Consider a pure exchange economy with a representative agent with additively separable utility receiving a stochastic endowment. This agent can choose to consume this endowment or invest in a portfolio of j–period bonds, $\{B_{j,t} : \quad j = 1, \ldots, k\}$. The bonds are assumed to be default free. Expectations at time t are conditioned on the information set $\mathbf{F}_t$ – which contains all the information about the environment available at time t. Consumption C_t is required to be measurable at t with respect to $\mathbf{F}_t$. The consumer maximizes the following objective:

$$\max_{\{C_t, B_{j,t}\}_{t=0}^{\infty} \quad j=1,\ldots,k} \quad \sum_{t=0}^{\infty} \delta^t E_0 U(C_t); \qquad 0 < \delta < 1,$$

subject to:

$$C_t^N + \sum_{j=1}^{k} B_{j,t} \leq Y_t^N + \sum_{j=1}^{k} B_{j,t-j}(1 + R_{j,t-j}^N), \tag{A.1}$$

where C_t is the agent's real consumption, C_t^N represents nominal consumption, $R_{j,t-j}^N$ is the nominal yield on a j–period bond bought at time $t-j$, E_t is the expectation operator conditioned on information $\mathbf{F}_t$, Y_t^N is the nominal endowment[1] and δ is the consumer's constant time discount factor. Form the Lagrangian L:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \left\{ U(C_t) + \lambda_t (Y_t P_t + \sum_{j=1}^{k} B_{j,t-j}(1 + R_{j,t-j}) - C_t P_t - \sum_{j=1}^{k} B_{j,t}) \right\} \tag{A.2}$$

The first-order conditions are:

[1] The conditional expectation $E_t[Y_{t+j}^N]$ is assumed to exist for all $j \geq 1$.

$$C_t : \qquad U'(C_t) - P_t \lambda_t = 0$$

$$B_{j,t} : \qquad -\lambda_t + \delta^j E_t \lambda_{t+j}(1 + R^N_{j,t}) = 0 \qquad j = 1, \ldots, k.$$

Solve the first condition for λ_t and substitute this into the second:

$$\frac{U'(C_t)}{P_t} = \delta^j E_t \frac{U'(C_{t+j})}{P_{t+j}}(1 + R^N_{j,t})$$

$$1 = \delta^j E_t \frac{U'(C_{t+j})}{U'(C_t)} \frac{P_t}{P_{t+j}}(1 + R^N_{j,t})$$

Substitute the real interest rate, $R_{j,t}$ times the inflation rate on the RHS and the Euler equation (2.2) of chapter 2 obtains:

$$E\left[\delta^j \frac{U'(C_{t+j})}{U'(C_t)}(1 + R_{j,t}) - 1 \,\middle|\, \mathbf{F}_t\right] = 0, \qquad \text{for} \quad j = 1, \ldots, k, \qquad (A.3)$$

The consumer's problem does not restrict the agent from selling the bonds before maturity. $B_{j,t}$ can be positive or negative. Other assets can also enter into the problem. They will not affect the basic first-order conditions. Note that to keep the consumer's problem well behaved, the arbitrary rolling over of debt is ruled out. Essentially, a limitation on the terminal value of the debt is imposed.