A firm is considering two “research and development” projects. The rate of interest is 10% per period and the projects have the following attributes:

<table>
<thead>
<tr>
<th></th>
<th>Project # 1</th>
<th>Project # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$15</td>
<td>$20</td>
</tr>
<tr>
<td>Duration</td>
<td>1 period</td>
<td>2 periods</td>
</tr>
<tr>
<td>Reward</td>
<td>$100</td>
<td>$240</td>
</tr>
<tr>
<td>Probabilities</td>
<td>.5</td>
<td>.2</td>
</tr>
<tr>
<td>Probabilities</td>
<td>.5</td>
<td>.8</td>
</tr>
<tr>
<td>Expected Present Value</td>
<td>$55.5(^a)</td>
<td>$19.7(^b)</td>
</tr>
</tbody>
</table>

\(^a\) $55.5 = -15 + \frac{1}{1.1}[.5 \times 100 + .5 \times 55]

\(^b\) $19.7 = -20 + \frac{1}{1.1}[.2 \times 240 + .8 \times 0]

Note that it takes 1 period to develop #1 and 2 periods to develop #2. The costs of $15 and $20, respectively, are measured in “$ at the beginning of the development period”. At the end of the development period the reward for the project is revealed and a decision can be made to accept this reward or to develop the other project. This reward is the payoff of an “accepted project” and is measured in “$ at the time of acceptance”. The projects are mutually exclusive in the sense that it is not possible to obtain a reward from #1 and #2. Thus #1 could be researched and, depending upon the discovered value of #1’s reward, a decision could be made regarding whether or not to research #2. Having researched both #1 and #2 a decision could be made to accept the largest of the (now known) rewards, et cetera.

• Which project should be researched first? Is this the project with the highest expected present value?

The expected values of researching the two projects are

\[ E_1 = -15 + \frac{1}{1.1}(.5 \times 100 + .5 \times 55) \]
\[ = 55.5 \]
\[ E_2 = -20 + \frac{1}{1.1}^2(.2 \times 240 + .8 \times 0) \]
\[ = 19.7 \]

The expected value of researching 2 if 1 is researched first and turns out to be worth 55 is

\[ -20 + \frac{1}{1.1}^2(.2 \times 240 + .8 \times 55) = 56 \]

Since this is greater than 55 it pays to research 2 in the event that 1 pays 55. Since

\[ -20 + \frac{1}{1.1}^2(.2 \times 240 + .8 \times 100) = 85.8 \]

it would not pay to research 2 if 1 pays 100. The expected value of 1 first is thus

\*This example and the optimal rule are due to Weitzman, “Optimal Search”, *Econometrica*, 47 (May, 1979) pages 641–54.
\[-15 + \frac{1}{1,1} \left\{ .5(100) + .5 \left( -20 + \frac{1}{1,1^2} [.2(240) + .8(55)] \right) \right\} = 55.9 \]

and the expected value of researching 2 first is

\[-20 + \frac{1}{1,1^2} \left\{ .2(240) + .8 \left( -15 + \frac{1}{1,1} [ .5(100) + .5(55) ] \right) \right\} = 56.3 \]

The greatest expected payoff is thus associated with researching 2 first.

Consider the following shadow prices. $v_1$ is a sure payment which would make the firm indifferent between accepting $v_1$ and researching the first project if these were the only two alternatives and if $v_1$ could be accepted either before or after the reward from the first project is discovered. Let $v_2$ be the corresponding shadow price for the second project. Thus

\[
v_1 = -15 + \frac{1}{1,1} \left[ .5 \times 100 + .5 \times v_1 \right] \\
v_2 = -20 + \frac{1}{1,1^2} \left[ .2 \times 240 + .8 \times v_2 \right]
\]

Solving gives

\[
v_1 = 55.83 \\
v_2 = 58.049
\]

Note that the following rule for researching projects is optimal: research that project first which has the largest shadow price. Stop if the actual reward from the project researched first exceeds the shadow price of the other project; otherwise research the other project and accept the largest discovered reward.