Linear Quadratic Gaussian Controller - Design Equations

To design a controller using the LQG approach, assuming $R_{12} = 0$ and $V_{12} = 0$:

Solve two Riccati equations for $P$ and $Q$:

$$0 = A^T P + PA - P \Sigma P + R_1,$$
where $\Sigma = BR_2^{-1}B^T$ (left hand)

$$0 = A Q + QA^T - Q \Sigma Q + V_1,$$
where $\Sigma = C^T V_2^{-1}C$ (right hand)

Then compute the matrices of the LQG compensator

$$C_c = -R_2^{-1} B^T P$$
$$B_c = QC_2 V_2^{-1}$$
$$A_c = A + BC_c - B_c C - B_c D C_c$$

Note that the Matlab command $X = \text{are}(A, B, C)$ returns the stabilizing solution (if it exists) to the continuous-time left hand matrix Riccati equation, assuming $B$ is symmetric and non-negative definite and $C$ is symmetric.

Going from a left hand Riccati equation to a right hand Riccati equation is the same as substituting $A'$ for $A$ in the \text{are} command.

$X = \text{are}(A, B, C)$ solves $0 = A^T X + X A - X B X + C$

$Y = \text{are}(A', B, C)$ solves $0 = A Y + Y A^T - Y B Y + C$

The total cost is

$$J(A_c, B_c, C_c) = tr(Q R_1 + P Q \Sigma Q) = tr(P V_1 + Q P \Sigma P)$$

The state cost is

$$J_s(A_c, B_c, C_c) = tr(Q_1 R_1)$$

The control cost is

$$J_c(A_c, B_c, C_c) = tr(Q_2 C_c^T R_2 C_c)$$