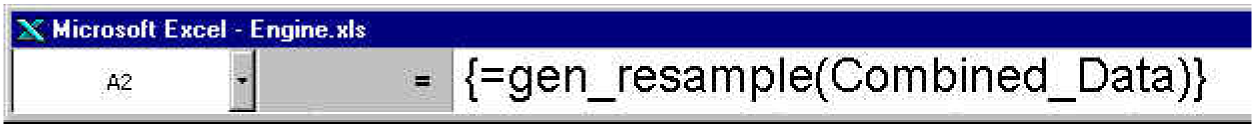
brackets will now appear around your formula. Array formulas control multiple cells at once. When gen_resample is used as an array formula, it assures that the random sample taken from the temperature data in cell A2, is paired with its associated pressure data in B2. NOTE: It was not necessary to enter the two data ranges individually, separated by ",". It would also have been possible to enter the single combined range as an argument.




9. Run a simulation of the failure indicator with the dependence preserved, and you should find a much higher probability of failure of around 5.5%. That's about eight times higher than the previous result.

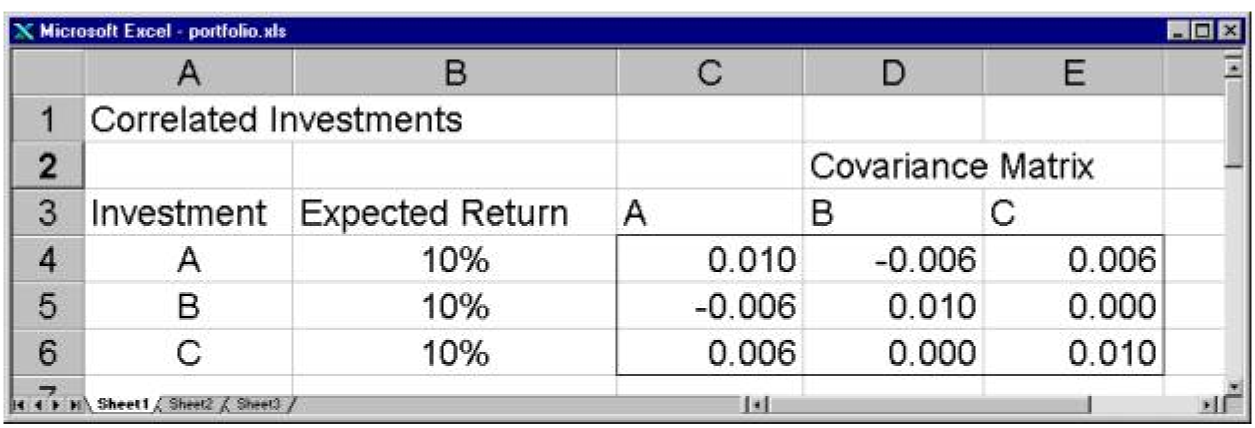
This tutorial has demonstrated:

- The use of indicator formulas to estimate the probability of an event.
- The use of gen_resample to simulate a distribution form past data.
- The array version of gen_resample to simulate multiple random inputs with dependence.

## Simulating Investment Portfolios

Perhaps the area in which the modeling of statistical dependence has been most institutionalized is that of investment portfolios. Because the distributions of many investment instruments are either normal or log normal, an elegant alternative exists to resampling. Instead, the mean returns and covariance matrix may be calculated from past data. For an algebraic definition of covariance see any text on statistics. For an intuitive geometric definition of covariance, see chapter three of INSIGHT.xla.

Consider three potential investments with expected annual returns and covariance matrix shown below in file Portfolio.xls.

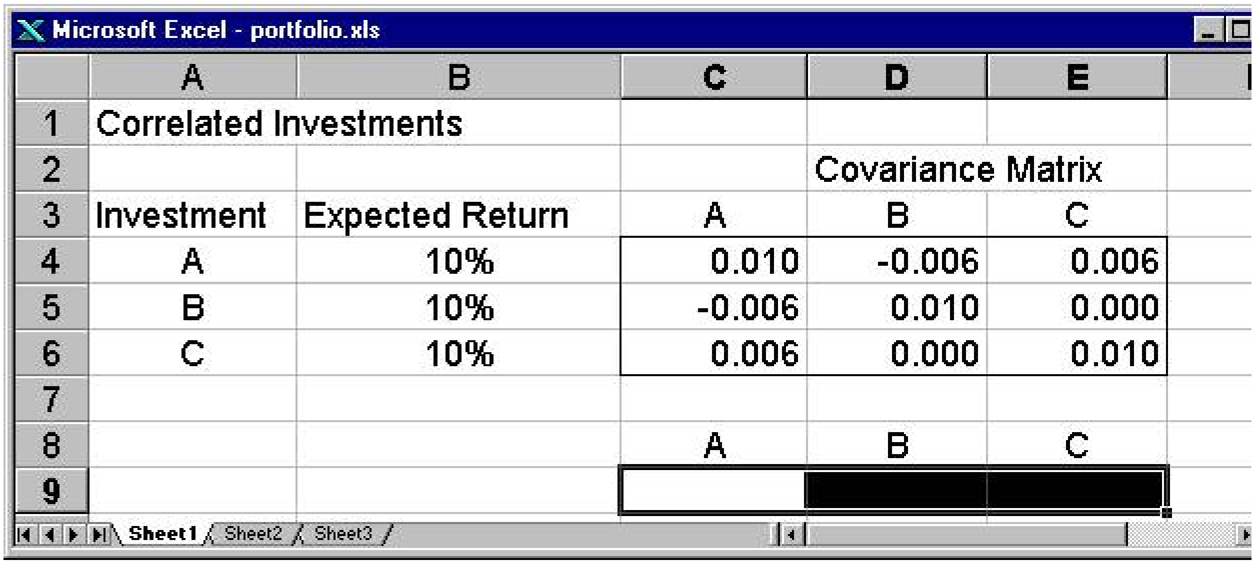


| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Correlated Investments | | | | |
| 2 | | | | Covariance Matrix | |
| 3 | Investment | Expected Return | A | B | C |
| 4 | A | 10% | 0.010 | -0.006 | 0.006 |
| 5 | B | 10% | -0.006 | 0.010 | 0.000 |
| 6 | C | 10% | 0.006 | 0.000 | 0.010 |

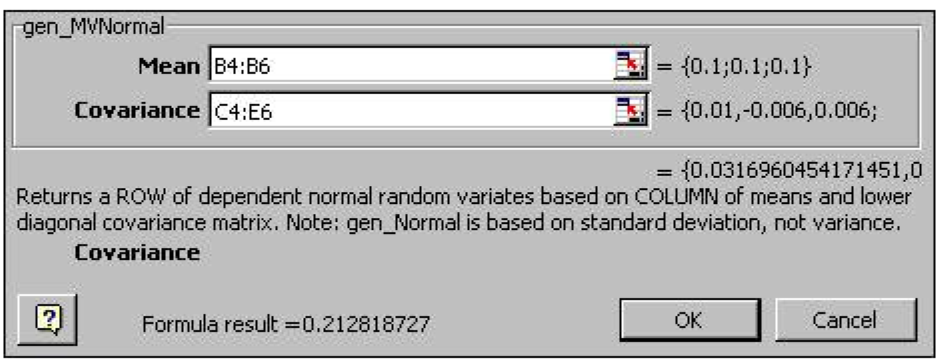
Note that all three potential investments have the same expected return and variance. However, A and B are negatively correlated, A and C are positively correlated while B and C are uncorrelated. In the following tutorial we will simulate three different portfolios comprised of the following investments: 50% of the portfolio in A and 50% in B (we will call this "A+B"), and the other two 50/50 combinations, "A+C" and "B+C". This will demonstrate the well-known correlation effect at the heart of modern portfolio theory (see Reference [2]).

## Tutorial 4b

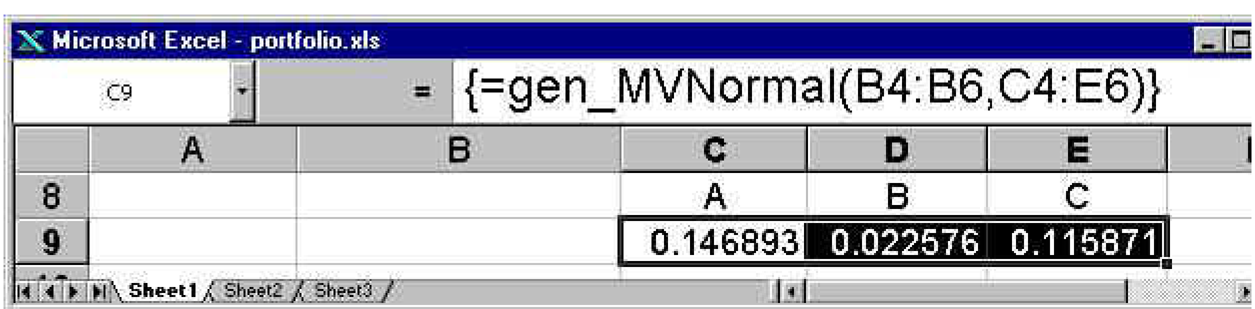
1. Open the file Portfolio.xls. Use the Simulation, Thaw command to make the formulas live. The first step is to simulate the three correlated investments. This will be done with the gen_MVNormal function as follows. Begin by selecting cell C9:E9 as shown.



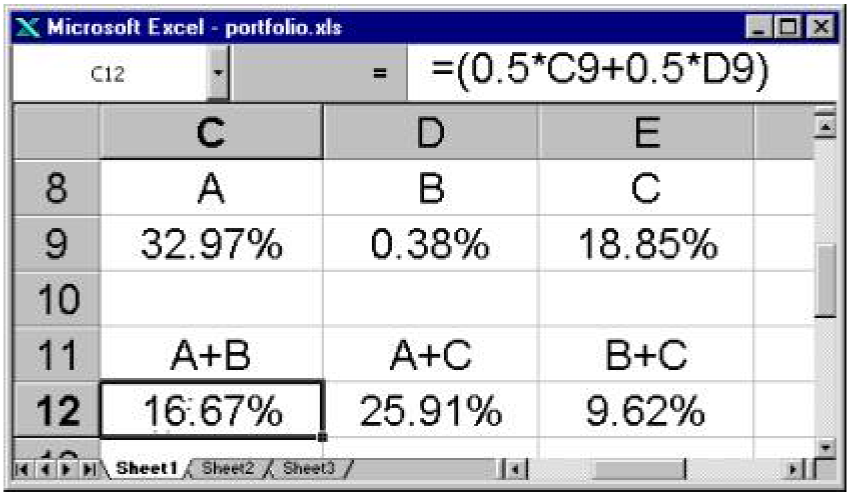

2. Next use the function wizard to insert gen_MVNormal. The arguments are B4:B6 for the mean, and C4:E6 for the covariance matrix. Once the arguments are selected, DO NOT CLICK OK. Instead simultaneously hold down <Shift> and <Ctrl>, then press <Enter>.




3. Curly brackets should appear around the formula as shown. Press the <F9> key a few times to make sure everything works. Be sure to see gen_Functions.xls for demonstrations of all the random number generators



4. Next we will model the three portfolios described earlier. The return of the A+B portfolio for example is (.5*C9+.5*B9) as shown below.



Microsoft Excel - portfolio.xls

C12 = =(0.5*C9+0.5*D9)

| | C | D | E |
|---|---|---|---|
| 8 | A | B | C |
| 9 | 32.97% | 0.38% | 18.85% |
| 10 | | | |
| 11 | A+B | A+C | B+C |
| 12 | 16.67% | 25.91% | 9.62% |

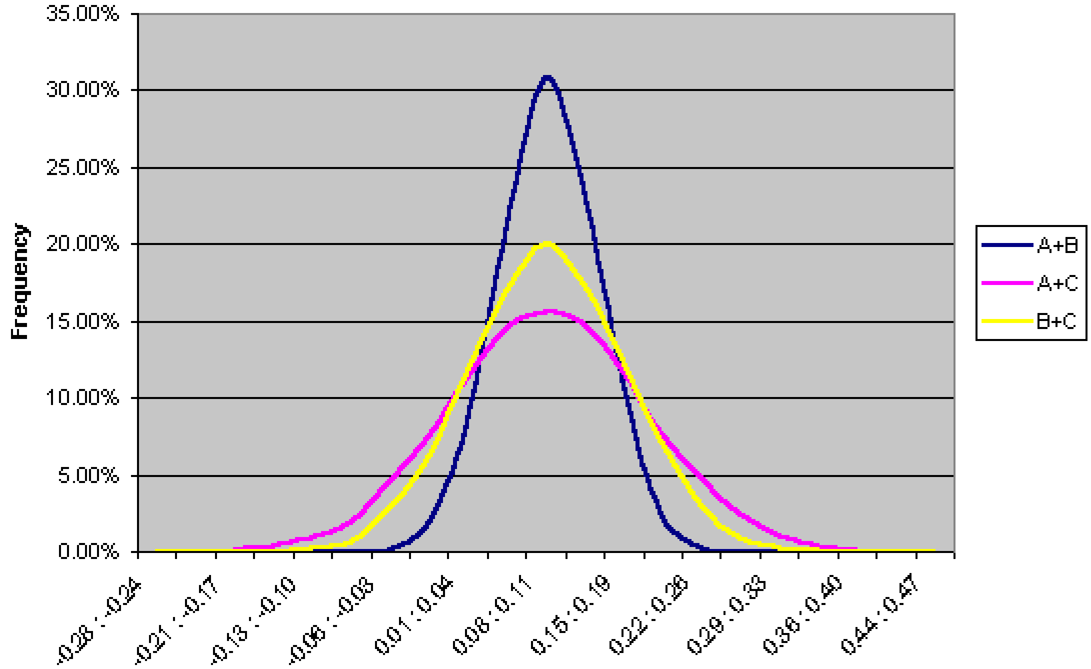
Sheet1 / Sheet2 / Sheet3

**PUZZLE:** Before proceeding to the next step, what do you think the expected return will be for each portfolio? How would you expect the shapes of the three distributions to differ?

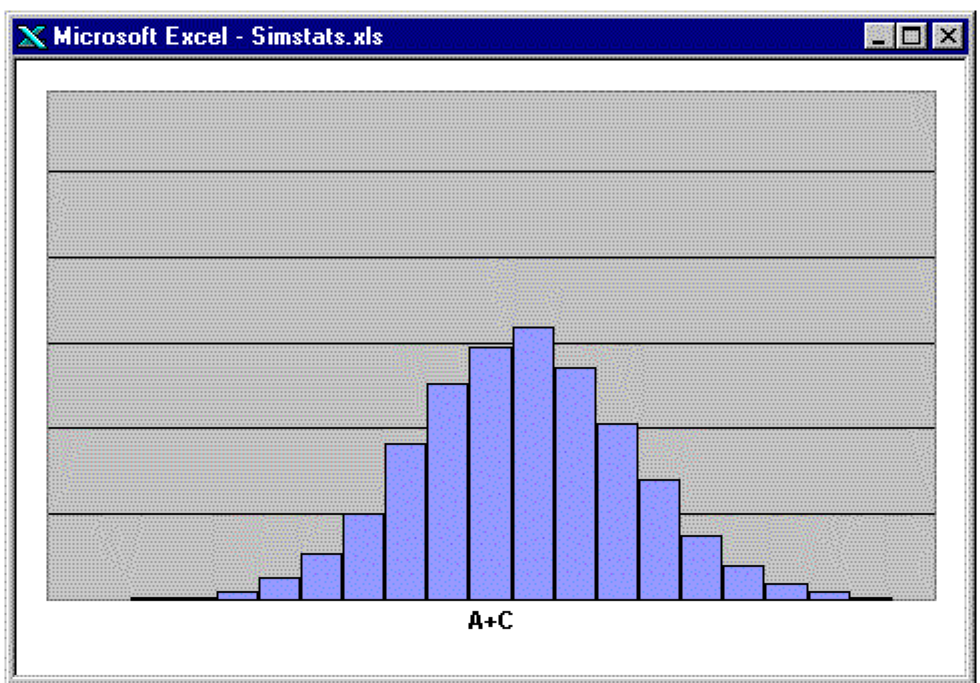
5. Run a simulation with one output cell for each of the three portfolios. The notice that the expected returns of the three portfolios are nearly identical at roughly 10%. The 5th and 10th percentiles display the value at risk at these levels. For example there is a 5% chance that A+B will return 2.75% or less, A+C will lose 4.6% or more, and B+C will lose 1.7% or more.

| | A+B | A+C | B+C |
|---|---|---|---|
| Average | 0.1006 | 0.1007 | 0.1004 |
| Std Dev | 0.0445 | 0.0897 | 0.0707 |
| Std Err | 0.0004 | 0.0009 | 0.0007 |
| Max | 0.2498 | 0.4398 | 0.352 |
| Min | -0.073 | -0.29 | -0.159 |
| | | | |
| Percentiles | | | |
| 5% | 0.0275 | -0.046 | -0.017 |
| 10% | 0.0441 | -0.012 | 0.0095 |

Next, create a common histogram of each of the portfolios using Smoothed 2D Lines. This clearly shows the effects of the correlation between the original investments. With A+B, the correlation is negative, with changes in one likely to be cancelled out by opposite changes in the other. This has the effect of narrowing the distribution of the return, and is desirable if you are risk averse. This was the central point of the pioneering work of Harry Markowitz in the early 1950's (see Reference [2]). With the A+C portfolio, the correlation is positive, with changes in one likely to be amplified by similar changes in the other, hence the wide distribution. This is the least desirable for the risk averse investor. There is no correlation between B+ C, resulting in a distribution that is between the other two.




It is also instructive to use the interactive common histogram. Use <Ctrl><PgUp> and <Ctrl><PgDn> to cycle through the histograms.




## Summary

- In this section we introduced the concept of an indicator variable whose average estimated the probability that an event would occur.

- We used resampling to simulate a distribution based on historical data and demonstrated the importance of capturing statistical dependence if present.

- Finally we showed how to generate Multivariate Normal random variables using their means and covariance matrix.