Solution Set 1

As in my JEDC paper, the region of convergence corresponds to

\[ r \equiv \beta \exp \left[ (1 - \gamma)\mu + \frac{1}{2}(1 - \gamma)^2 \frac{\sigma^2}{(1 - \rho)^2} \right] < 1. \tag{1} \]

Hence I wrote a program that solved for the \( \beta \) such that \( r = 1 \) for any choice of \( \gamma \), i.e.

\[ \beta^* = \exp \left[ -(1 - \gamma)\mu - \frac{1}{2}(1 - \gamma)^2 \frac{\sigma^2}{(1 - \rho)^2} \right]. \tag{2} \]

All values of \( \beta \) below \( \beta^* \) are ones for which the model is solveable. The results are plotted in the first figure.

As \( \rho \) becomes larger (more positive) the region becomes smaller, whereas, as \( \rho \) becomes more negative the region becomes larger. This is obvious from the fact that

\[ \frac{\partial \beta^*}{\partial \rho} = -\beta^*(1 - \gamma)^2 \sigma^2 (1 - \rho)^{-3} < 0 \text{ for } \gamma \neq 1 \text{ and } = 0 \text{ for } \gamma = 1. \]

I think the intuitive argument for this is that the more positively serially correlated the consumption growth process is the more the MRS spends time in one part of the state space or another. The extreme form of this would be if we went to the limit and had both states being approximately absorbing states. In this world the behavior of the MRS at the mean is less important than the behavior of the MRS in the tails.

For the second part I just got my program to find the largest eigenvalue of the \( A \) matrix for any combination of \( \beta \) and \( \gamma \). I used bisection to find the \( \beta \) (given any \( \gamma \)) such that the largest eigenvalue of \( A \) is 1. This is \( \beta^* \) for the approximated model. You can see, from the second figure, that the \( \beta^* \) corresponding to the approximated model is close to \( \beta^* \) for the exact model with just \( n = 4 \) points.

My results for the means and standard deviations are in the attached table. People seemed to get the means right but the variances incorrect. You should have found that the variances converged as the number of points rose. Please check your code versus mine. I have attached my code and posted it on the web.
Region of Convergence for Different Values of $\rho$

- $\rho = -0.95$
- $\rho = -0.6$
- $\rho = -0.14$
- $\rho = 0$
- $\rho = 0.14$
- $\rho = 0.6$
- $\rho = 0.95$
Region of Convergence with \( n \) Point Rule

- \( n \) = 2
- \( n \) = 3
- \( n \) = 4
- Exact Model
### Assignment 1

**Summary Statistics of the Price Dividend Ratio, Risk Free Rate and Equity Return**

<table>
<thead>
<tr>
<th>No of Points</th>
<th>Price-Dividend Ratio</th>
<th>Risk-Free Rate</th>
<th>Equity Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>2</td>
<td>26.0778</td>
<td>1.3735</td>
<td>0.0227</td>
</tr>
<tr>
<td>3</td>
<td>27.8188</td>
<td>1.5544</td>
<td>0.0168</td>
</tr>
<tr>
<td>4</td>
<td>27.8701</td>
<td>1.5616</td>
<td>0.0166</td>
</tr>
<tr>
<td>5</td>
<td>27.8713</td>
<td>1.5618</td>
<td>0.0166</td>
</tr>
<tr>
<td>6</td>
<td>27.8714</td>
<td>1.5618</td>
<td>0.0166</td>
</tr>
<tr>
<td>7</td>
<td>27.8714</td>
<td>1.5618</td>
<td>0.0166</td>
</tr>
<tr>
<td>8</td>
<td>27.8714</td>
<td>1.5618</td>
<td>0.0166</td>
</tr>
</tbody>
</table>

Uses $\beta=1.125$ and $\gamma=14$. 
% asset pricing example with serially correlated shocks

clear
clc

% set the parameters of the law of motion of consumption growth
% calibrated to annual U.S. data from Mehra and Prescott 1985

mu=0.0179;
rho=-0.139;
sig=0.0348;

% graph the regions where the true solution is well-defined

gammax=(0:0.1:50)';
rhov=[-0.95 -0.6 -0.139 0 0.139 0.6 0.95]';

for j=1:7
    betacrit(:,j)=exp((gammax-1)*mu-0.5*(gammax-1).^2/((1-rhov(j,1))^2));
    betadet=exp((gammax-1)*mu);
end

figure(1)
plot(gammax,[betadet betacrit],'LineWidth',3)
legend('Nonstochastic','\{\rho\}=-0.95','\{\rho\}=-0.6','\{\rho\}=-0.14','\{\rho\}=0','\{\rho\}=0.14','\{\rho\}=0.6','\{\rho\}=0.95','Location','NorthWest')
title('Region of Convergence for Different Values of \{\rho\}','FontSize',16)
xlabel('{\gamma}','FontSize',14)
ylabel('{\beta}','FontSize',14)
ylim([0.9 1.4])

ng=101; % number of gammas to use in the approximate convergent solution graph

gammax=(0:50/(ng-1):50)';
betaex=exp((gammax-1)*mu-0.5*(gammax-1).^2/((1-rho)^2));

% load the quadrature data

load ghquad.dat

for n=2:4 % number of quadrature points to use ( must be <= 10 )

    % get the relevant submatrix of data for the number of quadrature points
    ymat=ghquad(0.5*(n-1)*n+1:0.5*n*(n+1),2); % ymat is the vector of abscissas (points)
    wmat=ghquad(0.5*(n-1)*n+1:0.5*n*(n+1),3); % wmat is the vector of weights
    zmat=ghquad(0.5*(n-1)*n+1:0.5*n*(n+1),4); % zmat is the vector of upper end points of intervals
    zmat2=[-100; zmat(1:n-1,1)]; % zmat2 is the vector of lower end points of the intervals

    % Now transform the abscissas and end points given the law of motion
    ymat=mu+ymat*sig;
    zmat=mu+zmat*sig;

zmat2 = mu + zmat2 * sig;

% generate the transition matrix
pim = gettrans(ymat, wmat, mu, rho, sig);

% get solution for price dividend ratio
for j = 1:ng
    g = gammax(j, 1);
    blow = 0.75; bup = 1.25;
    eps = 1;
    while eps > 1e-10
        b = (blow + bup) / 2;
        mrs = (b * exp(ymat(:, ones(n, 1)) * (1 - g)))';
        pm = pim .* mrs; % Euler equation can be rewritten as (I-pm)*v = pm*1 for a unit vector 1.
        L = eig(pm);
        differ = max(L) - 1;
        eps = abs(differ);
        if differ < 0
            blow = b;
        else
            bup = b;
        end
        end
    betax(j, n-1) = b;
end

figure(2)
plot(gammax, [betax betaex], 'LineWidth', 3)
legend({'\(n\)=2','\(n\)=3','\(n\)=4','Exact Model','Location','NorthWest'}
title('Region of Convergence with\(n\) Point Rule','FontSize',16)
xlabel('\(\gamma\)','FontSize',14)
ylabel('\(\beta\)','FontSize',14)
ylim([0.75 1.25])

b = 0.95; g = 4;
for n = 2:8 % number of quadrature points to use ( must be \(\leq\) 10 )
    % get the relevant submatrix of data for the number of quadrature points
    ymat = ghquad(0.5*(n-1)*n+1:0.5*n*(n+1), 2); % ymat is the vector of abscissas (points)
    wmat = ghquad(0.5*(n-1)*n+1:0.5*n*(n+1), 3); % wmat is the vector of weights
    zmat = ghquad(0.5*(n-1)*n+1:0.5*n*(n+1), 4); % zmat is the vector of upper end points of intervals
    zmat2 = [-100; zmat(1:n-1, 1)]; % zmat2 is the vector of lower end points of the intervals
    % Now transform the abscissas and end points given the law of motion
    ymat = mu + ymat * sig;
    zmat = mu + zmat * sig;
    zmat2 = mu + zmat2 * sig;
% generate the transition matrix
pim=gettrans(ymat,wmat,mu,rho,sig);
pix=null(eye(n)-(pim'));
pix=pix/sum(pix); % unconditional probabilities of the states

% get solution for price dividend ratio
v=getprice(ymat,pim,b,g);
q=getqbond(ymat,pim,b,g);
rf=ones(n,1)./q-1;
re=(ones(n,1)*(1+v')).*(ones(n,1)*exp(ymat'))./(v*ones(1,n))-1;

mv=pix'*v; sv=sqrt(pix'*((v-mv).^2));
mrf=pix'*rf; srf=sqrt(pix'*((rf-mrf).^2));
mre=pix'*((re.*pim)*ones(n,1)); sre=sqrt(pix'*((re-mre).*pim)*ones(n,1).^2));

sts(n-1,:)=[ n mv sv mrf srf mre sre ];
end

disp(’Moments of the Variables’)
disp(’ n  Mean(v)  Std(v)  Mean(rf)  Std(rf)  Mean(re)  Std(re)’)
disp(sts)
function q=getqbond(ymat,pim,bet,gam);
% function to solve the euler equation
% ymat is the vector of quadrature points
% pim is the transition matrix
% bet is beta
% gam is gamma

[n,n0]=size(pim);   % determines the number of points in state space = n
mrs=(bet*exp(ymat(:,ones(n,1))*(-gam)))';    % defines the mrs
pm=pim.*mrs;       % creates the element product of pim and mrs
q=pm*ones(n,1);