Indifference Surfaces for Lotteries

Needs "Graphics'ContourPlot3D"
Needs "Graphics'ImplicitPlot"

Consider an individual whose vonNeuman utility function is equal to the natural log of the payoff

\[ u[w_] := \log[1 + w] \]

where \( w \) is measured in millions of dollars and let \((x,y,p)\) denote a lottery that pays $x with probability \( p \) and $y with probability \( 1-p \). It follows that this person, let’s call him Nat Log, would be indifferent between a certain prize of $2 and any lottery \((x,y,p)\) for which

\[ pu[x] + (1 - p) u[y] == u[2] \]

or, equivalently, for which

\[ \text{indiff} = pu[x] + (1 - p) u[y] - u[2] \]

is equal to zero. This surface in \( x,y,p \) space can be plotted as follows where the origin is in the lower left hand corner, the \( x \) and \( y \) axes form the two edges of the bottom of the shallow box and the \( p \) axis forms the third (vertical) edge.

\[ \text{ContourPlot3D} \left[ \text{indiff}, \{x, 0, 4\}, \{y, 0, 4\}, \{p, 0, 1\}, \text{PlotPoints} \to 5, 5, \text{PlotLabel} \to \text{"Indifference Surface\"} \right]; \]

Representative cross-sections corresponding to different values of \( p \) are illustrated below.

\[ \text{ImplicitPlot} \left[ \text{Table} \left[ \text{indiff} == 0/.p \to i, \{i, 0, 1.2\} \right], \{x, 0, 4\}, \{y, 0, 4\}, \text{PlotPoints} \to 50, \text{PlotStyle} \to \text{Table} \left[ \text{Hue}[i], \{i, 0, 1.2\} \right] \right]; \]
Note that each is (weakly) convex. The horizontal line corresponds to $p=0$ - when there is no chance of winning $x$ - and the vertical line to $p=1$ - when there is no chance of winning $y$. In between, the indifference curves "twist" clockwise as $p$ increases from 0 to 1. Each of these curves is, of course, a cross-section of the three-dimensional indifference surface illustrated before.