Provide brief explanations as well as your answers.

1. Fighting an opponent of unknown strength. Two people are involved in a dispute. Person 1 does not know whether Person 2 is strong or weak; she assigns probability $\alpha$ to person 2’s being strong. Person 2 is fully informed. Each person can either fight or yield. Each person’s preferences are represented by the expected value of a Bernoulli payoff function that assigns the payoff of 0 if she yields (regardless of the other person’s action) and a payoff of 1 if she fights and her opponent yields; if both people fight, then their payoffs are $(-1, 1)$ if person 2 is strong and $(1, -1)$ if person 2 is weak. Formulate this situation as a Bayesian game, show the strategic form of the game and find its Nash equilibria when
   (a) $\alpha < 1/2$
   (b) $\alpha > 1/2$

2. Reporting a crime with an unknown number of witnesses. There are two players and three possible states of nature:
   1) There is a crime and Player 1 is the only witness
   2) There is a crime and Player 2 is the only witness
3) There is a crime and both Player 1 and Player 2 are witnesses
4) There is no crime and thus there are no witnesses

Information partitions:
Player 1. \{\{1, 3\}, \{2, 4\}\}
Player 2. \{\{2, 3\}, \{1, 4\}\}

Conditional on a crime occurring, each player believes that the probabilities of the states are
\[(p_1, p_2, p_3) = \left(\frac{\pi}{1 + \pi}, \frac{\pi}{1 + \pi}, \frac{1 - \pi}{1 + \pi}\right)\]

Note that a player only gets to act if the player is a witness. Conditional on being a witness, each player thus believes that the probability of being the only witness is \(\pi\).

Each player attaches the value \(v\) to the police being informed and bears a cost \(c\) of calling the police where \(v > c > 0\).

(a) Model this situation as a Bayesian game and find a condition on \(\pi\) under which the game has an equilibrium in which each player chooses \textit{Call} when witnessing the crime.

(b) When the condition for a Perfect Bayesian equilibrium in pure strategies is violated, find the symmetric equilibrium in mixed strategies.

(c) What happens when \(\pi = 0\)?

3. An exchange game. Each of two individuals receives a ticket on which there is an integer from 1 to \(m\) indicating the size of a money prize she may receive. The individuals’ tickets are assigned randomly and independently and the probability of an individual’s receiving
each possible number is positive. Individuals are simultaneously given the option of ex-
changing their own prize for the other’s prize. The exchange occurs only if both choose to
exchange — otherwise each keeps her own prize. Model this situation as a Bayesian game
and show that in any Bayesian Nash equilibrium, the highest prize that either individual is
willing to exchange is the smallest possible prize.

4. Adverse selection. Firm $A$ (the “acquirer”) is considering taking over firm $T$ (the “target”).
It does not know firm $T$’s value but believes that when $T$ is controlled by its own manage-
ment, this value is at least $0$ and at most $100$ with each of the 101 dollar values in this
range equally likely. $T$ will be worth 50% more under $A$’s management than under its own.
Suppose that $A$ bids $y$ to take over $T$ and that $T$ is worth $x$ under its own management.
Then if $T$ accepts $A$’s offer, the payoffs for $A$ and $T$ would be $3/2x - y$ and $y$. If $T$ rejects
the offer then the payoffs would be 0 and $x$. Model this situation as a Bayesian game in
which $A$ decides how much to offer and $T$ decides how low an offer to accept. Find the
Nash equilibria of this game and explain the significance of the term adverse selection.

5. Groucho Marx once remarked that he would never join any club that would have him as a
member. Create a Bayesian game in which the equilibrium is consistent with his remark.