

Banks as Producers of Financial Services

Wilbur John Coleman II and Christian Lundblad*

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Abstract

This paper documents that a rise in government debt is associated with a rise in traditional banking and a fall in shadow banking, which is explained in a model where banks are valued for the financial services they offer rather than the safety of their deposits. Government debt does not compete directly with banks in providing financial services and hence is not viewed as a substitute for bank deposits. Government debt nonetheless offers a liquidity premium due to the high cost of default on financial services and the consequent demand for government debt by banks to manage their exposure to risk. A rise in government debt and the associated fall in liquidity premium is estimated to disproportionately benefit traditional banks so a rise in government debt leads to a rise in traditional banks at the expense of shadow banks. An optimal government debt policy leads both types of banks to hold sufficient amounts of government debt to become default free.

*Duke University (coleman@duke.edu), and The University of North Carolina (Christian.Lundblad@kenan-flagler.unc.edu). We wish to thank Ravi Bansal for many helpful comments.

1 Introduction

We begin this paper by documenting that a rise in government debt is associated with a rise in traditional/commercial banking assets and a fall in shadow banking assets. The curious positive association between government debt and traditional banking poses a challenge to a variety of banking models that posit banks' value stem primarily from their production of safe, liquid assets. If safety per se receives special value in an economy, and banks are primarily involved in the production of safe assets, then government debt should be a substitute for all types of banking, including traditional banking. Models of banking that primarily stress the role of banks in producing safe, liquid assets include Gorton and Pennacchi (1990), Holmström and Tirole (1998, 2011), Krishnamurthy and Vissing-Jorgenson (2012, 2015), Stein (2012), Sunderam (2014), Greenwood, et. al. (2015), and Diamond (2019) among many others.¹ In this paper we argue that banks' value stem primarily from their production of a menu of financial services. Safe, government debt thus does not compete directly with bank deposits. Instead, we argue that the high cost of default on financial services leads to a demand by banks for safe, government debt that in turn leads to a liquidity premium on government debt. A rise in the supply of government debt leads to a fall in the liquidity premium, which essentially lowers the cost of providing financial services and thereby affects the size of the banking sector. We estimate that a fall in the liquidity premium benefits traditional banks more so than shadow banks, especially in an environment with deposit insurance and capital regulation, and thus an expansion of government debt leads to an expansion of traditional banks at the expense of shadow banks.

Focusing on financial services provides a natural distinction between traditional and shadow banks in terms of the menu of financial services each produces, a distinction that is important to understanding why they respond differently to an expansion of government debt. Traditional banks offer a full menu of check-clearing and electronic transaction services and are meant to accommodate high-volume transaction accounts, whereas shadow banks offer more limited check-writing features and are meant to satisfy occasional liquidity needs. This distinction may also motivate why the cost of default on financial services offered by a traditional bank may be higher than that for a shadow bank. To understand how these features affect how each type of financial intermediary responds to an expansion of government debt, consider first an optimal

¹To be sure, Diamond and Dybvig (1983) stress the safe, liquid aspect of bank deposits, but in their setup banks offer risk sharing that differentiates their deposits from government debt from the perspective of depositors. Diamond and Rajan (2001) also stress the safe, liquid aspect of bank deposits, but use this as a mechanism to encourage appropriate behavior on lenders to avoid a bank run.

debt policy characterized by a supply of government debt that eliminates the liquidity premium. With a positive cost of default and no cost to holding government debt, all financial institutions would become riskless (Greenwood, et. al., 2015, explore a related result whereby an expansion of government debt crowds out socially excessive issuance of private, short-term debt), and if all financial institutions are riskless, then the relative size of shadow to traditional banking would depend entirely on the relative cost and value of the financial service they offer. Any substitution of traditional banking for shadow banking as government debt expands from relatively low levels thus depends on a comparison of how each is able to compete in an environment where they must absorb risk versus an environment where each becomes riskless. An important aspect of managing risk is holdings of riskless government debt, which is costly to hold due to a liquidity premium on government debt. An expansion of government debt that lowers the liquidity premium thus disproportionately benefits financial institutions that choose to hold large amounts of government debt to manage their exposure to risk. Deposit insurance and capital adequacy regulation change this margin for traditional banks, but the essential idea that a reduction in the liquidity premium benefits financial institutions that hold large amounts of government debt is still valid, whether these large holdings are by choice or a consequence of capital regulation. Indeed, we estimate that deposit insurance and bank capital regulation are the chief reason why traditional banks expand at the expense of shadow banks during an expansion of government debt.

This model is also consistent with a variety of other features of banking, including how traditional and shadow banks respond in different ways to a rise in uncertainty. Specifically, with deposit insurance the model predicts that a rise in uncertainty leads to a shift towards traditional banks and away from shadow banks, but without deposit insurance the model predicts that a rise in uncertainty leads to a shift away from both traditional and shadow banks. We validate this prediction by comparing the behavior of banking during the Great Depression to the behavior during the 2007/08 Financial Crisis. In this sense, we argue that the flight to safety currently associated with traditional banking is less an intrinsic feature of banking and more simply a feature of deposit insurance.

While we explore the importance of deposit insurance in determining the size of traditional banking to shadow banking, we argue that the emergence of shadow banking, defined generally as financial institutions that compete with traditional financial institutions of the day, predates deposit insurance,² so their continual emergence seems

²As discussed below, prior to the creation of the Federal Reserve or federal deposit insurance, in the late 19th and early 20th century state-chartered Trust Companies emerged to compete with traditional national

to be more fundamental to banking than just a reaction to deposit insurance. As argued here, banking fundamentally involves a variety of services that differ by cost and value, thus alternative forms of banking may emerge to offer a limited menu of financial services at lower cost. In modern parlance, institutional investors may not require high-cost checkable deposits, and may be better served by lower cost Money Market Mutual Funds that mostly offer some form of liquidity. Part of the technological innovation spurring the expansion of Fintech today may be thought of in a similar manner (see Buchak, et. al., 2017). We view this perspective as complementary to that of Gorton and Metrick (2012), who argue that securitized lending through shadow banks emerged to compete with traditional banks due to the limits of deposit insurance (see also Chrétien and Lyonnet, 2019, who explore the consequences of a combination of regulatory costs and deposit insurance to encourage the expansion of shadow banks, or Gennaioli, et. al, 2013, who explore the consequences of misperception of risks of securitized lending for the emergence of shadow banking). Surely the securitized lending feature of shadow banks is an important part of their development, but the emergence of shadow banks prior to deposit insurance suggests that other factors may be involved as well.

This paper has a close connection to Bansal and Coleman (1996) and the earlier literature that attempts to model the special value of money in an economy that goes beyond any pecuniary rate of return money may (or may not) offer (see the discussion in Feenstra, 1986). Many of the issues involving the role of banking can be cast in terms of defining what is meant by money. In Lucas and Stokey (1987) money is currency and hence banking has no special role in an economy. In Bansal and Coleman (1996) money includes checkable deposits and hence banks play a central role in the creation of money that has special value in an economy. In their model, banks offer financial services, but banks were assumed to face an infinite cost of bankruptcy, thereby leading them to be 100 percent backed by safe, government debt. Stein (2012) and Hanson, et. al., (2015) think of money as any riskless asset and hence banks compete with other producers of riskless assets in the creation of money. An essential difference is that Bansal and Coleman (1996) think of money as being backed up by riskless assets whereas Stein (2012) and Hanson, et. al., (2015) think of money directly as any riskless asset. This difference in perspective has important implications regarding the role of banks as either uniquely offering financial services or simply as creators of riskless assets, which is crucial to thinking how banks respond to a change in the supply of government debt. The current paper contributes to this literature in two important

banks. To be sure, though, some states offered state-level insurance.

ways. First, while the model developed here continues to assume that money's special role flows from the services offered by financial institutions, it is assumed that the cost of bankruptcy for financial institutions is finite so that banks have a non-trivial portfolio decision regarding how much risk to absorb in pursuing a high return on their portfolio. Along these lines, Bansal and Coleman (1996) would not be able to explain the flight from commercial banking during the Great Depression, but the model developed here explains this as a natural response to a rise in the riskiness of commercial banks. Second, here we include shadow banks as offering a different menu of financial services that is associated with a different cost of bankruptcy. This setup thus allows us to explore issues related to the interaction of traditional and shadow banks that was not possible in Bansal and Coleman (1996).

Krishnamurthy and Vissing-Jorgenson (2015) provide empirical support for the argument that government debt competes with the financial sector in the production of safe assets, hence an increase in the supply of government debt crowds out financial sector lending. As in Stein (2012) and Hanson, et. al., (2015), they propose a model in which all safe assets, whether produced by banks or supplied by the government, enter directly in preferences.³ We re-examine the empirical evidence presented by Krishnamurthy and Vissing-Jorgenson (2015) and find indeed that a rise in the supply of government debt is associated with a fall in the size of shadow banking but, as our model predicts, a rise in the size of traditional banking.

Recent papers have continued to explore models in which banks play a special role in creating liquidity that leads to a liquidity premium in safe assets. DeAngelo and Stultz (2015) explore a model in which banks create value by supplying liquidity in a world with liquidity shocks. Moreira and Savov (2017) also develop a model in which banks primary role is to finance liquidity shocks, but one in which shadow banks create assets that are safe enough to offer liquidity in periods of low uncertainty, but too risky to offer liquidity in periods of high uncertainty. The focus of both models on liquidity shocks as opposed to financial services lead to important differences in the behavior of the model as it relates to the expansion of government liquidity, especially for the behavior of traditional banks. Diamond (2019) develops a model in which households have special preferences for safe assets created by banks, which again imparts a liquidity premium to safe assets (although it would seem this approach would share the feature of Krishnamurthy and Vissing-Jorgenson (2015) that bank assets compete with government debt). Lenel, Piazzesi, and Schneider (2019) develop

³In a related approach, Nagel (2016) places bank deposits and government debt in preferences as imperfect substitutes and estimates a high degree of substitutability between money and near money assets.

a reduced-form specification that allows them to estimate the liquidity premium in short-term government debt and thereby document empirical relationships between this liquidity premium and overall bank balance sheets over the business cycle. The view that banks have a special role in creating liquidity and are thereby central to understanding the relationship between the supply and rate of return of various types of assets is leading to an emerging literature that seems to be yielding important insights.

The next section of this paper highlights the characteristics of Bank Trusts and Money Market Mutual Funds that will be the foundation for thinking of them as shadow banks in this paper. The section after that summarizes U.S. time series data on shadow and traditional banking, which in particular distinguishes between the period prior to deposit insurance and period during deposit insurance. The next part develops a model of shadow and traditional banking in a competitive market without deposit insurance. Following the presentation of the model we show, first through a simple example and then via simulations of a more general specification, that the model exhibits key features that characterize the data. We then extend the model to include a discussion of deposit insurance and contrast the behavior of the model with and without deposit insurance. We show that the model without deposit insurance can explain key aspects of the behavior of shadow and traditional banks during the Great Depression, and the model with deposit insurance can explain key aspects of the behavior of shadow and traditional banks during the 2007/2008 Financial Crisis. We then conclude. All proofs are in the appendix.

2 Trusts and Money Market Funds as Shadow Banks

In the model below, we provide a fundamental, economic motivation for the existence of two types of financial intermediaries, one we call a shadow bank and one we call a traditional bank. By shadow bank we do not mean a bank-like financial intermediary that operates outside a regulatory environment imposed on traditional banks (although this could certainly be a feature of a particular banking environment). By shadow bank we mean a financial intermediary that offers a menu of financial services that share some characteristics of a traditional bank but are otherwise more limited in scope. As a consequence, these shadow banks face different costs of providing their service, different costs of default on this service and as well face different incentives for investing in risky

portfolios. With our focus more on financial services than maturity transformation, we have a somewhat narrower perspective on shadow banking than that described in the Financial Stability Board’s (2018) *Global Shadow Banking Monitoring Report*, which includes insurance corporations and pension funds in their measure of shadow banking.⁴ Here we provide some discussion along these lines of two types of shadow banks, trust companies that were prevalent in the late 19th and early 20th century, and Money Market Mutual Funds along with certain aspects of investment banks beginning in the second half of the 20th century.

2.1 Trust Companies

As described by Moen and Tallman (1992), trust companies are state-chartered institutions that emerged out of the Free Banking Era to compete with national banks, so that by 1907 trust companies in New York State had about 75 percent of the total assets of national banks. Both trust companies and national banks accepted deposits and cleared checks, so in this sense they were similar. However, as summarized by Moen and Tallman (1992, p.613):

Although trusts could perform many of the particular functions of banks, the general role of trusts in the New York financial market was different from that of national banks. While the volume of deposits at trust companies in New York City was comparable to that at national banks, trusts did much less clearing activity than national banks. The trusts had only 7 percent of the clearings of national banks, so were not like commercial banks that provided transactions services to the average depositor. According to George Barnett, trust deposit accounts served as “surplus funds of individuals and corporations deposited for income and pending investment,” and thus were not widely used as transactions or checking accounts, despite the fact that the deposits were demand deposits subject to check.

There is thus a strong sense in which trust companies were not in the business of creating high-volume transaction services, as would a national bank. The use of a trust company is highlighted in the following quote from Herrick (1908, p. 378) that is cited in Moen and Tallman (1992):

Banks and trust companies are not identified with each other in the popular mind. Banks are ancient, trust companies are modern. Banks deal primarily with merchants, trust companies with all classes, without distinction. Banks lend on personal credit, trust companies on the security of pledged

⁴See, e.g., *Global Shadow Banking Monitoring Report 2017*. See also Perotti (2013) and Pozsar, et. al., (2013) for a detailed discussion regarding the maturity transformation role of shadow banks.

collaterals. Banks take on the risk of the business success of mercantile enterprises, while trust companies incur only the risk of a decline in investment values. Banks actively promote commerce, while trust companies manage investments. What they have in common is that they both receive deposits....

Regarding investments by trust companies, Moen and Tallman (1992, p. 613-614) note: "... trusts were an important source of short-term business financing, actively underwriting and distributing securities. ... in contrast to national banks, trusts could own stock equity directly. They could also own real estate, although no more than 15 percent of their total assets could be in this form."

Thus, it seems that trust companies offered limited financial services to a clientele that did not require full-service checkable accounts. Presumably, trusts offered these serves at a lower cost than the full menu of services of a national bank, and as stated by Moen and Tallman (1992, p. 613), "... they generally offered higher interest rates on deposits than did national banks." Trusts were also freed to invest in a broader set of asset classes than traditional banks, and they chose to do so.

2.2 Money Market Mutual Funds

In contemporary banking, limited bank-like services would seem to correspond to services offered by Money Market Mutual Funds (MMMF). These funds typically offer limited free check-writing, automated electronic exchange services, telephone exchange and redemption, and are highly liquid and easily converted into cash on demand. Firms use them as part of their cash flow management system. Indeed, the initial MMMFs were created to be substitutes to commercial bank accounts that were limited in interest payments by Regulation Q. At their peak in 2008, MMMFs had over \$3.9 trillion in assets, which was 26 percent of GDP. MMMFs are also intimately connected to what is broadly thought of as the shadow banking sector. MMMFs are the largest holder of commercial paper issued or underwritten by investment banks, at their peak holding almost 50 percent of all open-market paper. In this sense, MMMFs are a key component in the demand for maturity transformation fueling the shadow banking system. Indeed, conceptually we will think of the shadow banking sector as reflected in an integrated balance sheet, with MMMFs offering limited financial services on the one end, and investment banks issuing commercial paper backed up by risky assets (such as mortgages) on the other end (to be sure, a significant fraction of the activity—M&A, IPOs, market making, etc.—of investment banks seems unrelated to what would reasonable be thought of as shadow banking activity).

Corresponding to our association of MMMFs with shadow banking, we will associate M2 minus the currency component of M2 minus retail money funds with traditional banking. Retail money funds are incorporated into M2 because they are considered cash equivalent and held in small amounts (institutional money funds are held in large amounts). However, retail money funds are not issued by commercial banks, rather they are issued by asset managers. Along these lines, they are not regulated by the FDIC or the Fed, but are regulated by the SEC. For these reasons, we move retail money funds out of M2 and keep them in our measure of MMMFs as we separate measures of traditional banking from shadow banking.

3 The Data

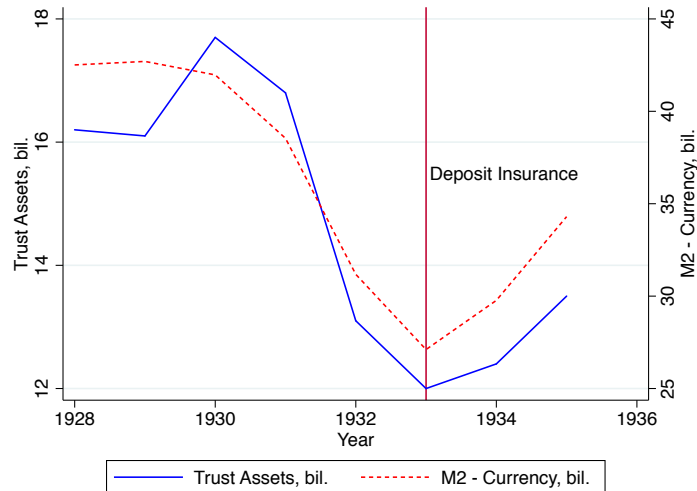
Here we conduct three empirical exercises. In the first exercise we contrast the behavior of shadow and commercial banking during the time period 1928-1935, which is a historical period without deposit insurance that includes the Great Depression, with the behavior during the time period 2006-2018, which is a period with deposit insurance that includes the Financial Crisis of 2007/2008. In the second exercise we examine the relation among shadow banking, commercial banking, and the supply of government debt from 1980-2018. We focus on data since 1980 because MMMFs, our measure of the size of shadow banking, only came into existence in 1971 and did not exist in substantial quantity prior to 1980. In the third exercise we examine the relation between commercial banking and the supply of government debt from 1960-2018. Here we cannot examine the behavior of MMMFs as they were not in existence in the early part of the sample, but we use this longer time period to validate that our findings regarding the behavior of commercial banking during the post 1980 sample also carry over to this longer time period. In this longer sample we also examine in more detail how commercial bank holdings of treasury securities are related to the supply of treasury securities and the size of the commercial banking sector.

3.1 Banking, Financial Crises, and Deposit Insurance

As mentioned, in this section we contrast the behavior of shadow and traditional banks with and without deposit insurance during a financial panic. Historical data on bank trust assets is obtained from various edition of the Annual Report of the Comptroller of the Currency. Historical data on M2 and Currency Held by the Public is obtained from Friedman and Schwartz (1963). For the period 1928 to 1935, Fig. 1 plots the

value of M2 net of currency held by the public as well as the value of trust assets. For the period 2006 to 2015, Fig. 2 plots the value of M2 minus currency held by the public minus retail money funds as well as the value of MMMF deposits. As can be seen from these figures, during the period without federal deposit insurance (up to 1933), the financial crisis (the Great Depression) was associated with a collapse of commercial banking and trust assets.⁵ During the period with deposit insurance, the financial crises (the 2007-2008 Financial Crisis) was associated with a continued rise in traditional banking and a fall in shadow banking. Based on this evidence, the flight to safety associated today with commercial banking seems due more to deposit insurance than an intrinsic feature of commercial banking.

Figure 1: Bank Assets During the Great Depression

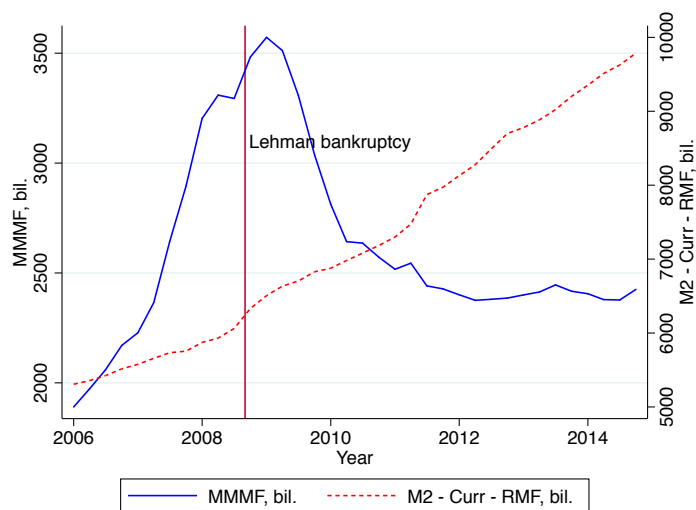


3.2 Shadow, Commercial Banking and Government Debt

Here we describe the relation between the size of traditional and shadow banks and the supply of government debt using U. S. quarterly data from 1980 to 2018. For the supply of safe government debt we use the supply of U.S. Treasury securities held by the public (net of any foreign holdings) obtained from the Federal Reserve’s *Flow of*

⁵Recall as well that commercial banking and trust companies collapsed during the 1907 Financial Panic. However, as argued by Tallman and Moen, 1995, the collapse of trust companies was due largely to the loss of clearinghouse privileges by trusts in New York, which is not a feature captured in this paper. Note, though, that trusts in Chicago maintained their clearinghouse privileges and as reported by Tallman and Moen, in Chicago during the financial panic deposits at trusts fell 6 percent while deposits at national banks fell 7 percent.

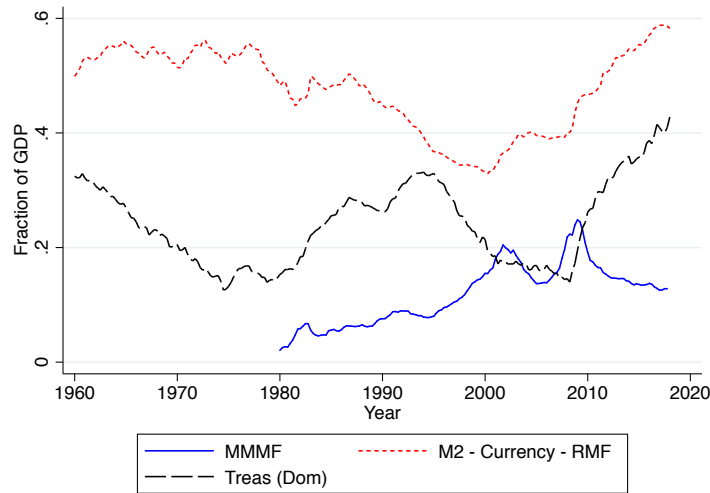
Figure 2: Bank Assets During the 2007/2008 Financial Panic



Funds accounts. For a measure of traditional bank size we use M2 minus currency held by the public (the component of M1) minus retail money funds, all obtained from St. Louis FRED. For a measure of the size of liquidity provided by shadow banks we use financial assets held by Money Market Mutual Funds, which includes retail and institutional money funds, also available from St. Louis FRED. All monetary aggregates are measured relative to nominal GDP, obtained from St. Louis FRED.

Fig. 3 displays the data on traditional bank size, shadow bank size, and government debt, all relative to GDP (data on traditional banks and government debt goes back to 1960, which is discussed in the next section). Table 1 displays results of regressing bank size on the supply of government debt and real per-capita GDP growth volatility (5-year smoothed). Due to the trend in MMMF, all regressions involving MMMF include a time trend. The first regression shows results regressing the ratio of traditional bank size to shadow bank size on government debt and GDP volatility. Here we see very clearly that a rise in government debt leads to a rise in traditional banking relative to shadow banking. We also see that a rise in uncertainty as captured in the volatility of GDP leads to a substitution towards traditional banking and away from shadow banking. The second regression shows results of regressing traditional bank size on government debt and GDP volatility and the third regression shows results of regressing shadow bank size on government debt and GDP volatility. Government debt enters both positive and significant in the traditional bank regression and both negative and significant in the shadow bank regression, reflecting the complementary

Figure 3: Bank Size and Government Debt



nature of government debt and traditional banking and the substitutability between traditional and shadow banking. Also, the coefficient on GDP volatility is positive and significant in the traditional bank regression and negative and significant in the shadow bank regression, reflecting a flight to quality during times of heightened uncertainty.

3.3 Commercial Banking and Government Debt

Here we describe the relation between the size of traditional banks and the supply of government debt using U. S. quarterly data from 1960 to 2018. The source of the data is the same as above. The earlier data is graphed as well in Fig. 3. The first regression in Table 2 displays results of regressing commercial bank size on the supply of government debt and real per-capita GDP growth volatility (5-year smoothed). The results are similar as for the regression for the shorter time period 1980 - 2018. The coefficient for the domestic supply of Treasury securities is positive and significant, although the magnitude is somewhat smaller for the longer time period.

Table 2 also examines more closely the role of commercial bank holdings of treasury debt in determining the relationship between the supply of treasury debt and the size of the commercial banking sector. The second regression in Table 2 shows results of regressing the holdings of treasury securities by commercial banks (as a fraction of total bank financial assets obtained from the Flow of Funds) on the domestic supply of treasury securities. Here we see that a rise in the domestic supply of treasury securities leads to an expansion of holdings of treasury securities by commercial banks. The third

Table 1: Shadow and Traditional Bank Regressions
(U.S. Quarterly Data: 1980.1 - 2018.1)

	$\frac{MMMF}{M2-Curr-RMF}$	$\frac{M2-Curr-RMF}{GDP}$	$\frac{MMMF}{GDP}$
Constant	-3.92*** (0.39)	-0.55*** (0.13)	-4.89*** (0.26)
Time	0.01*** (0.001)		0.01*** (0.001)
Treas (Dom)/GDP	-0.78*** (0.19)	0.31*** (0.08)	-0.50*** (0.14)
GDP Vol	-61.04*** (16.49)	23.54*** (5.36)	-26.70** (11.32)
<i>Adj.R</i> ²	0.71	0.43	0.80

All ratios are in log. HAC-adjusted standard errors in parentheses.

** denotes significance at 5 percent. *** denotes significance at 1 percent.

Table 2: Traditional Bank Regressions
(U.S. Quarterly Data: 1960.1 - 2018.1)

	$\frac{M2-Curr-RMF}{GDP}$	$\frac{Treas.(U.S.Banks)}{Assets}$	$\frac{M2-Curr-RMF}{GDP}$
Constant	-0.833*** (0.059)	-3.007*** (0.361)	-0.781*** (0.102)
Treas (Dom)/GDP	0.114*** (0.030)	0.722*** (0.272)	
Treas. (U.S. Banks) / Assets			0.052*** (0.016)
GDP Vol	31.444*** (3.260)	134.346*** (25.405)	23.463*** (7.580)
<i>Adj.R</i> ²	0.32	0.27	0.32

All ratios are in log. HAC-adjusted standard errors in parentheses.

*** denotes significance at 1 percent.

regression in Table 2 reports results of regressing the size of the commercial banking sector on the amount of treasury securities held by commercial banks. Here we see that a rise in the holdings of treasury securities held by commercial banks is associated with a rise in the commercial banking sector. A rise in the supply of treasury debt thus encourage banks to hold this debt and to expand. Note that both regressions control for uncertainty by including the volatility of GDP as an independent variable. For a given overall domestic supply of treasury securities, a rise in uncertainty leads to a rise in the holdings of treasury securities by commercial banks, which should be expected in an environment in which commercial banks manage their exposure to risk by adjusting their holdings of safe, government debt. Also, for a given share of treasury debt in commercial bank portfolios, higher uncertainty leads to an expansion of the commercial banking sector, which should be expected during a flight to safety. The relationship between the supply of treasury debt and the size of the commercial banking sector thus goes beyond a reaction of the supply of treasury debt and the size of the commercial banking sector to changing levels of uncertainty in an economy,

4 The Model

This section provides a detailed overview of the model and foreshadows some key results. The model consists of firms, a government, financial intermediaries, and households in a discrete-time, infinite-horizon, risk-neutral setting subject to uncertainty. Firms are represented by equity that is a claim to an exogenous endowment. The government issues risk-free one-period debt financed by lump-sum taxes or transfers. Financial intermediaries accept deposits and invest in a portfolio of safe government bonds and risky equity. There are two types of financial intermediaries, traditional commercial banks and shadow banks, which differ by the value of the transactions services associated with their deposits (as well as the cost of producing these transaction services) and the cost of default to households of their deposits. In comparison to shadow banks, traditional banks can be thought of as offering more transaction services, but default on traditional bank deposits imposes a higher cost on households. Government debt does not directly offer any transaction services, but the demand for safe government debt by banks to manage their default risk leads to a liquidity premium on government debt. The lower return on government debt acts like a tax on banks that choose to hold government debt. Due to the higher cost of default on traditional bank deposits, traditional banks tend to hold more government debt, and hence tend to be more affected by the liquidity premium on government debt. As government

debt expands the liquidity premium falls, which tends to favor traditional banks that hold more government debt, thus leading to an expansion of traditional banks at the expense of shadow banks. The model developed here makes these results explicit, and to some extent qualifies these statements, and provides a framework that can be solved and estimated so that simulations of the model can be compared to the data.

4.1 Firms

A large number of identical firms arise each period and exist for only one period. Production is an exogenous amount and is fully perishable at the end of the period. Denote the aggregate output of all firms in the current period by $y \geq 0$. Aggregate output y' in the next period is drawn from a conditional distribution $F(y', y)$.

Assumption 1. $F(y', y) > 0$ is a distribution function for every $y \geq 0$ with compact support $0 < y_{min} \leq y' \leq y_{max}$.

As we discuss the equilibrium, we will make further assumptions on F . Denote the expectation operator over next period's realizations of y' conditional on the current realization y as E_y and denote $\bar{y} = E_y[y']$. Denote also the gross growth rate of aggregate output by $g' = y'/y$.

For simplicity, the only claim on firms is a claim on a portfolio of all firms that trades one period prior to production. That is, owning z' claims in the current period entitles the owner to $z'y'$ units of aggregate output y' in the next period. All claims are initially owned by households and are traded one period prior to the production of output. Denote the price of a unit of such a claim in terms of current consumption by p .

4.2 Government

The government fully honors its existing debt obligations, issues new one-period debt, and levies a lump-sum tax or pays a lump-sum distribution. Denote the face value in units of current output of outstanding debt in the current period as B . The government issues new one-period debt B' at a per-unit price q_b . The difference $T = B - q_b B'$ is financed by a lump-sum tax (positive or negative) on households. Denote debt per unit of aggregate output as $b' = B'/y$. Assume the government implements a stationary policy to achieve an amount of debt per unit of aggregate output $b' = G(y)$ based on realizations of y .

Assumption 2. $G(y) > 0$ for every $y \geq 0$.

4.3 Financial Intermediaries

Households create and manage financial intermediaries. Financial intermediaries accept deposits from households and invest in a portfolio of government bonds and equities. There are two types of financial intermediaries, labeled as type i for $i \in \{0, 1\}$, which differ by the service value of their deposits to households and the cost of offering this service. The value to households will be more fully described in the section that describes households. We will think of and refer to financial intermediaries of type 0 as traditional banks and financial intermediaries of type 1 as shadow banks. It costs θ_i per unit of deposit in the current period to operate a financial intermediary (new or continuing) of type i .

Assumption 3. $0 < \theta_i < 1$, $i \in \{0, 1\}$.

Designate the fraction of a financial intermediary's portfolio invested in equities by α . In the current period, each financial intermediary of type i with portfolio α issues one-period debt (deposits) denoted by $\hat{d}'_i(\alpha) > 0$ at a per unit price $q_i(\alpha)$. Each financial intermediary is of fixed size normalized to one, by which is meant

$$q_i(\alpha)\hat{d}'_i(\alpha) = 1.$$

The overall size of the financial intermediation sector, and the relative size of shadow banking to traditional banking, will thus be entirely along an external margin that determines the number of banks and not the size of banks.

Financial intermediary (i, α) purchases government debt denoted by $\hat{b}'_i(\alpha)$ and equity denoted by $\hat{z}'_i(\alpha)$, so

$$q_b\hat{b}'_i(\alpha) + p\hat{z}'_i(\alpha) = 1,$$

where

$$p\hat{z}'_i(\alpha) = \alpha$$

and

$$q_b\hat{b}'_i(\alpha) = 1 - \alpha.$$

In the next period financial intermediary (i, α) will receive a cash flow of $\hat{b}'_i(\alpha) + \hat{z}'_i(\alpha)y'$, which is distributed to its debt holders and owners. Debt holders receive either the

face value of their debt or the entire cash flow, whichever is less. Let $x'_i(\alpha)$ denote the payout on their debt during the next period, so that every unit of face value gets paid $x'_i(\alpha)$. From the perspective of the current period the payout ratio $x'_i(\alpha)$ is a random variable, which is given by the relation $x'_i(\alpha)\hat{d}'_i(\alpha) = \min \left\{ \hat{d}'_i(\alpha), \hat{b}'_i(\alpha) + \hat{z}'_i(\alpha)y' \right\}$, and can be rewritten as

$$x'_i(\alpha) = \min \left\{ 1, \left(\alpha \frac{y'}{p} + (1 - \alpha) \frac{1}{q_b} \right) q_i(\alpha) \right\}.$$

The next-period payoff to owners of one financial intermediary can similarly be written as

$$\omega'_i(\alpha) = \max \left\{ 0, \alpha \frac{y'}{p} + (1 - \alpha) \frac{1}{q_b} - \frac{1}{q_i(\alpha)} \right\}.$$

Households begin a period with $N_i(\alpha)$ financial intermediaries of type $i \in \{0, 1\}$ holding portfolio $\alpha \in [0, 1]$ for various values of α and choose $N'_i(\alpha)$ financial intermediaries for the current period for various values of α . To clarify with an example, households may begin a period with $N_1(\alpha) = 0$ except for $N_1(.5) > 0$ and choose $N'_1(\alpha) = 0$ except for $N'_1(.75) > 0$, which means that households begin a period with all financial intermediaries of type 1 investing 50 percent of their portfolio in equities and for the next period choose all financial intermediaries of type 1 investing 75 percent of their portfolio in equities. This notation will be suitable for characterizing an equilibrium in which, for each i , there is an optimal portfolio mix α that all households choose, but in which this optimal portfolio mix may change over time.

4.4 Households

Risk-neutral households value the distribution for an infinite sequence of consumption $\{\hat{c}, \hat{c}', \hat{c}'', \dots\}$ via the expected utility function with constant discount factors

$$\hat{c} + \beta E[\hat{c}'] + \beta^2 E[\hat{c}'] + \dots \tag{1}$$

where $0 < \beta < 1$.

Transactions involving goods cost resources and financial intermediaries offer a service that reduces this transaction cost. This service is embodied in the debt or account \hat{d}'_i , such as various levels of check writing features (e.g., a traditional bank would allow essentially an unlimited number of checks to be written against the account whereas a money money mutual fund would allow limited check writing), instant liquidity, or some accounting/bookkeeping features. To simplify the notation, from the out-

set assume that for each type i financial intermediary, households choose to purchase debt from one portfolio type α'_i , but choose the portfolio type optimally. Later we will show that households will never have an incentive to deviate from such an equilibrium (that is, at any point in time there will never be two types of traditional banks or two types of shadow banks, although these types of banks can differ from each other and over time). Financial services are derived from the CES composite $(\sum_{j=0}^1 \gamma_j (q_j(\alpha'_j) \hat{d}'_j(\alpha'_j))^\epsilon)^{1/\epsilon}$, where $q_i(\alpha'_i) \hat{d}'_i(\alpha'_i)$ for $i \in \{0, 1\}$ is the value of debt holdings or deposits that a household holds of financial intermediaries of type i with portfolio α'_i (throughout the remainder of this paper, we will simply use Σ to denote the summation over j from 0 to 1). With this composite, transaction costs are given by

$$\nu \hat{c} + \frac{\psi}{\eta - 1} \hat{y}^\eta \left(\Sigma \gamma_j (q_j(\alpha'_j) \hat{d}'_j(\alpha'_j))^\epsilon \right)^{\frac{1-\eta}{\epsilon}},$$

where $\nu > 0$, $\psi > 0$, $\eta > 1$, $\epsilon \leq 1$, $\gamma_0 = 1$ (a normalization) and \hat{y} is per-capita firm output (which is exogenous to household decisions).

Regarding the form of the transaction-cost function, note that a rise in consumption and liquid assets leads to a less than proportionate rise in transaction costs, reflecting an economy of scale in transaction costs for wealthier households. Over time the rise in \hat{y} will ensure that as consumption and liquid assets rise, transaction costs will rise in proportion in equilibrium.

A key assumption underlying this model concerns the cost of default of a financial intermediary for a household. We have in mind that if a financial intermediary is relied on to provide a service based on a credible promise to maintain the value of deposits, such as the clearing of checks or providing other forms of liquidity, then to households there is a cost in addition to the lost value of the defaulted asset if the financial intermediary is not able to honor its debt (Bansal and Coleman, 1996, essentially assume this cost is infinite). Specifically, we assume that this additional cost of default is given by

$$\Sigma \xi_j (1 - x_j(\alpha_i)) \hat{d}_j(\alpha_j).$$

Note that $x_i(\alpha_i)$ is the payout rate on debt from financial intermediary (i, α_i) that matures in the current period, so $(1 - x_i(\alpha_i)) \hat{d}_i(\alpha_i)$ is the face value of debt that is not paid. The cost of default is the key mechanism by which the riskiness of an asset limits its usefulness in providing transaction or liquidity services. While this is an ex-post feature, certainly ex-ante assets that have a relatively high likelihood of default will have a lower expected value in terms of financial services. Note that the cost of default can differ by financial intermediary type, capturing the idea that different levels of

service may be associated with different levels of default cost.

Households own all the financial intermediaries and begin each period directly owning all the firms that yield output in the next period. Denote by \hat{z}_h the amount of equity directly owned by each household that is a claim to current output (a remaining amount, say \hat{z}_b , is owned by financial intermediaries). Denote by \hat{z}' the amount of equity owned by each household at the beginning of a period that is a claim to next period's output (which in aggregate equals one). During the period households sell claims on firms (to financial intermediaries), keeping equity \hat{z}'_h . Households can also purchase government debt directly, denoted by \hat{b}'_h , but in doing so do not receive any financial services (all financial services are provided by financial intermediaries). At the beginning of each period households also make or receive lump-sum tax payments to the government, denoted by \hat{T} . To distinguish between a choice of financial intermediary for deposits (as a consumer of financial services) from one for investment (as a producer of financial services), let α'_i denote a choice of a financial intermediary for deposits and $\tilde{\alpha}'_i$ denote a choice of financial intermediary for investment.

The flow budget constraint for households in the current period is given by

$$\begin{aligned} \hat{c} &+ \nu \hat{c} + \frac{\psi}{\eta - 1} \hat{y}^\eta \left(\sum \gamma_j (q_j(\alpha'_j) \hat{d}'_j(\alpha'_j))^\epsilon \right)^{\frac{1-\eta}{\epsilon}} + \sum \xi_j (1 - x_j(\alpha_j)) \hat{d}_j(\alpha_j) & (2) \\ &+ q_b \hat{b}'_h + \sum q_j(\alpha'_j) \hat{d}'_j(\alpha'_j) + p(\hat{z}'_h - \hat{z}') + \sum \hat{n}'_j(\tilde{\alpha}'_j) \theta_j + \hat{T} \\ &= \hat{b}_h + \sum x_j(\alpha_j) \hat{d}_j(\alpha_j) + \hat{z}_h y + \sum \hat{n}_j(\tilde{\alpha}_j) \omega_j(\tilde{\alpha}_j). \end{aligned}$$

Households must also satisfy various non-negativity conditions, such as $\hat{n}'_i \geq 0$, $\hat{d}'_i \geq 0$, $\hat{b}'_h \geq 0$ and $\hat{z}'_h \geq 0$.

4.5 Equilibrium Conditions

With free entry, if financial intermediary (i, α) is in operation, then owners should expect discounted profits to equal zero, which leads to the following relation under risk neutrality

$$\theta_i = \beta E_y \max \left\{ 0, \alpha \frac{y'}{p} + (1 - \alpha) \frac{1}{q_b} - \frac{1}{q_i(\alpha)} \right\}.$$

In general, the value $q_i(\alpha)$ is the lowest price a financial intermediary of type i with portfolio α will accept for its debt. It remains to determine if households will choose to hold debt from a financial intermediary of type i with portfolio α at the price $q_i(\alpha)$.

Households choose state-contingent sequences for \hat{c} , \hat{b}'_h , \hat{d}'_i , \hat{n}'_i , α'_i , $\tilde{\alpha}'_i$ and \hat{z}'_h to maximize preferences given by (1), subject to their flow budget constraints given by

(2). Denote the number of next-period financial intermediaries of type i with portfolio α per unit of aggregate output by $n'_i(\alpha) = N'_i(\alpha)/y$, and denote aggregate financial intermediary debt per unit of output by $d'_i(\alpha) = n'_i(\alpha)\hat{d}'_i(\alpha)$. Denote aggregate household consumption by c . Denote household aggregate debt holdings by B'_h and household aggregate debt holdings per unit of aggregate output by $b'_h = B'_h/y$. Note that initial aggregate household equity holdings equal 1. The first-order conditions, after imposing market-clearing conditions and transforming to aggregate variables, can be written as:

$$y = c + \nu c + \frac{\psi}{\eta - 1} (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}} y + \frac{y}{g} \Sigma \xi_j (1 - x_j) d_j + y \Sigma n'_j \theta_j, \quad (3)$$

$$q_b \geq \beta, \text{ w/eq. if } b'_h > 0, \quad (4)$$

$$q_i \geq \beta E_y[x'_i] + \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_i (q_i d'_i)^{\epsilon-1} q_i - \beta \xi_i (1 - E_y[x'_i]),$$

$$\text{w/eq. if } d'_i > 0, \quad (5)$$

$$q_b b' = q_b b'_h + \Sigma (1 - \alpha'_j) q_j d'_j, \quad (6)$$

$$p = \beta \bar{y}, \quad (7)$$

$$q_i d'_i = n'_i, \quad (8)$$

where “w/eq.” denotes “with equality,” where

$$E_y[x'_i] = E_y \left[\min \left\{ 1, \left(\alpha'_i \frac{y'}{\bar{y}} + (1 - \alpha'_i) \frac{\beta}{q_b} \right) \frac{q_i}{\beta} \right\} \right], \quad (9)$$

$$\theta_i = E_y \left[\max \left\{ 0, \alpha'_i \frac{y'}{\bar{y}} + (1 - \alpha'_i) \frac{\beta}{q_b} - \frac{\beta}{q_i} \right\} \right], \quad (10)$$

and the non-negativity constraints $d'_i \geq 0$ and $b'_h \geq 0$ hold. Further, as shown by considering deposits at financial intermediaries with alternative strategies for α'_i , optimality with respect to α'_i requires that

$$\hat{q}_i \geq \beta E_y[\hat{x}'_i] + \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_i (q_i d'_i)^{\epsilon-1} \hat{q}_i - \beta \xi_i (1 - E_y[\hat{x}'_i]), \text{ for any } 0 \leq \hat{\alpha}'_i \leq 1, \quad (11)$$

for $E_y[\hat{x}'_i]$ and \hat{q}_i defined by

$$E_y[\hat{x}'_i] = E_y \left[\min \left\{ 1, \left(\hat{\alpha}'_i \frac{y'}{\bar{y}} + (1 - \hat{\alpha}'_i) \frac{\beta}{q_b} \right) \frac{\hat{q}_i}{\beta} \right\} \right], \quad (12)$$

$$\theta_i = E_y \left[\max \left\{ 0, \hat{\alpha}'_i \frac{y'}{\bar{y}} + (1 - \hat{\alpha}'_i) \frac{\beta}{q_b} - \frac{\beta}{\hat{q}_i} \right\} \right]. \quad (13)$$

Eq. (3) is the aggregate flow budget constraint. Eq. (4) stipulates that if some government debt is not held to generate a liquidity value, then its rate of return must

equal the rate of time preference. Eq. (5), the key equation in this model, stipulates that the return on financial intermediary debt/deposits must reflect both the liquidity value of this debt as well as the costs of not fulfilling the promise of delivery on any promised financial service. Eq. (6) accounts for the various holdings of government debt. Eq. (7) relates the price of equity to its expected payoff in a risk-neutral setting. Eq. (8) relates the number of financial intermediaries (and the total cost of financial intermediation) to the overall volume of financial intermediation. Eqs. (9)-(10) summarize the expected payout on financial intermediary debt as well as the zero expected profit condition for financial intermediaries. Conditions (11)-(13) are conditions for optimality for the type of financial intermediaries and thereby rule out any excess profits for other types of financial intermediaries to enter. To simplify the notation, define $\theta = (\theta_0, \theta_1)$, and similarly for ξ , q , d' , n' , and α . For given parameters $(\theta, \psi, \xi, \eta)$, the system defined by eqs. (3)-(13) determines a recursive, stationary equilibrium comprised of endogenous variables $(c, q_b, q, p, d', n', \alpha')$ conditional on the state y .

To solve for the equilibrium, motivated by eq. (10), for fixed y define $H(a, e)$, $0 \leq a \leq 1$, $0 < e \leq 1$, such that

$$0 = E_y \left[\max \left\{ -e, a \left(\frac{y'}{\bar{y}} - 1 \right) - H(a, e) \right\} \right]. \quad (14)$$

Lemma 1: Under Assumptions 1 and 3, a unique function $H(a, e)$ exists such that eq. (14) holds for every $0 \leq a \leq 1$, $0 < e \leq 1$. The solution $H(a, e)$ is a continuous function of a and e . If $a \leq e$ then $H(a, e) = 0$, else $H(a, e) \geq 0$. If $e \leq \hat{e}$ then $H(a, e) \geq H(a, \hat{e})$ for every $0 \leq a \leq 1$.

For ease of notation, define $h_i(a) = H(a, \theta_i)$. In some sense $h_i(\alpha'_i)$ is a measure of the exposure to default of a financial intermediary of type i with investor capital θ_i and portfolio α'_i , as it is zero when there is no risk of default and positive if there is risk of default. From the definition of h_i and eq. (10) it follows that

$$\alpha'_i + (1 - \alpha'_i) \frac{\beta}{q_b} - \frac{\beta}{q_i} = \theta_i - h_i(\alpha'_i). \quad (15)$$

Combine eqs. (9) and (10) to derive

$$E_y[x'_i] = \left(\alpha'_i + (1 - \alpha'_i) \frac{\beta}{q_b} - \theta_i \right) \frac{q_i}{\beta}. \quad (16)$$

Substitute eqs. (15) and (16) into eq. (5) to derive the following relation that must

hold in equilibrium

$$(1 - \alpha'_i) \left(1 - \frac{\beta}{q_b}\right) + \xi_i h_i(\alpha'_i) + \theta_i \geq \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_i (q_i d'_i)^{\epsilon-1}, \text{ w/eq. if } d'_i > 0. \quad (17)$$

Similarly, the condition of optimality with respect to α'_i can be summarized as

$$(1 - \hat{\alpha}'_i) \left(1 - \frac{\beta}{q_b}\right) + \xi_i h_i(\hat{\alpha}'_i) + \theta_i \geq \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_i (q_i d'_i)^{\epsilon-1}, \text{ for any } \theta_i \leq \hat{\alpha}'_i \leq 1. \quad (18)$$

To summarize, an equilibrium can now be characterized as a $(q_b, q, b'_h, d', \alpha')$ that solve eqs. (4), (6), (15), (17) and (18), along with (c, n', p) that solve eqs. (3), (7) and (8), and in which the non-negativity constraints $d' \geq 0$ and $b'_h \geq 0$ hold.

The equilibrium can be characterized more sharply by assuming the conditional distribution $F(y', y)$ is continuous with density $f(y', y)$.

Assumption 4.

$F(y', y)$ is continuous with probability density function $f(y', y)$ for every $y \geq 0$.

Define $\Omega_i(a)$, $a > 0$, as

$$\Omega_i(a) = 1 + \frac{h_i(a) - \theta_i}{a}.$$

In addition to the results of Lemma 1, we can now establish the following properties of h_i .

Lemma 2: Under Assumptions 1-4, the function $h_i(a)$ that solves eq. (14) is a twice continuously-differentiable, concave function with derivatives given by

$$h'_i(a) = \frac{\int_{\bar{y}\Omega_i(a)}^{\infty} \left(\frac{y'}{\bar{y}} - 1\right) f(y', y) dy'}{\int_{\bar{y}\Omega_i(a)}^{\infty} f(y', y) dy'} \geq 0, \quad (19)$$

$$h''_i(a) = \left(\frac{ah'_i(a) - h_i(a) + \theta_i}{a}\right)^2 \frac{f(\bar{y}\Omega_i(a), y)\bar{y}}{a \int_{\bar{y}\Omega_i(a)}^{\infty} f(y', y) dy'} \geq 0. \quad (20)$$

Eq. (18) can now be rewritten using the following Lemma.

Lemma 3: Under Assumptions 1-4, eq. (18) holds if and only if

$$\xi_i h'_i(\alpha'_i) \leq 1 - \frac{\beta}{q_b}, \text{ w/eq. if } \alpha'_i < 1. \quad (21)$$

To summarize, an equilibrium is $(q_b, q, b'_h, d', \alpha')$ that solve eqs. (4), (6), (15), (17), and (21), along with (c, n', p) that solve eqs. (3), (7) and (8), and in which the non-negativity constraints $d' \geq 0$ and $b'_h \geq 0$ hold. It turns out that the existence of an equilibrium is remarkably easy to prove.

Theorem 4: Under Assumptions 1-4, there exists an equilibrium $(q_b, q, b'_h, d', \alpha')$ that solves eqs. (4), (6), (15), (17), and (21), along with (c, n', p) that solves eqs. (3), (7) and (8), and in which the non-negativity constraints $d' \geq 0$ and $b'_h \geq 0$ hold.

4.6 Equilibrium for a Simple Example

Suppose aggregate output can take on only two values, 0 with probability $1 - \pi$ and $\hat{y} > 0$ with probability π , $0 < \pi < 1$, and that the liquidity aggregate is linear in traditional and shadow bank deposits, achieved by setting $\epsilon = 1$. With this two-state distribution, the function h_i is given by

$$h_i(a) = \max \left\{ 0, (a - \theta_i) \frac{1 - \pi}{\pi} \right\}. \quad (22)$$

Note that this discrete distribution violates Assumption 4, hence we need to ensure eq. (18) holds instead of the condition in Lemma 3. Substitution of eq. (22) into (18) reveals that

$$\alpha'_i = \begin{cases} \theta_i, & \text{if } \frac{1-\pi}{\pi} \xi_i \geq 1 - \frac{\beta}{q_b}. \\ 1, & \text{otherwise.} \end{cases} \quad (23)$$

Hence, a financial intermediary either chooses $\alpha'_i = \theta_i$ and will thereby be risk-free, or $\alpha'_i = 1$ and thereby invests all of its assets in equities.

To simplify the exposition, we will make the following additional assumptions for this example.

Assumption 5.

$$\text{(i)} : \quad \frac{\gamma_1}{\theta_1} \leq \frac{1}{\theta_0}, \quad (24)$$

$$\text{(ii)} : \quad \gamma_1 < 1, \quad (25)$$

$$\text{(iii)} : \quad \frac{1 - \pi}{\pi} \xi_1 < 1 \leq \frac{1 - \pi}{\pi} \xi_0, \quad (26)$$

$$(27)$$

Assumption 5(i) is a key assumption that will lead to traditional banks dominating in the provision of liquidity if they are not constrained by the supply of safe assets. Assumptions 5(ii) and 5(iii) narrow down the equilibria to one in which traditional banks always choose to be safe and shadow banks, if they choose to enter and compete with traditional banks, choose a risky portfolio of assets.

To assist in the characterization of the equilibrium, define the function $w = W(D)$, $D > 0$ such that

$$1 - (1 - \theta_0)w = \psi \left(\frac{D}{(1 - \theta_0)w} \right)^{-\eta}. \quad (28)$$

Lemma 5: For any $D > 0$ there exists a unique w that solves eq. (28), $0 < w < 1/(1 - \theta_0)$, and both $W(D)$ and $D/W(D)$ are increasing functions of D .

Use W to define D_0^* such that

$$1 = W(D_0^*),$$

and to define D_1^* such that $D_1^* = 0$ if

$$\xi_1(1 - \theta_1) \frac{1 - \pi}{\pi} + \theta_1 \geq \gamma_1,$$

otherwise D_1^* solves

$$\xi_1(1 - \theta_1) \frac{1 - \pi}{\pi} + \theta_1 = [(1 - \theta_0)(1 - W(D_1^*)) + \theta_0] \gamma_1.$$

Note that with Lemma 5 and Assumption 5(i) it follows that

$$0 \leq D_1^* < D_0^*.$$

Proposition 6: Under Assumptions 1-3 and 5, the equilibrium is comprised of the following three regions:

(Region 1): If $\beta b' > D_0^*$, then the equilibrium is given by

$$\alpha'_i = \theta_i, \quad i \in \{0, 1\} \quad (29)$$

$$\frac{\beta}{q_b} = 1, \quad (30)$$

$$\frac{\beta}{q_i} = 1 - \theta_i, \quad i \in \{0, 1\} \quad (31)$$

$$q_0 d'_0 = \frac{D_0^*}{1 - \theta_0}, \quad (32)$$

$$q_1 d'_1 = 0, \quad (33)$$

$$q_b b'_h = \beta b' - D_0^*. \quad (34)$$

(Region 2): If $D_1^* < \beta b' \leq D_0^*$, then the equilibrium is given by

$$\alpha'_0 = \theta_0, \quad (35)$$

$$\alpha'_1 = \begin{cases} \theta_1, & \text{if } \frac{1-\pi}{\pi} \xi_1 \geq 1 - W(\beta b'). \\ 1, & \text{otherwise.} \end{cases} \quad (36)$$

$$\frac{\beta}{q_b} = W(\beta b'), \quad (37)$$

$$\frac{\beta}{q_0} = (1 - \theta_0)W(\beta b'), \quad (38)$$

$$\frac{\beta}{q_1} = \begin{cases} (1 - \theta_1)W(\beta b'), & \text{if } \frac{1-\pi}{\pi} \xi_1 \geq 1 - W(\beta b'). \\ \frac{1-\theta_1}{\pi}, & \text{otherwise.} \end{cases} \quad (39)$$

$$q_0 d'_0 = \frac{\beta b'}{(1 - \theta_0)W(\beta b')}, \quad (40)$$

$$q_1 d'_1 = 0, \quad (41)$$

$$q_b b'_h = 0. \quad (42)$$

(Region 3): If $0 < \beta b' \leq D_1^*$, then the equilibrium is given by

$$\alpha'_0 = \theta_0, \quad (43)$$

$$\alpha'_1 = 1 \quad (44)$$

$$\frac{\beta}{q_b} = W(D_1^*), \quad (45)$$

$$\frac{\beta}{q_0} = (1 - \theta_0)W(D_1^*), \quad (46)$$

$$\frac{\beta}{q_1} = \frac{1 - \theta_1}{\pi}, \quad (47)$$

$$q_0 d'_0 = \frac{\beta b'}{(1 - \theta_0)W(D_1^*)}, \quad (48)$$

$$q_1 d'_1 = \frac{1}{\gamma_1} \frac{D_1^* - \beta b'}{(1 - \theta_0)W(D_1^*)}, \quad (49)$$

$$q_b b'_h = 0. \quad (50)$$

When there is a large amount of government debt outstanding, specifically $\beta b' > D_0^*$, then the economy is in Region 1, which means that traditional banks completely crowd out shadow banks. With such high levels of government debt, the price of government debt does not reflect a liquidity premium. Hence, shadow banks cannot take advantage of a higher risk-adjusted return from shifting their portfolio to riskier securities (whose price never reflects a liquidity premium). Traditional and shadow banks thus compete only on cost θ_i and return γ_i characteristics, and by assumption traditional banks dominate in this comparison in the provision of liquidity.

As the supply of government debt falls, the economy enters Region 2 and then later Region 3. In Region 2 the price of government debt begins to reflect a liquidity premium, but still the advantage to shadow banks of shifting their portfolio to riskier securities does not encourage them to enter. As the economy enters Region 3 the price of government debt reflects a sufficient premium that both lowers the return on safe assets held by traditional banks and provides a sufficient advantage to shadow banks that can absorb the risk of investing in riskier assets, so that shadow banks begin to enter. In Region 3, a region in which both traditional and shadow banks enter, a rise in government securities b' leads to a rise in traditional banking and a fall in shadow banking. This matches a key feature of the data. In the model, a rise in government securities leads traditional banks to expand as they earn a higher return on their safe portfolio and leads shadow banks to contract as they are less able to exploit a liquidity

premium by investing in risky securities.

4.7 Some General Results

From eq. (17) it follows that if $q_0 d'_0 > 0$ and $q_1 d'_1 > 0$ (which will occur if $\epsilon < 1$), then

$$\frac{q_1 d'_1}{q_0 d'_0} = \left(\frac{\gamma_1 \frac{(1 - \alpha'_0) \left(1 - \frac{\beta}{q_b}\right) + \xi_0 h_0(\alpha'_0) + \theta_0}{(1 - \alpha'_1) \left(1 - \frac{\beta}{q_b}\right) + \xi_1 h_1(\alpha'_1) + \theta_1}}{\right)^{\frac{1}{1-\epsilon}}.$$

This equation captures five different features that determine the relative size of shadow to traditional banking. Three of these features are the substitutability between shadow and traditional banking as captured by ϵ , the value of shadow banking relative to traditional banking as reflected in γ_1 , and the cost of shadow banking relative to shadow banking as reflected in θ_1 relative to θ_0 . The term $(1 - \alpha'_i)(1 - \beta/q_b)$ captures the implicit cost of holding government debt, which is comprised of the amount of debt held, as reflected in $1 - \alpha'_i$, times the departure of the return on government debt from its fundamentals (the rate of time preference), as captured by $1 - \beta/q_b$. The final term $\xi_i h_i(\alpha'_i)$ captures the cost of default, which is comprised of the cost of default should it occur, as reflected in ξ_i , times the exposure to default risk, as captured by $h_i(\alpha'_i)$.

To study a specific example, consider two extremes, one in which government debt is so high that there is no liquidity premium on government debt, and one in which no government debt is issued. Suppose also that there is imperfect substitutability between traditional and shadow bank deposits ($\epsilon < 1$) and that there is a continuous distribution of shocks. Define ℓ_0 and ℓ_1 such that

$$\theta_i = \psi \left(\sum \gamma_j \ell_j^\epsilon \right)^{\frac{1-\eta}{\epsilon} - 1} \gamma_i \ell_i^{\epsilon-1}, \quad (51)$$

which means that

$$\ell_0 = \left(\frac{\theta_0}{\psi(1 + \gamma_1 \varphi^\epsilon)^{\frac{1-\eta}{\epsilon} - 1}} \right)^{-\frac{1}{\eta}}, \quad (52)$$

$$\ell_1 = \left(\frac{\theta_0}{\psi(1 + \gamma_1 \varphi^\epsilon)^{\frac{1-\eta}{\epsilon} - 1}} \right)^{-\frac{1}{\eta}} \varphi, \quad (53)$$

for

$$\varphi = \left(\frac{\gamma_1 \theta_0}{\theta_1} \right)^{\frac{1}{1-\epsilon}}.$$

Proposition 7: (a) If

$$\beta b' \geq \Sigma(1 - \theta_j)\ell_j$$

then the equilibrium is given by

$$\begin{aligned}\alpha'_i &= \theta_i, \\ q_b &= \beta, \\ \frac{\beta}{q_i} &= 1 - \theta_i, \\ q_b b'_h &= \beta b' - \Sigma(1 - \theta_j)\ell_j. \\ q_i d'_i &= \ell_i,\end{aligned}$$

(b) If $b' = 0$, then the equilibrium is given by

$$\begin{aligned}\alpha'_i &= 1, \\ \frac{\beta}{q_i} &= 1 - \theta_i + h_i(1), \\ b'_h &= 0,\end{aligned}$$

and $q_i d'_i$ for $i \in \{0, 1\}$ solve

$$\xi_i h_i(1) + \theta_i = \psi \left(\Sigma \gamma_j (q_j d'_j)^\epsilon \right)^{\frac{1-\eta}{\epsilon}-1} \gamma_i (q_i d'_i)^{\epsilon-1}.$$

From Proposition 7(a) it follows that as $q_b b'$ becomes very large, the size of shadow banking relative to traditional banking is determined by

$$\frac{q_1 d'_1}{q_0 d'_0} = \left(\frac{\gamma_1 \theta_0}{\theta_1} \right)^{\frac{1}{1-\epsilon}}. \quad (54)$$

Large values of $q_b b'$ thus do not lead to the complete elimination of any particular type of financial institution, but rather leads the relative size of financial institutions to be determined by their relative value as reflected in γ_1 and relative cost as reflection in θ_1/θ_0 . With large values of $q_b b'$, shadow banking is relatively larger when it has a higher relative value (γ_1) and a lower relative cost (θ_1/θ_0). As well, large values of $q_b b'$ affect the safety of financial institutions by raising the return on safe, government assets which in turn encourages financial intermediaries to invest in these types of safe assets. With sufficiently large values of $q_b b'$ financial intermediaries invest sufficiently

in government securities so that they become free of default risk.

From Proposition 7(b) it follows that as $q_b b'$ becomes very small, the size of shadow banking relative to traditional banking is determined by

$$\frac{q_1 d_1'}{q_0 d_0'} = \left(\frac{\gamma_1 (\xi_0 h_0(1) + \theta_0)}{\xi_1 h_1(1) + \theta_1} \right)^{\frac{1}{1-\varepsilon}}.$$

Since both types of financial intermediaries are now exposed to risk of default, the cost of default as captured by ξ_0 and ξ_1 is now important in determining their relative size. Shadow banking is relatively larger when the cost of default of shadow banking is relatively low (ξ_1) or the cost of default of traditional banking is relatively high (ξ_0). Their relative size is also determined by how each type of financial intermediary is exposed to default risk, as captured by $h_0(1)$ and $h_1(1)$. A high value of θ_0 relative to θ_1 will lead to low value of $h_0(1)$ relative to $h_1(1)$ as a higher value of θ_0 leads to a larger capital investment in traditional banks that makes them safer for depositors. Note that these effects tend to work in opposite directions, as a traditional bank may have a higher value of ξ_0 , but due potentially to a higher value of θ_0 , may have a lower value of $h_0(1)$.

In comparing the two extreme cases, $(q_1 b_1')/(q_0 d_0')$ falls as $q_b b'$ rises from 0 to a very large value if

$$\frac{\xi_1 h_1(1)}{\xi_0 h_0(1)} < \frac{\theta_1}{\theta_0}.$$

The left side can be thought of as the risk advantage of traditional banking relative to shadow banking and the right side can be thought of as capturing the cost advantage of traditional banking relative to shadow banking. If the cost advantage exceeds the risk advantage, then a rise in government debt will be associated with a rise in traditional banking relative to shadow banking. The reason is that a rise in government debt makes risk less important as a feature distinguishing all types of banking. In this range of parameter values traditional banking has a cost advantage, so as the equilibrium moves from one where relative size is determined by a risk advantage to one where relative size is determined by a cost advantage, traditional banking will rise relative to shadow banking. It is of course not at all clear how this comparison works out in actual banking environments, as it will depend on values of parameters that need to be estimated. It is thus not clear that a rise in government debt should be expected to lead to an expansion or contraction of traditional banking relative to shadow banking in an environment without deposit insurance.

4.8 Simulation: Lognormal Distribution for Output

To study the quantitative properties of the model, consider simulations of the model calibrated to U.S. data when the log of aggregate output is drawn from a Normal distribution with mean μ and variance σ^2 : $\ln y' \sim \mathcal{N}(\mu, \sigma^2)$. Let Φ denote the CDF for $\mathcal{N}(0, 1)$. Recall the definition of $\Omega_i(\alpha)$ and note that $\Omega_i(\alpha)$ solves

$$\Omega_i(\alpha) = \Omega_i(\alpha) \Phi \left(\frac{\ln(\Omega_i(\alpha))}{\sigma} + \frac{\sigma}{2} \right) - \Phi \left(\frac{\ln(\Omega_i(\alpha))}{\sigma} - \frac{\sigma}{2} \right) + \frac{\alpha - \theta_i}{\alpha}.$$

The solution $\Omega_i(\alpha)$ (and thereby $h_i(\alpha)$) will have to be computed numerically, which will be facilitated by the result that Φ has good approximations that are closed form. Note as well that

$$h'_i(\alpha) = \frac{1}{\alpha} \left(\frac{\theta_i}{1 - \Phi \left(\frac{\ln \Omega_i(\alpha)}{\sigma} + \frac{\sigma}{2} \right)} \right) + \Omega_i(\alpha) - 1.$$

We will choose parameter values so that the model matches various moments listed in Table 3.⁶ Since this version of the model does not include deposit insurance (deposit insurance will be included in the next section), the data in Table 3 largely covers the period 1900 - 1933, which is prior to the Banking Act of 1933 that created federal deposit insurance. Specifically, we will choose parameters so that the model matches the conditional variance of log real per-capita GDP, the real return to the stock market, real interest rates on short-term government debt, commercial paper, and traditional bank deposits, traditional bank assets as measured by M2 minus currency held by the public as a ratio to GDP, trust bank assets as a ratio to GDP, as well as the fraction of asset portfolios that commercial and trust banks invest in government debt or cash. Note that one moment we will not match is the ratio of government debt to GDP, but rather by construction we will match the ratio of government debt held by financial intermediaries to GDP. In the model they are the same (at least for the equilibria in which government debt offers a liquidity premium), but in reality these two ratios can be quite different. There are surely a variety of reasons for holding government debt that are not captured by this model, and here we focus mainly on the demand for government debt by financial intermediaries.

The model is assumed to operate at a quarterly frequency. The variance σ^2 of the conditional distribution of log income y is chosen to match the variance of the first difference of log per-capita quarterly real GDP, estimated as the annual variance

⁶Due to the difficulty of obtaining historical data, all of the variables do not begin in the same year.

Table 3: U. S. Annual Data, 1900 - 1933.

Description	Value
Standard Deviation diff. log real per-capita GDP	0.032
Real return value-weighted U. S. Equity	0.066
Real rate 30 day Treasury bill (1920 - 1933)	0.049
Real rate AA Commercial Paper (1920 - 1933)	0.062
Real rate Checking Accounts (1920 - 1933)	0.039
Gross Federal Debt / GDP	0.162
(M2 - Currency) / GDP (1914-1933)	0.409
Trust Assets / GDP (1914-1933)	0.145
Gov't Assets + Cash + Reserves / Assets for Commercial Banks (1928-1933)	0.150
Gov't assets + Cash + Reserves / Assets for Trust Banks (1928-1933)	0.149

Note: Standard Deviation diff. log real per-capita GDP is annual value of .063 divided by square root of 4. All rates are annual. *Source:* GDP and CPI is from Jordà, et. al. (2017), Treasury bill and Commercial Paper rate data is from NBER Macroeconomy database, equity market data is from Schwert (1990). Gross Federal Debt is series Y493 from Historical Statistics of the United States, Colonial Times to 1970, M2 and Currency is from Friedman and Schwartz (1963), data for Trusts and Commercial Banks is from Annual Report of the Comptroller of the Currency, 1863-1980.

divided by 4. The subjective discount factor β is chosen to match the real return on the stock market. The parameter θ_0 is chosen so that eq. (15) for $i = 0$ holds at values of α_0 , q_b , and q_0 from the data, and similarly for θ_1 . Given θ_0 , the value of ξ_0 is chosen to be consistent with eq. (21) for $i = 0$, and similarly for ξ_1 . Eq. (17) represents two equations in the remaining four unknown parameters η , ϵ , ψ , and γ_1 . Our approach will be to use eq. (17) to determine ψ and γ_1 for various choices of η and ϵ . In some sense the more interesting parameter is ϵ , which determines the substitutability between traditional and shadow bank deposits, so essentially we will be asking how the predictions of the model depend on the substitutability between these two types of assets (we will always assume $\eta = 2$). The values of the parameters are in Table 4. The solution to the model with these parameter values is presented in Table 5. Note that the solution to the model does not depend on the particular choice of ϵ , as ψ and γ_1 are chosen to maintain the calibration, but the response of the model to a change as the quantity of government debt or uncertainty may depend on the value of ϵ .

Consider how each type of bank responds to a rise in government debt. Fig. 4 shows that a rise in government debt leads to a fall in traditional banking and a rise in shadow banking. The possibility of this result is already explained above. For the parameter

Table 4: Parameter Values for Model Without Deposit Insurance

Parameter	Description	Value		
		$\epsilon = .85$	$\epsilon = .90$	$\epsilon = .95$
σ	income std. dev.	0.0320	0.0320	0.0320
β	time discount	0.9841	0.9841	0.9841
θ_0	unit cost trad. bnk.	0.0140	0.0140	0.0140
θ_1	unit cost shad. bnk.	0.0110	0.0110	0.0110
ξ_0	default cost trad. bnk.	0.1884	0.1884	0.1884
ξ_1	default cost shad. bnk.	0.1588	0.1588	0.1588
η	transaction cost curv.	2.0	2.0	2.0
ψ	transaction cost coef.	0.0047	0.0046	0.0046
γ_1	liq. value shad. bnk.	0.7075	0.7452	0.7849

Table 5: Model Solution Without Deposit Insurance

Variable	Description	Value
r_b	interest gov't debt	0.049
r_0	interest trad. bnk.	0.039
r_1	interest shad. bnk.	0.062
$q_b b'$	value gov't debt	0.083
$q_0 d'_0$	value trad. bnk.	0.400
$q_1 d'_1$	value shad. bnk.	0.145
$q_b b'_h$	value gov't debt holding pvt.	0.0
α'_0	frac. equity debt holding trad. bnk.	0.85
α'_1	frac. equity holding shad. bnk.	0.849

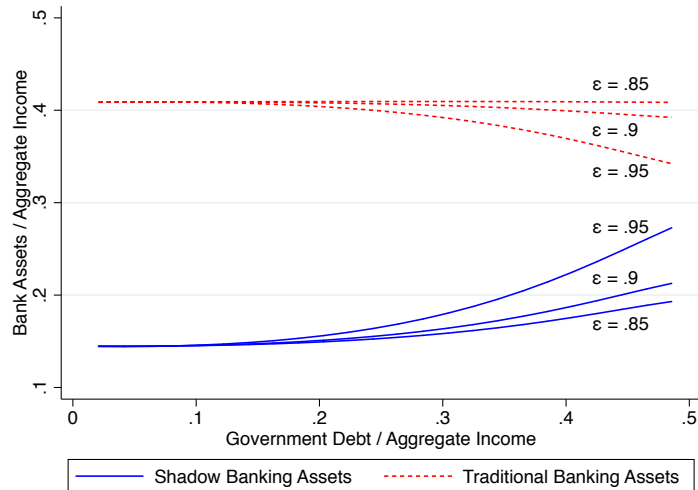
Note: All rates are annualized.

values in Table 4, due to the larger initial capital investment required of a traditional bank ($\theta_0 > \theta_1$), at low levels of government debt traditional banks have a risk advantage over shadow banks that is larger than the cost advantage of traditional banks.⁷ Thus, traditional banks dominate at low levels of government debt. As government debt expands, traditional banks lose this advantage and, at the estimated parameter values, shadow banks offer sufficient value so that they begin to replace traditional banks. As we will see, though, the situation is very different with deposit insurance.

Fig. 5 documents that the size of traditional and shadow banking both fall with a

⁷In the simulation $h_0(1) = 0.0116$ and $h_1(1) = 0.0147$, so that $(\xi_1 h_1(1))/(\xi_0 h_0(1)) = 1.07$ is higher than $\theta_1/\theta_0 = .79$, which leads to $\left(\frac{\gamma_1 \theta_0}{\theta_1}\right)^{\frac{1}{1-\epsilon}} = .91$ and $\left(\frac{\gamma_1 (\xi_0 h_0(1) + \theta_0)}{\xi_1 h_1(1) + \theta_1}\right)^{\frac{1}{1-\epsilon}} = 0.35$.

Figure 4: Bank Size and Government Debt
(No Deposit Insurance)



rise in uncertainty. Both types of banks choose to be exposed to risk, and the liquidity value of their deposits depends importantly on the safety of these deposits, so a rise in uncertainty leads to a fall in the expected value of both types of bank deposits. This behavior mimics the observed behavior of both types of banks during the Great Depression, which is a period of no deposit insurance.

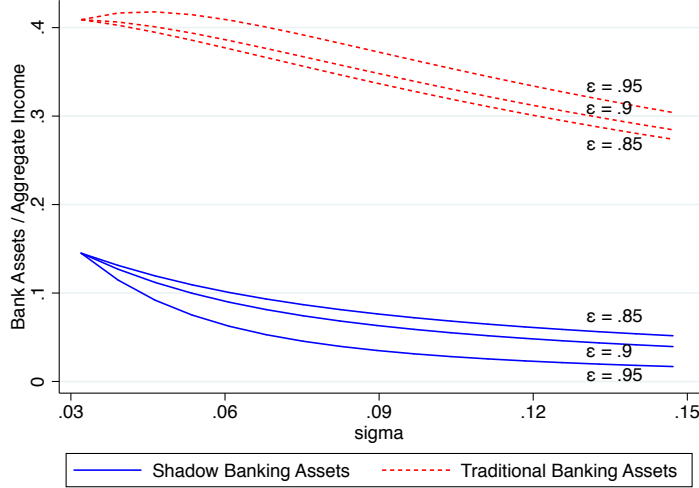
5 Deposit Insurance

In response to bank failures during the Great Depression, the U. S. passed the Banking Act of 1933 that established federal deposit insurance which protected depositors from loss of value should a bank enter bankruptcy. This section considers the consequence of deposit insurance for the competition between traditional and shadow banks.

5.1 Modifying the Model to Include Deposit Insurance

By deposit insurance we simply mean that the government guarantees the value of deposits at traditional banks. In the event of bankruptcy of a traditional bank, the government will make up any shortfall so that depositors receive the full value of their deposits. This transfer is financed by a lump-sum tax on the population. The owners of the bank receive no value once their bank enters bankruptcy. In return for deposit insurance, the regulators limit the risk exposure of banks by choosing a value of α'_0

Figure 5: Bank Size and Uncertainty
(No Deposit Insurance)



(the fraction of assets invested in risky securities). Since a choice of α'_0 limits the risk exposure of banks, it is meant to capture the effects of both reserve requirements and capital adequacy requirements. The set-up of the problem is as before, except with $x'_0 = 1$ (reflecting deposit insurance), and α'_0 given by the regulator. The Euler eq. (5) for $i = 0$ now becomes

$$q_0 \geq \beta + \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_0 (q_0 d'_0)^{\epsilon-1} q_0, \text{ w/eq. if } d'_0 > 0, \quad (55)$$

This leads to replacing eq. (17) for $i = 0$ by

$$(1 - \alpha'_0) \left(1 - \frac{\beta}{q_b}\right) - h_0(\alpha'_0) + \theta_0 \geq \psi (\Sigma \gamma_j (q_j d'_j)^\epsilon)^{\frac{1-\eta}{\epsilon}-1} \gamma_0 (q_0 d'_0)^{\epsilon-1}, \text{ w/eq. if } d'_0 > 0. \quad (56)$$

Proposition 8: Under Assumptions 1-4, and for any $\theta_0 \leq \alpha'_0 \leq 1$, there exists an equilibrium $(q_b, q, b'_h, d', \alpha'_1)$ that solves eqs. (4), (6), (15), (17) for $i = 1$, (21) for $i = 1$ and (56), along with (c, n', p) that solves eqs. (3), (7) and (8), and in which the non-negativity constraints $d' \geq 0$ and $b'_h \geq 0$ hold.

5.2 Some General Results

Using eq. (56) instead of eq. (17) for $i = 0$, it follows that if $q_0 d'_0 > 0$ and $q_1 d'_1 > 0$, then

$$\frac{q_1 d'_1}{q_0 d'_0} = \left(\gamma_1 \frac{(1 - \alpha'_0) \left(1 - \frac{\beta}{q_b}\right) - h_0(\alpha'_0) + \theta_0}{(1 - \alpha'_1) \left(1 - \frac{\beta}{q_b}\right) + \xi_1 h_1(\alpha'_1) + \theta_1} \right)^{\frac{1}{1-\varepsilon}}. \quad (57)$$

With deposit insurance the parameter ξ_0 is no longer relevant and the implicit subsidy to risk-taking by traditional banks introduces the term “ $-h_0(\alpha'_0)$ ” that tends lower the supply of shadow banks relative to traditional banks. For a supply of government debt that is so high that there is no liquidity premium on government debt, the equilibrium is the same as before since both banks acquire sufficient government debt to become default free (provided $\alpha'_0 = \theta_0$). With a fixed value of $\alpha'_0 > 0$ it is no longer possible to run the thought experiment of letting the supply of government debt go to zero. However, from eq. (57) we can see that a low required value of α'_0 acts like a tax in forcing traditional banks to hold low interest-bearing government debt, and thus a rise in the supply of government debt that raises the interest rate may disproportionately benefit traditional banks.

5.3 Simulation of the Model With Deposit Insurance

We again assume a log-normal distribution of output and calibrate the model in the similar manner as before, which leads to using the same method for choosing values of parameters for σ , β , θ_0 , θ_1 , ξ_1 , α_0 and α_1 , except based on data in Table 6 instead of Table 3. Table 6 is based on more recent data since 1980 that includes deposit insurance. We will choose parameters to match similar moments as before, although now traditional bank assets are measured by M2 minus currency held by the public minus retail money funds, and shadow bank assets are measured by financial assets held by money market mutual funds. Since depositors never lose any value of their deposits, the parameter ξ_0 is no longer relevant. The only parameters that required a new method for calibration are ψ and γ_1 , which are now based on eq. (56) for $i = 0$ and eq. (17) for $i = 1$. The new parameter values are listed in Table 7 and the new model solution is in Table 8. The essential difference is that the calibration leads to a significantly higher value of ξ_1 and a lower value of γ_1 , which suggests that shadow banks today offer a different value than shadow banks prior to the Great Depression.

Fig. 6 shows that with deposit insurance, a rise in government debt leads to a rise in traditional bank assets and a fall in shadow bank assets. This is supported in the

Table 6: U. S. Quarterly Data, 1980 - 2017.

Description	Value
Standard Deviation diff. log real per-capita GDP (1950-2017)	0.009
Real return value-weighted CRSP (1950 - 2017)	0.072
Real rate 30 day Treasury bill	0.022
Real rate AA Commercial Paper	0.029
Real rate Checking Accounts (6/2009-4/2018)	-0.015
Marketable U. S. Treasuries (domestic) / GDP	0.255
(M2 - Currency - RMF) / GDP	0.442
MMMF / GDP	0.118
Treasury bills plus Reserves / Assets for Commercial Banks	0.25
Treasury bills / Assets for Shadow Banks	0.10

Note: All rates are annualized based on quarterly data. *Source:* Equity return is value-weighted total return from CRSP. All other data is from St. Louis Federal Reserve Bank Fred Database.

data in Table 1 and is explained by the model. Government debt does not directly offer financial services and hence does not compete with banks in the provision of financial services. The only effect of changing government debt is thus through directly changing the profitability of banks. The demand for government debt by banks, either by regulation of traditional banks or by optimal risk management by shadow banks, leads to a liquidity premium in government debt that lowers the return on government debt. An expansion of government debt leads to a fall in its liquidity premium and thus a rise in its rate of return. Both types of banks benefit from this higher rate of return, but traditional banks benefit more so because they are required to hold more government debt than what shadow banks choose to hold, hence a rise in the return on government debt raises the relative profitability of traditional banks over shadow banks and thereby the size of traditional banks over shadow banks.

An important difference in the behavior of this model also regards the flight to safety in response to a rise in uncertainty. With deposit insurance, traditional bank assets are entirely safe assets, whereas shadow bank assets remain a risky form of liquidity. Hence, a rise in uncertainty leads to a rise in traditional bank assets and a fall in shadow bank assets, which is shown in . Fig. 7. As shown in Fig. 2, this feature is also documented during the financial panic of 2007/08.

Table 7: Parameter Values for Model With Deposit Insurance

Parameter	Description	Value		
		$\epsilon = .85$	$\epsilon = .90$	$\epsilon = .95$
σ	income std. dev.	0.0090	0.0090	0.0090
β	time discount	0.9832	0.9832	0.9832
θ_0	unit cost trad. bnk.	0.0177	0.0177	0.0177
θ_1	unit cost shad. bnk.	0.0098	0.0098	0.0098
ξ_1	default cost shad. bnk.	4.8919	4.8919	4.8919
η	transaction costs curv.	2.0	2.0	2.0
ψ	transaction cost coef.	0.0057	0.0056	0.0056
γ_1	liq. value shad. bnk.	0.5335	0.5699	0.6088

Table 8: Model Solution With Deposit Insurance

Variable	Description	Value
r_b	interest gov't debt	0.022
r_0	interest trad. bnk.	-.015
r_1	interest shad. bnk.	0.029
$q_b b'$	value gov't debt	0.1164
$q_0 d'_0$	value trad. bnk.	0.4420
$q_1 d'_1$	value shad. bnk.	0.1180
$q_b b'_h$	value gov't debt holding pvt.	0.0
α'_0	frac. equity holding trad. bnk.	0.75
α'_1	frac. equity holding shad. bnk.	0.9

Note: All rates are annualized.

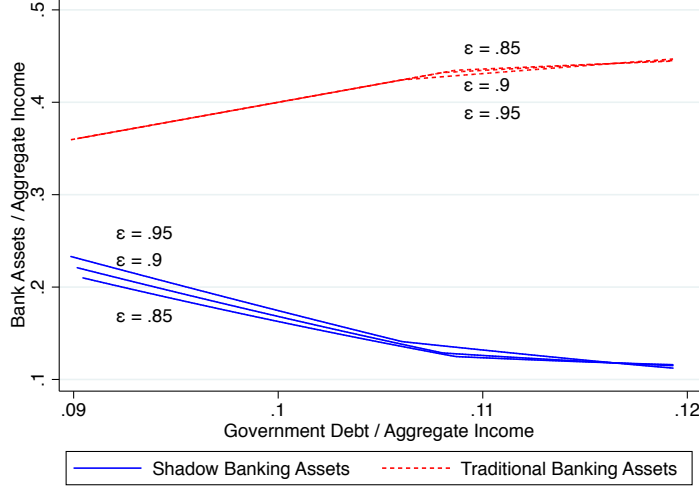
6 Welfare and Optimal Debt Policy

An optimal government policy is an amount of government debt outstanding to maximize consumption each period. From eq. (3) this amounts to a policy that minimizes the sum of transaction costs and banking costs. From Proposition 7(a) any debt level that satisfies

$$\beta b' \geq \Sigma(1 - \theta_j)\ell_j \quad (58)$$

will lead to the term $\Sigma\xi_j(1 - x_j)d_j$ achieving its minimum value of zero. Also according to Proposition 7(a), as long as debt exceeds the bound in (58) it follows that $q_i d'_i = \ell_i$.

Figure 6: Bank Size and Government Debt
(Deposit Insurance)



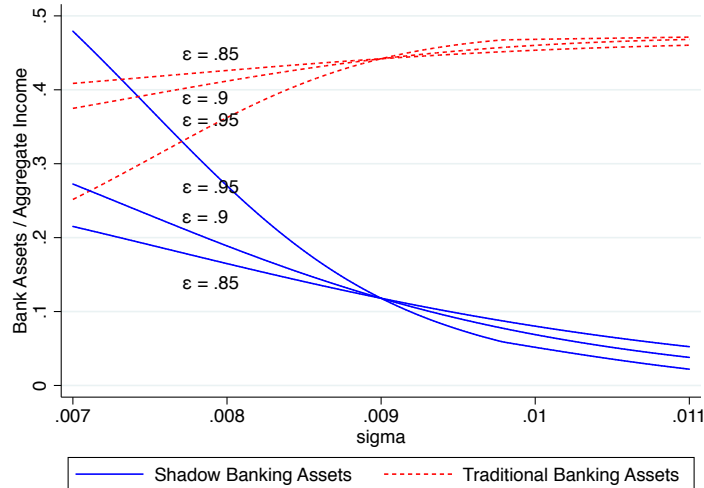
Since (ℓ_0, ℓ_1) is also the solution to

$$\min_{\ell_0, \ell_1} \frac{\psi}{\eta - 1} \left(\sum \gamma_j \ell_j^\epsilon \right)^{\frac{1-\eta}{\epsilon}} + \sum \ell_j \theta_j$$

it follows that any value of government debt that exceeds the bound in (58) achieves the maximum welfare for the model without deposit insurance. Since there is no default in this equilibrium, along with $\alpha'_0 = \theta_0$ this is also the optimal government policy with deposit insurance.

Optimal government policy is thus one with sufficient debt so that interest rates do not reflect a liquidity premium and all types of banks hold sufficient amount of government debt to be free of default risk. Both shadow and traditional banks would thus be free of default risk and would compete solely on value (γ) and cost (θ) considerations. Since both offer a different value/cost bundle and thus offer a value in such an equilibrium, neither is completely crowded out by the other. Note too that expanding the supply of government debt to eliminate the liquidity premium that encourages all banks to become free of default risk would obviating the need for deposit insurance.

Figure 7: Bank Size and Uncertainty
(Deposit Insurance)



7 Summary

This paper presents a model in which banks' value stem from the services they offer. Safety is an important aspect of banking, but not because banks are unique in offering safety, but rather that safety is a key dimension of the services they offer. An efficient payment mechanism, which is the *raison d'être* of banking, requires an assurance that payments will be honored and hence requires that banks are safe institutions. Government debt does not per se produce financial services, and hence does not directly compete with banks. Rather, banks view government debt as a source of safety and their demand for these safe assets imparts a liquidity premium to government debt.

The view of banks as offering financial services that rely on safety opens the door to different types of financial institutions offering different bundles of financial services and safety. Traditional banks are perhaps more full-service financial institutions that put a high premium on safety, whereas shadow banks are perhaps more limited-service financial institutions that put less of a premium on safety. The relationship between the supply of government debt and the size of traditional and shadow banks is thus a complex one. Since both value safety, with a sufficient supply of government debt that eliminates the liquidity premium on government debt, both should hold sufficient government debt to be free of default risk, and thus the size of traditional versus shadow banks should be entirely determined by the value and cost of their respective financial services. With a very limited supply of government debt, the size of traditional versus

shadow banks will depend on how each one is able to accommodate a higher exposure to risk. The parameter values determined in this paper suggests that traditional banks are better able to accommodate risk, which also implies that a rise in government debt should lead to a fall in the relative size of traditional to shadow banks. However, in a world with deposit insurance, the size of traditional banking depends more on the implicit tax of being required to hold low-interest government debt, so a rise in government debt and consequent rise in yield on government debt leads to a rise in traditional banking relative to shadow banking.

An important implication of this paper is that a rise in government debt that eliminates the liquidity premium on government debt will encourage all financial institutions, in particular traditional and shadow banks, to hold more government debt and thereby be less subject to default risk. With the absence of a liquidity premium that would otherwise hamper the development of either traditional or shadow banks, they would have to compete based on their fundamental value/cost bundles. It should not be expected or even desirable that shadow banking vanish due to a rise in the supply of government debt, as shadow banking may offer a value/cost bundle of services distinct from traditional banks. A rise in the supply of government debt that eliminated the liquidity premium would encourage all financial institutions to become safer, thereby achieving important welfare gains, whatever happens to the relative size of traditional and shadow banks.

8 Appendix

Proof of Lemma 1: Define $Z(a, e, h) = E_y[\max\{-e, a(y'/\bar{y} - 1) - h\}]$. Note that $Z(a, e, 0) \geq E[a(y'/\bar{y} - 1)] = 0$, $Z(a, e, a(y_{max}/\bar{y} - 1)) \leq 0$ since all terms inside the expectations operator are non-positive, and $Z(a, e, h)$ is a strictly-decreasing function of h in the range $0 \leq h \leq a(y_{max}/\bar{y} - 1)$. There thus exists a unique solution $H(a, e) \geq 0$ such that $Z(a, e, H(a, e)) = 0$. $Z(a, e, h)$ is a continuous function of a , e and h , so $H(a, e)$ is a continuous function of a and e . For $a \leq e$, note that $E_y[\max\{-e, ay'/\bar{y} - a\}] = E[a(y'/\bar{y} - 1)] = 0$ hence $H(a, e) = 0$ for $a \leq e$. The monotonicity of H with respect to e follows from the monotonicity of Z with respect to e and h . **Q.E.D.**

Proof of Lemma 2: The solution $h_i(a)$ can be written as

$$0 = -\theta_i F(\bar{y}\Omega_i(a), y) + \int_{\bar{y}\Omega_i(a)}^{\infty} \left(a \left(\frac{y'}{\bar{y}} - 1 \right) - h_i(a) \right) f(y', y) dy'.$$

The results follow from repeated use of the Fundamental Theorem of Calculus. Note that $h'_i(\theta_i) = 0$ is well defined, as the left and right limits agree. **Q.E.D.**

Proof of Lemma 3: Follows from the concavity of the left side of inequality (18) with respect to $\hat{\alpha}'_i$. **Q.E.D.**

Proof of Theorem 4: Define the function $T(q_b)$ as

$$T(q_b) = q_b b' - \Sigma(1 - \alpha'_j)q_j d'_j, \quad (59)$$

where the dependence of α'_i on q_b is given by eq. (21) and the dependence of $q_i d'_i$ on q_b is given by eq. (17) along with the dependence of α'_i on q_b just described. Note that T is a well-defined, continuous function. If $T(\beta) \geq 0$ then the solution is $q_b = \beta$, $q_b b'_h = T(\beta)$, $\alpha_i = \theta_i$, $q_i = \beta/(1 - \theta_i)$, and $q_i d'_i$ solves eq. (17) where the left side equals θ_i . Suppose $T(\beta) < 0$. Note that since $\theta_i > 0$, it follows that $\Sigma(1 - \alpha'_j)q_j d'_j$ as a function of q_b is bounded above, hence there exists a q_b^* such that $T(q_b) \geq 0$ for any $q_b \geq q_b^*$. There thus exists a value of q_b between β and q_b^* such that $T(q_b) = 0$. The associated value of (q, b'_h, d', α') defines an equilibrium. **Q.E.D.**

Proof of Lemma 5: These results are straightforward to verify. **Q.E.D.**

Proof of Proposition 6: (Region 1): Eq. (4) holds since $q_b = \beta$. Eq. (6) can be shown to hold just by substitution of the solution into this equation. To show that eq. (15) holds, note that $h_i(\theta_i) = 0$. Eq. (23) holds since $\alpha'_i = \theta_i$ and again since $q_b = \beta$. Eq. (17) holds with equality for $i = 0$ by construction of D_0^* , and holds with inequality for $i = 1$ as $\theta_1 \geq \theta_0 \gamma_1$ holds by assumption.⁸

(Region 2): To show that eq. (4) holds, note that $W(\beta b') \leq W(D_0^*) = 1$. Eq. (6) can again be shown to hold just by substitution of the solution into this equation. Through straightforward substitution, eq. (15) can be shown to hold for $i = 0$ and for $i = 1$ when $\frac{1-\pi}{\pi}\xi_1 \geq 1 - W(\beta b')$. When $\frac{1-\pi}{\pi}\xi_1 < 1 - W(\beta b')$ use the result that $h_1(1) = (1 - \theta_1)\frac{1-\pi}{\pi}$ to show that eq. (15) holds for $i = 1$. Eq. (17) for $i = 0$ holds by definition of W . Eq. (17) for $i = 1$ and $\frac{1-\pi}{\pi}\xi_1 \geq 1 - W(\beta b')$ holds if

$$1 - (1 - \theta_1)W(\beta b') \geq (1 - (1 - \theta_0)W(\beta b'))\gamma_1,$$

which holds since $0 \leq W(\beta b') \leq W(D_0^*) = 1$ and $\gamma_1 < 1$ and $\theta_1 \geq \theta_0 \gamma_1$ by assumption.

⁸Equilibria also exist for $\alpha'_0 < \theta_0$, but all prices and the supply of liquidity are the same for these equilibria.

Eq. (17) for $i = 1$ and $\frac{1-\pi}{\pi}\xi_1 < 1 - W(\beta b')$ holds if

$$(1 - \theta_1)\xi_1 \frac{1 - \pi}{\pi} + \theta_1 \geq ((1 - \theta_0)(1 - W(\beta b')) + \theta_0)\gamma_1,$$

which holds since in this region $W(D_1^*) \leq W(\beta b')$ and this inequality holds for $W(D_1^*)$ by construction of D_1^* . Eq. (23) holds for $i = 0$ since $\frac{1-\pi}{\pi}\xi_0 > 1$ by assumption (and thus $\frac{1-\pi}{\pi}\xi_0 > 1 - W(\beta b')$). Eq. (23) holds for $i = 1$ by construction, given that $\frac{\beta}{q_b} = W(\beta b')$.

(Region 3): Eq. (4) holds since $W(D_1^*) \leq 1$. Eq. (6) and (15) can be shown to hold by substitution of the solution into these equations. Eq. (17) for $i = 0$ holds by definition of W and eq. (17) for $i = 1$ holds by construction of D_1^* . Eq. (23) holds for $i = 0$ for the same reason as in Region 2. Eq. (23) holds for $i = 1$ if $\frac{1-\pi}{\pi}\xi_1 \leq 1 - W(D_1^*)$. From the definition of D_1^* , this inequality holds if

$$\frac{1 - \pi}{\pi}\xi_1(1 - \gamma_1) + \left(1 - \frac{1 - \pi}{\pi}\xi_1\right)(\theta_1 - \theta_0\gamma_1) \geq 0,$$

which holds since $1 - \gamma_1 > 0$, $\theta_1 - \theta_0\gamma_1 > 0$, and $1 - \frac{1-\pi}{\pi}\xi_1 > 0$ by assumption. **Q.E.D.**

Proof of Proposition 7: These results are straightforward to verify. **Q.E.D.**

Proof of Proposition 8: The proof is similar to that of Theorem 4. **Q.E.D.**

9 References

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