Provide brief explanations as well as your answers.

1. Frank buys a lottery ticket from Sam which promises a 7% chance of winning 700 dollars. Frank is risk-neutral and the price Frank pays for the ticket makes him indifferent between buying and not buying the ticket. The winning ticket is to be drawn momentarily, so time discounting is not necessary.
   (a) What price does Frank pay for the ticket?
   (a) 
   (b) Suppose that Sam, who is numerically challenged, has misreported to Frank the true chance of winning which is, in fact, only 5%.
     (a) If the numerical error were discovered before the winning ticket is drawn, how much would Sam have to pay Frank to make him as well off as if there had been no error?
     (a) 
     (b) Suppose instead that the error will not be discovered before the drawing. After the drawing, on the other hand, Frank will be sure to complain if he loses and the error will then be discovered. How much, to the nearest cent (2 decimal places), would Sam have to pay Frank after the drawing if he loses to make him as well off as if there had been no error?
     (b)
     Hint: Frank was promised the lottery ($700, $0; \frac{7}{100}, \frac{1}{100})$ but will, in fact, get the lottery ($700, z; \frac{5}{100}, \frac{1}{100})$ where $z$ is the compensation he will receive if he loses.

2. A risk averse individual is endowed with $1000 and is contemplating an action which would result in a 60% chance of having to pay a $400 fine. Let
   
   **Lottery A**: the lottery corresponding to a 40% increase in the chance of having to pay the fine (from 60% to 84%)
   
   **Lottery B**: the lottery corresponding to a 40% increase in the amount of the fine (from $400 to $560)
   
   Which of the following are correct?
   
   Choose one or more of the following.
Lottery A is a mean preserving spread of Lottery B.

Lottery B is a mean preserving spread of Lottery A.

A risk averse individual would prefer Lottery A.

A risk averse individual would prefer Lottery B.

There is insufficient information to determine which lottery a risk averse individual would prefer.

Lottery A would provide the stronger deterrent.

Lottery B would provide the stronger deterrent.

There is insufficient information to determine which lottery would provide the stronger deterrent.

3. Lucky Pierre is a risk averse prospector who has struck it rich. He has 216 ounces of gold dust, his only wealth, safely stashed away on his claim. He wants to get his gold from his claim to the big city where he hopes to spend it. The good news is that there are many competitive shippers each of whom is willing to transport any integer number of ounces of Pierre's gold for a fee of 5% of the gold shipped. The bad news is that for each of these shippers there is an independent probability of 1/7 that all the gold on any such trip will be stolen and a probability of 6/7 that all of the gold will be successfully delivered. A small consolation is that there is no shipping charge for gold that is stolen, i.e., shipping charges only apply to gold successfully delivered.

(a) Suppose Lucky is considering the following options:

One: make one shipment of 216 ounces.

Two: make two shipments of 108 ounces each.

i. The expected value of gold successfully delivered and net of shipping charges under One would be

ii. The expected value of gold successfully delivered and net of shipping charges under Two would be

[Hint. Four events must be considered: both shipments get delivered, both shipments are stolen, only the first shipment is stolen and only the second shipment is stolen.]

iii. Which of the following are correct?

Choose one or more of the following.

One is a mean-preserving spread of Two.
Two is a mean-preserving spread of One.

Two would be strictly preferred by Pierre.

One would be strictly preferred by Pierre.

Pierre would be indifferent between One and Two.

There is insufficient information to determine which option would be preferred by Pierre.

(b) Now suppose Pierre is considering making an arbitrary number of shipments but subject, of course, to the requirement that each shipment must have at least one ounce. How many shipments will he choose to make?

(b) __________

4. Two people with strictly concave utility functions (risk-averse) are endowed with the same independent lottery, L, which pays either 108 or 132 with probability 1/2 each. The fact that the lotteries are independent means that the event that both win 108 occurs with probability 1/4, the event that one wins 108 and one wins 132 occurs with probability 1/2 and the event that both win 132 occurs with probability 1/4.

(a) Let P denote the lottery that each would get if they entered into a binding agreement to split their actual winnings equally, e.g., if one won 108 and the other won 132 then both would get 120. What is the expected value of P?

(a) __________

Which of the following are correct?

Choose one or more of the following.

- P first-order stochastically dominates L.
- L first-order stochastically dominates P.
- P second-order stochastically dominates L.
- L second-order stochastically dominates P.
- P is a mean-preserving spread of L.
- L is a mean-preserving spread of P.

(b) Suppose a third person appears who is also endowed with L and let Q denote the lottery that each would get if all three entered into a binding agreement to split their actual winnings equally so that, e.g., if two won 108 and one won 132 then each would get 116.
What is the expected value of $Q$?

(b) __________

Which of the following are correct?

*Choose one or more of the following.*

- $P$ first-order stochastically dominates $Q$.
- $Q$ first-order stochastically dominates $P$.
- $P$ second-order stochastically dominates $Q$.
- $Q$ second-order stochastically dominates $P$.
- $P$ is a mean-preserving spread of $Q$.
- $Q$ is a mean-preserving spread of $P$.

(c) Any of the three people would rank

*Choose one of the following.*

- $L$ best and $P$ worst
- $L$ best and $Q$ worst
- $P$ best and $L$ worst
- $P$ best and $Q$ worst
- $Q$ best and $L$ worst
- $Q$ best and $P$ worst
- $L$, $P$ and $Q$ indifferent
- insufficient information to determine ranking

5. Russian Roulette. A rich man is forced to play a game of Russian Roulette. He is an expected utility maximizer who prefers living to dying and, provided that he lives (no bequest motive), prefers more wealth to less. The cylinder of a six-shot revolver containing *two* bullets is spun and pointed at his head. He is offered the opportunity of paying to have *both* bullets removed before the trigger is pulled and it turns out that the payment can be made as high as $10$ million before he becomes indifferent between paying and taking the chance of getting shot.

Hint. Only three “prizes” and the corresponding von Neumann utilities need be considered in analyzing the lotteries involved in this question: (W) living and paying nothing, (P) living and paying $10$ million and (L) dying (with no bequest motive, it doesn’t matter whether he pays or not).
(a) Suppose the cylinder contains *one* bullet. Is the most that he should be willing to pay to have the one bullet removed more than, equal to or less than $10 million? *Choose one or more of the following.*

- [ ] more than $10 million
- [ ] equal to $10 million
- [ ] less than $10 million
- [ ] insufficient information to determine

(b) Suppose the cylinder contains *four* bullets. Is the most that he should be willing to pay to have one of the bullets removed more than, equal to or less than $10 million? *Choose one or more of the following.*

- [ ] more than $10 million
- [ ] equal to $10 million
- [ ] less than $10 million
- [ ] insufficient information to determine