Omega Optimisation: A Portfolio of 3 Assets

In this note we illustrate the benefit of Omega analysis of portfolio allocations. We apply our simplest metric to the allocation of three assets with diverse characteristics (These are hedge funds of different 'style'.)

In contrast to the use of mean/variance optimisers, our analysis allows us to see the characteristics of the resulting distribution. In a mean/variance world, the underlying assumption is that returns are normally distributed, so of course there is nothing more to consider once the optimal Sharpe ratio has been identified. This means that the impact of features such as asymmetry and fat tails is ignored in the allocation decision. The Omega function of the portfolio distribution allows the investor to assess the Sharpe optimal allocation, or any other, for its risk/reward features

Figure 1 shows the Omega functions for the three assets. They are far from normal.

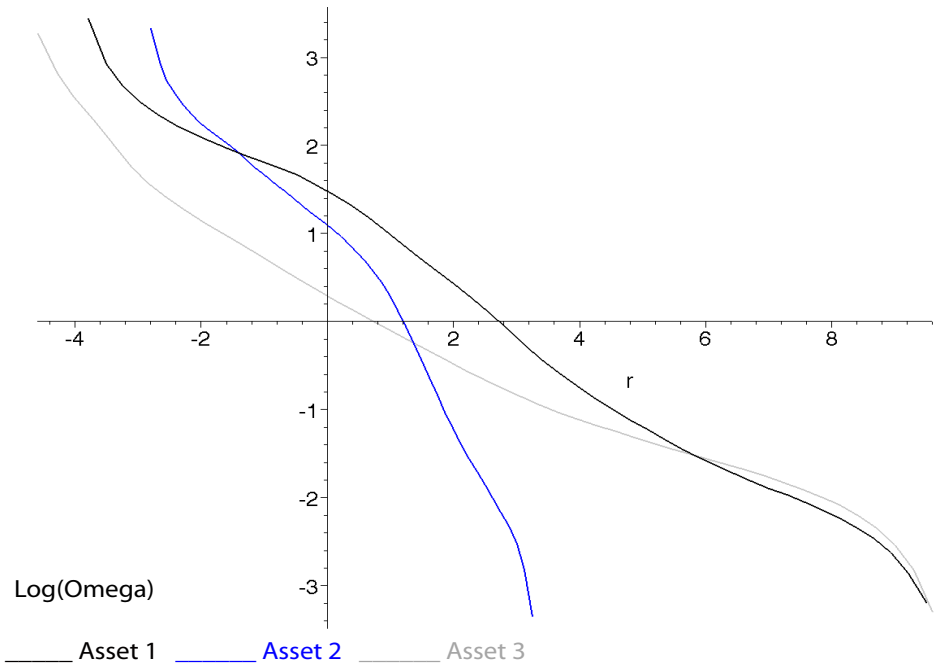

Figure 1 Log(Omega) for the 3 assets.

We compare the returns distribution of our allocation with the distribution of returns on the portfolio with the optimal Sharpe ratio in figure 2. The Omega allocation has a higher mean return and considerably more upside, both of which have been obtained by assuming a very small increase in downside risk relative to the Sharpe optimal allocation.

This is a trade off which a highly risk averse investor, or one whose target return was no higher than the Sharpe optimal's mean return, may well be unwilling to make. It would, on the other hand, obviously be attractive in a moderately aggressive portfolio. Using mean/variance analysis, there is no way to identify, much less decide, such issues.

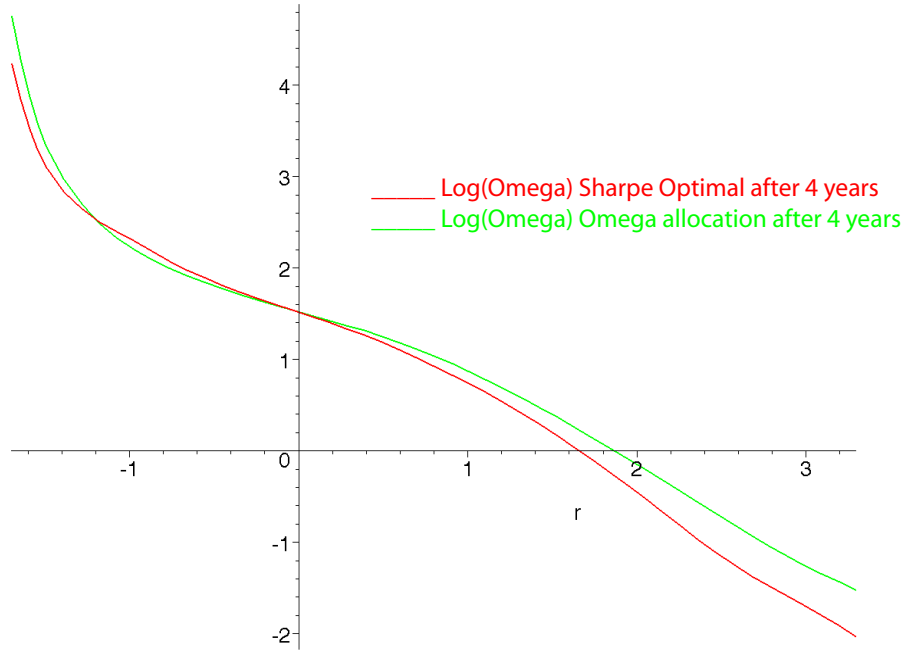

Figure 2. Log(Omega) for the Sharpe Optimal allocation: 33% Asset 1, 58% asset 2 , 9% asset 3 and for the Omega allocation: 46% asset1, 45% asset 2 and 9% asset 3, computed from 4 years of monthly data. Unlike the Sharpe optimal allocation, the Omega allocation allows us to trade off the attractive mean return and fat upside tail of Asset 1 against its higher downside risk relative to asset 2.

The difference between these two distributions is significant and there is a strong implication for expected terminal values of the two funds. Over the 5 year returns sample the terminal value of £1 invested in the the Omega allocation was £2.91 compared with only £2.62 from the Sharpe Optimal allocation. The time series of values of the two portfolios and the component assets is shown in Figure 3.

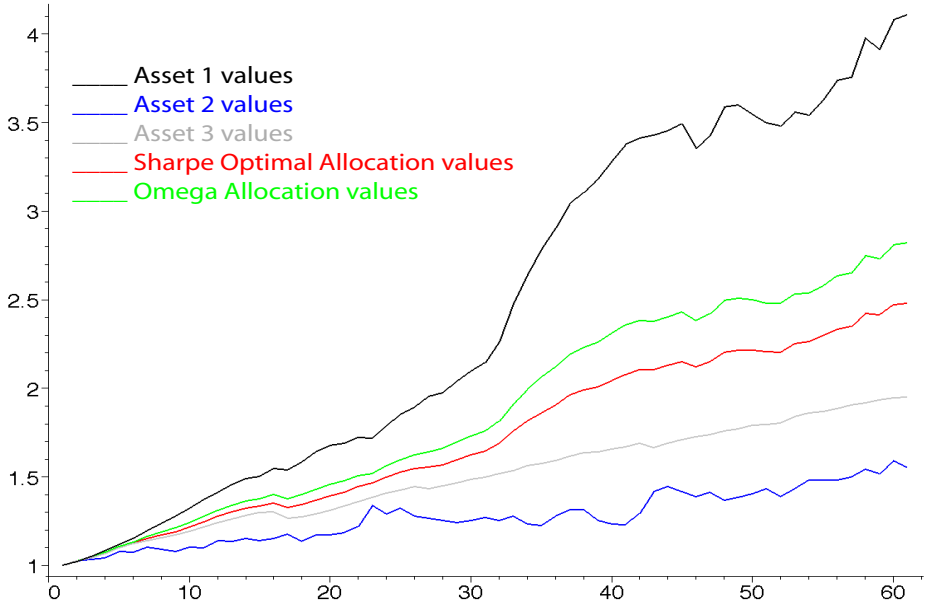

Figure 3. Time series of values of the component assets and the Sharpe optimal and Omega portfolios.