Extended Sensitivity Analysis for “Optimal Hiring and Retention Policies for Heterogeneous Workers who Learn”

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This online note contains a longer version of the paper’s Section 6.3. It includes the backing tables that support the paper’s reporting of the following results: variation in base-levels of performance, sampling variance, and training costs.

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Variance of base-level performance in prior distribution. We parameterize the employer’s uncertainty concerning the ability of untested workers using the prior standard deviation of A, $\widehat{\sigma}^2$. By varying $\widehat{\sigma}$ while holding σ constant, we can see how the optimal screening policy changes with worker heterogeneity. Here, we analyze three values of the prior standard deviation, 0.20, 0.40, and 0.80 (i.e., $\widehat{\sigma}^2 = 0.04, 0.36, 0.64$ respectively), and we discuss how they affect our results. All other parameters are as in Section 6.1.

Table 1 shows how the Gittins index, the fraction of terminated workers, and the long-run average service rate change with $\widehat{\sigma}^2$. The values obtained in the numerical example agree with the general idea that the Gittins index reflects an option value inherent in the ability to change arms, and it favors arms with more diffuse prior distributions. In our context this implies that, for a given $\widehat{\mu}$, an increase in the variation of ability across workers allows the employer to screen more strictly, thereby increasing termination rates, retaining relatively more capable employees, and lowering total costs.

Sampling variance. We then perform a sensitivity analysis with respect to the sampling variance, σ^2. The analysis is similar to that for the prior variance, but here we keep $\widehat{\sigma}$ constant as we let σ vary. The values of σ we consider are 0.60, 0.80, 1.00. The other parameters are fixed as in Section 6.1.

Table 1 **Simulation results with different prior variances** *(standard errors for the mean in parenthesis)*.

	Gittins	Fraction of terminated workers				Long-run average
$\widehat{\sigma}$	index	Day 1	Days 2–10	Days 11 –20	Total	service rate
0.2000	5,715.1	0.0000 *(.0000)*	0.0330 *(.0008)*	0.0351 *(.0008)*	0.1060 *(.0014)*	0.5204 *(.0064)*
0.4000	5,491.7	0.0196 *(.0006)*	0.2830 *(.0020)*	0.0557 *(.0010)*	0.3982 *(.0022)*	0.6417 *(.0138)*
0.8000	4,752.8	0.2406 *(.0019)*	0.2840 *(.0020)*	0.0439 *(.0009)*	0.5842 *(.0022)*	1.1357 *(.0488)*

Table 2 Simulation results with different sampling variances *(standard errors for the mean in parenthesis)*.

	Gittins	Fraction of terminated workers				Long-run average
σ	index	Day 1	Days 2–10	Days 11 –20	Total	service rate
0.6000	4,993.3	0.0057 *(.0003)*	0.1831 *(.0017)*	0.0595 *(.0011)*	0.2755 *(.0020)*	0.6626 *(.0111)*
0.8000	5,491.7	0.0196 *(.0006)*	0.2830 *(.0020)*	0.0557 *(.0010)*	0.3982 *(.0022)*	0.6417 *(.0138)*
1.0000	6,190.4	0.0534 *(.0010)*	0.3426 *(.0021)*	0.0517 *(.0010)*	0.4961 *(.0022)*	0.6104 *(.0160)*

Table 3 Simulation results with different training costs *(standard errors for the mean in parenthesis)*.

	Gittins	Fraction of terminated workers				Long-run average
c_h	index	Day 1	Days 2–10	Days 11 –20	Total	service rate
0	3,645.2	0.5849 *(.0022)*	0.2600 *(.0020)*	0.0153 *(.0005)*	0.8736 *(.0015)*	0.8316 *(.0153)*
15	4,833.7	0.0833 *(.0012)*	0.3844 *(.0022)*	0.0548 *(.0010)*	0.5546 *(.0022)*	0.6889 *(.0138)*
30	5,491.7	0.0196 *(.0006)*	0.2830 *(.0020)*	0.0557 *(.0010)*	0.3982 *(.0022)*	0.6417 *(.0138)*
45	6,028.9	0.0057 *(.0003)*	0.1949 *(.0018)*	0.0525 *(.0010)*	0.2880 *(.0020)*	0.6122 *(.0136)*
60	6,509.1	0.0017 *(.0002)*	0.1404 *(.0016)*	0.0415 *(.0009)*	0.2201 *(.0019)*	0.5949 *(.0134)*

Table 2 displays the increase in the Gittins index and the decrease in the long-run average service rate as σ increases. It also indicates that, for lower σ, the fractions of employees who are terminated are lower. Thus, reductions in within-period variability improve the selectivity and effectiveness of screening procedures, allowing the employer to reduce termination rates using the optimal policy.

Training costs. Section 6.1 studied a setting in which every time a new worker is employed, the employer incurs a training cost, $c_h = 30$. Here we perform a sensitivity analysis that studies how termination rates and total expected discounted costs vary with training costs. When different values of training costs are considered, the stopping boundaries in Figure 1 change as one would expect. The stopping boundary with respect to the posterior mean of A jumps up as training costs increase, and it retains its peculiar "cupped" shape. Thus, the same observations about two competing forces made in Section 6.1 hold here as well. Similarly, an increase in training costs also produces an upward shift of the stopping boundary with respect to $\mathbb{E}[Z_n]$. Naturally, when there are no training costs and $c_h = 0$, the initial jump disappears in both boundaries.

Table 3 shows how the Gittins indices, the fractions of terminated workers, and the long-run average service rates change when $c_h = \{0, 15, 30, 60\}$. It is interesting to note that when training costs are absent the screening process is very selective and terminates 58.49% of employees on day 1 and 87.36% overall. As training costs enter into the problem, the termination rates quickly drop, and the values of the Gittins indices and of the service rates follow, naturally, the opposite trend.