Eggs in a Basket:
Harry Markowitz's Contribution and How I Achieved Erdős 3

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KEY FINDINGS

- Markowitz's foundational contribution on portfolio selection included three key assumptions with the most obvious simplifying assumption being that risk is defined as variance. Research focusing on downside risk generalized his model by proposing a definition of optimal portfolios that takes both variance and skewness into account.
- The Markowitz model made a simplifying assumption of static probability beliefs. Considerable research attention has focused on relaxing this assumption and allowing for both dynamic risk as well as expected returns.
- Markowitz also assumed that investors had exact knowledge of the inputs to his model (expected returns, variances, and correlations). Key advances have been made by injecting uncertainty into the optimization problem either via bootstrapping or employing Bayesian portfolio selection.

ABSTRACT

How many times have you heard: "Don't put all your eggs in one basket" in reference to stock investments? That is Harry Markowitz's foundational contribution. Many of my research ideas were a direct result of his insights. In his famous 1952 paper, Markowitz realized that the mean–variance efficient portfolio was only optimal if there was no preference for skewness. This led to my work in the *Journal of Finance*, expanding the definition of risk to include skewness and estimating mean–variance-skewness efficient portfolios. The 1952 paper also assumes static probability beliefs. This inspired a line of research looking at dynamic probability beliefs with time-varying risk and expected returns. Finally, in the Markowitz framework, the expected returns, variances, and covariances are assumed to be exactly known. My work on Bayesian portfolio optimization relaxes this assumption and allows for uncertainty in the inputs. My research epitaph might read: He was a careful reader of Harry Markowitz's footnotes.

How many times have you heard: "Don't put all your eggs in one basket" when referring to stock investments? That is Harry Markowitz's contribution. I would like to make a few remarks on his insights and highlight his impact on my career.

"Portfolio Selection," Markowitz's 1952 paper in the *Journal of Finance,* is *the* foundational paper in finance. It is fascinating to look at that particular issue of the *Journal of Finance*. The journal was only seven years old and the style of papers it was publishing is so different from today. There were six papers. The first five papers were a mixture of discussions and policy proposals. The first five contained no equations.

For example, in that issue there was a paper discussing wage and price controls to deal with post-World War II inflation. Relegated to the final slot, Markowitz's paper was shockingly different. He had completed his PhD course work at the University of Chicago and was working at the RAND Corporation when his paper was published. It was a highly quantitative paper with over 20 equations. Other than the hand-drawn figures, the paper looks like a paper published in 2024—not 1952. The other papers in the issue have long since been forgotten. Markowitz developed a technique now called quadratic programming to optimize a portfolio, meaning for a target expected return, he showed how to calculate the weights on the assets to get the lowest possible variance.

While the idea of diversification was not new, Markowitz (1952) showed us how to diversify in an optimal way. This had fundamental impact on our field and pushed our finance in a more quantitative direction. The fact that it is so routine not to put all our eggs in one basket is a tribute to the impact of Harry Markowitz's contribution.

On a personal note, there are three research streams I pursued that were directly a result of this Nobel Prize–winning paper. I have carefully read Markowitz's 1952 paper many times in my career. As a student at the University of Chicago, it was obviously required reading. However, I did not fully appreciate the paper until many years later.

Some criticized this foundational paper as only offering an analytical technique rather than an economic model that could yield general insights. I think this reaction is a misreading of the contribution. The idea of efficient diversification is fundamental in all of our asset pricing models. Bill Sharpe's capital asset pricing model (Sharpe 1964) builds on Markowitz's contribution. In Sharpe, the risk of an asset is defined as that asset's contribution to the risk of a well-diversified portfolio.

Three ideas in the paper inspired my research. The first is a key assumption that Markowitz recognizes as a chief "limitation" of his framework. His approach assumes "static probability beliefs" (Markowitz 1952). One way to think of this is to use average values over a long history for the expected returns and correlations. I was much more interested (Harvey 1988, 1989; Ferson and Harvey 1991) in using the information available today to form forecasts for portfolio selection. In this setup, probability beliefs are dynamic and are "conditional" on the economic information the investor considers relevant (e.g., are we in a recession or not, are current interest rates above or below the average?).

Markowitz (1952) is very precise in defining the economic basis—the utility foundation—for his model. He derives an optimal portfolio, but there are many optimal portfolios; it depends on what gives you the highest utility, and that could vary across investors. A key assumption is mean–variance utility. Agents like higher expected returns but they dislike variance. This allows for a two-dimensional optimization whereby the investor prefers the minimum variance portfolio for any given level of portfolio expected return or the maximum expected return portfolio for any given level of variance, thereby creating an efficient frontier, another foundational element of Sharpe's (1964) CAPM.

There was something at the top of the last page of the paper that caught my eye (Markowitz 1952, p. 91). Again, Markowitz understood the limitations of his approach and he said that the portfolio that he mathematically derives is only optimal if investors have no preference for skewness ("$\partial U/\partial M_3$" in his notation where U is utility and M_3 is skewness, see pp. 90–91).

A preference for skewness means that you prefer a distribution of returns with a large positive tail over a distribution with a negative tail. The type of risk that Harry Markowitz is optimizing is a risk where the distribution of returns is symmetric, which is implicit in the definition of risk as variance.

So, I thought, what if you relax that assumption? (I was not the first to go in this direction.) It is well known that people actually prefer big positive returns to big negative returns, and this led to my *Journal of Finance* paper on conditional skewness (Harvey and Siddique 2000). We extend the mean–variance framework to one more dimension. Think of many portfolios with the same expected return and the same variance. What we proposed was of all those portfolios, investors want the one with the most positive skewness. In my setting, the skewness can change through time. My coauthor and I recently published an update showing the model has worked well in 25 years of out-of-sample evidence (Harvey and Siddique 2023).

There is another contribution I made that is directly related to this 1952 paper. Another key assumption that Markowitz makes, and he is very clear about this, is that the optimization assumes that you know the expected returns, the variances, and the correlations exactly. Many have noticed that when you perform these mean–variance optimizations, a seemingly small change in the expected return for an asset could drastically alter the allocation to that asset and other assets because of the assumption of exact knowledge of these quantities.

The idea of my paper in *Quantitative Finance* was to relax that assumption and to allow for a Bayesian-style portfolio selection, where you have a prior belief with some uncertainty over the inputs (Harvey et al. 2010). Building on my work on skewness, we applied the uncertainty to expected returns, covariances, and co-skewness of the portfolio.

All of these ideas are directly linked to Harry Markowitz's original contribution. My papers would not exist without his foundational contribution. It is important to notice that my ideas were based on a paper published before I was even born. My ideas examined avenues for improving portfolio theory, in ways that Markowitz had already predicted decades before. That is a tribute to the depth of his contribution. He was far ahead of his time.

I knew Harry for years via conferences such as those held by the American Finance Association and the Q Group. However, our bond was established over an academic dispute! Harry also had a paper relaxing the assumption that you exactly knew the inputs to the optimization (see Markowitz and Usmen 2003). He was proposing a bootstrapping technique. I knew this was useful but not optimal. I circulated a paper called "Bayes vs. Resampling" (Harvey, Liechty, and Liechty 2008a), and this got his attention. There was considerable back and forth critiquing my paper, and I published a rejoinder (Harvey, Liechty, and Liechty 2008b). All of this was very respectful. There was no name calling—both of us were seeking the best possible technique for investors.

One of the highlights of my career was coauthoring with Harry. An opportunity arose when a new journal, the *Journal of Financial Data Science*, was being launched and I proposed a paper with Harry and Rob Arnott. The idea was to develop a backtesting protocol in the era of machine learning. Harry, throughout the years, had been a critic of data mining and had published on the subject many different times (see Markowitz and Xu 1994), and this intersected with my interest (see Harvey 2017) and Rob Arnott's interest. So, the three of us developed a protocol to help researchers avoid the curse of overfitting the data (Arnott, Harvey, and Markowitz 2019). I am very proud of this paper.

This is also where the *Erdős Number* comes in.[1] The Erdős Number is a way to judge the closeness of an author to the late prolific mathematician, Paul Erdős (who published over 1,500 papers and had over 500 coauthors). Erdős 1 is reserved for coauthors of Paul Erdős. Erdős 2 means you collaborated with a coauthor of Erdős–and Harry Markowitz is Erdős 2. As such, I am Erdős 3!

While Harry is gone, his research legacy will live on for a very long time. I will miss him. I am so grateful that I had a chance to learn from him and to get to know him.

[1] Erdős Number Project: https://sites.google.com/oakland.edu/grossman/home/the-erdoes-number-project.

AUTHOR NOTE

I appreciate the comments of Rob Arnott.

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