Type II) exists in the market. When the value to social capital is high, we show that trust is increasing in fees at low fee premiums, but is decreasing at high fee premiums. The opposite relationship holds for markets in which the value to social capital is low. Throughout what follows, we relate our findings to the literature on agency theory and show where our findings depart from classic theory.

Consider that the potential for productivity in the market $q$ depends on how much of the investment $p_t$ is employed in the opportunity (fraction $1 - \phi$). If $\phi$ is higher, more money is paid to the agents who manage the investment, and less capital is employed for the good of the client. Therefore, the function $q(\phi)$ that we consider is twice continuously differentiable, strictly decreasing in $\phi$, and convex. The fact that $q''(\phi) > 0$ implies that there are economies of scale in the investment, but is only sufficient, not necessary, to derive the results which follow. To maintain tractability of the model, $\phi \in [\phi, \bar{\phi}]$ where $\phi > 0$ and $\bar{\phi} < 1$. The rest of the model defined in Section 2 remains unchanged and we assume that the level of government control $\epsilon$ is again given exogenously.

We begin by analyzing the case in which a Type I equilibrium exists. The following proposition characterizes the effects of $\phi$ on the level of trust $\tau$ that exists when $F(\cdot)$ is concave.

**Proposition 7.** In a Type I equilibrium, there exists a threshold $\phi_1^*$ such that

$$ \frac{d\tau}{d\phi} = \begin{cases} > 0 & \text{if } \phi < \phi_1^* \\ < 0 & \text{if } \phi > \phi_1^* \end{cases} $$
Proposition 7 implies that when fees are low ($\phi < \phi_1^*$), increasing the fraction of the investment that agents receive leads to increased trust in the market. However, once fees become relatively high, then public trust is strictly decreasing in $\phi$. To explain this relationship, we highlight three effects that fees have on the investment that is made by clients and the actions of the agents in the market. First, increasing $\phi$ has a direct negative effect on both $q$ and the investment difference $\Delta p$. As mentioned, increased fees lower the potential productivity of the investment $q$, which lowers the size of the pie there is to split. Further, since by Proposition 1, $\frac{\partial \Delta p}{\partial q} > 0$, increasing fees causes a decrease in $\Delta p$. Second, increasing $\phi$ generates higher incentives for the agents to become trustworthy. Because each agent keeps $\phi p_2$ (where $p_2 \in \{p_S, p_F\}$), as $\phi$ increases, agents have incentives to maximize the probability that they realize a success in the first period for their clients.

The third effect is due to the feedback effect that trust has on incentives, which highlights a novel feature of our model. Recall from Proposition 1 (and from Figure 2), that the relationship between $\Delta p$ and $\tau$ is hump-shaped. When trust is low, increasing trust leads to a higher investment difference. However, this relationship reaches a peak (at $\bar{\tau}$), and for trust levels greater than $\bar{\tau}$, $\frac{\partial \Delta p}{\partial \tau} < 0$. When all agents are trustworthy ($\tau = 1$), $\Delta p$ is indeed zero. Therefore, as $\phi$ initially increases, the benefit to becoming trustworthy comes from two sources: a higher investment in period 1 (because of higher trust) and a higher relative payoff when the investment succeeds. However, once $\tau$ becomes sufficiently high, the benefit from the second portion of this return diminishes. That is, when $\tau$ is sufficiently high, the relative reward for having a successful investment decreases (lower $\Delta p$), which drives down the incentives to become trustworthy.

Therefore, the predictions that this model generates differ from the effects that incentives have in standard agency models. Like a standard agency framework, higher powered incentives lead to a loss in total surplus. In the standard framework, this is a result of a risk transfer, whereas in this model we assume that it results from a decrease in potential productivity. The most notable difference, however, is that high-powered incentives (high $\phi$) may lead to a lower effort provision (trust) in the aggregate. The source of this difference is that the clients’ inference about any particular agent’s type depends on the actions of all of the other agents. This externality may cause the reward to becoming trustworthy to decrease even though the direct incentives represented by the fee are higher. Therefore, higher incentives (high $\phi$) may lead to a lower effort provision (decreased tendency to honor the fiduciary duty to clients), a decreased wage difference (through $\Delta p$), and a lower ability to rely on the agents for the provision of effort (lower trust $\tau$).
As already mentioned, the opposite relationship between fees and trust exist in a Type II equilibrium. We conclude this section with the following proposition which characterizes this relationship in a Type II equilibrium.

**Proposition 8.** In a Type II equilibrium, there exists a threshold $\phi^*_2$ such that

\[
\frac{d\tau^*_2}{d\phi} = \begin{cases} 
> 0 & \text{if } \phi > \phi^*_2 \\
< 0 & \text{if } \phi < \phi^*_2.
\end{cases}
\]

6 Conclusions

As pointed out by Fukuyama (1995), culture and social customs are important drivers of economic growth or the underperformance of markets. Despite the presence of many empirical studies to support this assertion, there is a paucity of economic theory on the subject.\footnote{Two notable exceptions are Zak and Knack (2001) and Glaeser, Laibson, and Sacerdote (2002).} This paper attempts to fill this void by studying the origins of trust formation in the market and the relationship between trust, the law, and economic growth. We take the underlying culture of a society as a primitive and analyze how public trust evolves in society and how it affects growth. We derive empirical predictions that appear to be consistent with existing empirical work, as well as provide predictions which may lead to new empirical investigation. Testing these new findings is the subject of future research.

In the paper, we derive conditions under which two types of generic trust equilibria may arise. In a Type I equilibrium, government regulation is a strict substitute for public trust and may inhibit economic growth. Also, in this case, the potential for productivity in the economy is a catalyst for public trust formation. Type II equilibria arise when agents have higher costs of becoming trustworthy. In this type of equilibrium, government intervention may add value because regulation complements public trust. In this case, however, the potential for productivity may decrease economic growth because the propensity for opportunism increases as growth is made possible.

We then analyze when it is optimal for a government to intervene in the market to protect investors. We show that when the value to social capital is relatively low and/or the growth potential in the economy is low, it is never optimal to institute a Coasian plan (absence of government...
regulation). We conclude our analysis by considering the effect that professional fees have on the trust that forms in the market.

We believe that this paper represents a plausible way to think about the effects of trust and the law on economic growth, and represents an important step to understanding the effect of culture on economic productivity.
Appendix A

Proof of Proposition 1

(i) Recall that
\[ \Delta p = (1 - \epsilon)q \left[ \frac{\tau}{\tau + \epsilon(1 - \tau)} - \frac{(1 - q)\tau}{(1 - q)\tau + (1 - eq)(1 - \tau)} \right] \]

Straight differentiation with respect to \( q \) yields:
\[
\frac{\partial \Delta p}{\partial q} = \frac{\Delta p}{q} + (1 - \epsilon)q \left[ -\frac{\tau[(1 - q)\tau + (1 - eq)(1 - \tau)] - (1 - q)\tau[-\tau - \epsilon(1 - \tau)]]}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} \right]
\]
\[
= \frac{\Delta p}{q} + \frac{(1 - \epsilon)q\tau}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} [(1 - q)\tau + (1 - eq)(1 - \tau) - (1 - q)\tau - \epsilon(1 - q)(1 - \tau)]]
\]
\[
= \frac{\Delta p}{q} + \frac{(1 - \epsilon)q\tau(1 - \tau)}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} [1 - eq - \tau(\epsilon - eq)]
\]
\[
> 0
\]

With respect to \( \epsilon \), straight differentiation yields:
\[
\frac{\partial \Delta p}{\partial \epsilon} = \frac{-\Delta p}{(1 - \epsilon)} + (1 - \epsilon)q \left[ -(1 - \tau)\frac{\tau}{[\tau + \epsilon(1 - \tau)]^2} - q(1 - \tau)\frac{(1 - q)\tau}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} \right]
\]
\[
< 0 \quad (13)
\]

(ii) First, consider the first derivative of \( \Delta p \) with respect to \( \tau \):
\[
\frac{\partial \Delta p(\tau)}{\partial \tau} = (1 - \epsilon)q \left\{ \frac{[\tau + \epsilon(1 - \tau) - \tau(1 - \epsilon)]}{[\tau + \epsilon(1 - \tau)]^2} - \frac{(1 - q)[(1 - q)\tau + (1 - eq)(1 - \tau)] - (1 - q)\tau[(1 - q) - (1 - eq)]}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} \right\}
\]
\[
= (1 - \epsilon)q \left\{ \frac{\epsilon}{[\tau + \epsilon(1 - \tau)]^2} - \frac{(1 - q)(1 - eq)}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^2} \right\} \quad (14)
\]
The first derivative is well-defined, continuous, and finite for all \( \tau \in [0, 1] \). The second derivative is then:
\[
\frac{\partial^2 \Delta p(\tau)}{\partial \tau^2} = -2(1 - \epsilon)^2q \left\{ \frac{\epsilon}{[\tau + \epsilon(1 - \tau)]^3} + \frac{(1 - q)(1 - eq)q}{[(1 - q)\tau + (1 - eq)(1 - \tau)]^3} \right\}
\]
\[
< 0 \quad (15)
\]
The second derivative of \( \Delta p \) is well-defined, continuous, and strictly negative for any value of \( \tau \in [0, 1] \), since we assumed that \( \epsilon > 0 \) and \( q < 1 \). As long as \( \epsilon \neq 1 \) and \( q \neq 0 \), it follows that \( \Delta p \) is globally strictly concave in \( \tau \).

(iii) From (ii) we know that \( \Delta p \) is globally strictly concave in \( \tau \). It follows that the function achieves a unique maximum at some value \( \bar{\tau} \) at which the first derivative (equation 14) equals zero. Several steps of algebra yield the value of \( \bar{\tau} \) as:

\[
\bar{\tau} = 1 + \sqrt{\frac{1 - q}{\epsilon(1 - \epsilon q)}}^{-1}
\]

Proof of Proposition 2

We first state and prove a lemma that will be useful in the proof of the Proposition, as well as later in the proof of Proposition 4:

**Lemma A.1.** The function \( h(\tau, v) \), where \( v = (q, \epsilon, \phi) \), is \( C^2 \), and, for all \( \tau > 0, \epsilon < 1, q > 0, \) and \( \phi > 0 \), it is strictly decreasing in \( q \) and \( \phi \), and strictly increasing in \( \epsilon \).

**Proof.** Recall that \( h(\tau, v) = \tau - F(\phi(1 - \epsilon)q\Delta p) \). As shown in Proposition 1, \( \Delta p \) is twice continuously differentiable in \( \tau \) at every \( \tau \in [0, 1] \). Because \( F \) is also \( C^2 \) by assumption, their composition is also \( C^2 \). Since then \( G(\tau, v) \) is twice continuously differentiable in \( \tau \), then so is \( h(\tau, v) \).

Consider now the partial derivative results. Since \( F \) is a CDF, it is increasing, and since we assumed that \( f(x) > 0 \) for all \( x \in (0, 1) \), \( F \) is strictly increasing. Using the properties of \( \Delta p \) derived in Proposition 1, this implies that \( \frac{\partial G(v, \tau)}{\partial q} > 0, \frac{\partial G(v, \tau)}{\partial \epsilon} < 0, \) and \( \frac{\partial G(v, \tau)}{\partial \phi} > 0 \) for \( \tau > 0 \). Since \( h(\tau, v) = \tau - G(\tau, v) \), the stated results follow immediately. ■

We now proceed by stating Proposition 8.3.1 from Mas-Colell (1985):

**Proposition 9.** Let \( F : N \times B \to \mathcal{R}^m, N \subset \mathcal{R}^n, B \subset \mathcal{R}^s \) be \( C^r \) with \( r > \max\{n - m, 0\} \). Suppose that \( \theta \) is a regular value of \( F \); that is, \( F(x, b) = 0 \) implies \( \text{rank} \, \partial F(x, b) = m \). Then, except for a set of \( b \in B \) of Lebesgue measure zero, \( F_b : N \to \mathcal{R}^m \) has \( \theta \) as a regular value.

Note that the notation \( F_b(x) \) refers to the function \( F(x, b) \) when the exogenous parameter \( b \) is held fixed for the moment. As such, we have the following mapping to the objects defined in Proposition 8.3.1:
• The function $F(x, b)$ is our function $h(\tau, v)$.
• The variable $x$ is our $\tau$, and therefore the set $N$ is the closed interval $[0, 1]$, and $n = 1$.
• The parameter $b$ is the triple $v = (q, \epsilon, \phi)$, and the set $B$ is the product $V = [0, 1) \times (0, 1] \times [0, 1]$, and $s = 3$.
• The function $h$ takes on values in the interval $[-1, 1]$, so $m = 1$.
• Since $n = m = 1$, the smoothness condition that is required by the proposition is that the function $h$ be at least $C^1$, which is satisfied by our function, as shown in Lemma A.1.

Let now $\tau^*$ be an equilibrium value, for a given set of parameters $v = (q, \epsilon, \phi)$. Restrict attention to $q > 0$ and $\epsilon < 1$ (this is without loss of generality, because at the excluded parameter values $\Delta p = 0$, and thus no positive trust equilibria exist). Consider the Jacobian matrix $Dh$:

$$Dh = \begin{pmatrix}
\frac{\partial h(\tau,v)}{\partial \tau} \\
\frac{\partial h(\tau,v)}{\partial q} \\
\frac{\partial h(\tau,v)}{\partial \epsilon} \\
\frac{\partial h(\tau,v)}{\partial \phi}
\end{pmatrix}$$

The following two statements are true:

• From Lemma A.1, if $\tau^* > 0$, then $\frac{\partial h(\tau,v)}{\partial q} < 0$, $\frac{\partial h(\tau,v)}{\partial \epsilon} > 0$, and $\frac{\partial h(\tau,v)}{\partial \phi} < 0$. Therefore, the rank of the jacobian matrix $Dh$ is 1 at any such equilibrium $\tau^*$.
• If $\tau^* = 0$, then all three partial derivatives w.r.t. the parameters are zero, because then $\Delta p = 0$, but $\frac{\partial h}{\partial \tau^*} > 0$, because we assumed $F'(0) = 0$. Therefore, the rank of the Jacobian is 1 at $\tau^* = 0$ as well.

As a consequence of these two points, the value 0 is a regular value of $h(\tau,v)$. Therefore, the proposition cited above applies directly, yielding the result that except for a set of parameter values $v$ of Lebesgue measure zero, the function $h(\tau)$ has 0 as a regular value, i.e. $\frac{\partial h}{\partial \tau^*} \neq 0$ for any equilibrium point $\tau^*$.

Because in a generic equilibrium $\frac{\partial h}{\partial \tau^*} \neq 0$, the Inverse Function Theorem implies that there exists a neighborhood around $\tau^*$ where the function $h$ is invertible. Therefore, in that neighborhood, $h$ is strictly monotonic, which implies in turn that such a neighborhood cannot contain another equilibrium. Each regular equilibrium is therefore locally unique (or isolated). The set of generic equilibria is therefore discrete. Moreover, because the function $h$ is continuous, the set of all
equilibria (including $\tau^* = 0$) for any parameter set $v$ is closed, and it is also bounded by virtue of being contained in the interval $[0, 1]$. By virtue of Theorem M.F.3 (p. 945) in Mass-Colell, Whinston and Green (1995), the set of all equilibria generated by a generic parameter set $v$ is finite, because it is a discrete and compact set. Excluding the point $\tau^* = 0$ from a finite set yields another finite set, $T_v^*$. Therefore, the set of regular equilibria $T_v^*$ is finite. ■

Proof of Proposition 3

First, notice that since $F$ does not have a mass point at 0, then $\tau^* = 0$ is always a solution to equation (5). We will find the conditions under which another solution exists. The plan is as follows:

(i) Show that $F(\phi (1 - \epsilon) q \Delta p)$ is concave in $\tau$.

(ii) Show that the slope of $F(\phi (1 - \epsilon) q \Delta p)$, as a function of $\tau$, is greater than 1 at 0, if $\epsilon < \bar{\epsilon}$.

(iii) Since $F(\phi (1 - \epsilon) q \Delta p)$ is increasing and concave in $\tau$ for $\tau < \bar{\tau}$, and $F = 0$ at $\tau = 1$, this establishes the existence and uniqueness of the non-zero fixed point of $F(\phi (1 - \epsilon) q \Delta p)$.

For part (i), we need to sign the second derivative of $F$:

$$\frac{\partial^2 F(\cdot)}{\partial \tau^2} = f'(\phi (1 - \epsilon) q \Delta p) \left( \frac{\partial \Delta p}{\partial \tau} \right)^2 (1 - \epsilon) q \Delta p \frac{\partial^2 \Delta p}{\partial \tau^2} (1 - \epsilon) \phi q$$

Under the assumption that $F''(y) \leq 0$ for all $y$, the first term is negative or zero. From part (ii) of Proposition 1, we know that $\Delta p$ is strictly concave in $\tau$, so $\frac{\partial^2 \Delta p}{\partial \tau^2} < 0$. The sum of the two terms must then be negative. We have thus proved part (i).

Now on to parts (ii) and (iii): showing that $F$ starts off at a slope greater than 1. We need to show that $\partial F/\partial \tau > 1$ at $\tau = 0$. Define:

$$s(\epsilon) \equiv \frac{\partial F}{\partial \tau} \bigg|_{\tau=0}$$

$$s(\epsilon) = f(\phi (1 - \epsilon) q \Delta p(0))(1 - \epsilon) \phi q \frac{\partial \Delta p(0)}{\partial \tau}$$

where with a slight abuse of notation $\Delta p(0)$ is taken to be the value of $\Delta p$ when $\tau = 0$ and $\epsilon, q$
take generic values. Then, using equation (14) and the fact that $\Delta p(0) = 0$, we have:

$$s(\epsilon) = f(0)(1 - \epsilon)\phi q(1 - \epsilon)q \left[ \frac{1}{\epsilon} - \frac{(1 - q)}{(1 - \epsilon q)} \right]$$

$$= f(0)\phi q^2(1 - \epsilon)^3 \phi q^2 \left[ 1 - \epsilon (1 - q) \right]$$

Since $f(0) > 0$ this is clearly positive, but we want to show that $s(\epsilon) > 1$. Clearly, this is true for values of $\epsilon$ close to 0, since $\lim_{\epsilon \to 0} s(\epsilon) = \infty$. Also clearly, this is not true for value of $\epsilon$ close to 1, since $\lim_{\epsilon \to 1} s(\epsilon) = 0$. Consider however how $s(\epsilon)$ changes with $\epsilon$:

$$\frac{ds(\epsilon)}{d\epsilon} = \frac{f(0)\phi q^2(1 - \epsilon)^2}{\epsilon^2(1 - \epsilon q)^2} \left[ -1 + 2\epsilon q - 2\epsilon(1 - q) \right]$$

$$< 0 \quad \text{since} \quad \epsilon^2 q < 1$$

This means that $s(\epsilon)$ is above 1 for low values of $\epsilon$, below 1 for high values of $\epsilon$, and decreasing - therefore there exists a value $\bar{\epsilon}$, defined by $s(\bar{\epsilon}) = 1$, above which the slope of $F(\cdot)$ is always less than 1, and hence $F(\cdot)$ does not intersect the 45-degree line at any point at which $\tau > 0$. For values of $\epsilon < \bar{\epsilon}$, the slope of $F(\cdot)$ is initially higher than 1, so $F$ must at some point intersect the 45-degree line, and since it is concave for the entire increasing portion, that intersection point is unique. We have thus established existence.

Applying the Implicit Function Theorem to equation (5), which defines $\tau^*_1$, we get:

$$\frac{d\tau^*_1}{d\epsilon} = \frac{\phi q f(\phi(1 - \epsilon)q\Delta p)[(1 - \epsilon)\frac{\partial \Delta p}{\partial \epsilon} - \Delta p]}{1 - f(\phi(1 - \epsilon)q\Delta p)\phi(1 - \epsilon)q\frac{\partial \Delta p}{\partial \epsilon}} \quad (18)$$

Recall that we showed that $\tau^*_1$ is the unique non-zero fixed point of $F(\phi(1 - \epsilon)q\Delta p)$, using the concavity of $F$ and the fact that its slope at 0 exceeds 1. This implies that at the fixed point, the slope of $F$ is less than 1, which implies that the denominator in RHS of the above equation is positive. From lemma 1, we know that $\frac{\partial \Delta p}{\partial \epsilon} < 0$, which makes the numerator negative and proves the desired result that $\frac{d\tau^*_1}{d\epsilon} < 0$.

The result that $\frac{d\tau^*_1}{d\epsilon} > 0$ follows immediately from equation (5) by, again, the Implicit Function
Theorem:

\[
\frac{d\tau_1^*}{dq} = \frac{f(\phi(1-\epsilon)q\Delta p)(1-\epsilon)\phi}{1-f(\phi(1-\epsilon)q\Delta p)\phi(1-\epsilon)q^2\Delta p \partial^2p}{(1-\epsilon)\phi(1-\epsilon)q\Delta p \partial^2p}
\]  

(19)

From Lemma 1, we know that the numerator is positive, and as already argued the denominator is positive. Hence, the fraction is also positive.

For the comparative statics on \( \bar{\epsilon} \), recall that \( \bar{\epsilon} \) solves \( s(\bar{\epsilon}) = 1 \), i.e.:

\[
f(0)\phi \frac{q^2(1-\bar{\epsilon})^3}{\bar{\epsilon}(1-\bar{\epsilon}q)} = 1
\]

(20)

Straightforward application of the Implicit Function Theorem yields the two results:

\[
\frac{d\bar{\epsilon}}{dq} > 0
\]

(21)

\[
\frac{d\bar{\epsilon}}{df(0)} > 0
\]

(22)

Next, we can calculate

\[
\frac{dp_1}{dq} = \tau_1^* + q \frac{d\tau_1^*}{dq} + (1-\tau_1^*)\epsilon - \epsilon q \frac{d\tau_1^*}{dq}
\]

\[
= \tau_1^* + (1-\tau_1^*)\epsilon + (1-\epsilon)q \frac{d\tau_1^*}{dq}
\]

\[
> 0
\]

since \( d\tau_1^*/dq > 0 \).

For the final result, consider the marginal effect of increasing government intervention:

\[
\frac{dp_1}{d\epsilon} = q \frac{d\tau_1^*}{d\epsilon} - \epsilon q \frac{d\tau_1^*}{d\epsilon} + (1-\tau_1^*)q
\]

\[
= (1-\epsilon)q \frac{d\tau_1^*}{d\epsilon} + (1-\tau_1^*)q
\]

(23)

Since \( \tau_1^* \) decreases with \( \epsilon \), economic growth will decrease in \( \epsilon \) for those values of it where

\[
\frac{d\tau_1^*}{d\epsilon} < -\frac{1-\tau_1^*}{1-\epsilon}
\]

(24)

\[\blacksquare\]
Proof of Proposition 4

Define the function

\[ G(v, \tau) \equiv F(\phi(1 - \epsilon)q\Delta p(\tau)). \]

The following lemma will be useful in the proof of Proposition 4.

**Lemma A2.** Let \( v \in V \) and \( F'(0) = 0 \). Then, there exists a \( t > 0 \) such that \( \forall \tau \in (0, t), G(v, \tau) < \tau \).

**Proof:** Consider the function \( h(\tau, v) = \tau - G(v, \tau) \).

\[
\frac{\partial h}{\partial \tau} = 1 - \frac{\partial G(v, \tau)}{\partial \tau} = 1 - F'(\phi(1 - \epsilon)q\Delta p) \frac{\partial \Delta p}{\partial \tau}.
\]

Evaluated at \( \tau = 0 \), we get

\[
\frac{\partial h}{\partial \tau} = 1 - F'(0) \frac{\partial \Delta p}{\partial \tau} = 1 \tag{26}
\]

since \( F'(0) = 0 \) and the derivative of \( \Delta p \) with respect to \( \tau \) is finite at 0. Moreover, the function \( \frac{\partial h}{\partial \tau} \) is continuous in \( \tau \) because \( F' \) and \( \frac{\partial \Delta p}{\partial \tau} \) are continuous in \( \tau \). Therefore, \( \frac{\partial h}{\partial \tau} \) must be positive in the neighborhood of zero, making \( h(\tau, v) \) increasing in that region. Since \( h(0) = 0 \), it follows that there exists some \( t > 0 \) such that \( h(\tau, v) > 0 \) for \( \tau < t \), i.e. that \( G(v, \tau) < \tau \) for \( \tau < t \). ■

We now proceed to proving Proposition 4. The proof will proceed in three parts. In Part 1, we establish the existence of the sets \( E \) and \( N \) and show that they comprise all of the points in \( V \), except for a two-dimensional manifold of zero measure. In Part 2, we use the results in Proposition 2 to establish the genericity of regular equilibria, and establish existence of Type I and Type II equilibria in the game. We also show in generic cases that the minimum (maximum) positive trust equilibrium is a Type II (Type I) equilibrium. Finally, in Part 3 we derive the properties of Type I (\( \tau_1^* \)) and Type II (\( \tau_2^* \)) equilibria.

**Part 1:** We know from the proof of Lemma A.1 that \( \frac{\partial G(v, \tau)}{\partial q} > 0 \), \( \frac{\partial G(v, \tau)}{\partial \epsilon} < 0 \), and \( \frac{\partial G(v, \tau)}{\partial \phi} > 0 \) for \( \tau > 0 \). Recall that \( \underline{v} \equiv (0, 1, 0) \) and \( \overline{v} \equiv (1, 0, 1) \), and the parameter space \( V \equiv [0, 1) \times (0, 1] \times [0, 1] \) where every point \( v \in V \) is a triple \((q, \epsilon, \phi)\).
(i) Pick any $\tau^0 \in (0, 1)$. We have that for $v \in V$,

$$
\lim_{v \to \overline{v}} G(v, \tau^0) = \lim_{v \to \overline{v}} F \left( \phi(1 - \epsilon)q \Delta p(\tau^0) \right) \\
= F(1) \\
= 1 \\
> \tau^0
$$

(27)

Since for all $v \in V$, $G(v, \tau^0)$ is continuous in $v$, there exists a point $v^* = (q^*, \epsilon^*, \phi^*)$ in the neighborhood of $\overline{v}$ such that $G(v^*, \tau^0) > \tau^0$. Moreover, given that $G$ is strictly increasing in $\phi$ and $q$ and strictly decreasing in $\epsilon$, it is also true that $G(v, \tau^0) > \tau^0$ for all $v = (q, \epsilon, \phi)$ such that $q \geq q^*$, $\epsilon \leq \epsilon^*$ and $\phi > \phi^*$. Hence, for any $v^*$ that satisfies $G(v^*, \tau^0) > \tau^0$, there exists a subset $C(v^*)$ of the parameter set $V$ (specifically, a cube) such that for all $v \in C(v^*$), $G(v, \tau^0) > \tau^0$.

Now choose any $v^*$ and pick a $v \in C(v^*)$. Consider now the value of $G(v, \tau)$ at $\tau = 1$:

$$
G(v, 1) = F \left( \phi(1 - \epsilon)^2q(1 - 1) \right) \\
= 0 \\
< 1
$$

(28)

Since $G(v, \tau)$ is continuous in $\tau$ over the entire interval $[0, 1]$, it follows that the function $G(v, \tau)$ crosses the 45-degree line from above at some point $\tau^*_1 > \tau^0$. This establishes the existence of a positive-trust equilibrium for any $v \in C(v^*)$.

Next, Lemma A1 implies that there exists some point $\tau'' < \tau^0$ arbitrarily close to 0 such that $G(v, \tau'') < \tau''$. Continuity of $G(v, \tau)$ in $\tau$ implies then that there exists a point $\tau^*_2 < \tau^0$ at which $G$ crosses the 45-degree line from below.

We have thus far shown for any $v^*$ (and its associated $C(v^*)$), that for all $v \in C(v^*)$, at least two positive trust equilibria exist: one in which the $G$ function crosses the 45-degree line from above, and one in which it crosses from below. This implies that there exists a non-empty parameter set $E$ for which both kinds of equilibria exist. While the set $E$ may be larger than any set defined by a particular $C(v^*)$, it is indeed strictly smaller than $V$, as we will show next.

(ii) Pick now an arbitrary point $\hat{v} \in V$. By Lemma A2, there exists some $t > 0$ such that
$G(\tilde{v}, \tau) < \tau$ for all $\tau \in (0, t)$. This means that no equilibrium can exist in the interval $(0, t)$. Moreover, because $G$ is increasing in $\phi$ and $q$ and decreasing in $\epsilon$, no equilibrium will exist in $(0, t)$ for any point $v$ on the segment connecting $\tilde{v}$ and $\underline{v}$, i.e. all points $v = \lambda \tilde{v} + (1 - \lambda)\underline{v}$, for $\lambda \in [0, 1]$. The next step is to show that for some values of $v$, no equilibrium can exist for $\tau \in [t, 1]$ either. To that end, define the set of all points $v$ on the straight line segment connecting $\tilde{v}$ and $\underline{v}$ as $W$ and note that $W$ is compact. Therefore, the set $Z \equiv W \times [t, 1]$ is also compact. The function $G(v, \tau)$ is well defined and continuous for all points $(v, \tau) \in Z$. Therefore, $G(v, \tau)$ is uniformly continuous since it is a continuous function over a compact space (e.g. Royden 1968). This means that for all points $(v, \tau)$ and $(v', \tau')$ with Euclidian norm $\|(v, \tau) - (v', \tau')\| < \delta$, we have that $|G(v, \tau) - G(v', \tau')| < \eta$. Let now $v = \underline{v}$ and $\tau' = \tau$. Uniform continuity implies that

$$\|(v, \tau) - (\underline{v}, \tau)\| < \delta \Rightarrow |G(v, \tau) - G(\underline{v}, \tau)| < \eta.$$ 

Since $G(\underline{v}, \tau) = 0$, we have thus shown that for any $\eta > 0$, there exists $\delta > 0$ such that if $\|(v, \tau) - (\underline{v}, \tau)\| < \delta$, then $G(v, \tau) < \eta$. Since this must be true for all $\eta$, it is true for $\eta < t$, which implies that for some $v^{**}$ sufficiently close to $\underline{v}$, $G(v, \tau) < \eta < t$ for all $\tau$. This implies that for all $\tau \in [t, 1]$, there exists a point $v^{**}$ such that $G(v^{**}, \tau) < \tau$. Since $G(v^{**}, \tau) < \tau$ for any $\tau \in (0, t]$ and $\tau \in [t, 1]$, then for the parameter value $v^{**}$, no positive trust equilibrium exists.

Given that $G$ is strictly increasing in $\phi$ and $q$ and strictly decreasing in $\epsilon$, it is also true that $G(v, \tau) < \tau$ for all $v = (q, \epsilon, \phi)$ such that $q \leq q^{**}$, $\epsilon \geq \epsilon^{**}$ and $\phi \leq \phi^{**}$. Hence, for any $v^{**}$ that satisfies $G(v^{**}, \tau) < \tau$, there exists a subset $M(v^{**})$ of the parameter set $V$ (specifically, a cube) such that for all $v \in M(v^{**})$, $G(v, \tau) < \tau$ for all $\tau$.

As before, the non-existence set $N$ is non-empty, but may be larger than any particular set $M(v^{**})$. In the final step, we now show that the existence set $E$ and the non-existence set $N$ comprise all of the points in $V$, except for a two-dimensional manifold of zero measure.

(iii) We have shown that there exists a non-empty set of parameters $E$ for which at least two positive trust equilibria exist, and that there exists a non-empty set $N$ for which no positive trust equilibrium exists. Let $I$ denote the set of elements of $V$ that do not belong to either set, i.e. $I = V \setminus (E \cup N)$. We will show that the set $I$ cannot be “thick”, i.e. that for any point $v \in I$ and any distance $\eta > 0$, the ball $N_\eta(v)$ contains points that lie outside $I$.

Pick any point $v = (q, \epsilon, \phi) \in I$. Since $v \notin E$, $G(v, \tau) \leq \tau$ for all $\tau > 0$. Since $v \notin N$, $\exists \tau^0 > 0$
such that $G(v, \tau^0) = \tau^0$. In any $\eta$-ball around $v$, we can find a point $v' = (q', \epsilon, \phi)$ with $q' > q$. Since $G$ is strictly increasing in $q$, $G(v', \tau^0) > G(v, \tau^0)$. But then $G(v', \tau^0) > \tau^0$, and $v' \in E$. Conversely, pick a point $v'' = (q'', \epsilon, \phi)$ with $q'' < q$. Since $G$ is strictly increasing in $q$, for all points $\tau^0$ where $G(v, \tau^0) = \tau^0$, $G(v'', \tau^0) < G(v, \tau^0)$, and hence $G(v'', \tau^0) < \tau^0$. But then $G(v'', \tau) < \tau$ for all $\tau$, and $v'' \in N$.

Analogous reasoning applies to the $\epsilon$ and $\phi$ dimensions, yielding the result that if $v \in I$, then every positive measure neighborhood of $v$ contains elements that lie outside $I$, and thus the set $I$ has Lebesgue measure zero in $V$.

**Part 2**

In principle, the existence set $E$ could contain parameter values for which an infinity of equilibria arise, and in particular could contain equilibria for which the partial derivative of the function $h(\tau, v)$ with respect to $\tau$ is zero at the equilibrium. However, Proposition 2 implies that the set of parameter values $v$ for which such pathologic equilibria can arise has Lebesgue measure zero. Therefore, the existence set $E$ contains with probability 1 points $v$ that generate at least two regular equilibria.

Restricting attention to generic cases, we now know that at least two positive trust equilibria exist, and at any equilibrium point $\tau^*$, $\partial h(\tau^*, v)/\partial \tau \neq 0$. Since $h(\tau, v)$ is differentiable with respect to $\tau$, either $\partial h(\tau^*, v)/\partial \tau < 0$ or $\partial h(\tau^*, v)/\partial \tau > 0$. According to our definition, this implies that every equilibrium is either a generic Type I or a generic Type II equilibrium. In particular, whenever the $G$ function crosses the 45-degree line from above, we have a generic Type I equilibrium ($\partial h(\tau^*, v)/\partial \tau > 0$), while whenever the $G$ function crosses the 45-degree line from below, we have a generic Type II equilibrium ($\partial h(\tau^*, v)/\partial \tau < 0$). This last statement is true because all generic equilibria are locally unique; hence, there exists a neighborhood around each equilibrium $\tau^*$ in which no other equilibrium lies. When the function $h$ is crossing the x-axis from above (i.e. when $G$ crosses the 45-degree line from below), this means that $h(\tau, v) > 0$ for all $\tau < \tau^*$, and $h(\tau, v) < 0$ for all $\tau > \tau^*$. It then follows from the fact that $h$ is differentiable at $\tau^*$ that $\partial h(\tau^*, v)/\partial \tau > 0$, for the opposite sign of the derivative would violate the existence of the derivative at $\tau^*$. The same argument works for the case when $h$ crosses from below.

Proposition 2 also states that in the generic case, the number of equilibria is finite. This implies that both a maximum-trust and a minimum-trust equilibrium exists. Denote these equilibria $\tau^*_{\text{max}}$ and $\tau^*_{\text{min}}$ respectively. It follows easily $\tau^*_{\text{max}}$ is a Type I equilibrium, while $\tau^*_{\text{min}}$ is a Type II equilibrium. As a proof, suppose that $\tau^*_{\text{max}}$ is not a Type I equilibrium. It must then be a Type II equilibrium. Therefore, $\partial h(\tau^*_{\text{max}}, v)/\partial \tau < 0$, and since by the Inverse Function Theorem the
function $h$ is locally invertible, and hence monotonic, around $\tau_{\text{max}}^*$, it follows that there exists a point $\tau' > \tau_{\text{max}}^*$, such that $h(\tau') < 0$. But then $G(v, \tau') > \tau'$, and because $G(v, 1) = 0$ and $G$ is continuous in $\tau$, there must exist another equilibrium $\tau^* > \tau'$, which also means that $\tau^* > \tau_{\text{max}}^*$. This contradicts the premise that $\tau_{\text{max}}^*$ was the maximum-trust equilibrium. Hence, the maximum-trust equilibrium must be a Type I equilibrium. The proof that the minimum-trust equilibrium $\tau_{\text{min}}^*$ is Type II is analogous.

**Part 3**

We now turn to the properties of our generic equilibria. Let $\tau_1^*$ denote any generic Type I equilibrium, and $\tau_2^*$ any generic Type II equilibrium. Because both are regular equilibria, the partial derivative of $h(\tau, v)$ with respect to $\tau$ exists and is non-zero, allowing us to apply the Implicit Function Theorem in order to derive comparative statics results. We first investigate the effect of changes in $\epsilon$ and $q$ on the two different equilibria. Then, we evaluate the effects that these changes have on the aggregate investment in each type.

(i) *Changes in $\epsilon$.* Consider any generic Type I equilibrium $\tau_1^*$. The Implicit Function Theorem yields:

$$\frac{d\tau_1^*}{d\epsilon} = -\frac{\partial h/\partial \epsilon}{\partial h/\partial \tau_1^*}$$

$$= \frac{\partial G/\partial \epsilon}{\partial h/\partial \tau_1^*}$$

(29)

Since $\partial G/\partial \epsilon < 0$, and in a generic Type I equilibrium $\partial h/\partial \tau_1^* > 0$, the sign of the expression is negative, proving the result that in a Type I equilibrium, an increase in $\epsilon$ causes a decrease in the equilibrium level of trust.

For a Type II equilibrium, $\partial h/\partial \tau_1^* < 0$, reversing the sign of the expression, and leading to the opposite comparative statics result.

(ii) *Changes in $q$.* The proof is analogous to that for $\epsilon$, but now since $\partial G/\partial q > 0$, signs are reversed, showing that, for each equilibrium type, changes in $q$ have the opposite effect of changes in $\epsilon$ on the equilibrium trust level.

(iii) *Aggregate Investment.* Now, we complete the proof by considering how aggregate investment changes based on changing $\epsilon$ and $q$ in each equilibrium.

(a) Type I Equilibrium ($\tau_1^*$)
i. *Changes in q*: Consider now the marginal effect of increasing productivity on economic growth:

\[
\frac{dp_1}{dq} = \tau_1^* + q \frac{d\tau_1^*}{dq} + (1 - \tau_1^*)\epsilon - \epsilon q \frac{d\tau_1^*}{dq} \\
= \tau_1^* + (1 - \tau_1^*)\epsilon + (1 - \epsilon)q \frac{d\tau_1^*}{dq} \\
> 0
\]

since \(d\tau_1^*/dq > 0\).

ii. *Changes in \(\epsilon\)*: Consider the marginal effect of increasing government intervention:

\[
\frac{dp_1}{d\epsilon} = q \frac{d\tau_1^*}{d\epsilon} - \epsilon q \frac{d\tau_1^*}{d\epsilon} + (1 - \tau)q \\
= (1 - \epsilon)q \frac{d\tau_1^*}{d\epsilon} + (1 - \tau)q
\] (30)

Since \(\tau_1^*\) decreases with \(\epsilon\) as shown above, economic growth will decrease in \(\epsilon\) for those values of \(\epsilon\) where

\[
\frac{d\tau_1^*}{d\epsilon} < -\frac{1 - \tau_1^*}{1 - \epsilon}
\] (31)

(b) Type II Equilibrium (\(\tau_2^*\))

i. *Changes in \(\epsilon\)*: Consider now the marginal effect of increasing government intervention on economic growth:

\[
\frac{dp_1}{d\epsilon} = q \frac{d\tau_2^*}{d\epsilon} - \epsilon q \frac{d\tau_2^*}{d\epsilon} + (1 - \tau_2)q \\
= (1 - \epsilon)q \frac{d\tau_2^*}{d\epsilon} + (1 - \tau_2)q
\] (32)

Since \(\tau_2^*\) increases with \(\epsilon\) as shown above, economic growth (and aggregate investment) will increase in \(\epsilon\), that is

\[
\frac{dp_1}{d\epsilon} > 0
\] (33)

ii. *Changes in q*: Next, consider the marginal effect of increasing productivity \(q\) on economic growth:
\[
\frac{dp_1}{dq} = \tau_2^s + q \frac{d\tau_2^s}{dq} + (1 - \tau_2^s)\epsilon - eq \frac{d\tau_2^s}{dq}
\]
\[
= \tau_2^s + (1 - \tau_2^s)\epsilon + (1 - \epsilon) q \frac{d\tau_2^s}{dq}
\]

Since \(\tau_2^s\) decreases with \(q\), economic growth will decrease in \(q\) when

\[
\frac{d\tau_2^s}{dq} < -\left[\frac{\tau_2^s}{q} + \frac{\epsilon}{(1 - \epsilon)q}\right].
\]
Proof of Proposition 5

As shown in the discussion of Proposition 5, the government’s loss-minimization problem is equivalent to the problem of maximizing economic growth net of costs required to implement regulation. The latter problem is

$$\max_\epsilon \quad p_t - c(\epsilon)$$

s.t. \hspace{1cm} \tau^* = F(\phi(1 - \epsilon)q\Delta p(\tau^*)),$$

$$p_t = \tau^* q + (1 - \tau^*)eq. \quad (34)$$

(i) In a Type II equilibrium, as shown in Proposition 4, economic growth increases strictly in the level of government intervention. Therefore, since \(c(\delta) = 0\), the government will exercise its free enforcement option. Further, at \(\epsilon = \delta\), the government’s FOC cannot hold (recall \(c'(\delta) = 0\)). Therefore, \(\epsilon^* > \delta\) as claimed.

(ii) In a Type I equilibrium, we know from Proposition 3 that growth can decrease with \(\epsilon\). If this is the case, then the government will not exercise its enforcement option and will choose to minimize \(\epsilon\). If, however, growth increases with \(\epsilon\), then some intervention may be optimal. Recall the condition under which \(\frac{\partial \Delta p}{\partial \epsilon} > 0\):

$$\frac{d\tau^*_1}{d\epsilon} > \frac{1 - \tau^*_1}{1 - \epsilon} \quad (35)$$

which, evaluated at \(\epsilon = \delta\), is

$$\frac{\phi q f(\phi(1 - \delta)q\Delta p)((1 - \delta)\frac{\partial \Delta p}{\partial \epsilon} - \Delta p)}{1 - f(\phi(1 - \delta)q\Delta p)\phi(1 - \delta)q\frac{\partial \Delta p}{\partial \tau}} > \frac{(1 - \tau^*_1)}{(1 - \delta)} \quad (36)$$

Consider now the behavior of the inequality as \(q \to 0\). From equation (1) it is clear that at \(q = 0, \Delta p = 0\). From equation (13), it is also clear that the partial derivative of \(\Delta p\) with respect to \(\epsilon\) is zero at \(q = 0\). From equation (14), \(\lim_{q \to 0} \frac{\partial \Delta p}{\partial \tau} = 0\). It then follows that the LHS of inequality (35) approaches 0 as \(q\) approaches 0. We also know that at \(q = 0\), the only equilibrium is \(\tau^* = 0\), so that the RHS equals \(-\frac{1}{1 - \delta}\). Therefore the inequality holds as \(q \to 0\). Since both the LHS and the RHS of the inequality are continuous in \(q\), there exists a neighborhood of 0 in which the inequality also holds, which proves the existence of a value \(\tilde{q} > 0\) for values below which \(\epsilon^* = \delta\) cannot be optimal.

\[\blacksquare\]
Proof of Proposition 6

When \( q \) is not less than \( \bar{q} \) in a Type I equilibrium, the government will not exercise its free option to implement at least a \( \delta \) level of enforcement. Therefore, when \( q \geq \bar{q} \), the level of enforcement in a Type I equilibrium will be clearly lower than that in any Type II equilibrium. Thus, in what follows, we only need to prove that the results in the proposition hold when \( q < \bar{q} \).

For ease of reference, recall the government’s problem as discussed above

\[
\begin{align*}
\max_{\epsilon} & \quad p_t - c(\epsilon) \\
\text{s.t.} & \quad \tau^* = F(\phi(1 - \epsilon)q\Delta p(\tau^*)) \\
& \quad p_t = \tau^* q + (1 - \tau^*)\epsilon q
\end{align*}
\]  

(i) Consider now the two positive trust equilibria that can arise under the conditions defined in Proposition 4. Let \( \tau^*_1 \) be the Type I equilibrium and \( \tau^*_2 \) be the Type II equilibrium. Let \( \epsilon^*_1 \) solve the FOC of the government’s problem in the Type I case:

\[
(1 - \tau^*_1)q + (1 - \epsilon^*_1)q \frac{d\tau^*_1}{d\epsilon} = c'(\epsilon^*_1)
\]  

Since \( \tau^*_2 < \tau^*_1 \), it follows that:

\[
1 - \tau^*_2 > 1 - \tau^*_1
\]

Since \( \frac{d\tau^*_1}{d\epsilon} < 0 < \frac{d\tau^*_2}{d\epsilon} \), we have:

\[
(1 - \epsilon^*_1)q \frac{d\tau^*_2}{d\epsilon} < (1 - \epsilon^*_1)q \frac{d\tau^*_1}{d\epsilon}
\]

As a result, the LHS of the FOC in the Type I equilibrium, evaluated in the Type II equilibrium, is always higher than in the Type I equilibrium. The RHS is the same, because it does not depend on \( \tau^* \). In other words, at the level of government intervention that is optimal in the Type I equilibrium, the marginal benefit of increasing \( \epsilon \) in the Type II equilibrium exceeds the marginal cost. Assuming the second-order condition holds:

\[
\epsilon^*_2 > \epsilon^*_1
\]

(ii) Consider the first-order condition of the government’s problem:

\[
(1 - \tau^*)q + (1 - \epsilon)q \frac{d\tau^*}{d\epsilon} = c'(\epsilon)
\]
Let $\epsilon_1^*$ be the interior solution to the problem in the Type I economy. Recall from Proposition 4 that in such an equilibrium $\frac{d\tau^*_1}{d\epsilon} < 0$. It is then immediately obvious that the LHS of equation (40) (marginal benefit of increasing $\epsilon$) is greater in the Type II economy, since in that case $\frac{d\tau^*_2}{d\epsilon} > 0$ and $(1 - \tau^*_2) > (1 - \tau^*_1)$, while the RHS (marginal cost) is the same. As a result, a Type II economy generates a higher optimal $\epsilon$, that is, $\epsilon_2^* > \epsilon_1^*$.


Proof of Proposition 7

Consider the effect of a change in $\phi$ on $G(v, \tau)$:

$$\frac{\partial G(v, \tau)}{\partial \phi} = (1 - \epsilon) \left[ q\Delta p + \frac{dq}{d\phi} \left( \phi\Delta p + \phi q \frac{\partial \Delta p}{\partial q} \right) \right]$$ (41)

For clarity, denote this partial derivative $H$, and consider the conditions under which $H$ is positive, that is,

$$H > 0$$

$$q\Delta p + \frac{dq}{d\phi} \left( \phi\Delta p + \phi q \frac{\partial \Delta p}{\partial q} \right) > 0$$

$$q + \frac{dq}{d\phi} \left( \phi + q \frac{\partial \Delta p}{\Delta p} \right) > 0$$

$$\frac{q}{\phi} + \frac{dq}{d\phi} \left( 1 + q \frac{\partial \Delta p}{\Delta p} \right) > 0$$ (42)

Notice at this point that the first term is positive, while the second is negative, allowing in principle for the LHS expression to have either sign. Denote the second quantity in parentheses by $K$

$$K \equiv \frac{q}{\Delta p} \frac{\partial \Delta p}{\partial q}$$

so that $H$ becomes

$$H = \frac{q}{\phi} + \frac{dq}{d\phi} (1 + K).$$

At this point, it is helpful to define $x \equiv \frac{1 - \tau}{\tau}$, noting that because we are interested in the effect of $\phi$ on positive trust equilibria, excluding $\tau = 0$ from consideration is without loss of generality. Then, using the analysis in the proof of Proposition 1 and substituting appropriately for $x$, we
obtain

\[ K = \frac{q}{\Delta p} \left\{ \frac{\Delta p}{q} + (1 - \epsilon)q \left[ \frac{\partial}{\partial q} \left( \frac{1 - \epsilon q}{1 - q} \right) \right] \right\} \]

\[ = 1 + \frac{q}{\Delta p}(1 - \epsilon)q \left[ \frac{1 - \epsilon}{(1 - q)^2} \left[ \frac{(1 - q)^2}{[1 - q + (1 - \epsilon q) x]^2} \right] \right] \]

\[ = 1 + \frac{q}{\Delta p}[1 - q + (1 - \epsilon q) x]^2 \]

\[ = 1 + \frac{q(1 - \epsilon)(1 + \epsilon x) \left( \frac{1 - \epsilon q}{1 - q + (1 - \epsilon q) x} \right)}{1 - q + (1 - \epsilon q) x} \]

\[ = 1 + \frac{q(1 - \epsilon)(1 + \epsilon x) \left( \frac{1 - \epsilon q}{1 - q + (1 - \epsilon q) x} \right)}{1 - q + (1 - \epsilon q) x} \]

\[ = 1 + \frac{q(1 + \epsilon x)}{1 - q + (1 - \epsilon q) x} \]

We then have that \( H > 0 \) iff

\[ \frac{q}{\phi} + \frac{dq}{d\phi} \left[ 2 + \frac{q(1 + \epsilon x)}{1 - q + (1 - \epsilon q) x} \right] > 0. \] (43)

Since \( q''(\phi) > 0 \) and \( \frac{\partial K}{\partial q} > 0 \), \( \frac{\partial H}{\partial q} > 0 \). Furthermore, as \( q \to 0 \), \( LHS \to -2 < 0 \), while as \( q \to 1 \), \( LHS \to \infty > 0 \), which shows that there exists a threshold \( \bar{q} \), such that for values below \( \bar{q} \), \( H \) is negative, while it is positive for higher values of \( q \). Therefore, there exists a threshold \( \phi^*_1 \) such that

\[ \frac{\partial G(v, \tau)}{\partial \phi} = \begin{cases} > 0 \text{ if } \phi < \phi^*_1 \\ < 0 \text{ if } \phi > \phi^*_1 \end{cases} \]

From here on, if we restrict attention to points \( \phi < \phi^* \), we can reason analogously to part 2(c) of Proposition 4 and show that \( \frac{d\tau^*_1}{d\phi} \) is either positive or \( +\infty \). Conversely, for \( \phi > \phi^* \), we can derive that \( \frac{d\tau^*_1}{d\phi} \) is either negative of \( -\infty \), allowing us, using the same notational convention employed thus far about infinite values of the derivative, that:

\[ \frac{d\tau^*_1}{d\phi} = \begin{cases} > 0 \text{ if } \phi < \phi^*_1 \\ < 0 \text{ if } \phi > \phi^*_1 \end{cases} \]
Proof of Proposition 8

The result follows along the same lines of reasoning as in the proof of Proposition 7.
References


