Cross-sectional alpha dispersion and performance evaluation☆

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Our paper explores the link between cross-sectional fund return dispersion and performance evaluation. The foundation of our model is the simple intuition that in periods of high return dispersion, which is associated with high levels of idiosyncratic risk for zero-alpha funds, unskilled managers can more easily disguise themselves as skilled. Rational investors should be more skeptical and apply larger discounts to reported performance in high dispersion environments. Our empirical results are consistent with this prediction. Using fund flow data, we show that a one standard deviation increase in cross-sectional return dispersion is associated with an 11% to 17% decline in flow-performance sensitivity. The effect is stronger for recent data and among outperforming funds.

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1. Introduction

After 50 years of research on the performance of active mutual funds, a remarkable consensus exists: the vast majority of active mutual funds do not outperform. The evidence is consistent across three separate strands of research: the hypothesis testing literature, the literature on classifying funds into different performance groups, and the Bayesian performance evaluation literature.1

Investors face the problem of trying to identify the funds that are expected to outperform from a population dominated by funds that are generally poorly performing. Investors do not want to invest in a poorly performing

1 See Kosowski et al. (2006), Barras et al. (2010), Fama and French (2010), Harvey et al. (2016), and Harvey and Liu (2017) for the hypothesis testing literature, (Barras et al., 2010; Ferson and Chen, 2017), and Harvey and Liu (2018) for the literature on classifying funds into performance groups, and Baks et al. (2001); Pástor and Stambaugh (2002); Jones and Shanken (2005); Avramov and Wermers (2006); Busse and Irvine (2006); Kosowski et al. (2007); Avramov et al. (2011), and Harvey and Liu (2018) for the Bayesian performance evaluation literature.
asset manager (Type I error), but neither do they want to miss an outperforming manager (Type II error).\(^2\) In this context, we show that cross-sectional dispersion of manager returns is an important state variable that influences the trade-off between Type I and Type II errors, and we argue that cross-sectional dispersion of returns is information investors use in making manager-selection decisions. In periods of high cross-sectional dispersion, unskilled managers can easily be disguised as skilled, leading investors to be more skeptical of outperforming funds and to consequently discount fund alphas more harshly.

Motivated by the literature that classifies funds into performance groups (Barras et al., 2010; Ferson and Chen, 2017; Harvey and Liu, 2018), we build a model that assumes fund alphas are drawn from several subpopulations, with one subpopulation being the zero-alpha population. In addition, we assume the majority of funds come from the zero-alpha population, consistent with the main finding of the literature. Within this context, zero-alpha funds can still generate a nonzero estimated alpha by taking on idiosyncratic risk over a given period of time. We show that the average level of idiosyncratic risk for zero-alpha funds, which is closely related to cross-sectional return dispersion, determines the amount of shrinkage that rational investors apply in discounting the alphas of outperforming funds.\(^3\)

Our model provides unambiguous predictions about how the average level of idiosyncratic risk among zero-alpha funds affects performance evaluation. We cannot directly observe this level because we do not know which funds are zero-alpha funds. We propose to represent the level of idiosyncratic risk with a simple metric: the interquartile range (IQR) of the cross section of all fund returns. We argue that IQR is intuitively appealing because it succinctly captures the range of performance among funds, most of which, given the prior literature, are mediocre performers at best. As such, IQR gives investors a sense of the variation in performance that should be expected based on luck alone.

We provide simulation-based evidence that shows a very high time series correlation between IQR and the average level of idiosyncratic risk of the zero-alpha funds. Using IQR as a proxy for return dispersion, we test the main prediction of our model by examining the relation between future fund flows and past fund performance, which is known as the flow-performance sensitivity, and show that return dispersion negatively affects this relation. The impact is economically significant. A one standard deviation increase in return dispersion reduces flow-performance sensitivity by 11% to 17%, depending on the benchmark model we use to estimate alpha. Hence, our paper provides direct flow-based evidence on the relevance of the inference problems highlighted by the three strands of literature from a revealed-preference perspective (Berk and van Binsbergen, 2016; Barber et al., 2016; Agarwal et al., 2018).

Our results are robust across different alpha estimation methods and are stronger for the more recent data. This is consistent with the view that investors have learned through time and that the fraction of non performing funds is larger today than in the past. We also show that convexity in the flow-performance relation does not explain our results. Instead, our results suggest that flows to outperforming funds are especially sensitive to return dispersion. Lastly, we study how percentile alpha rankings affect our results. While return dispersion remains important after controlling for alpha rankings, alpha rankings lose their power to predict fund flows. Our model of return dispersion provides an economic interpretation to the finding that alpha rankings help predict fund flows.

Our paper is related to and motivated by the recent literature on performance evaluation.

Our model can be interpreted within a hypothesis testing framework, following Kosowski et al. (2006), Barras et al. (2010), Fama and French (2010), Ferson and Chen (2017), and Harvey and Liu (2017). These papers study performance evaluation from a multiple testing perspective, arguing that outperforming funds must surpass a statistical threshold, which is tougher than the usual threshold, to declare significance due to the large number of funds in the cross section. We show that return dispersion affects this threshold. All else equal, a higher return dispersion makes it easier for a typical zero-alpha fund to be cloaked as a fund whose managers have skill, resulting in a higher Type I error (i.e., falsely identifying zero-alpha funds as good). As a result, rational investors should apply a tougher threshold to control the Type I error rate at a desired level.

We present our model within a Bayesian framework. Suppose investors view fund managers as coming from several subpopulations, with the zero-alpha population being one of the subpopulations. In this setup, we show that a Bayesian investor’s prior on the average level of idiosyncratic risk of zero-alpha funds, which is the main driver of cross-sectional return dispersion, is important for his or her decision making. A higher prior level of the average idiosyncratic risk leads to a lower posterior mean for funds with large (positive) alphas, implying a more aggressive discount of the alphas for these funds and thus a weaker flow-performance sensitivity. Our model thus differs from existing Bayesian models by focusing on the time-varying impact of cross-sectional return dispersion on perceived performance.\(^4\)

Our research also complements the literature that models the cross section of funds’ performance by classifying

\(^{2}\) In our paper, we sometimes refer to funds as managers, although we do not observe managers. Funds could have more than one manager or experience turnover in managers.

\(^{3}\) The shrinkage of fund alphas is the key insight highlighted in several existing papers, including Pástor and Stambaugh (2002), Jones and Shanken (2005), Cohen et al. (2005), Avcamov and Wermers (2006), Busse and Irvine (2006), Mamaysky et al. (2007), Barras et al. (2010), and Harvey and Liu (2018).

\(^{4}\) The existing Bayesian applications to performance evaluation (e.g., Baks et al., 2001; Pástor and Stambaugh, 2002; Jones and Shanken, 2005; Harvey and Liu, 2018) assume a constant idiosyncratic risk for each fund and use the time series history of fund returns to make inference on the fund’s idiosyncratic risk (together with the prior distribution on fund alphas). Our model highlights time-varying idiosyncratic risk and focuses on the problem of inferring a fund’s alpha and risk over a relatively short period of time (e.g., a year), consistent with the literature on flow-performance sensitivity.
funds into distinct performance groups. On the one hand, we build on the premise that the majority of funds have a zero alpha, which is consistent with the previous empirical consensus. On the other hand, recent research focuses on the estimation of the cross-sectional distribution of funds’ performance, while we pay particular attention to the role of return dispersion in influencing the classification of funds.

Instead of trying to infer the cross-sectional distribution of skill from an economist’s perspective, as the three strands of literature do, we follow the recent literature on revealed preference to test the predictions of our model. As such, we are able to ascertain the information that motivates investors from a real-world asset allocation perspective.

Our paper also adds to the recent literature that takes a closer look at flow-performance sensitivity. Franzoni and Schmalz (2017) study how uncertainty about risk loadings on benchmark factors affects investors’ capital-allocation decisions. Starks and Sun (2016) examine the implications of economic policy uncertainty on flow-performance sensitivity. We show that our results are robust to the inclusion of risk loading uncertainty and economic policy uncertainty.

Kim (2017) examines the impact of cross-sectional standard deviation of performance on the convexity of the flow-performance relation. She finds that convexity is reduced following periods of low cross-sectional standard deviations of performance. In contrast to Kim’s work, we show that the usual flow-performance sensitivity is strengthened following periods of low return dispersion.

Finally, although we focus on a single important variable, return dispersion, that appears to influence investors’ capital-allocation decisions, other candidate variables implied by the previous literature on performance evaluation potentially exist. Past research mainly focuses on what investors should do to evaluate fund managers, whereas our paper provides an example of what investors are doing.5

The remainder of the paper is organized as follows. Section 2 presents our model that links return dispersion to flow-performance sensitivity. Section 3 tests our model predictions by empirically examining flow-performance sensitivity. Some concluding remarks are offered in the final section.

2. Model

We present our model of cross-sectional return dispersion in Section 2.1. We calibrate our model in Section 2.2. Section 2.3 and 2.4 discuss issues on applying our model to the data.

2.1. A simple model of return dispersion

We present a model that links cross-sectional return dispersion to individual fund performance. We assume the population of fund managers consists of several subpopulations, each characterized by a specific value of alpha (Barras et al., 2010; Ferson and Chen, 2017; Harvey and Liu, 2018). This view of the population provides a convenient, yet realistic, way to classify funds into performance groups. Meanwhile, building on the insights from the literature that uses hypothesis testing to evaluate fund performance (Kosowski et al., 2006; Barras et al., 2010; Fama and French, 2010), we assume the majority of funds have an alpha indistinguishable from zero. Finally, following the Bayesian performance evaluation literature (Baks et al., 2001; Pástor and Stambaugh, 2002; Harvey and Liu, 2018), we study the inference problem faced by Bayesian investors who learn about manager skill by using both past fund performance as well as information about the cross section of fund returns.

We cast our model within a Bayesian framework. Similar to existing models that study flow-performance sensitivity (Berk and Green, 2004; Huang et al., 2012; Starks and Sun, 2016; Franzoni and Schmalz, 2017), we focus on our model’s implications for the posterior mean of alpha.

We begin by assuming that investors view fund managers as coming from two subpopulations: one with $\alpha_0 = 0$ and the other with $\alpha_0 > 0$, where $0$ represents unskilled or zero alpha and $h$ is high alpha. We further assume that by observing a fund’s return history over the past $T$ periods, investors infer whether the fund exhibits no skill (i.e., $\alpha_0 = 0$) or skill (i.e., $\alpha_0 > 0$). The prior masses of the two subpopulations are $\Pi_0$ and $\Pi_h$, with $\Pi_0 + \Pi_h = 1$. Given the priors and past performance, investors come up with a perceived alpha, which we refer to as the posterior mean of alpha.

Our model abstracts from two complications that arise in performance evaluation. First, although some disagreement exists over the number of subpopulations that best describe the cross section of funds, we choose to use the two-group classification. Our model can be easily extended to accommodate more than two groups of managers. Second, we abstract from issues related to the adjustment for benchmark risk factors.

The group of funds with no skill can still generate nonzero performance because of idiosyncratic risk. Moreover, intuitively, the cross-sectional dispersion of returns across all funds should be positively correlated with the mean level of idiosyncratic risk. While we examine the cross-sectional dispersion of returns in our empirical analysis, our model focuses on idiosyncratic risk for analytical tractability. We later provide simulation-based evidence showing that the cross-sectional dispersion of returns does have a strong positive correlation with the mean level of idiosyncratic risk. We thus view cross-sectional return dispersion and the average level of idiosyncratic risk as interchangeable in our framework.

To model the cross-sectional distribution of idiosyncratic risk conditional on $\alpha_0$, we assume that the prior distribution (from the investors’ perspective) of idiosyncratic variance is given by an inverse-gamma distribution, $\text{IG}(\lambda, \theta_0)$, where $\lambda$ is the shape parameter and $\theta_0$ is the scale parameter.6 The inverse-gamma distribution is a standard

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5 See Brown and Wu (2016) for evidence on investors’ learning across funds within fund families.

6 The probability density function for the inverse-gamma distribution $\text{IG}(\lambda, \theta_0)$ is $p(x|\lambda, \theta_0) = \frac{\lambda^\lambda}{\Gamma(\lambda)} x^{-\lambda-1} \exp(-\frac{\lambda}{x})$ for $x > 0$. While both $\lambda$ and
conjugate prior for the variance parameter in Bayesian analysis. We adopt it to obtain an analytically tractable posterior distribution. The main message of our model, that an increase in return dispersion is associated with a reduction in flow-performance sensitivity, applies when we use other distributions.\footnote{When a nonconjugate prior is used, the posterior distribution, which is no longer analytically tractable, can be obtained through Gibbs sampling (see, e.g., Jones and Shanken, 2005).}

We assume that the prior distribution of idiosyncratic risk conditional on $\alpha_h$ is given by another inverse-gamma distribution $\text{IG}(\lambda, \theta_h)$. For simplicity, we assume this inverse-gamma distribution shares the same shape parameter as the prior distribution under $\alpha_0 = 0$. This assumption allows us to focus on the scale parameters (i.e., $\theta_0$ and $\theta_h$) that affect the means of the inverse-gamma distributions.\footnote{Unlike a normal distribution, for which the mean and the standard deviation are parameterized separately and are independent of each other, the scale parameter $\theta$ for an inverse-gamma distribution affects both its mean and its standard deviation. Although we could use a different parameterization of the inverse-gamma distribution that would allow us to vary the mean of the distribution without affecting the variance, such a parameterization is inconsistent with the observation that an increase in the mean of idiosyncratic risk is often accompanied by an increase in the standard deviation. Our model’s implications are the same under such an alternative parameterization, which highlights the importance of the mean level of idiosyncratic risk impacting the performance evaluation. We therefore focus on the standard parameterization of the inverse-gamma distribution and examine the impact of $\theta$ on the mean of the distribution.}

Throughout our analysis, we fix $\lambda$ and explore how the variations in $\theta_0$ and $\theta_h$ affect performance evaluation.

We choose to model $\theta_0$ and $\theta_h$ separately for several reasons. First, zero-alpha funds and positive-alpha funds, on average, can choose different levels of risk (Huang et al., 2011), and modeling $\theta_0$ and $\theta_h$ separately allows for this possibility. Second, although we are using a proxy (i.e., cross-sectional alpha dispersion) for $\theta_0$ in our empirical analysis (and we do not have a proxy for $\theta_h$), we believe studying the related theoretical implications for $\theta_h$ can provide valuable information. We later show different implications of $\theta_0$ and $\theta_h$ on flow-performance sensitivity. Third, the main prediction of our model (i.e., how return dispersion affects flow-performance sensitivity) is still valid if we assume a common factor drives both $\theta_0$ and $\theta_h$, as we discuss when we calibrate our model.

Finally, we assume that a fund’s returns follow an independent and identically distributed normal distribution with mean $\alpha$ and standard deviation $\sigma$. As such, the conditional likelihood function of a fund’s returns (conditional on $\alpha$ and $\sigma$) over the past is given by

$$p(R | \alpha, \sigma) = (2\pi \sigma^2)^{-T/2} \exp\left(-\frac{\sum_{t=1}^{T} (R_t - \alpha)^2}{2\sigma^2}\right),$$  \hspace{1cm} (1)

where $R = (R_1, R_2, \ldots, R_T)$ is the vector of fund returns (we suppress the fund subscript because we are considering a generic fund).

In contrast to the research that uses the Bayesian framework to study performance evaluation, our model highlights the impact of idiosyncratic risk on the inference of alpha. A strand of literature on unconditional performance evaluation also uses large samples to estimate the idiosyncratic risk of a fund. Assuming a constant idiosyncratic risk over the lifetime of a fund, this unconditional performance evaluation approach allows researchers to estimate idiosyncratic risk (i.e., second moment) with a much higher precision than they are able to estimate alpha (i.e., first moment). Our model, differing from that approach, is best understood within a conditional performance evaluation framework, where $T$ is a relatively short period of time (e.g., one year).\footnote{For more information on conditional performance evaluation, see, e.g., Ferson and Schadt (1996), Ferson and Warther (1996), and Christopherson et al. (1998).}

Adopting such a framework makes our model compatible with the literature on mutual fund flows for which the flow-performance sensitivity is shown to be much higher over the short run than over the long run.

Given our focus on the relatively short-run performance, inferring a fund’s idiosyncratic risk is challenging. Mutual funds can substantially change their risk levels over time (Huang et al., 2011; 2012), making it difficult for investors to reliably infer a fund’s idiosyncratic risk in a timely fashion.\footnote{See Ang (2014) (Chapter 16) for an illustration of time-varying fund risk taking.}

More fundamentally, the fact that a stock’s idiosyncratic risk is time-varying is well documented (see, e.g., Fu, 2009 and, in addition, has a common factor (Herskovic et al., 2016). Lastly, fund investors with limited resources and limited attention could simply rely on the most recent quarterly or annual shareholder reports to learn about a fund’s performance.\footnote{Although mutual fund holdings data are available (at least for the more recent sample), investors could still care more about actual performance than holdings-implied performance given the gap between them that is potentially caused by unobserved actions of mutual funds (Kacperczyk et al., 2006).}

The limited return information offered by these reports creates additional challenge in inferring a fund’s risk. To address the challenge of inferring a fund’s idiosyncratic risk, our model assumes that Bayesian investors use information in the cross section to aid in this endeavor. They use cross-sectional return dispersion, which our model links to the average idiosyncratic risk of zero-alpha funds, to elicit a prior on idiosyncratic risk.

We can summarize the assumptions in our model as follows. An investor has a dichotomous prior on a fund’s alpha (i.e., $\alpha_0 = 0$ versus $\alpha_0 > 0$), with the corresponding prior probability masses of $\Pi_0$ and $\Pi_1$. Conditional on the level of alpha, the prior distribution of $\sigma^2$ is either $\text{IG}(\lambda, \theta)_{\alpha}$, if $\alpha_0$, or $\text{IG}(\lambda, \theta_h)$, if $\alpha_h$. In addition, an investor observes a fund’s returns over the past $T$ periods and infers a fund’s alpha by combining information from both the fund’s recent financial data and the investor’s prior on the fund’s idiosyncratic risk.\footnote{Papers in the Bayesian performance evaluation literature entertain different priors that reflect alternative sources of information that investors could use to better estimate alphas. For example, Jones and Shanken (2005) assume fund alphas are drawn from a time-invariant normal distribution while being agnostic about fund idiosyncratic risk (i.e., a non-informative prior). As another example, Pastor and Stambaugh (2002) present risk as an unconditional normal distribution with mean zero and standard deviation. As a result, we model the alpha as a function of the idiosyncratic risk and the investor’s prior on the fund’s idiosyncratic risk.}
Based on the preceding assumptions, we first derive the (posterior) distribution for a fund’s alpha, established by Proposition 1.

**Proposition 1.** The posterior probability distribution of a fund’s alpha is given by

\[
p(\alpha|R) = \begin{cases} 
\frac{1}{1 + \left( \frac{\theta_0}{\theta_h} \right)^2 \left( \frac{\text{IVOL}^2 + (\bar{R} - \alpha_h)^2 + 2\theta_h}{\text{IVOL}^2 + \bar{R}^2 + 2\theta_h} \right)^{\frac{\alpha}{\theta_h}} \prod_{j=1}^{T} \frac{\theta_h}{\Pi_j}, & \text{if } \alpha = \alpha_h, \\
1 - p(\alpha_h|R), & \text{if } \alpha = \alpha_0,
\end{cases}
\]

where \( \bar{R} = \frac{\sum_{t=1}^{T} R_t}{T} \) is the fund’s mean return and \( \text{IVOL} = \sqrt{\sum_{t=1}^{T} (R_t - \bar{R})^2} \) is the fund’s standard deviation. The posterior mean \( \{\text{i.e., } E(\alpha|R)\} \) is therefore \( \alpha_h \times p(\alpha_h|R) \).

**Proof.** See Appendix Section A.1. □

Because the basic presentation of our model has no risk adjustment, \( \text{IVOL} \) is the same as the standard deviation of returns.

To gain more insight into Proposition 1, we consider a special case in which \( \theta_0 = \theta_h \), that is, we assume the same prior dispersion of returns across the two groups of funds. We can show in a straightforward manner that \( p(\alpha_h|R) \) is decreasing in \( \text{IVOL} \) if \( \bar{R} > \alpha_h/2 \) and increasing in \( \text{IVOL} \) if \( \bar{R} < \alpha_h/2 \). This result can be interpreted in a hypothesis testing framework, such that, when \( \bar{R} > \alpha_h/2 \), an investor likely classifies the fund as a good fund. Simultaneously, the investor is worried about a Type I error or falsely classifying a zero-alpha fund as a good fund. An increase in the fund’s \( \text{IVOL} \) would exacerbate this concern and lead the investor to reduce \( p(\alpha_h|R) \). When \( \bar{R} < \alpha_h/2 \), an investor is likely to classify the fund as a zero-alpha fund but is also concerned about a Type II error. An increase in the fund’s \( \text{IVOL} \) would make an investor less confident about classifying the fund as a zero-alpha fund, resulting in an increase in \( p(\alpha_h|R) \).

In the context of our paper, we are more interested in the comparative statics related to \( \theta_0 \) and \( \theta_h \). Proposition 2 establishes these.

**Proposition 2.** For each level of \( \text{IVOL} \), a lower bound on \( \bar{R} \) (denoted as \( \bar{R}^* \)) exists such that, when \( \bar{R} > \bar{R}^* \), the following relations result:

\[
\frac{\partial E(\alpha|R)}{\partial \theta_0} < 0, \quad \frac{\partial E(\alpha|R)}{\partial \theta_h} > 0.
\]

**Proof.** See Appendix Section A.2. □

Because \( E(\alpha|R) \) is simply \( \alpha_h \times p(\alpha_h|R) \) in our model, we plot \( p(\alpha_h|R) \) against \( \theta_0 \) and \( \theta_h \) in Fig. 1 to illustrate Proposition 2. We focus on an outperforming fund and fix its mean \( \bar{R} = 10\% \) and standard deviation \( \text{IVOL} = 15\% \). We also fix \( \alpha_h \) at 5\% and \( \Pi_0 \) at 80\%. We examine how changes in \( \theta_0 \) and \( \theta_h \) influence the probability of classifying the fund as a good fund, \( p(\alpha_h|R) \).

For Panel A of Fig. 1, in which we fix \( \theta_h \) at 0.12, when \( \theta_0 \) increases, \( p(\alpha_h|R) \) changes from around 0.8 to less than 0.1. When \( \sqrt{\theta_0} \) is very low (i.e., around 5\%), the expected value of idiosyncratic risk for zero-alpha funds is low, making it difficult for a zero-alpha fund to achieve a high average return (i.e., \( \bar{R} = 10\% \)). As a result, Bayesian investors attribute the good performance to skill and assign a high probability to the manager as being skilled. When \( \sqrt{\theta_0} \) is very high (i.e., around 20\%), the expected value of idiosyncratic risk for zero-alpha funds is high, making it easier for a zero-alpha fund to achieve the average return of 10% by drawing a higher risk from the inverse-gamma distribution. Consequently, investors assign a lower probability to the manager as being skilled.

When \( \sqrt{\theta_0} \) is large (i.e., around 20\%), \( p(\alpha_h|R) \) can be as low as 0.1, even if \( \bar{R} \) is closer to \( \alpha_h = 5\% \) than to zero. The intuition for this result is as follows. Bayesian investors are comparing the probabilities of two scenarios. One scenario is the case of a zero-alpha fund drawing a very large idiosyncratic variance, with a mean of 0.2², from the inverse-gamma distribution and achieving 10\% (\( = 10\% - 0\% \)) outperformance. The second scenario is the case of a good fund (i.e., \( \alpha_h = 5\% \)) drawing a still somewhat large idiosyncratic variance from the inverse-gamma distribution, with a mean of 0.1², and achieving 5\% (\( = 10\% - 5\% \)) outperformance. We find that the first scenario is more likely to occur than the second scenario, which, combined with the fact that the majority of funds are believed to be zero-alpha funds (i.e., \( \Pi_0 = 0.8 \)), leads investors to assign a small probability to the manager as being skilled. Our results, therefore, highlight the implication that in a high dispersion environment, a seemingly outperforming fund, whose performance can exceed the benchmark performance of managers that are regarded as skilled, can still be classified as a zero-alpha fund instead of as a skilled fund.

Similar arguments can be used to explain Panel B of Fig. 1, in which we fix \( \theta_0 \) at 0.12. When \( \theta_0 \) is fixed, a higher \( \theta_h \) makes it more likely for a good fund to achieve 5\% (\( = 10\% - 5\% \)) outperformance, leading Bayesian investors to assign a higher probability to the manager as being skilled. Although interesting from a theoretical perspective, we cannot test the implications of Panel B empirically because we do not have a proxy for \( \theta_h \). We focus on the implications of Panel A in our empirical work.

In addition to interpreting Proposition 2 from an analytical (i.e., Bayesian) perspective, we provide a simpler explanation using the intuition from the hypothesis testing framework.

For the population of funds with a zero alpha, a higher \( \theta_0 \) is associated with a higher dispersion of fund returns in the cross-section, which creates difficulty for investors to tell the good from the bad. All else being equal, a higher \( \theta_0 \) makes it easier for a typical zero-alpha fund to be disguised as a fund with skill, resulting in a higher Type I error (i.e., identifying a zero-alpha fund as good) for
investors if they do not take into account the change in \( \theta_0 \). Assuming that investors have information about \( \theta_0 \) and incorporate this information into their priors, they discount fund alphas more harshly [i.e., a lower \( E(\alpha|\mathbf{R}) \)] when \( \theta_0 \) becomes higher to reduce Type I error. This is the main hypothesis we test in our empirical work.

A similar intuition applies to \( \theta_h \). For the population of funds with a positive alpha (i.e., \( \alpha_h \)), a higher \( \theta_h \) results in a higher dispersion of positive-alpha fund returns, which means that a larger fraction of these funds will have bad luck and generate a return closer to \( \alpha_0 = 0 \) than to \( \alpha_h \). As a result, all else being equal, a higher \( \theta_h \) results in a higher Type II error (i.e., identifying a good fund as having zero alpha) for investors if they do not take into account the change in \( \theta_h \). If investors have information about the change in \( \theta_h \), they discount fund alphas less harshly [i.e., a higher \( E(\alpha|\mathbf{R}) \)] corresponding to an increase in \( \theta_h \) to reduce Type II error.

Lastly, we establish the comparative statics for \( IVOL \).

**Proposition 3.** Fixing \( \theta_0 \) and \( \theta_h \), a lower bound on \( \tilde{\alpha} \) (denoted as \( \tilde{\alpha}^* \)) exists such that, when \( \tilde{\alpha} > \tilde{\alpha}^* \).

\[
\frac{\partial E(\alpha|\mathbf{R})}{\partial IVOL} < 0.
\]

**Proof.** See Appendix Section A.3. □

The intuition for this result was previewed in our discussion of Proposition 1. For funds with a sufficiently large \( \hat{\alpha} \), although perhaps inclined to classify them as \( \alpha = \alpha_h \), investors concerned about a Type I error lower \( E(\alpha|\mathbf{R}) \) if \( IVOL \) increases.

**Proposition 3** is different from the usual result that the appraisal ratios and t-statistics, commonly used performance metrics that take risk into account, are negatively correlated with \( IVOL \) (Sirri and Tufano, 1998; Huang et al., 2012). Our result in Proposition 3 makes a statement about the perceived alpha level, as opposed to the estimated alpha scaled by \( IVOL \), as in appraisal ratios. In our empirical analysis, in which we study flow-performance sensitivity, we are not able to distinguish between Proposition 3 and the risk channel (i.e., fewer flows go to funds with a higher level of \( IVOL \), holding alpha constant). We therefore list Proposition 3 for completeness and point out that the impact of \( IVOL \) on alpha level, as shown in Proposition 3, can reinforce the risk channel (i.e., investors discount funds with a lower appraisal ratio, holding the estimated alpha constant), which we may not be able to distinguish empirically.

Overall, our model links return dispersion to idiosyncratic risk. If idiosyncratic risk for zero-alpha funds is high, then return dispersion is high (because the majority of funds have a zero alpha). This is represented by a high \( \theta_0 \).

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14 The appraisal ratio divides alpha estimate by idiosyncratic risk, whereas the t-statistic divides alpha estimate by its standard error, which takes into account the covariance structure among the regressors.
which is the average idiosyncratic variance among zero-alpha funds.

Our model focuses on one representation that links return dispersion to idiosyncratic risk. In a more general framework, the amount of idiosyncratic risk is endogenously chosen by funds, with skilled and unskilled managers potentially having different incentives for selecting a level of risk due to, for example, the convex flow-performance relation and manager career concerns (see Brown et al., 1996; Chevalier and Ellison, 1997; Huang et al., 2011, and the references therein). Our model abstracts from this endogenous choice problem by focusing on how return dispersion (or, equivalently, the average idiosyncratic risk of unskilled managers) affects capital allocation from an investor’s perspective.

2.2. Model calibration

We perform a calibration exercise on our model by setting the scale parameters of \( \theta_0 \) and \( \theta_h \) at different levels and examining how changes in the levels influence the relation between \( E(\alpha|R) \) and \( \bar{R} \), that is, the sensitivity of the perceived performance (i.e., \( E(\alpha|R) \)) to the estimated performance (i.e., \( \bar{R} \)). Assuming that investors allocate funds based on perceived performance, this relation is directly related to the flow-performance sensitivity analysis we conduct in our empirical analysis.

The baseline values for \( \theta_0 \) (mean level of idiosyncratic variance for zero-alpha funds) and \( \theta_h \) (mean level of idiosyncratic variance for positive-alpha funds) are both set at 0.1\(^2\), indicating an annualized idiosyncratic risk of 10% for the average fund in either group of funds, which is consistent with the historical data.\(^\text{15}\) Fig. 2 shows how changes in \( \theta_0 \) and \( \theta_h \) (changed \( \theta_0 \) and \( \theta_h \) are denoted by \( \tilde{\theta}_0 \) and \( \tilde{\theta}_h \)) affect the sensitivity of \( E(\alpha|R) \) (i.e., perceived performance) to \( \bar{R} \) (i.e., estimated performance).

Panels A and B of Fig. 2 show that the perceived performance becomes much less sensitive to the estimated performance when \( \theta_0 \) is raised from 0.1\(^2\) to 0.2\(^2\), whether \( \Pi_0 \) is set at 0.8 (Panel A) or 0.9 (Panel B). In our empirical analysis, we use cross-sectional return dispersion (the IQR for the cross section of alphas) to proxy for \( \sqrt{\theta_0} \). The peak and trough levels for the time series of our measure are 38% and 4%, respectively, suggesting that a 100% (=0.2/0.1) change in \( \sqrt{\theta_0} \) is not unusual.\(^\text{16}\) Given this increment in \( \theta_0 \), the change in sensitivity of the perceived performance to the estimated performance is substantial. For example, when \( \Pi_0 = 0.8 \), using the slope of a straight line that connects the two endpoints of the perceived performance curve to approximate sensitivity, the sensitivity of the perceived performance to the estimated performance changes from 0.007/0.15 (at \( \theta_0 = 0.1^2 \)) to 0.004/0.15 (at \( \tilde{\theta}_0 = 0.2^2 \)), implying a 43% (=0.003/0.007) reduction in slope.\(^\text{17}\)

A closer inspection of Panels A and B of Fig. 2 reveals that while both the reduction of large alpha estimates (e.g., observed performance around 0.15) and the markup of small alpha estimates (e.g., observed performance around zero) contribute to the change in slope, the former represents a more important contribution, highlighting Proposition 2, in particular, \( \frac{\partial E(\alpha|R)}{\partial \theta_0} < 0 \). This depends on the range of the alpha estimates we include in the graphs. Given that a 15% annual alpha estimate is not an uncommon observation in the cross section of funds, we use 15% as the limit for the alpha estimate in our graphs.\(^\text{18}\) The inclusion of even larger alpha estimates would make our result (i.e., the reduction in slope) stronger.

Panel C shows that when both \( \theta_0 \) and \( \theta_h \) are increased by the same factor, the sensitivity of the perceived performance to the observed performance still moves lower (slope flattens), and Panel D shows that when only \( \theta_0 \) is increased, this sensitivity generally increases. The pattern shown in Panel D is consistent with Proposition 2, in particular, \( \frac{\partial E(\alpha|R)}{\partial \theta_0} > 0 \).

Panel C shows that when \( \theta_0 = \theta_h \), an increase in \( \theta_0 \) (and, hence, an equal increase in \( \theta_h \)) lowers the sensitivity of the perceived performance to the estimated performance. This result holds true in general in our model (i.e., \( \frac{\partial E(\alpha|R)}{\partial \theta_0} < 0 \) when \( \theta_0 = \theta_h \)), although we do not formally establish this as a proposition in Section 2.1.

While our calibration exercise in Fig. 2 assumes a small fraction of outperforming funds, which is consistent with recent papers such as Barras et al. (2010), Persion and Chen (2017), and Harvey and Liu (2018), we also explore alternative parameterizations of our model in Appendix B. We show in Fig. B.1 that our calibration results are robust when a larger fraction of fund managers are skilled but less so (i.e., \( \alpha_h = 2.5\% \)) than what we assume in Fig. 2 (i.e., \( \alpha_h = 5\% \)).

Overall, our calibration results show the economic importance of \( \theta_0 \) and \( \theta_h \) in affecting the sensitivity of the perceived performance to the observed performance. The results also highlight the differential impact of \( \theta_0 \) and \( \theta_h \) on this sensitivity. While an increase in \( \theta_0 \) lowers the sensitivity, an increase in \( \theta_h \) implies the opposite pattern. In our empirical analysis, although we do not have an empirical measure of \( \theta_h \), we use a measure of cross-sectional return dispersion to proxy for \( \theta_0 \). Given the differential impact of \( \theta_0 \) and \( \theta_h \) on flow-performance sensitivity, it is crucial to ensure that our measure of \( \theta_0 \) is not contaminated by \( \theta_h \). We later provide simulation evidence that shows our measure of cross-sectional return dispersion has near-perfect correlation with \( \theta_0 \) and a low correlation with \( \theta_h \) from a time series perspective.

\(^{15}\) The average level of return dispersion across all funds ranges from 7.4% to 11.0%, depending on the benchmark factor model used. We thus set \( \sqrt{\theta_0} \) and \( \sqrt{\theta_h} \) at 10%.

\(^{16}\) These levels are based on the capital asset pricing model (CAPM) adjusted alpha.

\(^{17}\) Let the perceived performance at observed performance = 0 and observed performance = 0.15 be \( p_0 \) and \( p_1 \), respectively. Then, the slope of the straight line that connects the two endpoints of the perceived performance curve is given by \( \frac{p_1 - p_0}{\sqrt{\theta_0}} \).

\(^{18}\) About 4.5% of fund-month observations in our sample have an annual CAPM-adjusted alpha estimate of no less than 15%. 
2.3. Measuring return dispersion

Our model shows that investors’ predictions about alpha are influenced by cross-sectional dispersion of returns and are different across \(\theta_0\) and \(\theta_h\). A larger \(\theta_0\) implies a more aggressive discount of funds with large alphas (and hence a lower flow-performance sensitivity), and a larger \(\theta_h\) implies the opposite. Crucially, therefore, our measure of \(\theta_0\) must not be contaminated by \(\theta_h\). Achieving this requirement is a challenge. Ideally, we group funds into performance categories and calculate the within-group IVOL, but how do we make the initial classification?

We propose a simple proxy for \(\sqrt{\theta_0}\) based on the cross section of alphas. We use the IQR of the cross section of alphas. Later, we define alpha as either the mean excess return over a simple benchmark (either the risk-free rate or the market) or the factor model adjusted alpha over the past year.\(^{10}\) Our focus on annual alpha is consistent with the literature on flow-performance sensitivity. Compared with higher-frequency alphas, a one-year focus also allows a sufficient amount of time for investors to absorb information in the cross section. To summarize, at the end of each month \(t\), we obtain the IQR of the cross section of alphas estimated over the past year. We use this monthly measure of return dispersion to predict future flow-performance sensitivity.

\(^{10}\) See Section 3.1 for details.
We believe that the return dispersion of mutual funds is the type of information investors consider in making fund-selection decisions. Further, given previous evidence that the vast majority of funds are unskilled, the dispersion measured by the IQR is largely representing the unskilled funds.

In Appendix C, we provide simulation-based evidence to further justify our use of the IQR. We simulate the panel of returns for a cross section of funds that are drawn from two (or three) subpopulations, with the idiosyncratic risk for funds within each subpopulation following a separate inverse-gamma distribution. Moreover, the means of these inverse-gamma distributions are time-varying. When the majority of funds are drawn from the zero-alpha population, we show that the IQR for the entire cross section of funds closely tracks the time-varying mean level of idiosyncratic risk (i.e., $\sqrt{\theta_0}$) for funds with a zero alpha. The contemporaneous correlation is close to one under various parameterizations of our model. In contrast, our measure of return dispersion has a very low time series correlation with $\sqrt{\theta_0}$.\footnote{The cross-sectional standard deviation of alphas (e.g., Kim, 2017) has a much lower correlation with the mean level of idiosyncratic risk for zero-alpha funds.}

The intuition for the performance of the IQR in tracking the average level of idiosyncratic risk for zero-alpha funds is straightforward. As Alizadeh et al. (2002) demonstrate, range-based statistics provide a good approximation to time-varying volatilities. In our context, extreme returns are more likely to be generated by funds with nonzero alphas, so range-based statistics that depend on extreme returns (e.g., maximum minus minimum) are not very useful in approximating the average level of idiosyncratic risk for zero-alpha funds.\footnote{We use two methods to simulate the panel of returns in Appendix C. The first approach (Panel A and Panel B) assumes a constant $\alpha_i$ (alpha for skilled managers) but allows $\theta_0$ (the mean idiosyncratic variance among zero-alpha funds) and $\theta_1$ (the mean idiosyncratic variance among skilled funds) to vary across time. The second approach allows $\alpha_i$ to vary across time by assuming that $\alpha_i$ is perfectly correlated with the time-varying $\theta_1$, capturing the idea that skilled managers could create more value when they take more risk.} Range-based statistics that rule out extreme returns (e.g., IQR) are more likely to be representative, because the majority of funds are assumed to have a zero alpha.

### 2.4. Model implications

We cast our model within a Bayesian framework in which Bayesian investors, endowed with priors (governed by $\theta_0$ and $\theta_1$), on the levels of idiosyncratic risk taken by zero-alpha and positive-alpha funds, estimate fund alphas through their posterior distributions. How do we test our model predictions with the data?

Because it is well documented that mutual fund investors have a certain degree of recency bias (i.e., their investment decisions are sensitive to recent fund alphas), we construct a time-varying measure of return dispersion, $\theta_{0t}$.\footnote{In Appendix C, range-based statistics that depend on extreme returns have a low correlation with the average level of idiosyncratic risk for zero-alpha funds.}

Based on recent fund alphas. Our model implies that investors should take $\theta_{0t}$ into account when adjusting fund alphas, resulting in a certain degree of flow-performance sensitivity (termed $s_{t+1}$) that can be estimated using future data (i.e., data at time $t+1$). Our empirical analysis therefore studies whether $\theta_{0f}$ predicts $s_{t+1}$ (how the interaction between $\theta_{0f}$ and past alpha predicts $s_{t+1}$).

We are taking an empirical Bayes perspective to test our model’s implications in that we are assuming investors directly learn about key parameters of their priors from the data. This is not a crucial assumption of our model, and we can cast this learning within a frequentist framework in which investors simultaneously estimate individual fund alphas and the distributions of idiosyncratic risk.\footnote{For more details on the application of empirical Bayes methods to performance evaluation, see Harvey and Liu (2018).} But, given the close connection between return dispersion and the distributions of idiosyncratic risk, the main prediction of such a frequentist framework would be the same as that of our model. We, therefore, rely on our Bayesian framework to provide insights, while pointing out that our model’s implications hold more generally.

### 3. Flow-performance sensitivity conditional on return dispersion

We provide summary statistics for the data and the variables we use in our empirical analysis in Section 3.1. We present our main results in Section 3.2. Additional results and regression specifications are discussed in Section 3.3.

#### 3.1. Data and variables

Our data are drawn from the Center for Research in Security Prices (CRSP) Survivor-Bias-Free Mutual Fund Database from January 1980 to December 2016. Following the previous literature, we focus on domestic equity mutual funds and exclude sector funds using the CRSP objective code. We also exclude index funds and funds that, on average, have less than 80% of their holdings in stocks. We further apply several filters to mitigate omission bias (Elton et al., 2001) and incubation bias (Evans, 2010). We exclude fund return observations reported prior to the year of fund organization. We also exclude fund observations before a fund passes the $5 million threshold for assets under management (AUM). All subsequent observations, including those that fall under the $5 million AUM threshold in the future, are included.

We obtain fund expense ratio, turnover rate, fund age, and total net assets (TNA) directly from the CRSP database. Combined with fund returns, we calculate annual percentage flow between month $t+1$ and month $t+12$ as

$$Flow_{t+1 \rightarrow t+12} = \frac{TNA_{t+12} - TNA_{t} (1 + R_{t+1 \rightarrow t+12})}{TNA_{t}}.$$  \hspace{1cm} (2)

where $TNA_{ti}$ is fund i’s TNA at the end of month $t$ and $R_{t+1 \rightarrow t+12}$ is fund i’s annual return (i.e., between month $t+1$ and month $t+12$). We focus on annual fund flows to avoid seasonal (in particular, quarterly) fluctuations in
fund flows (Kamstra et al., 2017). While this introduces autocorrelation in regression residuals in our panel regressions, for which we have overlapping dependent variables (i.e., annual fund flows) at the monthly frequency, we focus on standard errors clustered by both fund and time (i.e., double-clustered standard errors).

Consistent with the standard practice detailed in the literature, we focus our empirical analysis on alphas estimated over the past year, although many alternatives are promulgated for how to best estimate alpha. Although simple benchmarks seem to be most consistent with investor flows (Berk and van Binsbergen, 2016) and produce a less biased assessment of fund performance (Cremers et al., 2012; Pástor et al., 2015), papers in the performance evaluation literature routinely use the Fama–French three-factor or the Fama–French–Carhart four-factor model.

We use a spectrum of alpha estimates to ensure that our results are not driven by the benchmark model we use. For our main results, we focus on the simple market-adjusted alpha (i.e., fund return minus market return), the CAPM-adjusted alpha, and the Fama–French–Carhart four-factor model. In Appendix D, we report results for the risk-free–adjusted alpha (i.e., fund excess return) as well as for alphas based on the Fama–French three-factor model. For the simple market-adjusted alpha and the risk-free–adjusted alpha, we use fund returns for the past year (i.e., between month \( t - 11 \) and \( t \)) to estimate fund alpha at time \( t \). For CAPM and multi-factor models, we first estimate risk loadings based on data for the past five years. We then adjust fund returns in the past year to the benchmark factors using the estimated risk loadings.

Let the estimated alpha for fund \( i \) by time \( t \) be \( \alpha_{i,t} \). Return dispersion at time \( t \) (\( Disp_t \)), which is our main variable of interest, is defined as the IQR for the cross section of \( \alpha_{i,t} \) at time \( t \).

We include several other controls in our regression analysis. We define the ordinary least squares (OLS) standard error for fund \( i \)’s alpha estimate as \( \text{Std}(\alpha_{i,t-11\ldots t})_t \). When the simple market-adjusted alpha model or the risk-free–adjusted alpha model is used, \( \text{Std}(\alpha_{i,t-11\ldots t})_t \) is defined as the standard deviation of the adjusted fund returns over the past year. When CAPM or multi-factor models are used, \( \text{Std}(\alpha_{i,t-11\ldots t})_t \) is the standard error of the alpha estimate over the past five years. In our regression analysis, we interact \( \text{Std}(\alpha_{i,t-11\ldots t})_t \) with fund alpha to control for the uncertainty in estimating fund \( i \)’s alpha based on its time series information alone. Sirri and Tufano (1998) and Huang et al. (2012) explore the implications of this fund-specific uncertainty on flow-performance sensitivity.

The other controls we include are consistent with the existing literature, namely, Franzoni and Schmalz (2017) and Starks and Sun (2016). Fund volatility (\( \text{Vol}_t \)) is the standard deviation of fund excess returns over the past year, and style flow (\( \text{StyleFlow}_t \)) is the average flow of the investment objective class. We also control for expense ratio (\( \text{ExpRatio} \)), turnover (\( \text{Turnover} \)), log TNA (\( \text{LogTNA} \)), and log fund age (\( \text{LogAge} \)). All variables involving fund returns [i.e., \( \alpha_{i,t} \), \( \text{Std}(\alpha_{i,t-11\ldots t})_t \), and \( \text{Vol}_t \)] are annualized. We also follow the literature and winsorize the fund flows and the control variables (i.e., \( \text{ExpRatio} \), \( \text{Turnover} \), \( \text{LogTNA} \), \( \text{LogAge} \), \( \text{Vol} \), and \( \text{StyleFlow} \)) at the 1st and 99th percentiles.

To control for the variables proposed by Franzoni and Schmalz (2017) and by Starks and Sun (2016), we follow them in constructing the market state (\( \text{State} \); Franzoni and Schmalz, 2017) and policy uncertainty (\( \text{PolicyUncer} \); Starks and Sun, 2016). We define a quarterly indicator variable that equals one if the quarterly market excess return is considered mediocre (i.e., the absolute value is no larger than 5%) and zero otherwise. We then take the average of the indicator variable over the past year to obtain the annual market state. Following (Starks and Sun, 2016), we obtain the Economic Policy Uncertainty Index (EPU) from Baker et al. (2016).

Table 1 reports the summary statistics for the variables we include in our regression analysis. The statistics are consistent with those presented in the recent literature.

### 3.2. Results

We begin our empirical analysis by studying the time series properties of return dispersion in Section 3.2.1. We present our main regression results in Section 3.2.2. We take a deeper look at our results in Section 3.2.3–3.2.5.

#### 3.2.1. Time series of return dispersion

The time series of our return dispersion measures are presented in Fig. 3 along with the widely followed Volatility Index (VIX), and the National Bureau of Economic Research (NBER) recession periods. We plot two series of return dispersion. One is based on CAPM-adjusted alpha, and the other corresponds to the Fama–French–Carhart four-factor–adjusted alpha.

Fig. 3 shows that VIX is different than our return dispersion measures, despite its positive correlation with them. The correlation coefficients between the VIX and the CAPM-adjusted return dispersion and the four-factor model–adjusted return dispersion are 0.35 and 0.32, respectively. Although the market was highly volatile in 1987 and in 2008 as measured by the VIX, our measures of return dispersion are relatively low during these same periods compared with the rest of the sample.

Two interesting patterns emerge in the time series of return dispersion. First, the level for the four-factor model–adjusted return dispersion is lower than that for the CAPM-adjusted dispersion, which can be explained by the fact that differences in fund loadings on the three benchmark factors (other than the market factor) in the Fama–French–Carhart four-factor model contribute to the cross-sectional return dispersion under CAPM. Second, the return dispersion during the 1999–2001 tech bubble is much higher than in other periods for the CAPM-adjusted dispersion.

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24 Following previous research, we require that a fund has at least 36 monthly observations.

25 Both the Fama–French factors and the momentum factor are obtained from the Kenneth French online data library.

26 Because we plot the lagged one-year average VIX at month \( t \), the peak in market volatility that occurred in 1987 and 2008 appears to show up in 1988 and 2009 in Fig. 3.
Table 1

Summary statistics.

Summary statistics are represented for the variables used in our main regression analysis. Panel A reports the summary statistics for alphas. ‘Excess Return’ is fund return minus the risk-free rate, and ‘Market-Adjusted Return’ is fund return minus the market return, both defined over the previous year. ‘CAPM Alpha’, ‘Three Factor Alpha’, and ‘Four Factor Alpha’ are obtained by first estimating risk loadings (on market return, Fama–French three factors, and Fama–French–Carhart four factors, respectively) over the past five years and then adjusting fund returns in the previous year to benchmark factors using the estimated risk loadings. Panel B reports summary statistics for control variables that include flow [annual percentage flow computed as the annual change in total net assets (TNA)] minus the dollar return on assets under management over the year and divided by the TNA of the prior year-end, total net assets (TNA, in millions of dollars), fund turnover ratio, fund age (the number of months since the first appearance in CRSP), expense ratio, and fund volatility (return volatility over the prior 12 months). p(10) and p(90) represent the 10th and 90th percentiles of the variable distribution. The sample period is January 1980 to December 2016. All variables reported are at the monthly frequency. The total number of monthly observations is 1,555,734.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Median</th>
<th>p(10)</th>
<th>p(90)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Alphas (percent annualized)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess Return</td>
<td>4.83</td>
<td>19.76</td>
<td>7.45</td>
<td>−20.27</td>
<td>25.34</td>
</tr>
<tr>
<td>Market-Adjusted Return</td>
<td>−1.51</td>
<td>10.16</td>
<td>−1.75</td>
<td>−10.86</td>
<td>7.98</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>−0.98</td>
<td>9.03</td>
<td>−1.38</td>
<td>−9.49</td>
<td>7.85</td>
</tr>
<tr>
<td>Three Factor Alpha</td>
<td>−1.58</td>
<td>6.56</td>
<td>−1.54</td>
<td>−7.95</td>
<td>4.64</td>
</tr>
<tr>
<td>Four Factor Alpha</td>
<td>−1.51</td>
<td>6.88</td>
<td>−1.57</td>
<td>−8.18</td>
<td>5.08</td>
</tr>
<tr>
<td><strong>Panel B: Other variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow (per annum)</td>
<td>0.24</td>
<td>1.06</td>
<td>−0.03</td>
<td>−0.32</td>
<td>0.87</td>
</tr>
<tr>
<td>Total Net Assets (millions of dollars)</td>
<td>450.91</td>
<td>1996.88</td>
<td>63.00</td>
<td>5.60</td>
<td>862.20</td>
</tr>
<tr>
<td>Turnover</td>
<td>0.86</td>
<td>1.59</td>
<td>0.62</td>
<td>0.17</td>
<td>1.64</td>
</tr>
<tr>
<td>Fund Age (months)</td>
<td>88.81</td>
<td>76.49</td>
<td>68.00</td>
<td>13.00</td>
<td>194.00</td>
</tr>
<tr>
<td>Expense Ratio (percent)</td>
<td>1.35</td>
<td>0.80</td>
<td>1.25</td>
<td>0.69</td>
<td>2.11</td>
</tr>
<tr>
<td>Fund Volatility (percent annualized)</td>
<td>15.60</td>
<td>7.68</td>
<td>14.12</td>
<td>7.79</td>
<td>25.20</td>
</tr>
</tbody>
</table>

Fig. 3. Time series of return dispersion, January 1980–December 2016. For each month \( t \) in our data, we calculate return dispersion as the interquartile range of the cross section of fund alphas for the previous year (i.e., from month \( t - 11 \) to month \( t \)). Fund alpha is calculated as either the CAPM–adjusted alpha (solid line) or the Fama–French–Carhart four-factor–adjusted alpha (dashed-dotted line). We also plot the time series of Volatility Index (VIX), which is calculated as the average of the daily VIX over the same period (i.e., from month \( t - 11 \) to month \( t \)). Shaded areas are NBER recession dates.
alpha, but not as evident for the Fama–French–Carhart–adjusted alpha. Despite some of the differences in the time series of the two measures of return dispersion, we study both (and a few other alternative measures) in our empirical analysis and show that our results are robust to the benchmark model we use to estimate alphas.

Some evidence also exists for a time trend for return dispersion in Fig. 3. Intuitively, a gradual decline is evident in return dispersion that is accompanied by the general decline in performance for an average fund. When we test for a time trend by regressing the time series of return dispersion on a constant and a time trend, we find significantly negative coefficients on the time trend for our measures of return dispersion.27 We therefore make sure that we always include time fixed effects in our empirical analysis. In Section 3.3.3, we discuss the impact of a time trend in more detail.

3.2.2. Main results

The main prediction of our model is that flow-performance sensitivity is decreasing in return dispersion. We test this prediction by estimating the regression

\[
\text{Flow}_{i,t+1-t_{12}} = b_1 \alpha_{i,t} + b_2 \text{Disp}_{i,t} \times \alpha_{i,t} \\
+ b_3 \text{Std}(\alpha_{i,t-1-t_{12}}) \times \alpha_{i,t} + b_4 \text{Other}_t, \\
\times \alpha_{i,t} + \text{Controls}_{i,t} + \epsilon_{i,t+1-t_{12}},
\]

(3)

where \(\text{Flow}_{i,t+1-t_{12}}\) is the annual percentage fund flow in the following year, \(\text{Disp}_{i,t}\) is cross-sectional alpha dispersion measured over the past year, \(\alpha_{i,t}\) and \(\text{Std}(\alpha_{i,t-1-t_{12}})\) are the alpha estimate and its standard error, \(\text{Other}_t\) includes either market state (State, as in Franzoni and Schmalz, 2017) or policy uncertainty (PolicyUncer, as in Starks and Sun, 2016), and \(\text{Controls}_{i,t}\) include log TNA, expense ratio, turnover ratio, log fund age, volatility of fund returns, and the average flow of the investment objective class.28 All independent variables are measured using data in the past except for the average flow of the investment objective class, which is measured contemporaneously with fund flow (consistent with previous literature). In our regression analysis, we include both fund and month fixed effects. Standard errors are also clustered by both fund and month.

Table 2 reports the regression results when alpha is calculated as the difference between fund return and the market return. Table 3 reports the results for the CAPM-adjusted alpha, and Table 4 reports the results for the Fama–French–Carhart four-factor model.

In the first column of Table 2, the positive coefficient on \(\alpha\) is consistent with the previous literature that finds a positive relation between past alphas and future flows. By interacting return dispersion with past alpha, the second column shows a negative relation between return dispersion and flow-performance sensitivity, which is highly significant, both statistically (\(t\)-statistic = \(-7.26\)) and economically. To quantify its economic significance, the mean and standard deviation of return dispersion for the model in Table 2 are 11.02% and 5.97%, respectively. This implies a mean flow-performance sensitivity of \(1.87 - 2.99 \times 11.02\% = 1.54\) and a decrease of \(2.99 \times 5.97\% = 0.18\) corresponding to a one standard deviation increase in return dispersion. Hence, a one standard deviation increase in return dispersion reduces flow-performance sensitivity by \(0.18/1.54 = 12\%\). The reduction in the flow-performance sensitivity is somewhat lower in the model that includes all standard controls (i.e., 8% as in the fourth column of Table 2).

When additional controls are introduced, our results are robust. When we add the interaction term between a fund’s alpha estimate and its standard error [i.e., \(\text{Std}(\alpha)\)], \(\text{Std}(\alpha)\) negatively impacts flow-performance sensitivity. This result is consistent with our Proposition 3 and can also be explained by the fact that investors care about not only alpha, as measured by the mean, but also the standard error of alpha, as measured in appraisal ratios or \(t\)-statistics. While we are agnostic on the interpretation, our main results on return dispersion are robust to the inclusion of \(\text{Std}(\alpha)\). Our results also survive when market state or policy uncertainty is included in our regressions.

Tables 3 and 4 show similar results to Table 2 when we use alternative alpha estimates.

When we use CAPM-adjusted alphas in Table 3, the results are similar to Table 2. To calibrate the economic significance, the mean and standard deviation of return dispersion for the model in Table 3 are 9.94% and 4.51%, respectively. This implies a mean flow-performance sensitivity of \(2.22 - 4.10 \times 9.94\% = 1.81\) and a decrease of \(4.22 \times 4.51\% = 0.19\) corresponding to a one-standard-deviation increase in return dispersion. Hence, a one-standard-deviation increase in return dispersion reduces flow-performance sensitivity by \(0.19/1.81 = 11\%\), which is very close to the results reported in Table 2.

For Table 4, when more factors are added to the benchmark model, the average idiosyncratic variance across funds decreases, leading to a decrease in the level as well as the variance of return dispersion. As a result, the coefficient estimates associated with return dispersion are larger in magnitude for multi-factor models than those for the CAPM or market-adjusted alpha models. Nonetheless, a similar calibration shows that a one standard deviation increase in return dispersion reduces flow-performance sensitivity by 13%.29

In Appendix D, we report additional results that correspond to the simple excess-return alpha (fund return minus risk-free rate; Table D.1) and the Fama–French three-factor–adjusted alpha (Table D.2).

Our results are consistent. Overall, across different benchmark factor models, the economic impact of a one standard deviation increase in return dispersion

\[\text{The mean and standard deviation of return dispersion are, respectively, 7.41\% and 2.28\% for the Fama–French–Carhart four-factor–adjusted alpha. Given these summary statistics, the impact of a one standard deviation increase in return dispersion reduces flow-performance sensitivity by 13\%}\]
Table 2

Regression results are presented on flow-performance sensitivity when alpha is estimated as the mean difference between fund returns and market returns. The dependent variable is the annual percentage fund flow between month \( t + 1 \) and \( t + 12 \). Explanatory variables are estimated fund alpha \((\alpha)\), cross-sectional return dispersion \((\text{Disp})\), standard error for alpha \((\text{Std}(\alpha))\), expense ratio \((\text{ExpRatio})\), turnover ratio \((\text{Turnover})\), log TNA \((\text{TNA})\), log fund age \((\text{LogAge})\), return volatility \((\text{VOL})\), and average flow of the investment objective class \((\text{StyleFlow})\). Market state \((\text{Franzoni and Schmalz, 2017})\) and policy uncertainty \((\text{Starks and Sun, 2016})\) are defined in Section 3.1. All explanatory variables at time \( t \) are based on data before \( (\text{including}) \) month \( t \), except for the average flow of the investment objective class \((\text{StyleFlow})\), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the \( t \)-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>1.33***</td>
<td>1.87***</td>
<td>2.23***</td>
<td>2.12***</td>
<td>2.02***</td>
<td>1.89***</td>
<td>2.59***</td>
<td>2.57***</td>
</tr>
<tr>
<td>( \text{Disp} \times \alpha )</td>
<td>(-2.99***)</td>
<td>(-1.88***)</td>
<td>(-2.43***)</td>
<td>(-1.60***)</td>
<td>(-2.13***)</td>
<td>(-1.99***)</td>
<td>(-2.41***)</td>
<td></td>
</tr>
<tr>
<td>( \text{Std}(\alpha) \times \alpha )</td>
<td>(-4.33***)</td>
<td>(-3.05***)</td>
<td>(-4.30***)</td>
<td>(-3.01***)</td>
<td>(-4.34***)</td>
<td>(-3.08***)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{State} \times \alpha )</td>
<td>0.10*</td>
<td>0.11*</td>
<td>( \alpha )</td>
<td>1.87</td>
<td>2.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{PolicyUncer} \times \alpha )</td>
<td>(-0.20**)</td>
<td>(-0.29**)</td>
<td>(-0.10)</td>
<td>(-0.28)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ExpRatio} )</td>
<td>-14.29***</td>
<td>-14.27***</td>
<td>-15.69***</td>
<td>(-5.16)</td>
<td>(-5.16)</td>
<td>(-5.53)</td>
<td></td>
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<tr>
<td>( \text{Turnover} )</td>
<td>0.02*</td>
<td>0.02*</td>
<td>0.01</td>
<td>1.94</td>
<td>1.92</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{LogTNA} )</td>
<td>(-0.27***)</td>
<td>(-0.27***)</td>
<td>(-0.28***)</td>
<td>(-34.18)</td>
<td>(-34.18)</td>
<td>(-32.45)</td>
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<tr>
<td>( \text{LogAge} )</td>
<td>(-0.29***)</td>
<td>(-0.29***)</td>
<td>(-0.28***)</td>
<td>(-23.57)</td>
<td>(-23.57)</td>
<td>(-22.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{VOL} )</td>
<td>0.23*</td>
<td>0.21</td>
<td>0.19*</td>
<td>1.77</td>
<td>1.59</td>
<td>1.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{StyleFlow} )</td>
<td>0.01***</td>
<td>0.01***</td>
<td>0.01***</td>
<td>(-3.71)</td>
<td>(-3.70)</td>
<td>(-3.68)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fund fixed effects</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
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<td>1,204,770</td>
<td>1,204,767</td>
<td>1,075,586</td>
<td>1,204,767</td>
<td>1,075,586</td>
<td>1,105,169</td>
<td>993,442</td>
</tr>
<tr>
<td>Adjusted ( R^2 ) (percent)</td>
<td>21.1</td>
<td>21.2</td>
<td>21.3</td>
<td>30.0</td>
<td>21.3</td>
<td>30.0</td>
<td>22.1</td>
<td>30.8</td>
</tr>
</tbody>
</table>

on flow-performance sensitivity is a reduction of 11% to 17%.\(^{30}\)

In our Internet Appendix, we explore alternative regression specifications and variable constructions. In Table IA1, we do not include fund fixed effects and re-estimate our model. In Table IA2 and IA3, instead of measuring annual percentage fund flow using the end-of-the-year assets under management, we explore an alternative construction that aggregates monthly dollar flows to calculate the annual percentage flow. Lastly, in Table IA4 and IA5, we use the monthly percentage fund flow (instead of the annual flow as in our main analysis) as the dependent variable and rerun our analysis.\(^{31}\) We show that our main results are robust to these alternative specifications.

\(^{30}\) The means and standard deviations of return dispersion are, respectively, 11.02% and 5.97% (risk-free-adjusted alpha) and 7.78% and 2.48% (Fama–French three-factor-adjusted alpha). Given these summary statistics, the impact of a one standard deviation increase in return dispersion reduces flow-performance sensitivity by 11% (Table D.1) and 17% (Table D.2), respectively.

\(^{31}\) We use annual fund flows as the dependent variables in our main analysis to avoid seasonal fluctuations in fund flows. This raises a concern about the potential bias in inference that is caused by overlapping observations in both the independent and dependent variables. We therefore explore a specification in which we use the monthly fund flow as the dependent variable.

3.2.3. Subsample analysis

We next examine whether the impact of return dispersion varies through time. We conjecture that the impact of return dispersion increases through time for two reasons. First, mutual funds performed better, on average, in the early years of our sample, making encountering a zero-alpha fund, which is important to rational investors in our model, less of a worry for investors. Second, investors could have become more sophisticated in the later years of our sample, as they have had more time to learn, data as well as the tools to analyze the data have become more widely available, and the number of third-party analysts providing advice has grown. To test this hypothesis, we create two subsamples: 1980–1999 and 2000–2016. Table 5 summarizes the key regression results across different factor models, and Tables IB1 to IB8 in the Internet Appendix report the detailed results.\(^{32}\)

\(^{32}\) Hereafter, we choose a few model specifications to report summary tables (i.e., Tables 5–7) in the main text, while leaving results on alternative model specifications to the Internet Appendix. Correspondingly, our calibrations of the economic impact of return dispersion are based on the tables reported in the main text. Alternative calibrations can be performed by using the more detailed tables reported in the Internet appendix.
When interpreting our results, there are two caveats. First, substantially fewer observations are available for the 1980–1999 subsample than for the full sample. The early subsample represents only approximately 15% of total observations. Second, the size of the coefficient estimate on Disp × α in Table 5 does not directly reflect the economic magnitude of the importance of return dispersion because our measure of the economic impact of return dispersion, the percentage change in flow-performance sensitivity corresponding to a one standard deviation increase in return dispersion, also depends on the size of the coefficient estimate on α and the mean and the standard deviation for return dispersion over the particular sample we examine. As a result, although the size of the coefficient estimate on Disp × α, as reported in Table 5, seems smaller post-2000 (Panel B) than pre-2000 (Panel A), the economic significance of return dispersion is higher post-2000 than pre-2000.

The results in Table 5 (as well as those in the Internet Appendix) are consistent with our conjecture.\textsuperscript{33} The impact of return dispersion is less significant, both statistically and economically, for the 1980–1999 sample than for the 2000–2016 sample. For example, a one standard deviation increase in return dispersion reduces flow-performance sensitivity by 5% pre-2000 and 18% post-2000 for market-adjusted alpha and by 4% pre-2000 and 16% post-2000 for CAPM-adjusted alpha.\textsuperscript{34}

We also perform another subsample analysis by excluding the sample period between January 1998 and December 2002 because return dispersion seems particularly high for this period (especially for return dispersion based on the CAPM-adjusted alpha). We report the results in Table IB9 in the Internet Appendix. Our results are robust to the exclusion of the 1998–2002 sample period.

\textsuperscript{33} The estimates for the coefficient of interest for the Fama–French three-factor model and the Fama–French–Carhart four-factor model are substantially larger (in magnitude) than those for the other two models.

\textsuperscript{34} When alpha is calculated as the difference between fund return and market return, the mean and the standard deviation for return dispersion are 11.45% and 3.65%, respectively, pre-2000 and 10.52% and 7.84%, respectively, post-2000. For CAPM-adjusted alpha, the mean and the standard deviation for return dispersion are 10.55% and 2.92%, respectively, pre-2000 and 9.24% and 5.77%, respectively, post-2000. These numbers can be used, together with the results in Table 5, to calculate the percentage change in flow-performance sensitivity corresponding to a one standard deviation change in return dispersion.
Table 4

Regression results are presented on flow-performance sensitivity when alpha is estimated with the Fama–French–Carhart four-factor model. The dependent variable is the annual percentage fund flow between month \( t + 1 \) and \( t + 12 \). Explanatory variables are estimated fund alpha (\( \alpha \)), cross-sectional return dispersion (Disp), standard error for alpha (\( \text{Std}(\alpha) \)), expense ratio (ExpRatio), turnover ratio (Turnover), log TNA (LogTNA), log fund age (LogAge), return volatility (Vol), and average flow of the investment objective class (StyleFlow). Market state (Frazoni and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time \( t \) are based on data before (including) month \( t \), except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the t-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>2.32***</td>
<td>3.23***</td>
<td>3.85***</td>
<td>3.92***</td>
<td>4.29***</td>
<td>4.09***</td>
<td>4.35***</td>
<td>4.06***</td>
</tr>
<tr>
<td>Disp ( \times \alpha )</td>
<td>12.60***</td>
<td>5.70**</td>
<td>13.57***</td>
<td>7.03***</td>
<td>14.08***</td>
<td>7.05***</td>
<td>14.14***</td>
<td></td>
</tr>
<tr>
<td>( \text{Std}(\alpha) \times \alpha )</td>
<td>-26.59***</td>
<td>-20.41***</td>
<td>-26.38***</td>
<td>-20.32***</td>
<td>-26.04***</td>
<td>-19.81***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>State ( \times \alpha )</td>
<td>-0.20***</td>
<td>0.08</td>
<td>-0.35</td>
<td>-3.148</td>
<td>-0.27**</td>
<td>-0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PolicyUncer ( \times \alpha )</td>
<td>-13.58***</td>
<td>-13.60***</td>
<td>-15.00***</td>
<td>-4.79</td>
<td>-4.80</td>
<td>-5.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ExpRatio</td>
<td>0.02***</td>
<td>0.02***</td>
<td>0.02**</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turnover</td>
<td>(2.76)</td>
<td>(2.76)</td>
<td>(2.48)</td>
<td>0.25***</td>
<td>0.26***</td>
<td>0.07***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LogTNA</td>
<td>-30.41</td>
<td>-30.38</td>
<td>-29.19</td>
<td>-30.35</td>
<td>-0.36***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LogAge</td>
<td>-21.67</td>
<td>-21.66</td>
<td>-20.77</td>
<td>-1.92</td>
<td>-1.85</td>
<td>-1.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VOL</td>
<td>0.01***</td>
<td>0.01***</td>
<td>0.01***</td>
<td>(3.27)</td>
<td>(3.25)</td>
<td>(3.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>StyleFlow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fund fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Number of observations</td>
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<td>1,035,675</td>
<td>1,035,675</td>
<td>952,089</td>
<td>1,035,675</td>
<td>952,089</td>
<td>943,735</td>
<td>876,550</td>
</tr>
<tr>
<td>Adjusted R(^2) (percent)</td>
<td>20.7</td>
<td>20.7</td>
<td>20.8</td>
<td>30.0</td>
<td>20.9</td>
<td>30.0</td>
<td>22.0</td>
<td>31.1</td>
</tr>
</tbody>
</table>

3.2.4. Convexity

A large literature shows a convex flow-performance relation.\(^{35}\) Under-performing funds do not experience capital outflows to the same degree as outperforming funds attract capital inflows. To ensure our results are not driven by the misspecification of the flow-performance relation, we investigate a piecewise linear function of fund returns (Frazoni and Schmalz, 2017) and examine how return dispersion affects the slopes of the piecewise linear regression.

We define \( \alpha_{it}^+ \) and \( \alpha_{it}^- \) as

\[
\alpha_{it}^+ = \max(\alpha_{it}, 0) \quad \text{and} \quad \alpha_{it}^- = \min(\alpha_{it}, 0).
\]

In other words, \( \alpha_{it}^+ \) (\( \alpha_{it}^- \)) equals \( \alpha_{it} \) if \( \alpha_{it} \) is greater (less) than zero and zero otherwise. This implies that \( \alpha_{it} = \alpha_{it}^+ + \alpha_{it}^- \). We then modify Eq. (3) by separating the impact of \( \alpha_{it}^+ \) and \( \alpha_{it}^- \) such that

\[
\begin{align*}
\text{Flow}_{it+1-t+12} & = b_1\alpha_{it}^+ + b_2\alpha_{it}^- + b_3\text{Disp}_i \times \alpha_{it}^+ \\
& \quad + b_4\text{Disp}_i \times \alpha_{it}^- + b_5\text{Std}\left(\alpha_{i1-t} \right)_{it} \\
& \quad \times \alpha_{it} + \text{Control}_{it} + e_{it+1-t+12},
\end{align*}
\]

To save space, we do not include market state or policy uncertainty [i.e., \( \text{Other}_{it} \) in Eq. (5)] in the preceding regression specifications. Our results are similar if we include them.

We report the key regression results in Table 6. More detailed regression results are reported in Tables IC1 and IC2 in the Internet Appendix.

The results in Table 6 show that the loading on \( \alpha_i^+ \) is much higher than on \( \alpha_i^- \), which is consistent with existing evidence on the convex flow-performance relation. When we focus on the loading on the interaction between return dispersion and \( \alpha_i^- \), we see a negative estimate across all specifications, which is consistent with our model predictions.

Our results on the impact of return dispersion on flow-performance sensitivity for \( \alpha_i^+ \) are stronger (both statistically and economically) than those for \( \alpha_i^- \), which is consistent with previous findings that fund flows respond more to outperformance than to under-performance. For example, a one standard deviation increase in return dispersion reduces flow-performance sensitivity for \( \alpha_i^+ \) by 18% for CAPM-adjusted alpha and by 21% for Fama–French three-factor adjusted alpha and for \( \alpha_i^- \) by 7% and 4%, respectively.

3.2.5. Percentile rankings

Previous research shows that percentile alpha rankings can predict future fund flows, above and beyond the ability

\(^{35}\) See, e.g., (Ippolito, 1992; Goetzmann and Peles, 1997; Gruber, 1996; Lynch and Musto, 2003), and Spiegel and Zhang (2013).
Table 5  

A summary of key regression results is presented on flow-performance sensitivity for the 1980–1999 and 2000–2016 subsamples. See Table I1B to I1B in the Internet Appendix for more detailed regression outputs. Alphas are estimated four ways: the market-adjusted alpha (fund return minus market return), the CAPM-adjusted alpha, the three-factor alpha (adjusted for the Fama–French three-factor model), and the four-factor alpha (adjusted for the Fama–French–Carhart four-factor model). The dependent variable is the annual percentage fund flow between month t + 1 and t + 12. Explanatory variables are estimated fund alpha (α), cross-sectional return dispersion (Disp), standard error for alpha [Stdev(α)], expense ratio (Expense), turnover ratio (Turnover), log TNA (LogTNA), log fund age (LogAge), return volatility (Vol), and average flow of the investment objective class (StyleFlow). ‘Controls’ include expense ratio, turnover ratio, log TNA, log fund age, return volatility, and average flow of the investment objective class. Market state (Franzoni and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time t are based on data before (including) month t, except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the t-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The total sample period is January 1980 to December 2016.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Market-adjusted alpha</th>
<th>CAPM-adjusted alpha</th>
<th>Three-factor alpha</th>
<th>Four-factor alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>α</td>
<td>2.91***</td>
<td>1.37***</td>
<td>3.08***</td>
<td>1.85***</td>
</tr>
<tr>
<td></td>
<td>(7.56)</td>
<td>(3.66)</td>
<td>(8.07)</td>
<td>(4.88)</td>
</tr>
<tr>
<td></td>
<td>(–1.60)</td>
<td>(2.12)</td>
<td>(–1.51)</td>
<td>(1.72)</td>
</tr>
<tr>
<td></td>
<td>(–3.65)</td>
<td>(0.66)</td>
<td>(–4.87)</td>
<td>(1.80)</td>
</tr>
<tr>
<td>Controls</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Number of observations</td>
<td>180,997</td>
<td>156,690</td>
<td>157,853</td>
<td>139,879</td>
</tr>
<tr>
<td>Adjusted R² (percent)</td>
<td>38.6</td>
<td>48.7</td>
<td>40.1</td>
<td>49.5</td>
</tr>
</tbody>
</table>


| α        | 1.87***                | 1.78***              | 2.39***           | 2.19***          | 3.65***         | 3.32*** | 3.68*** | 3.88*** |
| Disp × α | –1.42**                | –2.46**              | –2.49**           | –3.34**          | –11.32**        | –10.33**| –7.02** | –18.16** |
|          | (–4.15)                | (–6.72)              | (–3.89)           | (–3.54)          | (–6.33)         | (–4.72) | (–2.63) | (–5.97) |
|          | (–8.42)                | (–7.00)              | (–6.64)           | (–5.16)          | (–6.57)         | (–5.90) | (–7.45) | (–5.51) |
| Controls | N                      | Y                    | N                 | Y                | N               | Y    | N    | Y    |
| Number of observations | 1,020,585             | 915,782              | 984,814           | 897,392          | 900,819         | 831,901 | 900,819 | 831,901 |
| Adjusted R² (percent) | 23.3                  | 31.6                 | 22.9              | 33.0             | 22.9            | 32.6   | 23.0   | 32.6   |

of alphas to do so. In this section, we provide an analysis of the relations among alpha, percentile alpha ranking, and return dispersion.

The percentile alpha ranking is empirically relevant, but it lacks economic foundation (e.g., Berk and Green, 2004; Pástor and Stambaugh, 2012). Our paper provides an economic interpretation as to why percentile rankings could predict future flows.

Our thought process is as follows. Suppose a fund produces a 10% alpha at two times, t1 and t2. Return dispersion is higher at t2 than at t1. Although a 10% alpha can look very attractive, in comparison with other funds, when cross-sectional return dispersion is low at t1, it could become less attractive at t2 when cross-sectional return dispersion is high. As such, a higher cross-sectional return dispersion could be correlated with the lowering of the percentile alpha ranking, which would lead to lower flows in the future. In our model, high return dispersion predicts a lower perceived alpha, because zero-alpha funds, on average, take more idiosyncratic risk. Our model predicts that the time-varying cross-sectional dispersion interacting with alphas is what explains fund flows, not the percentile alpha ranking.

While return dispersion can be correlated with the change in the percentile ranking for certain funds (holding alpha constant), it does not necessarily imply a change in ranking. For example, suppose a fund earns an alpha of 20% (per annum) at two times. Given this high alpha, the fund is likely ranked very high at both times, regardless of the change in return dispersion. Assuming the fund is ranked at the 95th percentile at both times, both alphas and the percentile rankings of alphas suggest that future fund flows should be the same following the two times. In contrast, our model predicts that future fund flows should be lower following the time that has the higher return dispersion. This example illustrates how we can disentangle our model’s predictions from the previously shown results for percentile alpha rankings.

Table 7 includes the percentile alpha rankings and provides a summary of the key results. More detailed regression results are reported in Tables ID1 and ID2 in the Internet Appendix.

Overall, the results in Table 7, which control for percentile alpha rankings, are consistent with our model predictions. While percentile alpha rankings do provide some explanatory power of fund flows to alphas when only
Table 6
Flow-performance sensitivity under convexity: a summary

A summary of key regression results is presented on flow-performance sensitivity under convexity. See Tables IC1 and IC2 in the Internet Appendix for more detailed regression outputs. Alphas are estimated four ways: the market-adjusted alpha (fund return minus market return), the CAPM-adjusted alpha, the three-factor alpha [adjusted for the Fama–French three-factor model], and the four-factor alpha [adjusted for the Fama–French–Carhart four-factor model]. The dependent variable is the annual percentage fund flow between month t+1 and t+12. Explanatory variables are estimated fund alpha ($\alpha$), $\alpha^* = \max(\alpha, 0)$, $\alpha^- = \min(\alpha, 0)$, cross-sectional return dispersion ($Disp$), standard error for alpha ($Std(\alpha)$), expense ratio ($ExpRatio$), turnover ratio ($Turnover$), log TNA ($LogTNA$), log fund age ($LogAge$), return volatility (VOL), and average flow of the investment objective class (StyleFlow). ‘Controls’ include expense ratio, turnover ratio, log TNA, log fund age, return volatility, and average flow of the investment objective class. Market state (Franzonzi and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time t are based on data before (including) month t, except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the t-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Market-adjusted alpha</th>
<th>CAPM-adjusted alpha</th>
<th>Three-factor alpha</th>
<th>Four-factor alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\alpha^+$</td>
<td>2.82***</td>
<td>2.55***</td>
<td>3.78***</td>
<td>3.16***</td>
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<tr>
<td></td>
<td>(14.00)</td>
<td>(14.06)</td>
<td>(15.42)</td>
<td>(14.69)</td>
</tr>
<tr>
<td>$\alpha^-$</td>
<td>1.96***</td>
<td>1.91*</td>
<td>1.59***</td>
<td>1.64***</td>
</tr>
<tr>
<td></td>
<td>(17.76)</td>
<td>(17.37)</td>
<td>(10.51)</td>
<td>(10.50)</td>
</tr>
<tr>
<td>$Disp &gt; \alpha^+$</td>
<td>-2.94***</td>
<td>-3.60***</td>
<td>-7.77***</td>
<td>-6.27***</td>
</tr>
<tr>
<td></td>
<td>(-3.80)</td>
<td>(-5.43)</td>
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<td>(-5.62)</td>
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<tr>
<td>$Disp &lt; \alpha^-$</td>
<td>-0.80</td>
<td>1.08</td>
<td>2.15**</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(-1.16)</td>
<td>(-1.41)</td>
<td>(-1.90)</td>
<td>(0.63)</td>
</tr>
<tr>
<td>$Std(\alpha) \times \alpha$</td>
<td>-5.28***</td>
<td>-3.77***</td>
<td>-8.78***</td>
<td>-6.63***</td>
</tr>
<tr>
<td></td>
<td>(-8.68)</td>
<td>(-7.33)</td>
<td>(-5.57)</td>
<td>(-4.67)</td>
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<tr>
<td>Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1,204,767</td>
<td>1,075,586</td>
<td>1,145,655</td>
<td>1,040,191</td>
</tr>
<tr>
<td>Adjusted $R^2$ (%)</td>
<td>21.3</td>
<td>30.0</td>
<td>20.8</td>
<td>30.7</td>
</tr>
</tbody>
</table>

Table 7
Flow-performance sensitivity: controlling for percentile rankings

A summary of key regression results is presented on flow-performance sensitivity, controlling for percentile rankings of alphas. See Tables ID1 and ID2 in the Internet Appendix for more detailed regression outputs. Alphas are estimated four ways: the market-adjusted alpha (fund return minus market return), the CAPM-adjusted alpha, the three-factor alpha [adjusted for the Fama–French three-factor model], and the four-factor alpha [adjusted for the Fama–French–Carhart four-factor model]. The dependent variable is the annual percentage fund flow between month t+1 and t+12. Explanatory variables are the percentile ranking of alpha ($\text{rank}(\alpha)$), estimated fund alpha ($\alpha$), cross-sectional return dispersion ($Disp$), standard error for alpha ($Std(\alpha)$), expense ratio ($ExpRatio$), turnover ratio ($Turnover$), log TNA ($LogTNA$), log fund age ($LogAge$), return volatility (VOL), and average flow of the investment objective class (StyleFlow). ‘Controls’ include expense ratio, turnover ratio, log TNA, log fund age, return volatility, and average flow of the investment objective class. Market state (Franzonzi and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time t are based on data before (including) month t, except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the t-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

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<thead>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\text{rank}(\alpha)$</td>
<td>0.06*</td>
<td>0.10**</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(1.75)</td>
<td>(2.81)</td>
<td>(0.68)</td>
<td>(1.53)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.01***</td>
<td>1.71***</td>
<td>2.86**</td>
<td>2.15**</td>
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<tr>
<td></td>
<td>(8.73)</td>
<td>(7.63)</td>
<td>(10.28)</td>
<td>(8.58)</td>
</tr>
<tr>
<td>$Disp \times \alpha$</td>
<td>-1.29**</td>
<td>-1.60***</td>
<td>-2.14</td>
<td>-1.65**</td>
</tr>
<tr>
<td></td>
<td>(-2.64)</td>
<td>(-3.66)</td>
<td>(-2.33)</td>
<td>(-2.12)</td>
</tr>
<tr>
<td>$Std(\alpha) \times \alpha$</td>
<td>-4.59***</td>
<td>-2.93***</td>
<td>-11.77**</td>
<td>-7.59**</td>
</tr>
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<td></td>
<td>(-6.64)</td>
<td>(-4.97)</td>
<td>(-8.46)</td>
<td>(-6.04)</td>
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<tr>
<td>Controls</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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<tr>
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<td>30.7</td>
</tr>
</tbody>
</table>

The rankings and alphas are included in the regressions (consistent with the previous literature; see our results in Tables ID1 and ID2 in the Internet Appendix), they are on longer significant [under the CAPM, three-factor, and four-factor models] or only marginally significant [under market-adjusted alpha] after we include return dispersion. Hence, our results not only show that the impact of return dispersion is robust to the inclusion of alpha rankings, but also suggest that the well-documented impact of alpha rankings is subsumed by our model of return dispersion.
3.3. Additional results

We present three sets of additional results to investigate alternative explanations for our results. Section 3.3.1 presents results that control for alternative proxies for market conditions. Section 3.3.2 examines quadratic flow-performance sensitivity. We further reconcile our findings with the existing literature in Section 3.3.3.

3.3.1. Time-varying market conditions

We include in our main analysis the variables in Franzoni and Schmalz (2017) and Starks and Sun (2016). These variables affect the flow-performance sensitivity and are related to time-varying market or economic conditions. We also construct three market-related variables that could be related to the flow-performance sensitivity. They are the average market excess return, the standard deviation of the market excess return, and the absolute value of the average market excess return, all measured over the past year.

We report in Tables IE1–IE4 in the Internet Appendix our regression results that control for the market-related variables. We find that although some evidence shows that the average market excess return influences the flow-performance sensitivity, it seems to depend on the benchmark factor model we use. We find no evidence that the other two variables that are related to market uncertainties affect the flow-performance sensitivity. Our results on the impact of cross-sectional return dispersion among mutual funds on the flow-performance sensitivity are robust to the inclusion of market-related state variables. Overall, our evidence suggests that it is cross-sectional return dispersion among mutual funds, not market uncertainty in general, that is driving our results.

3.3.2. Quadratic flow-performance sensitivity

In our main analysis, we use a piecewise linear function to capture the convex flow-performance relation. We explore alternative functional forms in the Internet Appendix. We use a quadratic function to approximate the nonlinear flow-performance relation, following Barber et al. (2005), Sensoy (2009), and Kim (2017). We focus on the impact of our measure of return dispersion on both the linear and the quadratic term for fund performance.

Tables IC3 and IC4 in the Internet Appendix report our results. Consistent with the previous literature (e.g., Chevalier and Ellison, 1997), we find a significant quadratic term in capturing the convexity in the flow-performance sensitivity.36 Consistent with our previous results, we find a significant impact of return dispersion on the linear flow-performance sensitivity across all the models we examine. Return dispersion also seems to dampen the quadratic flow-performance sensitivity and does so significantly across three out of four benchmark models we examine (with the exception of the four-factor adjusted alpha). Overall, our results on the quadratic flow-performance sensitivity provide an additional robustness check of our model’s predictions.

3.3.3. Relating to existing papers

Several existing papers also study the role of return uncertainty or market uncertainty in driving the flow-performance sensitivity. Huang et al. (2012) argue that investors’ learning of a fund’s skill has implications on the fund’s time series return volatility impacting the flow-performance relation. Different from their work, our paper focuses on how cross-sectional return variability affects the flow-performance sensitivity. Nonetheless, we include the interaction between a fund’s time series volatility and its past performance in our empirical analysis to control for the learning channel proposed by Jun et al. (2014) study how market uncertainty affects the flow-performance relation for Chinese mutual funds. Their uncertainty measure is different from our measure of return dispersion. Moreover, our results in Table IE1–IE4 show that market uncertainty is not driving our results for US mutual funds.

Perhaps most closely related to our paper is Kim (2017), which, among other things, empirically studies the impact of industry return dispersion (as measured by the standard deviation of the cross section of fund returns) on the flow-performance sensitivity. Contrary to our results, she finds that an increase in return dispersion increases the convexity (as measured by the coefficient on the quadratic term of past performance) of the flow-performance relation. We reconcile her findings with ours in Table IC3 in the Internet Appendix. We replicate her findings when we do not include time fixed effects.37 When we include the time fixed effects, which is the specification we use throughout our empirical analysis, the pattern is reversed and we find a negative and significant impact of return dispersion on convexity.38 Given the gradual decline in return dispersion displayed in Fig. 3 (which, not surprisingly, coincides with the decline in performance for an average fund),39 we believe it important to include the time fixed effects when measuring the impact of return dispersion on the flow-performance sensitivity. Our main results on how return dispersion affects the linear flow-performance relation are robust to the exclusion of the time fixed effects.

4. Conclusion

Over the 50 years of modern financial research on performance evaluation, much has been learned. First, luck

36 This is inconsistent with the results in Kim (2017), who finds an insignificant role for the quadratic term in driving the flow-performance sensitivity. We reconcile her findings with ours in Section 3.3.3.

37 We show that her measure of return dispersion positively (albeit insignificantly) affects the convexity of the flow-performance relation. Note that Kim (2017) first orthogonalizes her measure of cross-sectional standard deviation with respect to market volatility and then studies the impact of the orthogonalized variable. In contrast, we directly control for market volatility as well as time-fixed effects in our regression models.

38 We show in Table IC4 that our results are consistent across alternative benchmark models. Kim (2017) focuses only on the market-adjusted alpha (i.e., the difference between fund return and market return).

39 We find strong evidence for a time trend for both series of return dispersion used in Table IC3. We test for a time trend by regressing the time series of return dispersion on a constant and a time trend. The t-statistics for the coefficients on time trends are -5.92 for our measure of return dispersion and -9.95 for the Kim (2017) measure of return dispersion.
needs to be taken into account and, when it is, the majority of fund managers appear unskilled. Second, it is a useful paradigm to think about a two-group categorization (skilled and unskilled) to study Type I and Type II errors. Third, the Bayesian performance evaluation literature has made an important point, that is, a particular fund’s performance cannot be evaluated without using the information in the cross section of fund returns.

Given the extensive literature on what investors should do, we ask a different question: Are investors taking the above issues into account in their capital allocation decisions? To answer this question, we have presented a model that provides a straightforward link between return dispersion and investor actions. Periods of high return dispersion should make it especially difficult for investors to separate good (skilled) funds from bad (unskilled) funds.

Our empirical results suggest that investors take dispersion of idiosyncratic risk into account in making manager-selection decisions. Investors are much more likely to be skeptical of good performance when return dispersion across funds is high. Our results are robust to different factor models used to estimate alphas. Our results are also stronger using more recent data, which is consistent with investors learning through time. In addition, convexity in the flow-performance relation, as highlighted by the previous literature, does not account for our findings. Instead, our results allow for a new interpretation of this convexity. Flows to outperforming funds are especially sensitive to cross-sectional return dispersion. Finally, our results are robust to the inclusion of percentile alpha rankings. Our analysis shows that the predictive power of percentile rankings vanishes once we include return dispersion.

Appendix A. Proofs

A.1. Proposition 1

\[ p(\alpha | R) = \frac{p(R)}{p(\alpha) \Pi_h} = \frac{p(R)}{p(\alpha) \Pi_h + p(R) \Pi_1} \]  \hspace{1cm} (A3)

and that

\[ \sum_{i=1}^{T} R_i^2 = \sum_{i=1}^{T} (R_i - \bar{R})^2 + T \bar{R}^2 \]  \hspace{1cm} (A4)

and

\[ \sum_{i=1}^{T} (R_i - \alpha_h)^2 = \sum_{i=1}^{T} (R_i - \bar{R})^2 + T (\bar{R} - \alpha_h)^2, \]  \hspace{1cm} (A5)

we have

\[ p(\alpha_h | R) = \frac{1}{1 + \left( \frac{\theta_0}{\theta_h} \right)^{\lambda} \left( \frac{IVOL^2 + (\bar{R} - \alpha_h)^2 + 2\theta_h/T}{IVOL^2 + R^2 + 2\theta_0/T} \right)^{\frac{\lambda}{2}} \Pi_0 / \Pi_h} \]  \hspace{1cm} (A6)

A.2. Proposition 2

\[ \frac{dE(\alpha | R)}{d\theta_0} \] has the opposite sign of \[ \frac{d\psi(\theta_0)}{d\theta_0} \], where \[ \psi(\theta_0) \] is defined as

\[ \psi(\theta_0) = \lambda \log \left( \frac{\theta_0}{\theta_h} \right) - \left( \frac{T}{2} + \lambda \right) \log \left( IVOL^2 + \frac{2\theta_0}{T} \right) \]  \hspace{1cm} (A7)

When \[ \frac{dE(\alpha | R)}{d\theta_0} < 0 \], we have

\[ \frac{d\psi(\theta_0)}{d\theta_0} = \frac{\lambda}{\theta_0} \left[ 1 + \frac{2\lambda/T}{IVOL^2 + R^2 + 2\theta_0/T} \right] > 0. \]  \hspace{1cm} (A8)

Solving the above inequality, we have

\[ IVOL^2 + \bar{R}^2 > \frac{\theta_0}{\lambda}. \]  \hspace{1cm} (A9)

Similarly, \[ \frac{dE(\alpha | R)}{d\theta_h} \] has the opposite sign of \[ \frac{d\psi(\theta_h)}{d\theta_h} \], where \[ \psi(\theta_h) \] is defined as

\[ \psi(\theta_h) = \lambda \log \left( \frac{\theta_0}{\theta_h} \right) + \left( \frac{T}{2} + \lambda \right) \]  \hspace{1cm} (A10)

\[ \times \log \left( IVOL^2 + (\bar{R} - \alpha_h)^2 + \frac{2\theta_h}{T} \right) . \]

Requiring \[ \frac{d\psi(\theta_h)}{d\theta_h} < 0 \], we have

\[ IVOL^2 + (\bar{R} - \alpha_h)^2 > \frac{\theta_h}{\lambda}. \]  \hspace{1cm} (A11)

Eqs. (A9) and (A11) must be simultaneously satisfied if \[ \bar{R} \] is large enough. Let \[ \bar{R}^* \] be the minimum level of \[ \bar{R} \] such that Eqs. (A9) and (A11) are satisfied if \[ \bar{R} > \bar{R}^* \]. When \[ \bar{R} > \bar{R}^* \], we have \[ \frac{dE(\alpha | R)}{d\theta_0} < 0 \] and \[ \frac{dE(\alpha | R)}{d\theta_h} > 0 \].
A.3. Proposition 3

\[ \frac{\partial E(\alpha | R)}{\partial \xi(IVOL)} \] has the opposite sign of \( \frac{\partial E(\alpha | R)}{\partial IVOL} \), where \( \xi(IVOL) \) is defined as

\[ \xi(IVOL) = \log(IVOL^2 + (\bar{R} - \alpha_h)^2 + 2\theta_h/T) - \log(IVOL^2 + \bar{R}^2 + 2\theta_h/T). \]  

(A12)

Requiring \( \frac{\partial E(\alpha | R)}{\partial \xi(IVOL)} > 0 \) (hence, \( \frac{\partial E(\alpha | R)}{\partial IVOL} < 0 \)), we have

\[ \bar{R} > \frac{\alpha_h}{2} + \frac{\theta_h - \theta_0}{T \alpha_h}. \]  

(A13)

Hence, when \( \bar{R} > \bar{R}^* \), we have \( \frac{\partial E(\alpha | R)}{\partial IVOL} < 0 \).

Appendix B. Model calibration under alternative parameterizations

Panel A: Fraction of zero-alpha funds \( \Pi_0 = 0.6 \)

Panel B: Fraction of zero-alpha funds \( \Pi_0 = 0.5 \)

Panel C: Fraction of zero-alpha funds \( \Pi_0 = 0.6 \)

Panel D: Fraction of zero-alpha funds \( \Pi_0 = 0.5 \)

Fig. B1. Alternative model calibration: sensitivity of perceived performance \([E(\alpha|R)]\) to observed performance \((\bar{R})\). We plot the perceived performance against the observed performance based on our model presented in Section 2.1 under various specifications of \( \theta_0 \) (mean level of idiosyncratic variance for zero-alpha funds), \( \theta_h \) (mean level of idiosyncratic variance for positive-alpha funds), and \( \Pi_0 \) (fraction of zero-alpha funds). The corresponding specifications for Panel A are solid line \((\theta_0 = 0.1^2, \theta_h = 0.1^2, \Pi_0 = 0.6)\) and dotted line \((\theta_0 = 0.2^2, \theta_h = 0.1^2, \Pi_0 = 0.6)\); for Panel B, solid line \((\theta_0 = 0.1^2, \theta_h = 0.1^2, \Pi_0 = 0.5)\) and dotted line \((\theta_0 = 0.2^2, \theta_h = 0.1^2, \Pi_0 = 0.5)\); for Panel C, solid line \((\theta_0 = 0.1^2, \theta_h = 0.1^2, \Pi_0 = 0.6)\) and dotted line \((\theta_0 = 0.2^2, \theta_h = 0.2^2, \Pi_0 = 0.6)\); and for Panel D, solid line \((\theta_0 = 0.1^2, \theta_h = 0.1^2, \Pi_0 = 0.5)\) and dotted line \((\theta_0 = 0.2^2, \theta_h = 0.2^2, \Pi_0 = 0.5)\). \( \alpha_h \) is set at 2.5% per annum.
Appendix C. Return dispersion as a proxy for $\sqrt{\theta}$

Table C1
Simulated correlations between $\sqrt{\theta_0}$ ($\sqrt{\theta_1}$) and different measures of return dispersion.

For Panel A, we simulate $M = 30$ years of monthly data. Assuming a two-group structure for the cross section of funds (with a probability mass of $\Pi_0$ for zero-alpha funds and $\Pi_1$ for positive-alpha funds), at the beginning of year $m$, $\theta_{m}$ (mean idiosyncratic risk for zero-alpha funds) and $\theta_{m}$ (mean idiosyncratic risk for positive-alpha funds) are randomly and independently generated by $0.1 \times (1 + x)$, where the variable $x$ is uniformly distributed on $(-0.5, 0.5)$, Conditional on $\theta_{m}$ and $\theta_{m}$, we first simulate idiosyncratic risks for the cross section of funds (assuming $N = 1,000$ funds in the cross section) by drawing independently from the inverse-gamma distribution $\Gamma(\lambda = 2, \theta_{m})$ for zero-alpha funds and $\Gamma(\lambda = 2, \theta_{m})$ for positive-alpha funds. Conditional on the simulated idiosyncratic risks for the cross section of funds, we then simulate individual fund returns over the year by drawing independently from a normal distribution with a mean of zero for zero-alpha funds and $\alpha_h$ for positive-alpha funds. We then calculate three statistics to measure cross-sectional return dispersion for year $m$: inter-quartile range ($\text{iqr}_m$), standard deviation ($\text{stdev}_m$), and range between the maximum and the minimum ($\text{range}_m$). Finally, we calculate the time series correlation between $\sqrt{\theta_{m}}$ ($\sqrt{\theta_{m}}$) and the three dispersion statistics. We simulate one thousand times and report the average correlations. A similar procedure is applied to the case with three fund groups (with a probability mass of $\Pi_0$ for zero-alpha funds, $\Pi_{-1}$ for negative-alpha funds, and $\Pi_1$ for positive-alpha funds) in Panel B. In Panel C, we assume $\alpha_h$ is time-varying and positively correlated with $\theta_{m}$: $\alpha_h$ at the beginning of year $m$ is generated by $2\% + (5\% - 2\%) \times (\theta_{m}/0.1 - 0.5)$. As such, it has a correlation of 100% with $\theta_{m}$ and is bounded between 2% and 5%. The remaining steps of the simulation procedure for Panel C are the same as for Panel A.

Panel A: Two groups ($\Pi_0$ and $\Pi_1$)

$\Pi_0 = 0.8$, $\Pi_1 = 0.2$

<table>
<thead>
<tr>
<th>$\alpha_h = 2%$</th>
<th>$\alpha_h = 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>iqr</td>
</tr>
<tr>
<td>$\sqrt{\theta_0}$</td>
<td>0.978</td>
</tr>
<tr>
<td>$\sqrt{\theta_1}$</td>
<td>-0.034</td>
</tr>
</tbody>
</table>

$\Pi_0 = 0.9$, $\Pi_1 = 0.1$

| $\sqrt{\theta_0}$ | 0.986 | 0.559   | 0.477 | 0.987 | 0.600   | 0.492 |
| $\sqrt{\theta_1}$ | 0.027 | 0.078   | 0.101 | 0.002 | 0.046   | 0.049 |

Panel B: Three groups ($\Pi_0$, $\Pi_{-1}$, and $\Pi_1$)

$\Pi_0 = 0.8$, $\Pi_{-1} = 0.1$, $\Pi_1 = 0.1$

<table>
<thead>
<tr>
<th>$\alpha_{-1h} = 2%$</th>
<th>$\alpha_{-1h} = 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>iqr</td>
</tr>
<tr>
<td>$\sqrt{\theta_0}$</td>
<td>0.987</td>
</tr>
<tr>
<td>$\sqrt{\theta_{-1}}$</td>
<td>-0.017</td>
</tr>
<tr>
<td>$\sqrt{\theta_1}$</td>
<td>-0.040</td>
</tr>
</tbody>
</table>

$\Pi_0 = 0.9$, $\Pi_{-1} = 0.05$, $\Pi_1 = 0.05$

| $\sqrt{\theta_0}$  | 0.987 | 0.641 | 0.512 | 0.987 | 0.596 | 0.417 |
| $\sqrt{\theta_{-1}}$ | 0.002 | 0.041 | 0.046 | 0.004 | 0.020 | 0.038 |
| $\sqrt{\theta_1}$  | 0.009 | 0.028 | 0.032 | 0.005 | 0.056 | 0.086 |

Panel C: Two groups and time-varying $\alpha_h$

$\Pi_0 = 0.8$, $\Pi_1 = 0.2$

<table>
<thead>
<tr>
<th>$2% \leq \alpha_h \leq 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$\sqrt{\theta_0}$</td>
</tr>
<tr>
<td>$\sqrt{\theta_1}$</td>
</tr>
</tbody>
</table>

$\Pi_0 = 0.9$, $\Pi_1 = 0.1$

| $\sqrt{\theta_0}$  | 0.986 | 0.421 | 0.492 |
| $\sqrt{\theta_1}$  | 0.023 | 0.594 | 0.127 |
### Appendix D. Additional results

#### Table D1
Flow-performance sensitivity: alpha = fund return - risk-free rate

Regression results are presented on flow-performance sensitivity when alpha is calculated as the mean fund excess return. The dependent variable is the annual percentage fund flow between month t + 1 and t + 12. Explanatory variables are estimated fund alpha (\(\alpha\)), cross-sectional return dispersion (Disp), standard error for alpha [Std(\(\alpha\))], expense ratio (ExpRatio), turnover ratio (Turnover), log TNA (LogTNA), log fund age (LogAge), return volatility (Vol), and average flow of the investment objective class (StyleFlow). Market state (Franzoni and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time t are based on data before (including) month t, except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the \(t\)-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

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<td>0.01***</td>
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#### Table D2

Regression results are presented on flow-performance sensitivity when alpha is estimated with the Fama–French three-factor model. The dependent variable is the annual percentage fund flow between month t + 1 and t + 12. Explanatory variables are estimated fund alpha (\(\alpha\)), cross-sectional return dispersion (Disp), standard error for alpha [Std(\(\alpha\))], expense ratio (ExpRatio), turnover ratio (Turnover), log TNA (LogTNA), log fund age (LogAge), return volatility (Vol), and average flow of the investment objective class (StyleFlow). Market state (Franzoni and Schmalz, 2017) and policy uncertainty (Starks and Sun, 2016) are defined in Section 3.1. All explanatory variables at time t are based on data before (including) month t, except for the average flow of the investment objective class (StyleFlow), which is constructed contemporaneously with the fund flow. For all regression models, we include both fund and month fixed effects. Standard errors are clustered by fund and month. We present the \(t\)-statistics in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively, assuming a single hypothesis is tested. The sample period is January 1980 to December 2016.

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<th>(5)</th>
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(continued on next page)
Table D2 (continued)

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References