The Corruption-Tax Tradeoff

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Abstract

This paper develops a mechanism grounded in supply-side economics by which corrupt countries become trapped in low rates of economic development. As with explicit taxation to finance public spending, rent extraction by corrupt governments tends to reduce the private incentive to create wealth. A novel result is that corrupt countries find it optimal to tax at low rates, as rent extraction exacerbates the distortion due to explicit taxation. This paper documents and explains the observed negative relation between corruption and tax rates across countries, the robust negative effects of corruption and tax rates on per-capita GDP, and the lack of a robust negative cross-country correlation between tax rates and per-capita GDP without controlling for corruption.

JEL classification: H21, O43, P16

Keywords: Economic growth, Corruption, Optimal taxation

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1 Introduction

This paper develops a mechanism grounded in supply-side economics by which corrupt countries become trapped in low rates of economic development. Thinking of the effects of corruption through the lens of supply-side economics provides a simple and empirically robust mechanism through which corruption destroys wealth, but remarkably also provides a framework that strengthens the cross-country empirical results highlighting the general importance of supply-side issues in relating tax rates to GDP. Both corruption and excessive taxation destroy wealth, which comes out clearly in cross-country data once the effects of both are taken into account. The symmetric nature by which corruption and taxation destroy wealth also explains why cross-country regressions of GDP on tax rates are misspecified and do not generally reveal a robust negative effect of tax rates on GDP. Corruption has similar distortionary effects as explicit taxation, so higher levels of corruption in a sense crowd out explicit taxation, which leads corrupt governments to impose low tax rates. Low tax rates are often associated with poor, corrupt countries, so without controlling for corruption, it does not appear that low tax rates promote GDP. Modeling corruption and taxation jointly from a supply-side economics perspective provides a tractable, empirically-robust set-up that explains the importance of both in destroying wealth.

The key implications of the model are verified in the data. Corrupt countries tend to have low rates of taxation and low rates of economic development. Moreover, in the model rent extraction and bribery income are used as a means to extract resources for the enjoyment of the group in power, and the consequent distortion creates an incentive to reduce revenue from explicit taxation. Explicit taxation is used to finance public goods – such as infrastructure and public education – so corruption crowds out public goods that enhance productivity. The model thereby explains why corrupt countries experience low government spending as a fraction of GDP, in particular low spending on public education and infrastructure. Market output is depressed due both to the allocation of labor away from market production and lower productivity stemming from fewer public resources directed to investments that enhance productivity. A key result of the paper is that corruption will be worse when a small fraction of the population is able to form a coalition to extract resources from a larger group. Some evidence is presented that countries with a large degree of ethnolinguistic fractionalization (ELF) tend to be more corrupt. The relation between corruption and ELF is significant, even though ELF is likely a poor indicator of coalition formation grounded in rent extraction.
For any study involving corruption, a challenge is measuring corruption. As in many other studies, I use survey-based measures on the perception of corruption. This understandably suffers from a criticism that perceptions of corruption may simply be a reflection of other economic difficulties, thereby biasing the relation between economic development and corruption. To address this issue, I use ELF as an instrument for corruption. ELF is correlated with corruption, as argued above, and it seems far less likely that ELF is responding to economic development. All the empirical results of the paper hold up using ELF as an instrument for corruption.

This paper studies the effects of corruption and the interaction between corruption and explicit taxation, and determines why a group in power chooses a level of corruption and a rate of taxation, but the paper takes as given the underlying political economy determining which coalition becomes the group in power as well as the size of this coalition relative to the general population. This paper will start from the premise that all groups in power have the ability to extract resources from the general population to enhance its own welfare, so the question addressed in this paper is why governments with this ability choose different levels of corruption. The underlying permissibility of corruption and the accountability of government is not addressed in this paper. The results of this paper may shed some new light on how to approach this important topic.

2 Literature Review

It is beyond the scope of this paper to provide an extensive review of the empirically-oriented literature studying the effects of changes in tax rates on economic growth. Such an empirical review has been recently provided by Gale and Samwick (2016). This literature is vast and has a long history, but just to briefly summarize, the empirical literature relating tax rates and income is at best a mixed bag. For example, cross-country studies of Easterly and Rebelo (1993) and Piketty, Saez, and Stantcheva (2014) find no strong evidence linking high tax rates to low growth. Using a structural vector-autoregression setup for post-war U.S. data, Blanchard and Perotti (2002) find a small effect of taxation on output, but Romer and Romer (2010, 2014) and Mertens and Ravn (2014) find a large effect.

corruption leads to a lower quality of public infrastructure, and Mauro (1998) found that corruption is associated with lower levels of government spending on education. Del Monte and Papagni (2001) and Aghion, et. al., (2016) developed models with growth, taxation, public infrastructure, and corruption, which Del Monte and Papagni applied to Italian data and Aghion, et. al., applied to U.S. state data. An insightful alternative approach is Acemoglu (2005), who modeled the relation between tax rates, government spending, and the likelihood of a government remaining in power.

3 Data Summary

Various agencies attempt to measure the degree of corruption in the public sector in different countries based on surveys that measure the perception of various forms of public corruption, ranging from bribery, diversion of public funds, using public office to pursue private gain, lack of transparency and accountability, and a wide variety of other features broadly associated with corruption. Transparency International aggregates 13 of these surveys into one Corruption Perception Index (TiCPI). The index is scaled from 0 to 100, with 100 being the least corrupt country. For ease of interpretation, especially as it relates to measures of corruption in the model, in the statistical analysis I will refer to a reverse Corruption Perception Index (rCPI), which is simply 100 - TiCPI. With this scale, a higher value of the index is associated with more corruption. The rCPI in 2018 ranges from 12 (Denmark) to 90 (Somalia), with a median of 62 (the value for the U.S. is 29).

Income tax rates are estimates of the top income tax rate obtained from the Heritage Foundation’s Index of Economic Freedom Database. The Heritage Database also includes estimates of per-capita GDP PPP in constant 2011 international dollars, which they obtain from the International Monetary Fund’s World Economic Outlook Database. Data on Government Spending as a fraction of GDP and Public Education Spending as a fraction of GDP are obtained from the World Bank’s World Development Indicators Database.

Estimating the appropriate marginal tax rate is a difficult exercise. The progressive nature of most tax policies means there is no one marginal tax rate appropriate for everybody, and countries rely on a variety of tax rates other than a tax rate on personal income to raise revenue. Indeed, eleven countries in the Heritage Foundation dataset report a top income tax rate of zero. To partially address this issue, I will trim the data of extreme values on both ends, which effectively means removing data where the top personal income tax rate equals zero (five countries for which other data is present) and where the top rate is greater than
50 percent (another five countries). All regression results will be reported for non-trimmed and trimmed data. Some alternative tax rates are considered in a following section.

Table 1 displays results of regressing the income tax rate, government spending as a fraction of GDP, and public education spending as a fraction of GDP on the rCPI index of corruption. Higher levels of corruption are associated with lower rates of income taxation, lower levels of government spending, and in particular lower levels of spending on public education.

Ethnolinguistic Fractionalization (ELF) is a measure of the ethnic/linguistic diversity of a country. Roughly, it measures the probability that two individuals chosen at random belong to different ethnic groups. Drazanova (2019) has created a historical index of ELF for most countries in the world from 1945 to 2013. I will use the latest estimate from 2013. Table 2 displays results of regressing the rCPI index of corruption on ELF. Higher levels of ELF are associated with higher levels of corruption.

Table 3 displays results of regressing real, per-capita GDP on the income tax rate and corruption and Table 4 displays results of this regression where ELF is used as an instrument for the rCPI corruption perception index. The use of ELF as an instrument for corruption was similarly used by Mauro (1995) in his study of the relation between growth and corruption. By itself the income tax rate is not significantly related to GDP. However, once the rCPI is included, both variables become significant. Higher rates of income taxation are associated with lower levels of GDP, and higher levels of corruption are also associated with lower levels of GDP. The relation between GDP and corruption holds up even when ELF is used as an instrumental variable for the corruption index.

Table 5 summarizes the magnitude of the relationships documented in Table 3 (Panel B). Quantitatively, real per-capita GDP depends significantly on both corruption and income tax rates. For an income tax rate of 40 percent, as the rCPI index rises from 10 to 90 (roughly the range in the data), real per-capita GDP falls from $76,153 to $1,635. For an rCPI of 50, as the income tax rate rises from 20 to 60 percent, real per-capita GDP falls from $18,057 to $6,897. Both income tax rates and corruption seem to be related to real per-capita GDP in an important way, although getting a clearly sense of exactly how will require building a model of the dependence of GDP on corruption and taxation.
Table 1: Corruption, Taxes, and Spending

Panel A: non-trimmed data

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Income Tax (1)</th>
<th>Govt Spending (2)</th>
<th>Educ Spending (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rCPI</td>
<td>-0.201***</td>
<td>-0.261***</td>
<td>-0.035***</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.051)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.405***</td>
<td>0.478***</td>
<td>0.064***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.031)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Observations</td>
<td>138</td>
<td>138</td>
<td>117</td>
</tr>
<tr>
<td>R²</td>
<td>0.083</td>
<td>0.162</td>
<td>0.150</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.076</td>
<td>0.156</td>
<td>0.143</td>
</tr>
</tbody>
</table>

Panel B: trimmed data

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Income Tax (1)</th>
<th>Govt Spending (2)</th>
<th>Educ Spending (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rCPI</td>
<td>-0.223***</td>
<td>-0.240***</td>
<td>-0.031***</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.056)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.421***</td>
<td>0.463***</td>
<td>0.062***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.034)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Observations</td>
<td>128</td>
<td>128</td>
<td>109</td>
</tr>
<tr>
<td>R²</td>
<td>0.128</td>
<td>0.127</td>
<td>0.109</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.121</td>
<td>0.120</td>
<td>0.101</td>
</tr>
</tbody>
</table>

Note: All variables are measured as a fraction, not a percent. rCPI = reverse Corruption Perception Index. Income Tax = highest personal income tax rate. Govt Spending = total government spending as a fraction of GDP. Educ Spending = government spending on education as a fraction of GDP. Trimmed data excludes 5 countries with Income Tax = 0 and 5 countries with Income Tax > .5. Data Sources: Heritage Foundation, Transparency International, and World Bank World Development Indicators. *p<0.1; **p<0.05; ***p<0.01.
Table 2: ELF (2013) and Corruption

Panel A: non-trimmed data

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>rCPI</td>
<td></td>
</tr>
<tr>
<td>ELF</td>
<td>0.196***</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.484***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
</tr>
</tbody>
</table>

Observations 138  
R² 0.070  
Adjusted R² 0.063

Panel B: trimmed data

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>rCPI</td>
<td></td>
</tr>
<tr>
<td>ELF</td>
<td>0.179***</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.503***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
</tr>
</tbody>
</table>

Observations 128  
R² 0.066  
Adjusted R² 0.058

Note: rCPI =reverse Corruption Perception Index, ELF = Ethnolinguistic Fractionalization, Data Sources: Transparency International and Drazanova (2019). *p<0.1; **p<0.05; ***p<0.01.
Table 3: Regression on Income Tax and Corruption

Panel A: non-trimmed data

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDPpc (log)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IncomeTax</td>
<td>$-0.785$</td>
<td>$-2.819^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.795)</td>
<td>(0.571)</td>
</tr>
<tr>
<td>rCPI</td>
<td>$-4.927^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.399)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$9.490^{***}$</td>
<td>$12.908^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.252)</td>
<td>(0.326)</td>
</tr>
<tr>
<td>Observations</td>
<td>138</td>
<td>138</td>
</tr>
<tr>
<td>R²</td>
<td>0.007</td>
<td>0.534</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>-0.0002</td>
<td>0.527</td>
</tr>
</tbody>
</table>

Panel B: trimmed data

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDPpc (log)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IncomeTax</td>
<td>$0.350$</td>
<td>$-2.406^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.934)</td>
<td>(0.730)</td>
</tr>
<tr>
<td>rCPI</td>
<td>$-4.801^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.455)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$9.077^{***}$</td>
<td>$12.683^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.290)</td>
<td>(0.402)</td>
</tr>
<tr>
<td>Observations</td>
<td>128</td>
<td>128</td>
</tr>
<tr>
<td>R²</td>
<td>0.001</td>
<td>0.472</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>-0.007</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Note: GDPpc = per capita real GDP, IncomeTax = personal income tax, rCPI = reverse Corruption Perception Index, Data Sources: Heritage Foundation and Transparency International. *p<0.1; **p<0.05; ***p<0.01.
Table 4: Regression on Income Tax and Corruption (IV)

Panel A: non-trimmed data

<table>
<thead>
<tr>
<th>Dependent variable: GDPpc (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IncomeTax</td>
</tr>
<tr>
<td>rCPI</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

| Observations | 138 |
| R$^2$ | 0.264 |
| Adjusted R$^2$ | 0.253 |

Panel B: trimmed data

<table>
<thead>
<tr>
<th>Dependent variable: GDPpc (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IncomeTax</td>
</tr>
<tr>
<td>rCPI</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

| Observations | 128 |
| R$^2$ | 0.102 |
| Adjusted R$^2$ | 0.087 |

Note: ELF is an instrument for rCPI. GDPpc = per capita real GDP, IncomeTax = personal income tax, rCPI = reverse Corruption Perception Index. Data Sources: Heritage Foundation, Transparency International and Drazanova (2019).

*p<0.1; **p<0.05; ***p<0.01.
Table 5: Real, per-capita GDP Predictions
(from fitted regression, Table 3 (Panel B))

<table>
<thead>
<tr>
<th>Income Tax Rate</th>
<th>rCPI 20%</th>
<th>rCPI 40%</th>
<th>rCPI 60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>123,217</td>
<td>76,153</td>
<td>47,066</td>
</tr>
<tr>
<td>50%</td>
<td>18,057</td>
<td>11,160</td>
<td>6,897</td>
</tr>
<tr>
<td>90%</td>
<td>2,646</td>
<td>1,635</td>
<td>1,011</td>
</tr>
</tbody>
</table>

3.1 Alternative Measures of Corruption

Transparency International’s Corruption Perception Index is but one of many indices measuring corruption. To check the robustness of the results to some other measures of corruption, Tables 6 and 7 report regression results using nine other corruption indices. Three of the nine are the following from the Heritage Foundation database: Property Rights, Judicial Effectiveness, and Government Integrity. The remaining six are the following from the World Banks’ Worldwide Governance Indicators: Voice and Accountability, Political Stability and Absence of Violence/Terrorism, Government Effectiveness, Regulatory Quality, Rule of Law, and Control of Corruption. Table 6 reports results for the Heritage Foundation corruption indices and Table 7 reports results for the World Bank Worldwide Governance Indicators. The regression results reported are only for the regression of per-capita real GDP on a measure of corruption and a measure of taxation. In all of the regression results, the coefficient on the corruption index is statistically significant (and the correct sign), and just as important, for each regression the coefficient on the measure of taxation is negative and statistically significant.

3.2 Alternative Measures of Taxation

As an additional robustness exercise, I also experimented with alternative measures of taxation. The Heritage Foundation database includes the corporate income tax rate, and the World Bank World Development Indicators reports a value-added tax. SumTax1 is simply the sum of the personal income tax rate and the corporate income tax rate. As an attempt to take into account the importance of each, consider also the following method to include the effects of a corporate income tax. Let $W$ represent aggregate wage and salary income, and let $Z$ represent aggregate corporate income. Denote the personal income tax by $\tau_y$ and the corporate income tax by $\tau_z$, then total after-tax income can be written as
Table 6: Regression using Alternative Corruption Measures (1)

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>GDPpc (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>IncomeTax</td>
<td>$-1.806^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.528)</td>
</tr>
<tr>
<td>PropertyRights</td>
<td>$0.046^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>JudicialEffect</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>GovtIntegrity</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$7.422^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.226)</td>
</tr>
<tr>
<td>Observations</td>
<td>138</td>
</tr>
<tr>
<td>R²</td>
<td>0.575</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.569</td>
</tr>
</tbody>
</table>

Note: IncomeTax = personal income tax, PropertyRights = Property Rights, JudicialEffect = Judicial Effectiveness, GovtIntegrity = Government Integrity. Data Source: Heritage Foundation. *p<0.1; **p<0.05; ***p<0.01.
### Table 7: Regression using Alternative Corruption Measures (2)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong> GDPpc (log)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IncomeTax</td>
<td>$-2.710^{***}$</td>
<td>$-1.265^*$</td>
<td>$-2.227^{***}$</td>
<td>$-1.967^{***}$</td>
<td>$-2.530^{***}$</td>
<td>$-2.756^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.738)</td>
<td>(0.663)</td>
<td>(0.446)</td>
<td>(0.554)</td>
<td>(0.545)</td>
<td>(0.581)</td>
</tr>
<tr>
<td>VoiceAccount</td>
<td>0.707^{***}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PoliticalStability</td>
<td></td>
<td>0.772^{***}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.096)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GovtEffect</td>
<td></td>
<td></td>
<td>1.073^{***}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.060)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RegulatoryQuality</td>
<td></td>
<td></td>
<td></td>
<td>0.920^{***}</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.074)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RuleofLaw</td>
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<td></td>
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<td></td>
<td>0.987^{***}</td>
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</tr>
<tr>
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<td>(0.074)</td>
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<tr>
<td>ControlofCorruption</td>
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<td>0.930^{***}</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>(0.077)</td>
</tr>
<tr>
<td></td>
<td>(0.238)</td>
<td>(0.213)</td>
<td>(0.142)</td>
<td>(0.176)</td>
<td>(0.175)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>Observations</td>
<td>137</td>
<td>137</td>
<td>137</td>
<td>137</td>
<td>137</td>
<td>137</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.279</td>
<td>0.328</td>
<td>0.705</td>
<td>0.541</td>
<td>0.570</td>
<td>0.523</td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.268</td>
<td>0.318</td>
<td>0.700</td>
<td>0.534</td>
<td>0.564</td>
<td>0.516</td>
</tr>
</tbody>
</table>

Note: GDPpc = real per-capita GDP, IncomeTax = personal income tax, VoiceAccount = Voice and Accountability, PoliticalStability = Political Stability and Absence of Violence/Terrorism, GovtEffect = Government Effectiveness, RegulatoryQuality = Regulatory Quality, RuleofLaw = Rule of Law, ControlofCorruption = Control of Corruption. Data Sources: Heritage Foundation and World Bank World Governance Indicators. *p<0.1; **p<0.05; ***p<0.01
\[(1 - \tau)(W + Z) = (1 - \tau_y)(W + (1 - \tau_z)Z),\] from which can be derived \[\tau \approx \tau_y + \frac{Z}{W + Z}\tau_z,\] where the approximation ignores the interaction term \(\tau_y\tau_z.\) In the U.S., the Bureau of Economic Analysis estates that wage and salary income for 2020 is about \$8.5 trillion and corporate income is about \$9 trillion, so assuming roughly \(w = z,\) the overall tax rate is approximately \(\tau = \tau_y + \frac{1}{3}\tau_z,\) which is SumTax2 in Table 8. SumTax3 is SumTax2 plus the value added tax converted to an equivalent labor income tax (which is VAT/(1 + VAT)). Table 8 reports regression results using each of these other tax rates. Again, the regression results reported are only for the regression of per-capita real GDP on a measure of corruption and a measure of taxation. In all of the regression results, the coefficient on the measure of taxation is negative and statistically significant and the coefficient on the reverse Corruption Perception Index (rCPI) remains negative and statistically significant.

4 The Model

4.1 Setup

This is an overlapping-generations model with two groups of people, Groups A and B, in which each person lives for two periods. Group B is the group in power (or a collection of groups that collaborate and share power), by which is meant members of this group choose government policy. Group A comprises all other members of society. For each group \(i \in \{A, B\},\) each period is comprised of \(N^i\) young and \(N^i\) old, so the total population is \(2(N^A + N^B).\)

Young and old are endowed with one unit of time each period that they can allocate to market and non-market production. Both markets produce the same good, but non-market production is less efficient than market production. Allocating \(n\) units of time to market production yields

\[y = An\]  
units of a perishable consumption good, where \(A > 0,\) and allocating \(m\) units of time to non-market production yields \(A(m - (\nu/2)m^2)\) units of the same good, where \(0 < \nu \leq 1, n + m = 1\) and \(0 \leq n, m \leq 1.\)

\(^1\)The upper value \(\nu \leq 1\) is imposed to ensure that marginal productivity of non-market production is never negative. Thus, total production from one unit of time with allocation \((n, m)\) is \(A(1 - (\nu/2)m^2).\)

\(^1\)A following section considers an extension with a more general non-market production function \(A(1 - \frac{\nu}{\alpha}m^\alpha),\) for \(\alpha > 1.\)
### Table 8: Regression using Alternative Taxation Measures

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable:</th>
<th>GDPpc (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>SumTax1</td>
<td>$-2.186^{***}$</td>
<td>(0.361)</td>
</tr>
<tr>
<td>SumTax2</td>
<td>$-2.546^{***}$</td>
<td>(0.447)</td>
</tr>
<tr>
<td>SumTax3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rCPI</td>
<td>$-4.592^{***}$</td>
<td>(0.370)</td>
</tr>
<tr>
<td>Constant</td>
<td>$13.056^{***}$</td>
<td>(0.308)</td>
</tr>
</tbody>
</table>

| Observations   | 138                 | 138         | 98           |
| R²             | 0.568               | 0.557       | 0.501        |
| Adjusted R²    | 0.561               | 0.550       | 0.490        |

Note: GDPpc = real per-capita GDP, SumTax1 = personal income tax + corporate income tax, SumTax2 = personal income tax + 0.5*corporate income tax, SumTax3 = SumTax2 + value-added tax/(1 + value-added tax), rCPI = reverse Corruption Perception Index. Data Sources: Heritage Foundation, Transparency International and World Bank World Development Indicators. *p<0.1; **p<0.05; ***p<0.01.
Workers pay a proportional tax $\tau$ on market income. In addition to this payment, workers must also pay a bribe $\xi$ that is proportional to their income. Both decrease disposable income the same way, so a time allocation $(n, m)$ yields disposable income

$$A \left( (1 - \tau - \xi) n + m - \frac{\nu}{2} m^2 \right).$$

Denote market labor supply by each young person from Group $i$ by $n^i_y$ and old by $n^i_o$. Total tax revenue is thus given by

$$R = \Sigma_i \tau N^i A(n^i_y + n^i_o),$$

and total bribery revenue is given by

$$B = \Sigma_i \xi N^i A(n^i_y + n^i_o).$$

The essential difference between the tax rate $\tau$ and the bribery rate $\xi$ is that tax revenue generated by $\tau$ must be used for public spending, but bribery income is diverted for personal gain by Group $B$. Importantly, it is assumed that a corrupt government cannot steal from general tax revenue for their own personal gain (although clearly this has occurred in various countries). Assume that bribery income is divided evenly to all members of Group $B$ in a lump-sum fashion. Consumption in the current period by members of each group is thus given by

$$c^A_y = A((1 - \tau - \xi)n^A_y + m^A_y - (\nu/2)(m^A_y)^2),$$

$$c^A_o = A((1 - \tau - \xi)n^A_o + m^A_o - (\nu/2)(m^A_o)^2),$$

$$c^B_y = A((1 - \tau - \xi)n^B_y + m^B_y - (\nu/2)(m^B_y)^2) + B/(2N^B),$$

$$c^B_o = A((1 - \tau - \xi)n^B_o + m^B_o - (\nu/2)(m^B_o)^2) + B/(2N^B).$$

Each young person from Group $i$ consumes $c^i_y$ today and $c^i_o'$ tomorrow and has preferences

$$v_y = u(c^i_y) + \beta u(c^i_o'),$$

where $0 < \beta < 1$ and $u$ is a strictly-increasing, continuously-differentiable, concave function. Each old person consumes $c^i_o$ today and has preferences

$$v_o = u(c^i_o).$$
4.2 Optimal Time Allocation

Optimal time allocation by the old generation is to maximize $u(c^i_o)$, which leads to

$$m^i_o = \frac{\tau + \xi}{\nu},$$

and

$$n^i_o = 1 - \frac{\tau + \xi}{\nu}.$$ 

Define

$$\varphi = \frac{N^B}{N^A}.$$ 

A small value of $\varphi$ means that the group in power, Group $B$, is able to extract resources from a much larger group not in power, Group $A$. Consumption for a member of the old generation at their optimal time allocation for a given tax rate thus becomes (anticipating the bribery-income transfer for Group $B$)

$$c^A_o = A \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} \right),$$

$$c^B_o = A \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi \left( 1 - \frac{\tau + \xi}{\nu} \right) \right).$$

Similarly, for a forecast of $A'$, $\tau'$, and $\xi'$ while old, the young expect to enjoy consumption while old of

$$c^A'_o = A' \left( 1 - (\tau' + \xi') + \frac{(\tau' + \xi')^2}{2\nu} \right),$$

$$c^B'_o = A' \left( 1 - (\tau' + \xi') + \frac{(\tau' + \xi')^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi' \left( 1 - \frac{\tau' + \xi'}{\nu} \right) \right).$$

The young choose a current time allocation to maximize their preferences, knowing what choices they will make when old, which also yields

$$m^i_y = \frac{\tau + \xi}{\nu},$$

and

$$n^i_y = 1 - \frac{\tau + \xi}{\nu}.$$
Consumption for a member of the young generation in equilibrium for a given tax rate becomes

\[ c_y^A = A \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} \right), \]
\[ c_y^B = A \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi \left( 1 - \frac{\tau + \xi}{\nu} \right) \right). \]

### 4.3 Government Policy

Per-capita labor supply is given by

\[ n = 1 - \frac{\tau + \xi}{\nu}. \tag{2} \]

Per-capita public spending from tax revenue can be written as

\[ A\tau \left( 1 - \frac{\tau + \xi}{\nu} \right). \]

This spending is on activities that enhance private productivity, such as public education, an efficient legal system, infrastructure, etc., so that \( A' \) is assumed to evolve according to

\[ A' = G \left( A\tau \left( 1 - \frac{\tau + \xi}{\nu} \right) \right), \]

for some function \( G \) that satisfies Assumption 1.

**Assumption 1:** \( G : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is a strictly-increasing, continuously-differentiable, concave function such that \( G(0) = 0, G'(0) = \infty, \) and \( G'((\infty) = 0. \)

Given an expectation of \( \tau' \) and \( \xi' \) by the government in the next period, the current government is assumed to choose \( \tau \) and \( \xi \) to maximize the welfare only of members of Group B, with equal weights for each generation, given by

\[ W = \frac{1}{2} N^B v^B_y + \frac{1}{2} N^B v^B_o, \]

or

\[ W = N^B u \left( A \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi \left( 1 - \frac{\tau + \xi}{\nu} \right) \right) \right). \]

16
\[+ N^B \beta^* u \left( G \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) \right) \left( 1 - (\tau' + \xi') + \frac{(\tau' + \xi')^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi' \left( 1 - \frac{\tau' + \xi'}{\nu} \right) \right) \right),\]

where

\[\beta^* = \beta/2.\]

4.4 Equilibrium

By imposing the steady-state conditions \( A' = A, \tau' = \tau, \) and \( \xi' = \xi, \) the equilibrium, steady-state conditions that determine \( A, \tau, \) and \( \xi \) can be written as, provided \( \xi \geq 0,^2 \)

\[A = G \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) \right), \quad (3)\]

\[
\frac{1 - \frac{\tau + \xi}{\nu} + \frac{1 + \varphi}{\varphi} \frac{\xi}{\nu}}{(1 - \frac{2\tau + \xi}{\nu}) \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi \left( 1 - \frac{\tau + \xi}{\nu} \right) \right)} = \beta^* G' \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) \right), \quad (4)\]

and

\[
\frac{1 - \frac{\tau + \xi}{\nu} - \frac{1 + \varphi}{\varphi} \left( 1 - \frac{\tau + 2\xi}{\nu} \right)}{-\frac{\tau}{\nu} \left( 1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi}{\varphi} \xi \left( 1 - \frac{\tau + \xi}{\nu} \right) \right)} = \beta^* G' \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) \right). \quad (5)\]

Define

\[\delta = \frac{1}{2 + \varphi}.\]

The left sides of eqs. (4) and (5) can be equated to derive

\[\frac{\tau + \xi}{\nu} = \delta. \quad (6)\]

Substituting this result back into eqs. (3) and (4) yields

\[A = G \left( A \tau \left( 1 - \delta \right) \right) \quad (7)\]

and

\[
1 - \delta = \beta^* G' \left( A \tau \left( 1 - \delta \right) \right) \left( 1 - \tau \right) \left( 1 - \delta \right)^2 - \left( 1 - \frac{\nu}{2} \right) \delta^2. \quad (8)\]

Eq. (7) can be used to derive the relation \( A = \Gamma(\tau(1 - \delta)) \), which is an increasing

\[^2\text{The condition } \xi = 0 \text{ is discussed in the appendix.}\]
function of \( \tau \) such that \( \Gamma(0) = 0 \). To see this, define

\[
Z(A) = \frac{G(A\tau_1(1-\delta))}{A} - 1
\]

and note that the solution \( A = \Gamma(A) \) is such that \( Z(A) = 0 \).

Lemma 1: Under Assumption 1, there exists one, and only one, value of \( A \) such that \( Z(A) = 0 \).

Proof: \( Z \) is a strictly-decreasing function that becomes unbounded as \( A \) approaches zero and approaches -1 as \( A \) becomes unbounded. Q.E.D.

Fig. 1 is a graph of \( G \) that incorporates these results.

Substitute \( \Gamma(A) \) into eq. (8) to derive an equation only in \( \tau \):

\[
1 - \delta = \beta^* G'(\Gamma(\tau(1-\delta))\tau(1-\delta)) \left( (1-\tau)(1-\delta)^2 - \left( 1 - \frac{\nu}{2} \right) \delta^2 \right). \tag{9}
\]

Proposition 2 established the existence of a unique value of \( \tau \) that satisfies eq. (9).

Proposition 2: Under Assumption 1, there exists one, and only one, value of \( \tau \) that satisfies eq. (9).
Proof: The right side of eq. (9) is a strictly-decreasing function of \( \tau \), becomes arbitrarily large as \( \tau \) approaches zero, and approaches zero as \( \tau \) approaches \( 1 - (1 - \nu/2)(\delta/(1 - \delta))^2 \). Q.E.D.

The remaining variables can easily be derived from the solution \( \tau \): eq. (6) determines \( \xi \), the function \( \Gamma \) determines \( A \), per-capita labor supply \( n \) is determined by eq. (2), and per-capita income \( y \) is determined by eq. (1).

4.5 An Example

Suppose

\[ G(x) = D^{1-\gamma}x^{\gamma}, \]

for \( 0 < \gamma < 1 \). From eq. (7),

\[ A = D (\tau(1 - \delta))^{\frac{1}{1-\gamma}}, \]

so that

\[ \beta^* G' (A\tau (1 - \delta)) = \frac{\beta^* \gamma}{\tau(1 - \delta)}. \]

This leads to the solution

\[ \tau = \frac{\beta^* \gamma}{1 + \beta^* \gamma} \left( 1 - \left( 1 - \frac{\nu}{2} \right) \left( \frac{\delta}{1 - \delta} \right)^2 \right) \]

and

\[ \xi = \nu \delta - \frac{\beta^* \gamma}{1 + \beta^* \gamma} \left( 1 - \left( 1 - \frac{\nu}{2} \right) \left( \frac{\delta}{1 - \delta} \right)^2 \right). \]

Per-capita market income is given by

\[ y = D(1 - \delta)^{\frac{1}{1-\gamma}} \left( \frac{\beta^* \gamma}{1 + \beta^* \gamma} \left( 1 - \left( 1 - \frac{\nu}{2} \right) \left( \frac{\delta}{1 - \delta} \right)^2 \right) \right)^{\frac{1}{1-\gamma}}. \]

With this solution, a rise in \( \delta \) (fall in \( \varphi \)) leads to a fall in \( \tau \), a rise in both \( \xi \) and \( \tau + \xi \), and a fall in \( y \). That is, the larger is Group \( A \) that Group \( B \) can extract resources from, the more Group \( B \) will choose to rely on bribery to increase its welfare, with a consequent fall in the tax rate to mitigate the rise in the overall distortion, and with a cost of reducing per-capita market income \( y \).
5 Model Extension: Non-productive public goods

5.1 Setup

The model just derived will have difficulty matching a negative relation between observed tax rates and GDP, as taxes are chosen optimally only to finance a productive public good. In the example above, e.g., a rise in \( \delta \) leads to a fall in both \( \tau \) and \( y \), which contributes to a positive relation between tax rates and income. Here the model is extended to include a non-productive public good, such as income equality, military expenditure, and environmental protection. Countries that put a high value on non-productive public goods may thus be observed to choose a high tax rate to finance this expenditure, at the cost of overall GDP.

Denote the per-capita quantity of a non-productive public good by \( h \). To simplify the derivation of results and thereby the exposition, the discussion will be limited to a log utility function. The utility function \( u \) now includes a non-productive public good, so that

\[
u = \log(c) + \eta \log(h),
\]

where \( c \) is the consumption of the private good. The coefficient \( \eta > 0 \) captures preferences for the non-productive public good. Since \( h \) is spent on the non-productivity public good, tax revenue directed to the productive public good is

\[
A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) - h,
\]

so that \( A \) evolves according to

\[
A' = G \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) - h \right).
\]

5.2 Equilibrium

The government is again assumed to choose a policy that maximizes the welfare only of members of Group \( B \), but now the choice involves directing part of tax revenue to a non-productive public good. The steady-state equilibrium conditions are similar to those before, except now reflecting an optimal choice of \( h \):

\[
A = G \left( A \tau \left( 1 - \frac{\tau + \xi}{\nu} \right) - h \right), \quad (10)
\]
\[
\frac{1 - \frac{\tau + \xi}{\nu} + \frac{1 + \varphi \xi}{\varphi \nu}}{1 - \frac{2(\tau + \xi)}{\nu} \left(1 - \frac{\tau + \xi}{\nu} + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi \xi}{\varphi \nu} \left(1 - \frac{\tau + \xi}{\nu}\right)\right)} = \beta^*G' \left(A\tau \left(1 - \frac{\tau + \xi}{\nu}\right) - h\right), \tag{11}
\]

\[
\frac{1 - \frac{\tau + \xi}{\nu} - \frac{1 + \varphi \xi}{\varphi \nu}}{1 - (\tau + \xi) + \frac{(\tau + \xi)^2}{2\nu} + \frac{1 + \varphi \xi}{\varphi \nu} \left(1 - \frac{\tau + \xi}{\nu}\right)} = \beta^*G' \left(A\tau \left(1 - \frac{\tau + \xi}{\nu}\right) - h\right), \tag{12}
\]

and

\[
\frac{\eta A}{\bar{h}} = \beta^*G' \left(A\tau \left(1 - \frac{\tau + \xi}{\nu}\right) - h\right). \tag{13}
\]

The simplification due to a log utility specification is evident in eq. (13), as with a more general utility function \(u(c, h)\), the left side of this equation would become \(Au_h(c, h)/cu(c, h)\).

The left sides of eqs. (11) and (12) are the same as the basic model, so eq. (6) continues to hold. Define \(\hat{h} = h/A\). Eqs. (10) and (13) can be written as

\[
A = G \left(A\tau (1 - \delta) - \hat{h}\right), \tag{14}
\]

\[
\frac{\eta}{\hat{h}} = \beta^*G' \left(A\tau (1 - \delta) - \hat{h}\right). \tag{15}
\]

As stated in the following proposition, these two equations can be used to derive functions \(A = \Omega_A(\tau(1 - \delta))\) and \(\hat{h} = \Omega_h(\tau(1 - \delta))\) that solve for \(A\) and \(\hat{h}\) as functions of \(\tau\).

**Proposition 3:** Under Assumption 1, there exist one, and only one, pair of values \((\tau, \hat{h})\) that solve eqs. (14) and (15). The solution \(\hat{h} = \Omega_h(\tau(1 - \delta))\) is an increasing function of \(\tau\).

**Proof:** Eq. (14) defines an inverse relation between \(A\) and \(\hat{h}\), eq. (15) defines a positive relation, and these two relations intersect at a unique point. This result is depicted in Fig. 2. Both curves in Fig. 2 shift to the right with a rise in \(\tau\), hence \(\Omega_h(\tau(1 - \delta))\) is an increasing function of \(\tau\). Q.E.D.

Since \(\Omega_h(\tau(1 - \delta))\) is an increasing function of \(\tau\), both \(\eta/\Omega_h(\tau(1 - \delta))\) and

\[
G' \left(\Omega_A(\tau(1 - \delta)) (\tau (1 - \delta) - \Omega_h(\tau(1 - \delta)))\right)
\]

are decreasing functions of \(\tau\). In a manner similar to above, substitute these results into eq. (11) to derive that \(\tau\) must satisfy

\[
1 - \delta = \beta^*G' \left(\Omega_A(\tau(1 - \delta)) (\tau (1 - \delta) - \Omega_h(\tau(1 - \delta)))\right) \left((1 - \tau)(1 - \delta)^2 - \left(1 - \frac{\nu}{2}\right) \delta^2\right). \tag{16}
\]
Figure 2

Steady State $A$ and $\hat{h}$

$A = G(A(\tau(1 - \delta) - \hat{h}))$

$\frac{\eta}{\hat{h}} = \beta^* G'(A(\tau(1 - \delta) - \hat{h}))$

**Proposition 4:** Under Assumption 1, there exists one, and only one, value of $\tau$ that solves eq. (16).

**Proof:** Given Proposition 3, the right side is a strictly-decreasing function of $\tau$ that become unbounded as $\tau$ approaches zero, and approaches zero as $\tau$ approaches $1 - \left(1 - \frac{\delta}{2}\right)\left(\frac{\delta}{1 - \delta}\right)^2$.

Q.E.D.

Given $\tau$, the remaining variable $\xi$ follows easily from eq. (6), provided the solution yields $\xi > 0$. If not, then the solution is $\xi = 0$ and the solution for the remaining variables is given in the appendix.

5.3 An Example

Suppose

$G(x) = D^{1-\gamma}x^\gamma$.

Eqs. (10) and (13) can be used to derive

$\frac{h}{A} = \frac{\eta}{\beta^*\gamma + \eta} \tau(1 - \delta), $
and

\[ A = D \left( \frac{\beta^* \gamma}{\beta^* \gamma + \eta} \tau(1 - \delta) \right)^{\frac{1}{1 - \gamma}}, \]

so that

\[ \beta^* G' (A \tau (1 - \delta) - h) = \frac{\beta^* \gamma + \eta}{\tau(1 - \delta)}. \]

Use this result in eqs. (11) and (12) to derive the solution

\[ \tau = \frac{\beta^* \gamma + \eta}{1 + \beta^* \gamma + \eta} \left( 1 - \left(1 - \frac{\nu}{2}\right) \left( \frac{\delta}{1 - \delta} \right)^2 \right) \tag{17} \]

and

\[ \xi = \nu \delta - \frac{\beta^* \gamma + \eta}{1 + \beta^* \gamma + \eta} \left( 1 - \left(1 - \frac{\nu}{2}\right) \left( \frac{\delta}{1 - \delta} \right)^2 \right). \]

Per-capita market income \( y \) is given by

\[ y = D(1 - \delta)^{\frac{1}{1 - \gamma}} \left( \frac{\beta^* \gamma}{1 + \beta^* \gamma + \eta} \left( 1 - \left(1 - \frac{\nu}{2}\right) \left( \frac{\delta}{1 - \delta} \right)^2 \right) \right)^{\frac{1}{1 - \gamma}}. \tag{18} \]

Thus, a higher preference for the non-productive public good as reflected in a higher value of \( \eta \) leads to a higher tax rate \( \tau \) and a lower value of \( y \). Differences in \( \eta \) across countries will thus contribute to a negative relation between tax rates \( \tau \) and income \( y \), especially conditional on a level of corruption as measured by \( \xi \).

### 6 Model Extension with Production Concavity

Eq. (6) presents an empirical problem, as with \( \delta \leq 1/2 \) and \( \nu < 1 \), it constrains

\[ \tau + \xi \leq \frac{1}{2}. \tag{19} \]

Observed tax rates often exceed 1/2, as they do in the dataset using in the empirical work for this paper. Such tax rates are inconsistent with this model, which will lead to problems in estimating the model’s parameters. Adding some measure of rent extraction as reflected in \( \xi \) only compounds this issue.

This problem can be resolved by generalizing the production process such that after-
tax/bribery income is
\[ A \left( (1 - \tau - \xi)n + m - \frac{\nu}{\alpha} m^\alpha \right), \]
where \( \alpha > 1 \). In the prior quadratic specification \( \alpha = 2 \). With this specification of the production function, all households choose a time allocation such that
\[
\begin{align*}
n &= 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}}, \\
m &= \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}}. 
\end{align*}
\]
It then follows that
\[
c^B = 1 - (\tau + \xi) + \left( \frac{\alpha - 1}{\alpha} \right) \nu \left( \frac{\tau + \xi}{\nu} \right)^{\frac{\alpha}{\alpha - 1}} + \left( \frac{1 + \varphi}{\varphi} \right) \xi \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}} \right).
\]
Government tax revenue directed to the productive public good is now
\[ A\tau \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) - h, \]
so that \( A \) evolves according to
\[ A' = G \left( A\tau \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) - h \right). \]
A detailed discussion of the equilibrium conditions for this model is in the appendix, including a closed-form solution for the specific function \( G \) used in previous examples. As it relates to the empirical issue that motivated this extension, define
\[ \Delta = \frac{\alpha - 1}{\alpha + \varphi}. \]
The equation corresponding to eq. (6) is then
\[ \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha - 1}} = \Delta. \]
Note that, via L'Hopital’s Rule, \( \Delta^{\alpha - 1} \) converges to 1 as \( \alpha \) converges to 1. As an example,
for $\nu = 1$, the model can be consistent with any $\tau + \xi \leq 1$.

7 Country Heterogeneity

To capture the different experiences of countries, suppose countries differ by values of $\varphi$, $\eta$, and $D$. Variation in $\varphi$ captures differences in the ability of a small group in power to extract resources from the country. Variation in $\eta$ captures heterogeneity in preferences for non-productive public goods. Variation in $D$ captures various sources of differences in income not captured by the model, such as geography or natural resources. To model this variation, suppose countries receive a random draw according to the following stochastic processes:

$$
\log(\varphi) = \log(\bar{\varphi}) + \epsilon_\varphi,
$$

$$
\log(\eta) = \log(\bar{\eta}) + \epsilon_\eta,
$$

$$
\log(D) = \log(\bar{D}) + \epsilon_D,
$$

where $\epsilon_\varphi$ is drawn from a Normal$(0, \sigma^2_\varphi)$ distribution, $\epsilon_\eta$ is drawn from a Normal$(0, \sigma^2_\eta)$ distribution, and $\epsilon_D$ is drawn from a Normal$(0, \sigma^2_D)$ distribution. The random draws are independent, both between variables and across countries.

8 Bribery and Measures of Corruption

Transparency International constructs an index of perceptions of corruption that is used to construct $rCPI$ for each country and the model captures corruption by a bribery rate $\xi$. Although it is reasonable to conjecture they are monotonically related, there is no reason to expect they are measured in the same units (or even linearly related). To partially address this issue, I will assume they are at least log-linearly related so that

$$
\xi = rCPI^\omega
$$

for some parameter $\omega$ that is constant across countries. The hypothesis is that agents observe $\xi$ and report $\xi^{1/\omega}$ as $rCPI$ in surveys.
9 Algorithm for Estimating/Calibrating Parameters

The model contains 11 parameters whose values are unknown: $\beta$, $\nu$, $\gamma$, $\alpha$, $\omega$, $\bar{\varphi}$, $\bar{\eta}$, $\bar{D}$, $\sigma^2_{\varphi}$, $\sigma^2_{\eta}$, $\sigma^2_D$. I will essentially use the first and second moments of the log of GDP, the income tax rate, and the corruption index to guide the selection of parameters. In total this amounts to 9 moments, which may require reducing the number of free parameters. To start, though, it will be convenient to divide the estimation into two parts, with one part estimating $\beta$, $\nu$, $\gamma$, $\alpha$, and $\omega$, and the other part estimating $\bar{\varphi}$, $\bar{\eta}$, $\bar{D}$, $\sigma^2_{\varphi}$, $\sigma^2_{\eta}$, and $\sigma^2_D$ conditional on values of the first set of parameters. This separation is convenient because the latter set of parameters can be estimated with a closed-form, non-iterative method.

Given values of $\beta$, $\nu$, $\gamma$, $\alpha$, and $\omega$, the remaining parameters can be estimated with the following method. In the following description of the algorithm, to distinguish between variables that are assumed constant across countries and variables that vary, all variables that vary will be designated with a subscript $i$. To avoid complications from $IncomeTax_i = 0$, I will only use the trimmed data.

Step 1: estimate $\bar{\varphi}$, and $\sigma^2_{\varphi}$. Use observed values of $IncomeTax_i$ and $rCPI_i$ to compute implied values of $\Delta_i$ from (recall $\xi_i = rCpi\omega$)

$$\Delta_i = \left(\frac{IncomeTax_i + rCPI_i\omega}{\nu}\right)^{\frac{1}{\alpha-1}}.$$

and use these values to compute

$$\varphi_i = \frac{\alpha - 1 - \alpha\Delta_i}{\Delta_i}.$$

Use these implied values of $\varphi_i$ to compute $\bar{\varphi}$, and $\sigma^2_{\varphi}$.

Step 2: estimate $\bar{\eta}$ and $\sigma^2_{\eta}$. Use observed values of $IncomeTax_i$ and implied values of $\Delta_i$ along with the relation in eq. (38) written with country heterogeneity as

$$\frac{(1 - \Delta_i)^2(\alpha - 1)IncomeTax_i}{\alpha - 1 - \alpha\Delta_i} = \beta*\gamma + \eta_i$$

$$1 - \nu\Delta_i^{\alpha-1} + \frac{\alpha - 1}{\alpha}\nu\Delta_i^\alpha + \frac{(1 - \Delta_i)^2(\alpha - 1)}{\alpha - 1 - \alpha\Delta_i} \nu\Delta_i^{\alpha-1} - IncomeTax_i$$

For some parameter values, estimates of $\varphi_i$ could become negative, so I will truncate $\varphi_i$ to some very small quantity (.0001).
to compute implied values of $\eta_i$. Use these implied values of $\eta_i$ to compute $\bar{\eta}$ and $\sigma^2_\eta$.

Step 3: estimate $\bar{D}$ and $\sigma^2_D$. Use observed values of GDP$_i$ and Income$\text{Tax}_i$ and implied values of $\Delta_i$ and $\eta_i$ to compute $D_i$ from eq. (18) written with country heterogeneity as

$$GDP_i = D_i(1 - \Delta_i)^{1/\gamma} \left( \frac{\beta^* \gamma_{i}^{\gamma}}{\beta^* \gamma_{i}^{\gamma} + \eta_i} \right)^{1/\gamma}.$$  

Use these implied value of $D_i$ to compute $\bar{D}$ and $\sigma^2_D$.

For a choice of parameter values for $\beta, \nu, \gamma, \alpha$, and $\omega$ (along with the related values for $\bar{\varphi}, \bar{\eta}, \bar{D}, \sigma^2_{\varphi}, \sigma^2_{\eta}$, and $\sigma^2_D$), the model can be solved numerically and simulated. In solving the model, the restriction that $\xi$ cannot be negative will be imposed. To compute various numerical properties of the model, I will simulate the model for 10,000 time periods. Each simulation is a draw ($\epsilon_{\varphi}, \epsilon_{\eta}, \epsilon_D$) from a Normal distribution with mean zero and variances determined by ($\sigma^2_{\varphi}, \sigma^2_{\eta}, \sigma^2_D$) (covariances are set to zero).\footnote{For numerical stability, only one simulation from a Normal distribution with mean zero and unit variance is drawn. Denote this simulation as $(u_1, u_2, u_3)$, where 10,000 values of this triple are drawn. The simulation for $(\epsilon_{\varphi}, \epsilon_{\eta}, \epsilon_D)$ is then $(\epsilon_{\varphi}, \epsilon_{\eta}, \epsilon_D) = (\sigma_{\varphi} u_1, \sigma_{\eta} u_2, \sigma_D u_3)$. The simulation for $(u_1, u_2, u_3)$ is held fixed as different values of $(\sigma_{\varphi}, \sigma_{\eta}, \sigma_D)$ are evaluated.}

To evaluate the fit of the model to the data, I will evaluate how close simulations of the model are able to match three regression coefficients observed in the data: in a regression of the income tax on the index of corruption, (1) the coefficient on corruption, and in a regression of real per-capita GDP (log) on the income tax and the measure of corruption, (2) the coefficient on the income tax and (3) the coefficient on the measure of corruption. Denote these three coefficients estimated from the data as $m = (m_1, m_2, m_3)$ expressed as a row vector and the corresponding coefficients based on simulated data as $\hat{m}$. Specifically, the data regression coefficients are obtained from regressions using observed data of Income$\text{Tax}_i$ on $rCpi_i$ and log$GDP_i$ on Income$\text{Tax}_i$ and $rCpi_i$. The model regression coefficients are obtained from regressions using simulated data of $\tau$ on $\xi^{\frac{1}{\omega}}$ and $y$ on $\tau$ and $\xi^{\frac{1}{\omega}}$. For a $3 \times 3$ weighting matrix $W$, the closeness of fit is determined by

$$(\hat{m} - m)W(\hat{m} - m)'.$$  

Following McFadden (1989), the weighting matrix $W$ will be $\Sigma^{-1}$, where $\Sigma$ is an estimate of the variance-covariance matrix of the estimator for $m$. To compute $\Sigma$, I use the following bootstrap method. Let $n$ be the sample size in the data. 10,000 different times, draw a
random sample of size $n$ from the data (with replacement) and compute 10,000 different values of $m$. $\Sigma$ is the variance-covariance matrix computed using these 10,000 values of $m$.

To reduce the number of free parameters to be estimated, I will assume a 10 year horizon with an annual discount rate of 5 percent to set $\beta = .6$. To relate aggregate productivity $A$ to the investment in productivity-enhancing public goods $h$, I will use the square root function, so $\gamma = .5$. Lastly, I will choose $\nu = 1$. As already discussed, low values of $\nu$ make it difficult for the model to match some observations with high tax rates, so I will start by choosing the highest possible value of $\nu$, which ranges from 0 to 1.\(^6\) The two remaining parameters to be estimated are $\alpha$ and $\omega$. A benchmark choice for the cost of non-market production would seem to be a quadratic as reflected in $\alpha = 2$, but as already discussed lower values of $\alpha$ are necessary for the model to predict some high tax rates observed in the data, hence as mentioned I will estimate this parameter. I have no clear sense of how to relate values of the corruption perception index to the bribery rate as captured by the parameter $\omega$, so as mentioned I will estimate this parameter as well.

10 Estimation/Calibration Results

The results of this estimation/calibration are reported in Table 9. Using these parameter estimates, the model is simulated for 10,000 time periods (using the same random sample from the estimation phase). These simulations are used to compute various statistics, including the regression coefficients used to estimate the model, which are compared to corresponding statistics from the data. Regressions from model-generated simulated data are in Tables 10 and 11. Table 10 reports results from regressing the tax rate $\tau$ on the index of corruption $\xi^{1/\omega}$ and Table 11 reports results from regressing market per-capita income $y$ on tax rates $\tau$ and the index of corruption $\xi^{1/\omega}$. For the regression of tax rates on corruption, in the simulation the coefficient on corruption is $-0.400$ and in the data (Table 1, Panel B) it is $-0.223$. For the regression of market income on tax rates and corruption, in the simulation the coefficient on tax rates is $-5.928$ and in the data (Table 3, Panel B) it is $-2.406$, and in the simulation the coefficient on corruption is $-1.436$ and in the data it is $-4.801$. The estimated model matches all the signs of the coefficients, and the magnitudes of the coefficients are all quantitatively large. The model tends to ascribe more importance to tax rates in reducing market income than is evident in the data (however, note that the coefficient in

\(^6\) All attempts at estimating this parameter led to the boundary value $\nu = 1$.\)
Table 9: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>time preference (10 years)</td>
<td>0.6†</td>
</tr>
<tr>
<td>$\nu$</td>
<td>non-market production cost</td>
<td>1†</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>efficiency curvature parameter</td>
<td>0.5†</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>non-market curvature</td>
<td>1.30</td>
</tr>
<tr>
<td>$\omega$</td>
<td>bribery to survey parameter</td>
<td>3.21</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>determines aver. size of group in power</td>
<td>0.42</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>determines aver. pref. for non-prod. public good</td>
<td>0.21</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>determines aver. production efficiency</td>
<td>177,940</td>
</tr>
<tr>
<td>$\sigma_\varphi$</td>
<td>std. dev. of log($\varphi$)</td>
<td>3.78</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>std. dev. of log($\eta$)</td>
<td>2.33</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>std. dev. of log($D$)</td>
<td>1.04</td>
</tr>
</tbody>
</table>

†Not estimated.

the IV regression (Table 4, Panel B) is -4.850, which is much closer to the estimated value for the model), and less to corruption. The greater importance of corruption in the data is perhaps because in addition to the supply-side channel by which corruption affects aggregate income that is the focus of this paper, there may be other mechanisms by which corruption affects aggregate income in the data. The effects of corruption are surely more complex than what is captured in this model, but the model clearly captures the quantitatively-significant dimension of the interplay between tax rates and corruption in determining market income.

11 Some Experiments

At the estimated parameter values, Fig. 3 shows the dependence in the model of market output $y$ (Panel A), tax rate $\tau$ (Panel B), and bribery rate $\xi$ (Panel C) on the determinant of corruption $\varphi$. Recall that a low value of $\varphi$ means that a small group in power is able to extract rents from a large group not in power. Here we see that as the group in power is able to extract resources from a larger group (going from right to left in the figures), the bribery rate rise, the tax rate falls, and market output falls. The magnitudes are quantitatively significant: as $\varphi$ falls and the bribery rate rises from 15 percent to 45 percent, the tax rate falls from about 25 percent to 17 percent and market output falls by roughly half. Market output falls because a larger fraction of revenue extracted from the private sector is diverted to financing consumption goods for the political elite.
Table 10: Tax Rate Regression, Simulated Data

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tau</td>
<td></td>
</tr>
<tr>
<td>$\xi^{(1/\omega)}$</td>
<td>$-0.400^{\ast\ast\ast}$</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.491$^{\ast\ast\ast}$</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Observations 10,000  
$R^2$ 0.510  
Adjusted $R^2$ 0.510

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 11: Output Regression, Simulated Data

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yLog</td>
<td></td>
</tr>
<tr>
<td>tau</td>
<td>$-5.928^{\ast\ast\ast}$</td>
<td>(0.095)</td>
</tr>
<tr>
<td>$\xi^{(1/\omega)}$</td>
<td>$-1.436^{\ast\ast\ast}$</td>
<td>(0.053)</td>
</tr>
<tr>
<td>Constant</td>
<td>11.343$^{\ast\ast\ast}$</td>
<td>(0.052)</td>
</tr>
</tbody>
</table>

Observations 10,000  
$R^2$ 0.311  
Adjusted $R^2$ 0.311

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Figure 3: **Simulation of Model.**  

**Panel A:** Plot of market output $y$ versus determinant of corruption $\phi$ (low $\phi$ means a small group in power can extract rents from a large group not in power.  

**Panel B:** Plot of tax rate $\tau$ against $\phi$.  

**Panel C:** Plot of bribery rate $\xi$ against $\phi$.  


Fig. 4 shows the dependence in the model of market output $y$ (Panel A), tax rate $\tau$ (Panel B), and bribery rate $\xi$ (Panel C) on the preference for non-productive public good as measured by $\eta$. Here we see that a lower value for the public good (again going from right to left in the figures), the tax rate falls, the bribery rate rises, and market output rises. That is, as less revenue is required to finance the non-productive public good, tax rates fall. The fall in tax rate leads to a rise in the bribery rate, although the overall effect is still for market output to rise. The magnitudes are quantitatively significant: as the preference for the non-productive good falls and the tax rate falls from about 60 percent to 10 percent, the bribery rate rises from zero to 50 percent and market output more than triples. Market output rises because even though the bribery rate rises, as a larger fraction of tax revenue is directed to financing public goods that enhance private productivity (the fraction of tax revenue directed to financing public goods that enhance private productivity is given by $\beta^* \gamma / (\beta^* \gamma + \eta)$).

12 Summary

Quantitatively, this paper has bolstered the evidence that supply-side forces are important determinants for wealth creation. This paper documents a robust negative relation across countries between rates of taxation and per-capita income. In documenting this relation, a principal argument of this paper is that it’s important to control for levels of corruption. Corrupt countries tend to have low levels of per-capita income and tend to impose low rates of taxation, thereby confounding a simple bi-variate relation between taxation and per-capita income. A robust empirical relation documented in this paper is that both excessive taxation and corruption are negatively associated with per-capita income.

To explain the effects of corruption and the relation between corruption and taxation documented in the data, this paper viewed the effects of corruption through a similar supply-side lens as explicit taxation. Rent are extracted by corrupt governments through the use of bribes that have a similar private disincentive effect as explicit taxation. Corrupt governments impose excessive rates of bribery, which destroys wealth. Moreover, the use of bribery competes with explicit taxation, so corrupt governments that impose high rates of bribery find it optimal to impose low rates of taxation. The model thus provides a simple supply-side argument whereby corruption destroys wealth and explains why corruption is inversely associated with explicit rates of taxation.

While much of the political economy is taken as given, the essential aspect of why
Figure 4: **Simulation of Model.** Panel A: Plot of market output $y$ versus determinant of preference for public good $\eta$. Panel B: Plot of tax rate $\tau$ against $\eta$. Panel C: Plot of bribery rate $\xi$ against $\eta$. 
some governments choose high levels of corruption and others do not is endogenous in the model. The model takes as given that all governments can choose their level of corruption, at least in terms of extracting resources though bribery, without fear of public revolt or other legal consequence. Self-interested governments choose a level of corruption to enhance the welfare of the group in power. This choice involves a tradeoff between the destruction of wealth against the reallocation of wealth. A key determinant is the relative size of the group in power, as the dilution of reallocated wealth sometimes leads governments to choose benevolence as their best option. From this perspective, the value of political accountability is the dilution of rents, which may be sufficient to discourage rent-seeking behavior by the government.
References


A Appendix

A.1 Basic Model with $\xi = 0$

If $\xi = 0$, the equilibrium, steady-state conditions that determine $A$ and $\tau$ can be written as

$$A = G \left( A\tau \left( 1 - \frac{\tau}{\nu} \right) \right)$$

(23)

and

$$\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \beta^* G' \left( A\tau \left( 1 - \frac{\tau}{\nu} \right) \right)$$

(24)

As in the main text, define $A = \Gamma \left( \tau \left( 1 - \frac{\xi}{\nu} \right) \right)$ and substitute this result into eq. (24) to show that $\tau$ must solve

$$\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \beta^* G' \left( \Gamma \left( \tau \left( 1 - \frac{\xi}{\nu} \right) \right) \tau \left( 1 - \frac{\tau}{\nu} \right) \right).$$

(25)

**Proposition A1:** Under Assumption 1, there exists one, and only one, value of $\tau$ that solves eq. (25).

**Proof:** The left side is a strictly-increasing function of $\tau$ for $0 < \tau < \nu/2$, equals 1 for $\tau = 0$ and becomes unbounded as $\tau$ approaches $\nu/2$. The right side is a strictly-decreasing function of $\tau$ for $0 < \tau < \nu/2$, and becomes unbounded as $\tau$ approaches zero. Q.E.D.

**Example:** Suppose

$$G(x) = D^{1-\gamma}x^\gamma$$

From (23),

$$A = D \left( \tau \left( 1 - \frac{\tau}{\nu} \right) \right)^{\frac{1}{1-\gamma}}.$$

and from (24), $\tau$ solves

$$\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \frac{\beta^*\gamma}{\tau \left( 1 - \frac{\tau}{\nu} \right)}.$$

A.2 Extended Model with $\xi = 0$

If $\xi = 0$, the equilibrium, steady-state conditions that determine $A$, $\tau$ and $h$ can be written as

$$A = G \left( A\tau \left( 1 - \frac{\tau}{\nu} \right) - h \right)$$

(26)
\[
\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \beta^* 1 + \frac{\phi}{\varphi} G'(A\tau (1 - \frac{\tau}{\nu}) - h)
\]

(27)

and

\[
\frac{\eta A}{h} = \beta^* G' \left( A\tau (1 - \frac{\tau}{\nu}) - h \right).
\]

(28)

Define \( \hat{h} = h/A \) and use the same function \( \Omega \) defined in the text to solve \( A = \Omega_A(\tau(1 - \tau/\nu)) \) and \( \hat{h} = \Omega_h(\tau(1 - \tau/\nu)) \). Substitute these functions into eq. (27) to show that \( \tau \) must solve

\[
\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \beta^* 1 + \varphi \left( \Omega_A \left( \tau \left(1 - \frac{\tau}{\nu} \right) \right) \left(1 - \frac{\tau}{\nu} \right) - \Omega_h \left( \tau \left(1 - \frac{\tau}{\nu} \right) \right) \right).
\]

(29)

**Proposition A2:** Under Assumption 1, there exists one, and only one, value of \( \tau \) that solves eq. (29).

**Proof:** The left side is a strictly-increasing function of \( \tau \) for \( 0 < \tau < \nu/2 \), equals 1 for \( \tau = 0 \) and becomes unbounded as \( \tau \) approaches \( \nu/2 \). The right side is a strictly-decreasing function of \( \tau \) for \( 0 < \tau < \nu/2 \), and becomes unbounded as \( \tau \) approaches zero. **Q.E.D.**

**Example:** Suppose

\[
G(x) = D^{1-\gamma}x^\gamma
\]

From eqs. (26) and (28) we can derive,

\[
\frac{h}{A} = \frac{\eta}{\beta^* \gamma + \eta} \tau \left(1 - \frac{\tau}{\nu} \right),
\]

and

\[
A = D \left( \frac{\beta^* \gamma}{\beta^* \gamma + \eta} \tau \left(1 - \frac{\tau}{\nu} \right) \right)^{\frac{1}{1-\gamma}},
\]

so that

\[
\beta^* G' \left( A\tau (1 - \delta) - h \right) = \frac{\beta^* \gamma + \eta}{\tau \left(1 - \frac{\tau}{\nu} \right)}.
\]

and from (27), \( \tau \) solves

\[
\frac{1 - \frac{\tau}{\nu}}{(1 - \frac{2\tau}{\nu})(1 - \tau + \frac{\tau^2}{2\nu})} = \frac{\beta^* \gamma + \eta}{\tau \left(1 - \frac{\tau}{\nu} \right)}.
\]

38
A.3 Extended Model with Production Concavity

A.3.1 Equilibrium with $\alpha > 1$ and $\xi > 0$

The equilibrium conditions can be written as

$$ A = G \left( A \tau \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} \right) - h \right), \quad (30) $$

$$ 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} + \frac{1 + \varphi}{\varphi} \frac{\nu}{\alpha - 1} \left( \frac{\tau + \xi}{\nu} \right)^{2-\alpha} $$

$$ \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} - \frac{\tau - 1}{\nu} (\frac{\tau + \xi}{\nu})^{\frac{2-\alpha}{\alpha-1}} \right) \left( 1 - (\tau + \xi) + \left( \frac{\alpha - 1}{\alpha} \right) \nu (\frac{\tau + \xi}{\nu})^{\frac{\alpha}{\alpha-1}} + \left( \frac{1 + \varphi}{\varphi} \right) \xi (1 - (\frac{\tau + \xi}{\nu})^{\frac{1}{\alpha-1}}) \right) $$

$$ = \beta^* G' \left( A \tau \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} \right) - h \right), \quad (31) $$

$$ \eta \frac{ \hat{h} }{ h } = \beta^* G' \left( A \tau \left( 1 - \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} \right) - h \right). \quad (32) $$

Define

$$ \Delta = \frac{\alpha - 1}{\alpha + \varphi}. $$

Equate the left sides of eqs. (31) and (32) to derive

$$ \left( \frac{\tau + \xi}{\nu} \right)^{\frac{1}{\alpha-1}} = \Delta. \quad (34) $$

Define again $\hat{h} = h/A$ and substitute the above relation into eqs. (30) and (33) to show that

$$ A = G \left( A (\tau (1 - \Delta) - \hat{h}) \right), \quad (35) $$

$$ \frac{\eta}{\hat{h}} = \beta^* G' \left( A (\tau (1 - \Delta) - \hat{h}) \right). \quad (36) $$
These two equations can again be used to derive functions \( A = \Omega_A(\tau(1 - \Delta)) \) and \( \hat{h} = \Omega_h(\tau(1 - \Delta)) \) that solve for \( A \) and \( \hat{h} \) as functions of \( \tau \), except with \( \Delta \) replacing \( \delta \).

This equation can be used in (31) to show that

\[
\frac{(1 - \Delta)(\alpha - 1)}{\alpha - 1 - \alpha\Delta} = \\
\beta^*G'(A\tau(1 - \Delta) - h) \left(1 - \nu\Delta^{a-1} + \frac{\alpha - 1}{\alpha} \nu\Delta^a + \frac{(1 - \Delta)^2(\alpha - 1)}{\alpha - 1 - \alpha\Delta} \nu \left(\Delta^{a-1} - \frac{\tau}{\nu}\right)\right) 
\]  

(37)

which can be used to solve for \( \tau \), with \( \xi \) given by eq. (34). This solution is valid if \( \xi > 0 \). If not, the solution is \( \xi = 0 \), with the remaining variables specified in the following subsection.

**Example with \( \alpha > 1 \) and \( \xi > 0 \):** Suppose

\[
G(x) = D^{1-\gamma}x^\gamma.
\]

From eqs. (30) and (33) we can derive,

\[
\frac{h}{A} = \frac{\eta}{\beta^*\gamma + \eta} \tau(1 - \Delta),
\]

and

\[
A = D \left(\frac{\beta^*\gamma}{\beta^*\gamma + \eta} \tau(1 - \Delta)\right)^{\frac{1}{\gamma}},
\]

so that

\[
\beta^*G'(A\tau(1 - \Delta) - h) = \frac{\beta^*\gamma + \eta}{\tau(1 - \Delta)}.
\]

Use this result above to show that \( \tau \) solves the equation (linear in \( \tau \))

\[
\frac{(1 - \Delta)^2(\alpha - 1)}{\alpha - 1 - \alpha\Delta} \frac{\tau}{\beta^*\gamma + \eta} = \\
\left(1 - \nu\Delta^{a-1} + \frac{\alpha - 1}{\alpha} \nu\Delta^a + \frac{(1 - \Delta)^2(\alpha - 1)}{\alpha - 1 - \alpha\Delta} \nu \Delta^{a-1} - \frac{\tau}{\nu}\right) \right)
\]  

(38)

Given \( \tau \), \( \xi \) solves

\[
\frac{\xi}{\nu} = \Delta^{a-1} - \frac{\tau}{\nu}.
\]
Per-capita market income \( y \) is given by

\[
y = D(1 - \Delta)^\frac{1}{1-\gamma} \left( \frac{\beta^* \gamma}{\beta^* \gamma + \eta} \right)^\frac{1}{1-\gamma}.
\]

### A.3.2 Equilibrium with \( \alpha > 1 \) and \( \xi = 0 \)

If \( \xi = 0 \), the equilibrium, steady-state conditions that determine \( A, \tau \) and \( h \) can be written as

\[
A = G \left( A \tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) - h \right) \tag{39}
\]

\[
1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \left( 1 - \frac{\alpha - 1}{\alpha} \nu \left( \frac{\tau}{\nu} \right)^{\frac{\alpha}{\alpha - 1}} \right) = \beta^* G' \left( A \tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) - h \right) \tag{40}
\]

and

\[
\frac{\eta A}{h} = \beta^* G' \left( A \tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) - h \right). \tag{41}
\]

Eqs. (39) and (41) define \( A \) and \( \hat{h} = h/A \) as functions of \( \tau \). Use these results in eq. (40) to define \( \tau \).

**Example with \( \alpha > 1 \) and \( \xi = 0 \):** Suppose

\[
G(x) = D^{1-\gamma} x^\gamma
\]

From eqs. (39) and (41) we can derive,

\[
\frac{h}{A} = \frac{\eta}{\beta^* \gamma + \eta} \tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right),
\]

and

\[
A = D \left( \frac{\beta^* \gamma}{\beta^* \gamma + \eta} \tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) \right)^\frac{\gamma}{1-\gamma},
\]

so that

\[
\beta^* G' \left( A \tau (1 - \delta) - h \right) = \frac{\beta^* \gamma + \eta}{\tau \left( 1 - \left( \frac{\tau}{\nu} \right)^{\frac{1}{\alpha - 1}} \right)}.
\]
and from (40), \( \tau \) solves

\[
\frac{1 - \left( \frac{\xi}{\nu} \right)^{\frac{1}{\alpha - 1}}}{\left( 1 - \frac{\alpha}{\alpha - 1} \left( \frac{\xi}{\nu} \right)^{\frac{1}{\alpha - 1}} \right) \left( 1 - \tau + \frac{\alpha - 1}{\alpha} \nu \left( \frac{\xi}{\nu} \right)^{\frac{\alpha}{\alpha - 1}} \right)} = \frac{\beta^* \gamma + \eta}{\tau \left( 1 - \left( \frac{\xi}{\nu} \right)^{\frac{1}{\alpha - 1}} \right)}.
\]