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**EMPIRICAL REGULARITIES IN THE  
DEUTSCH MARK FUTURES OPTIONS**

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### Abstract

This paper analyzes the Deutsche Mark futures options traded on the Chicago Mercantile Exchange, from January 24, 1984 to October 10, 1984. Transaction data were used to match the prices of the futures options and the underlying futures contracts.

In the data, there are very low rates of violations of early exercise boundary conditions and put-call parity conditions. This is indicative of well-synchronized markets and low transaction costs.

The analysis of implied volatilities turned up some interesting empirical regularities. (1) Implied volatilities are not constant over time. In fact, they appear to be serially correlated. (2) The moneyness bias is very strong, regardless of time to maturity. Deep-out-of-the-money options have the largest implied volatilities, and out-of-the-money options the second largest. In other words, the Black-Scholes model underprices deep-out-of-the-money options relative to at-the-money options. (The evidence is not conclusive for deep-in-the-money options, since they are not frequently traded.) (3) The maturity bias is also quite strong. Implied volatilities tend to rise as the options approach maturity. In other words, the Black-Scholes model underprices options which are closer to maturity relative to those which are further from maturity.

These systematic biases of the Black-Scholes model are unlikely to be caused by transaction costs and discontinuous trading. More likely, the distributional assumption of the Black-Scholes model is at fault. The cash price and futures price for DM do not appear to come from a Weiner diffusion process, as assumed in the Black-Scholes model. Three different candidates have been considered in the literature. One, the underlying process follows a mixture of a Weiner diffusion process and a Poisson jump process. This can explain both the maturity bias and moneyness bias. Two, the variance of the underlying futures price is increasing monotonically as the contract approaches maturity. Three, the variance of the underlying futures prices is stochastic, as in the case of an autoregressive conditional heteroscedasticity model. It appears that the third candidate may provide the most promising explanation.



## 1. Introduction

To date, most of the literature on foreign currency options primarily concern those written on cash currencies, such as those traded on the Philadelphia Stock Exchange. Theoretical pricing models for these options, developed along the lines of Black and Scholes [1973] and Black [1976], have appeared in Biger and Hull [1983], Garman and Kohlhagen [1983], Giddy [1983], and Grabbe [1983]. Empirical studies about these options have been published by Shastri and Tandon [1985, 1986], and Bodurtha and Courtadon [1986].

In this paper, we examine the options on the Deutsche mark (DM) futures contract traded on the Chicago Mercantile Exchange (CME). These options have been studied in Ball and Torous [1986], who use closing option prices and corresponding settlement futures prices from January 24, 1984 to March 1, 1985. Cox and Rubinstein [1985] expressed some reservations on the use of closing prices (p. 341). Hence in our work, we use transactions data obtained from the CME.

The motivation behind this study is twofold. First, we wish to examine whether the futures options behave differently than the cash options. Arbitrage restrictions for futures options should be tighter than those for cash options, because of side-by-side trading of the options and the underlying futures contract, and because of lower transaction costs in futures markets. The options on DM futures are traded in a pit on the CME only a few feet away from the pit where the underlying DM futures are traded. Floor traders can observe prices and conduct trades in both markets very easily. A priori, these two markets are "well synchronized," a point first brought up by Galai [1978] in discussing the link between stock options traded on the Chicago Board of Options Exchange and the underlying stocks traded on the major stock exchanges such as the New York and the American Stock Exchanges. The situation is very different on the Philadelphia Stock Exchange, where the options on cash currencies are traded, because the underlying cash currencies are traded in the interbank market. In order to arbitrage between the cash option and the cash currency, traders must use two different markets. It is therefore questionable whether the cash option and the cash currency markets are well synchronized.

In addition, the cost of trading in DM futures and futures options appear to be much smaller than that in DM cash and cash options. For example, Ball and Torous [1986] write: "Transaction costs to trading in futures contracts and futures option contracts are minimal. For floor traders in a particular contract, round trip transaction costs to a position in that instrument are on the order of 24¢ per contract. These costs include a service charge of 2¢ per contract per side payable to the exchange, and a clearing charge of 10¢ per contract per side payable to the clearing firm. However, for floor traders in a particular contract taking a position in a different instrument, say, a futures option trader taking a position in the underlying futures contract, an additional round trip service charge of approximately \$3 per contract is levied." (p. 864) The round trip transaction costs for arbitraging between the futures option and the underlying futures contract are less than \$3.50. On the other hand, Bodurtha and Courtadon [1986] estimate the market maker's transaction costs per contract to be:

|                                |                 |
|--------------------------------|-----------------|
| " OCC initial fee              | \$ 0.05         |
| PHLX \$2 broker fee            | \$ 2.00         |
| PHLX exchange fee              | \$ 0.05         |
| PHLX proportional value charge | 0.12%" (p. 157) |

In addition, they estimate the cost of each foreign exchange trade on the interbank market to be \$50.00. Therefore, the costs of arbitraging between the cash option and the interbank market are at least \$52.10, before adding the proportional value charge of 0.12%. These are substantially higher than the transaction costs on futures options and futures contracts.

Second, we are interested to examine whether the Black-Scholes model for option pricing provides a good description of the data. The low transaction costs and side-by-side trading of the option and the underlying futures contract are good approximations of the Black-Scholes model, which assumes zero transaction costs. On the other hand, we have reasons to doubt the appropriateness of the Black-Scholes model for DM futures options. Burt, Kaen and Booth [1977], Westerfield [1977], and Rogalski and Vinso [1978] have all found that the rate of change of spot currency prices are non-normal. In addition, Hsieh [1985] and Manas-Anton [1986] have found that these price changes are not independent and identically distributed, and

that they are better described by Engle's [1982] autoregressive conditional heteroscedasticity model (ARCH). In the present work, we concentrate on the systematic biases in the Black-Scholes model, from which we may obtain some further information about the distribution of these prices.

## 2. The Pricing of Options on Deutsche Mark Futures Contract

In this section, we review the literature on the pricing of options on futures contracts. For this purpose, we shall use the following notation. Let  $S$  denote the cash price of Deutsche Mark (DM) currency in terms of U.S. Dollars. Let  $i$  be the instantaneous U.S. riskless interest rate, and  $i^*$  the instantaneous German riskless interest rate. Both are assumed to be known and constant. There is no spread between borrowing and lending rates in both currencies. Furthermore, there are no transactions costs, and trading is conducted continuously over time.

Under these conditions, costless arbitrage ensures that the futures price of Deutsche Mark,  $F$ , must satisfy the following relation:

$$F = S \exp( (i-i^*) T' ),$$

where  $T'$  is the time to maturity of the futures contract, measured in the same time units as the interest rates.

A standard assumption for pricing option and futures contracts is to suppose that  $S$  can be described by the following continuous time stochastic process:

$$dS / S = \mu dt + \sigma dZ,$$

where  $Z$  is a standard Wiener diffusion process. Then  $F$  must also be characterized by:

$$dF / F = (\mu - i + i^*) dt + \sigma dZ.$$

Now, consider a European call option on the futures contract, with time to maturity  $T$  ( $< T'$ ), and strike price  $K$ . Then using a riskless hedge between the option and the futures contract, and noting that the carrying cost of the latter is zero, one obtains the Black [1976] formula for pricing a call option on futures contract:

$$C(F,T,K) = \exp(-iT) [ F N(d_1) - K N(d_2) ],$$

where  $d_1 = [ \log(F/K) + \sigma^2 T / 2 ] / [ \sigma \sqrt{T} ]$ ,

$$d_2 = d_1 - \sigma \sqrt{T},$$

and  $N(x) = [ \int_{-\infty}^x \exp(-u^2/2) du ] / 2\pi$ .

Similarly, one obtains the price of a put option:

$$P(F,T,K) = \exp(-iT) [ K N(-d_2) - F N(-d_1) ].$$

It is worthwhile to note here that, under the above assumptions, the forward price and the futures price of DM is the same, if the two contracts mature at the same time. Hence these pricing formulae for European futures options are identical to those for European cash options found in Biger and Hull [1983], Garman and Kohlhagen [1983], Giddy [1983], and Grabbe [1983].

The DM futures options traded on the CME are American options, which differ from European options in that their owners can exercise the option at any time prior to maturity. It can easily be shown that there are some circumstances in which the value of an American option would be greater than that of an identical European option. Consider the following. Suppose the futures price  $F$  is much higher than the strike price  $K$ . For a call option, the European formula will give a price approaching  $\exp(-iT) [F - K]$ . If the American call option is exercised immediately, the owner receives  $[F - K]$ . Hence the American option must command a higher price than its European counterpart. In fact, the American option must sell for a price no less than  $F - K$ , the value of early exercise.

Unfortunately, there is no simple closed form solution to evaluate the American option. Numerical methods or approximations must be used. In this paper, we use the technique described in Barone-Adesi and Whaley [1986]. They point out that the difference between the value of an American option and that of an identical European option is the early exercise premium of the former. Since the same partial differential equation (which results from the riskless hedge between the option and the underlying asset) applies to both the American and the European option, it also applies to the early exercise premium of the American option. Barone-Adesi and Whaley employ a quadratic approximation for this partial differential equation, and obtain a closed form (approximate) solution for the early exercise premium. When this is added to the value of the European option, it gives an approximation of the value of the American option. They show that this method not only gives accurate approximations of the value of the American option, but also is one of the fastest computational method available for pricing American options.



### 3. The Data

The data consist of quotes on the Deutsche Mark (DM) futures contract and quotes on the options on the DM futures contract, traded on the Chicago Mercantile Exchange (CME). Each DM futures contract is for the delivery of 125,000 DM. Each DM option contract is written on one DM futures contract. The futures contract was first traded on May 16, 1972, while the option did not start until January 24, 1984. The price of the futures contract as well as the options is quoted in U.S. Cents per DM. Each tick of the options and the futures equals \$12.50, which is the minimum price change for both contracts.

The data were provided by the CME, and are generally known as "quote capture" information. The data set contains the time and price of every transaction in which the price has changed from the previous transaction. In addition, bid and ask prices are also recorded if the bid price is above or the ask price is below the price of the previous transaction. There is no information regarding the number and volume of transaction at a particular price.

We eliminated all bid and ask quotes from the data, since they do not represent transaction prices. From the remaining quotes, we extracted a very small subsample of data. On each trading day, we found the last traded price for each option nearest to noon Chicago time, and then matched it to the price of the underlying futures contract which traded most closely preceding the option. On average, the time between an option transaction and the preceding futures transaction was only 152 seconds. We have a total of 1560 observations on call options and 824 on put options in our sample. The smaller number of observations on puts is due to the fact that puts are less frequently traded than calls. Table 1 gives a complete breakdown of these options by strike price and by maturity date.

We do not have any transaction data for interest rate. Instead, we use the three month Treasury bill rate in the Federal Reserve G.13 Release as the riskless U.S. interest rate ( $i$ ). This should not cause too much problem in pricing options, since interest rates were not as volatile as exchange rates, and option prices are not very sensitive to small changes in interest rates.

#### 4. Testing for Violations of Boundary Conditions

In this section, we test for violations of the early exercise boundary conditions and the put-call parity conditions in the data. These boundary conditions are of interest, since they are derived merely from arbitrage arguments and not from any particular model of option pricing. Furthermore, the low transaction costs and side-by-side trading of futures options and the underlying futures contract make these boundary conditions more likely to hold than cash options and cash currencies.

##### Early Exercise Boundary Conditions

As pointed out in section two, American options on futures contracts must sell for at least the value of early exercise, if there is to be no profitable arbitrage. These boundary conditions are:

$$C(F,K,T) \geq \text{Max}(0, F-K), \text{ and}$$

$$P(F,K,T) \geq \text{Max}(0, K-F).$$

Table 2 summarizes our findings of violations of the above boundary conditions. For call options, we found 16 violations out of 1560 observations, at a rate of 1.03%. These violations occurred only in the options maturing in March and June of 1984. In fact, all but one of these violations occurred before March 9, 1984. The size of the violations is quite small. The maximum violation is 4 ticks or \$50. For the calls maturing in March 1984, 4 violations involved the discrepancy of 1 tick, 2 violations involved 2 ticks, 2 violations involved 3 ticks, and 2 more involved 4 ticks. For the calls maturing in June 1984, 1 violation involved the discrepancy of 1 tick, another involved 2 ticks, 3 violations involved 3 ticks, and 1 involved 4 ticks.

For put options, we found 5 violations out of 824 observations, at a rate of 0.61%. All violations involved options maturing in June and September 1984. Here, too, the size of the violations is small. The maximum violation is 2 ticks or \$25.00. For the put options maturing in June 1984, 3 violations involved only 1 tick, and 1 violation involved 2 ticks. For the put options maturing in September 1984, there was only 1 violation involving 2 ticks.

These results contrast interestingly with those in previous studies. Ball and Torous [1986], using closing DM futures option prices and settlement DM futures prices from January 24, 1984 to March 1, 1985, found 2.70% violations for calls and 7.08% for puts. Shastri and Tandon [1985], using closing prices for cash options and cash currencies from December 14, 1982 to November 22, 1983, found 1.66% violation for calls and 17.60% for puts. Bodurtha and Courtadon [1986], using transactions prices for cash options and cash currencies from February 28, 1983 to September 14, 1984, found 0.77% violations in call observations, and 8.26% violations for puts. Thus, our results find a similar rate of violation in calls, but a substantially lower rate for puts.

We speculate that the high rate of violation for puts in Ball and Torous [1986] resulted from the use of closing prices. As mentioned earlier, puts are much less frequently traded than calls. Closing prices for puts are probably not synchronous with the corresponding settlement futures prices. This may also explain the high rate of violation for puts in Shastri and Tandon [1985]. But this cannot explain the high rate of violation for puts in Bodurtha and Courtadon [1986], who use transaction data. Bodurtha and Courtadon themselves show that the violations can be explained by transaction costs, which are much higher for arbitraging between cash options and the cash currency market than between futures options and the futures market.

#### Put-Call Parity Conditions

Besides the early exercise boundary conditions, there are other arbitrage restrictions between the prices of puts and calls. Ramaswamy and Sundaresan [1985] provides a relation between call and put options with similar contractual terms:

$$C(F,K,T) \leq P(F,K,T) + F - K b(T),$$

where  $b(T)$  is the price of a unit discount bond paying \$1 with time to maturity  $T$ . This relation is called "the upper put-call parity" in Ball and Torous [1986], and holds even if interest rates are stochastic. Stoll and Whaley [1986] provides a second relation:

$$C(F,K,T) \geq P(F,K,T) - F b(T) + K.$$

This condition is called the "lower put-call parity" in Ball and Torous [1986], and holds only if interest rates are constant over time.

We checked for violations of the two parity conditions in the data. We assumed a constant interest rate, and used the three month treasury bill rate ( $i$ ) to calculate the price of the discount bond as:

$$b(T) = \exp(-iT).$$

Table 3 summarizes our findings. For the lower put-call parity, there were only three violations out of 592 observations, a rate of 0.51%. Two violations occurred in the option with a strike price of 39¢ maturing on March 1984, when the options were one and two days prior to expiration. The maximum violation was only 3.98 ticks (\$49.75). The third violation occurred in the options with a strike price of 34¢, but it was only 1 tick (\$12.50).

For the upper put-call parity, there were only four violations out of 592 observations, a rate of 0.68%. The largest violation was 9.97 ticks (\$124.50). This appears to be sizable. However, it turns out that the put and the call traded 1 hour and 21 minutes apart, during which time, the futures price moved 11 ticks (\$137.50). The second largest violation was 6.14 ticks (\$76.75). In this case, the put and the call traded more than 4 hours apart, during which time the futures price moved 18 ticks (\$225.00). These two instances should not be considered violations of the upper put-call parity condition, since the puts and calls could not be traded simultaneously. The remaining two violations consist of 1.99 ticks (\$25.00) and .97 ticks (\$12.50).

Our results are similar to the findings of other authors. For the lower and upper put-call parity conditions, Ball and Torous [1986] found violation rates of 0.58% and 0.73%, respectively, while Bodurtha and Courtadon [1986] found violation rates of 0.05% and 0.32%, respectively. Shastri and Tandon [1985] found violation rates of 28.25% and 46.10% respectively, but Bodurtha and Courtadon argue that these results are caused by closing prices.

On the whole, we found that we have very low rates of violation of the early exercise boundary conditions and the put-call parity conditions. These results compare very favorably with those in previous work. For the few violations we found, we cannot attribute them to transaction costs and non-synchronized trading. The transaction costs of trading in futures options

and futures contracts are very minimal. In addition, the futures options and futures trade side-by-side. We do not believe that these violations are evidence of market inefficiency. Rather, we believe the violation may have other explanations, such as the presence of large bid-ask spreads and the quality of the data. It is possible that, when the DM futures options were first traded, there was little liquidity. Bid and ask prices may have been more than several ticks apart, causing some transactions to appear to violate the boundary conditions. As the liquidity in these options increased, the bid-ask spread narrowed, and so we do not observe these discrepancies any more. It is also possible that, when the options were first traded, the quote capture reporting procedures may not have been very accurate. The recorded time for an option price may be different from the actual trading time. We therefore observed some discrepancies which never existed in real time. The reporting procedures may have been improved at a later date, so we do not observe these violations any more.

## 5. Analysis of Implied Volatilities

In this section, we analyze whether the Black-Scholes method for pricing futures options describes the data adequately. For simplicity, let us denote the theoretical or model price of a call option on the DM futures contract as  $C(F,K,T;\sigma,i)$ , and that of a put option as  $P(F,K,T;\sigma,i)$ . We observe the variables  $F$  (the price of the underlying futures contract),  $K$  (the strike price),  $T$  (time to maturity),  $i$  (the riskless interest rate, assumed to be the three month Treasury bill rate), and the market value of the option. The only unknown is  $\sigma$  (the instantaneous standard deviation of the underlying contract).

There are two ways to proceed from here. One method is to estimate  $\sigma$  using past data, compute the model price (conditional on the estimated value of  $\sigma$ ), and compare it with the market price. This was done in Shastri and Tandon [1986]. Another method is to find the value of  $\sigma$  which equates the model price to the observed market price. We denote this value by  $V$ , which is known as the "implied volatility" of that option. As an example,  $V$  solves the following equation for a call option:

$$C(F,K,T;V,i) = \text{observed market price.}$$

This is a nonlinear equation, and is solved by an iterative Newton-Raphson procedure. This was done in Ball and Torous [1986], and is the method we follow. By examining the properties of the implied volatilities, we hope to gain some information about systematic biases of the Black-Scholes model, and about the distribution of the underlying futures contracts.

Under the assumptions of the Black-Scholes model, the two methods should produce the same results. The difference is that the first method requires an estimate of  $\sigma$ , which generally contains estimation error. Using the same estimate of  $\sigma$  for more than one option may produce systematic biases which would otherwise be absent. The second method computes an implied volatility for each option, and avoids the problem of estimation error altogether.

### Serial Correlation of Implied Volatilities

A frequently discussed issue concerning the Black-Scholes model is whether  $\sigma$  is constant over time. We cannot directly test this hypothesis, since we do not observe  $\sigma$ . Instead we use the implied volatility of the

option in its place. This procedure may contain some measurement errors, since the futures price, the option price, and the interest rate, may not be observed simultaneously. If the Black-Scholes model is correct, then the implied volatility should be different from the true volatility by a random measurement error  $\epsilon$ :

$$V = \sigma + \epsilon.$$

Unless the measurement errors have a systematic component,  $\epsilon$  should not be correlated across observations.

In Table 4, we present some time series statistics of the implied volatilities. For both puts and calls, it is quite clear that the first and second order serial correlation coefficients,  $\rho_1$  and  $\rho_2$ , are very different from zero. It therefore appears to be the case that the implied volatilities contain a systematic error term, contrary to the assumptions of the Black-Scholes model. This finding is consistent with a time-varying  $\sigma$ . The Black-Scholes model can be generalized, as in Merton [1973], to handle the case when  $\sigma$  is time-varying but nonstochastic. But there are no simple results when  $\sigma$  is stochastic, as in the case of an ARCH model.

#### Moneyiness Bias

Another frequently discussed problem of the Black-Scholes model is the "moneyiness" bias. The Black-Scholes model has been found to work well for at-the-money options, but often misprices deep-in-the-money and deep-out-of-the-money options. For our study, we created five categories of moneyiness. A call option is deep-out-of-the-money (DOM) if the ratio of the futures price to the strike price (F/K) is less than .95; it is out-of-the-money (OM) if F/K is between .95 and .98; it is at-the-money (AM) if F/K is between .98 and 1.02; it is in-the-money (IM) if F/K is between 1.02 and 1.05, and deep-in-the-money (DIM) if F/K is greater than 1.05. The moneyiness of put options is defined similarly, except reversing the order of the definition of moneyiness.

Table 5 summarizes our findings. In the first row, we compare the implied volatilities of deep-out-of-the-money calls (DOM) with those of the other four categories of call options. The comparison against the at-the-money calls (AM) is in the second column. In our data, there are 570 days

with observations for both categories. Of these, only 16 pairs (2.81%) contain at-the-money calls with implied volatilities greater than deep-out-of-the-money calls. The implied volatilities of the at-the-money calls are less than those of the deep-out-of-the-money calls at the 5 percent significance level, using the nonparametric test in Rubinstein [1985]. On average, the implied volatilities of the at-the-money calls are 10.0% smaller than those of the deep-out-of-the-money calls. Figure 2 contains a plot of the two implied volatilities for the calls maturing in September 1984, and shows that the deep-out-of-the-money calls consistently have higher implied volatilities than the at-the-money calls. A glance at Table 5 reveals that, as a rule, the deep-out-of-the-money calls have the largest implied volatilities, and the out-of-the-money calls the second largest. The rankings are not conclusive for the remaining calls, probably because these options are not traded as frequently.

The comparison for puts gives similar results. Table 5 shows that the deep-out-of-the-money puts have the largest implied volatilities, and the out-of-the-money puts the second largest. Again, the rankings are not conclusive for the remaining puts. Figure 3 contains a plot of the implied volatilities of the out-of-the-money puts and the at-the-money puts, and shows that the out-of-the-money puts consistently have higher implied volatilities than the at-the-money puts.

These findings differ somewhat with those in Whaley [1986], who finds that the model underprices out-of-the-money calls but overprices deep-out-of-the-money puts for options on the Standard and Poors 500 futures contract. Our results show that the model underprices out-of-the-money calls and puts relative to at-the-money options.

To check the robustness of our results, we performed regression tests using options which are identical except for strike prices. The results are consistent with those above. They show that options with lower strike prices have lower implied volatilities. (These results are not reported in our tables, but are available upon request.)

To check if the moneyness bias changes with time to maturity, we recomputed Table 5 for options with maturities of 0-2 weeks, 3-4 weeks, 5-6 weeks, 7-10 weeks, and 10 or more weeks. The results are in Appendix A-1 to A-5. They show very clearly that the moneyness bias continues to hold,



regardless of the time to maturity. In fact, the moneyness bias becomes more pronounced as time to maturity shortens. For example, the average difference in implied volatilities between out-of-the-money calls and deep-out-of-the-money calls is 7.1% when both options have 10 or more weeks until maturity. This difference rises to 11.0%, 21.7%, 23.1%, and 45.9% as the time to maturity falls to 7-10 weeks, 5-6 weeks, 3-4 weeks, and 0-2 weeks, respectively.

#### Maturity Bias

A third frequently discussed problem of the Black-Scholes model is the "maturity" bias. Ball and Torous [1986] observed that implied volatilities of DM futures options increased as the time to maturity decreased. We also find the same phenomenon. Figures 2 and 3 are representative time series plots of the implied volatilities in our data. In general, we find that (a) implied volatilities are not constant across time, and (b) implied volatilities tend to rise when the options approach maturity.

The first observation is consistent with the finding of heteroscedasticity in daily spot exchange rates, in Hsieh [1985] and Manas-Anton [1986]. Since our data contain only a few expiration series, we are not sure whether the rise in implied volatilities near maturity is related to a change in variances of the underlying futures contract during those dates, or whether there is a systematic maturity bias.

In order to discriminate between the two explanations, we compared the implied volatilities of pairs of options which are identical except that they mature three months apart. For example, we compared the implied volatility of the 38¢ call maturing in June with that of the 38¢ call maturing in September, as they move through time. In this way, we hold constant all other aspects except time to maturity. We performed the same analysis holding fixed moneyness as well. Under the assumptions of the Black-Scholes model, the implied volatilities should not differ between options with different times to maturity. The findings are in Table 6.

Let us first examine the effect of maturity alone. For call options, this is in the sixth row of the table, and for put options, in the last row. In our sample, there are 50 pairs of options which are identical except they mature three months apart, and the closer-to-maturity option has 0-2 weeks

remaining until expiration. There are 38 pairs (76%) in which the closer-to-maturity option has the larger implied volatility. The difference is 27.7 percent on average. The results are equally striking for those matched pairs in which the closer-to-maturity option has 3-4 weeks and 5-6 weeks remaining until expiration. The evidence is not conclusive for the remaining pairs.

For put options, when the closer-to-maturity option has 4 or fewer weeks remaining to expiration, it also tends to have a higher implied volatility than the further-to-maturity option. The evidence is not conclusive for the remaining pairs.

These results contradict the assumptions of the Black-Scholes model. The evidence strongly suggests that the implied volatility increases as an option nears expiration. This is consistent with the hypothesis that the distribution of the underlying futures price is a mixture of a Wiener diffusion process and a Poisson jump process, as suggested by Ball and Torous [1985]. This is also consistent with the Samuelson [1965] hypothesis that the variance of futures price increases monotonically as the contract approaches maturity, as discussed in Ball and Torous [1986].

To check if the maturity bias is affected by the moneyness of the options, we performed the same comparisons holding fixed moneyness. The first row in Table 6 shows that for deep-out-of-the-money calls, the closer-to-maturity option always has a higher implied volatility, regardless of time to maturity. This, however, does not hold for the other call options. The second row in Table 6 shows that for out-of-the-money calls, the closer-to-maturity option has a higher implied volatility only if it has less than 6 weeks to maturity. Similarly, the third row in Table 6 shows that for at-the-money calls, the closer-to-maturity option has a higher implied volatility only if it has less than 4 weeks to maturity. The evidence is inconclusive for in-the-money and deep-in-the-money calls. For put options, there are also too few observations to give any definitive conclusions about any effects moneyness has on the maturity bias.

## 6. Conclusion

In our analysis of the DM futures options, we found that futures options have very low rates of violations of early exercise boundary conditions and put-call parity conditions. This is indicative of well-synchronized markets and low transaction costs.

In our analysis of implied volatilities, we turned up some interesting empirical regularities.

- (1) Implied volatilities are not constant over time. In fact, they appear to be serially correlated.
- (2) The moneyness bias is very strong, regardless of time to maturity. Deep-out-of-the-money options have the largest implied volatilities, and out-of-the-money options the second largest. In other words, the Black-Scholes model underprices deep-out-of-the-money options relative to at-the-money options. (The evidence is not conclusive for deep-in-the-money options, since they are not frequently traded.)
- (3) The maturity bias is also quite strong. Implied volatilities tend to rise as the options approach maturity. In other words, the Black-Scholes model underprices options which are closer to maturity relative to those which are further from maturity.

These empirical regularities implies that there are systematic biases in the Black-Scholes model. We believe that these biases are unlikely to be caused by transaction costs and discontinuous trading. Trading costs are very low, and currencies can almost be traded around the clock. Rather, we believe that the biases of the Black-Scholes model results from an incorrect distributional assumption --- that the underlying asset price is a Wiener diffusion process. There are several alternatives.

The first candidate is that underlying process may be a mixture of a Weiner diffusion process and a Poisson jump process. Ball and Torous [1985] show that this can explain the mispricing behavior of the Black-Scholes model regarding deep-out-of-the-money options and close-to-maturity options. There is, however, additional evidence that contradicts this explanation. The mixture of a Weiner diffusion process and a Poisson jump process implies that incremental changes in the exchange rate are independent and identically distributed. Hsieh [1985] and Manas-Anton [1986] find that the

rates of change of spot exchange rates are not independent and identically distributed, and that the squared rates of change are serially correlated (even though the rates of change themselves are uncorrelated.)

The second candidate is that the Samuelson [1965] hypothesis that the variance of the futures price increases as the contract nears maturity. We do not believe that there is evidence in favor of this hypothesis. Figures 4 through 7 plot the daily highs and lows for the four DM futures contracts maturing in 1984. If the variance of futures prices is increasing monotonically over time, one should observe widening daily highs and lows as each contract nears maturity. Visual inspection shows that there is no strong evidence to support the Samuelson hypothesis. In addition, even if the Samuelson hypothesis can account for the maturity bias, it cannot explain the moneyness bias.

The third candidate is that the variance of the cash price, which is also the variance of the futures price, is changing systematically over time. If the variance is a nonstochastic function, then options can be priced using the method in Merton [1973]. It could explain the maturity bias, but does not explain the moneyness bias. If the variance is a stochastic function, such as the case in an ARCH model, then it may explain both the maturity bias and the moneyness bias, and still be consistent with the observations about the underlying cash process. There is, however, no simple way to price options when the underlying asset price follows an ARCH process. We leave this for future research.

Bibliography

Ball, Clifford A. and Walter N. Torous [1985], "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing," *Journal of Finance* 40: 155-173.

\_\_\_\_\_ [1986], "Futures Options and the Volatility of Futures Prices," *Journal of Finance* 41: 857-870.

Barone-Adesi, Giovanni and Robert E. Whaley [1986], "Efficient Analytic Approximation of American Option Values," University of Chicago, mimeo.

Biger, N., and J. Hull [1983], "The Valuation of Currency Options," *Financial Management* 12: 24-28.

Black, Fisher [1976], "The Pricing of Commodity Contracts," *Journal of Financial Economics* 3:167-179.

Black, Fisher and Myron Scholes [1972], "The Valuation of Option Contracts and a Test of Market Efficiency," *Journal of Finance* 27: 399-417.

\_\_\_\_\_ [1973], "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81: 637-654.

Bodhurta, James N. and George R. Courtadon [1986], "Efficiency Tests of the Foreign Currency Options Market," *Journal of Finance* 41: 151-162.

Burt, J., F.R. Kaen, and G.G. Booth [1977], "Foreign Market Efficiency Under Flexible Exchange Rates," *Journal of Finance* 32: 1325-30.

Cox, John C., Jonathen E. Ingersoll, Jr., and Stephen A. Ross [1981], "The Relation Between Forward Prices and Futures Prices," *Journal of Financial Economics* 9: 321-346.

Cox, John C. and Mark Rubinstein, Options Markets, New Jersey: Prentice-Hall, 1985.

Engle, Robert E. [1982], "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica* 50: 987-1007.

French, Dan W. [1984], "The Weekend Effect on the Distribution of Stock Prices: Implications for Option Pricing," *Journal of Financial Economics* 13: 547-559.

Galai, Dan [1978], "Empirical Tests of Boundary Conditions for CBOE Options," *Journal of Financial Economics* 6: 187-211.

Garman, Mark B. and Steven W. Kohlhagen [1983], "Foreign Currency Option Values," *Journal of International Money and Finance* 2: 231-238.

Giddy, Ian [1983], "Foreign Exchange Options," *Journal of Futures Markets* 3: 143-166.

- Grabbe, J. Orlin [1983], "The Pricing of Call and Put Options on Foreign Exchange," *Journal of International Money and Finance* 2: 239-254.
- Hsieh, David A. [1985], "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," University of Chicago, mimeo.
- Luft, Carl F. and Bruce D. Fielitz, "An Empirical Test of the Commodity Option Pricing Model Using Ginnie Mae Call Options," *Journal of Financial Research* 9: 137-151.
- Luft, Carl F. and Animesh Ghoshal, "Foreign Currency Option Values: A Comparison of Historical and Implied Volatilities," DePaul University, mimeo.
- Manas-Anton, Luis A. [1986], "Empirical Behavior of Flexible Exchange Rates: Statistical Analysis and Consistent Models," University of Chicago, Ph. D. dissertation.
- Merton, Robert C. [1973], "A Theory of Rational Option Pricing," *The Bell Journal of Economics and Management Science* 4: 141-183.
- Ramaswamy, Krishna, and Suresh M. Sundaresan [1985], "The Valuation of Options on Futures Contracts," *Journal of Finance* 40: 1319-1340.
- Rogalski, R.J., and J.D. Vinso [1978], "Empirical Properties of Foreign Exchange Rates," *Journal of International Business Studies*, 69-79.
- Rubinstein, Mark [1985], "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades on the 30 Most Active CBOE Option Classes from August 23, 1976 Through August 31, 1978," *Journal of Finance* 40: 455-480.
- Samuelson, Paul A. [1965], "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review* 6: 41-49.
- Shastri, Kuldeep and Kishore Tandon [1985], "Arbitrage Tests of the Efficiency of the Foreign Currency Options," *Journal of International Money and Finance* 4: 455-468.
- \_\_\_\_\_ [1986], "Valuation of Foreign Currency Options: Some Empirical Tests," *Journal of Financial and Quantitative Analysis* 21: 145-160.
- Stoll, Hans R. and Robert E. Whaley [1986], "New Option Instruments: Arbitrage Linkages and Valuation," *Advances in Futures and Options Research* 1.
- Westerfield, Janice M. [1977], "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," *Journal of International Economics* 7: 181-200.
- Whaley, Robert E. [1986], "Valuation of American Futures Options: Theory and Empirical Tests," *Journal of Finance* 41: 127-150.

Table 1

Number of Days with Traded Options: Jan. 23, 1984 - Oct. 10, 1984

| <u>Strike Price</u>  | <u>Maturity Date</u> |             |             |             |             |
|----------------------|----------------------|-------------|-------------|-------------|-------------|
|                      | <u>8403</u>          | <u>8406</u> | <u>8409</u> | <u>8412</u> | <u>8503</u> |
| <u>Call Options:</u> |                      |             |             |             |             |
| 30                   | -                    | -           | -           | -           | -           |
| 31                   | -                    | -           | -           | -           | -           |
| 32                   | -                    | -           | -           | 23          | 5           |
| 33                   | -                    | -           | -           | 28          | 21          |
| 34                   | 3                    | -           | 14          | 40          | 25          |
| 35                   | 9                    | 7           | 45          | 60          | 25          |
| 36                   | 25                   | 52          | 68          | 74          | 22          |
| 37                   | 31                   | 90          | 76          | 74          | 15          |
| 38                   | 22                   | 91          | 81          | 87          | -           |
| 39                   | 13                   | 72          | 70          | 39          | -           |
| 40                   | -                    | 58          | 75          | 19          | -           |
| 41                   | -                    | 37          | 39          | -           | -           |
| 42                   | -                    | 19          | 6           | -           | -           |
| Total                | 103                  | 426         | 474         | 444         | 113         |
| <u>Put Options:</u>  |                      |             |             |             |             |
| 30                   | -                    | -           | -           | 11          | -           |
| 31                   | -                    | -           | -           | 23          | 6           |
| 32                   | -                    | -           | -           | 29          | 5           |
| 33                   | -                    | -           | 7           | 36          | 2           |
| 34                   | -                    | -           | 40          | 37          | -           |
| 35                   | 9                    | 18          | 76          | 33          | -           |
| 36                   | 19                   | 59          | 67          | 24          | -           |
| 37                   | 18                   | 79          | 58          | -           | -           |
| 38                   | 7                    | 68          | 26          | -           | -           |
| 39                   | 6                    | 45          | 4           | -           | -           |
| 40                   | -                    | 12          | -           | -           | -           |
| 41                   | -                    | -           | -           | -           | -           |
| 42                   | -                    | -           | -           | -           | -           |
| Total                | 59                   | 281         | 278         | 193         | 13          |

Table 2

Violations of Early Exercise Boundary Conditions

|                                   | Maturity Date                     |             |             |             |             |
|-----------------------------------|-----------------------------------|-------------|-------------|-------------|-------------|
|                                   | <u>8403</u>                       | <u>8406</u> | <u>8409</u> | <u>8412</u> | <u>8503</u> |
| <u>Call Options:</u>              | $C \geq \text{Max}\{0, F-K\}$     |             |             |             |             |
| No. of Violations                 | 10                                | 6           | 0           | 0           | 0           |
| Percent of Violations             | 9.7%                              | 1.4%        | 0%          | 0%          | 0%          |
| Maximum Size of Violation (Ticks) | 4                                 | 4           | 0           | 0           | 0           |
| (Dollars)                         | \$50.00                           | \$50.00     | \$ 0        | \$ 0        | \$ 0        |
| Average Size of Violation (Ticks) | 2.20                              | 2.67        | 0           | 0           | 0           |
| (Dollars)                         | \$27.50                           | \$33.33     | \$ 0        | \$ 0        | \$ 0        |
| <br><u>Put Options:</u>           | <br>$P \geq \text{Max}\{0, K-F\}$ |             |             |             |             |
| No. of Violations                 | 0                                 | 4           | 1           | 0           | 0           |
| Percent of Violations             | 0%                                | 1.4%        | 0.4%        | 0%          | 0%          |
| Maximum Size of Violation (Ticks) | 0                                 | 2           | 2           | 0           | 0           |
| (Dollars)                         | \$ 0                              | \$25.00     | \$25.00     | \$ 0        | \$ 0        |
| Average Size of Violation (Ticks) | 0                                 | 1.25        | 2           | 0           | 0           |
| (Dollars)                         | \$ 0                              | \$15.63     | \$25.00     | \$ 0        | \$ 0        |



Table 3

Violations of Put-Call Parity Conditions

|   | Maturity Date |             |             |             |             |
|---|---------------|-------------|-------------|-------------|-------------|
|   | <u>8403</u>   | <u>8406</u> | <u>8409</u> | <u>8412</u> | <u>8503</u> |
| No. of Observations   | 55            | 219         | 183         | 131         | 4           |
| <u>Test of upper put-call parity:</u> $C(F,K,T) \leq P(F,K,T) + F - K b(T)$ , |               |             |             |             |             |
| No. of Violations   | 2             | 0           | 1           | 0           | 0           |
| Percent of Violations   | 3.6%          | 0%          | 0.5%        | 0%          | 0%          |
| Maximum Size of Violation (Ticks)   | 3.98          | 0           | 1           | 0           | 0           |
| (Dollars)   | \$49.75       | \$ 0        | \$12.50     | \$ 0        | \$ 0        |
| Average Size of Violation (Ticks)   | 2.5           | 0           | 1           | 0           | 0           |
| (Dollars)   | \$31.25       | \$ 0        | \$12.50     | \$ 0        | \$ 0        |
| <u>Test of lower put-call parity:</u> $C(F,K,T) \geq P(F,K,T) - F b(T) + K$ . |               |             |             |             |             |
| No. of Violations   | 1             | 3           | 0           | 0           | 0           |
| Percent of Violations   | 1.8%          | 1.4%        | 0%          | 0%          | 0%          |
| Maximum Size of Violation (Ticks)   | 6.14          | 9.96        | 0           | 0           | 0           |
| (Dollars)   | \$76.75       | \$124.50    | \$ 0        | \$ 0        | \$ 0        |
| Average Size of Violation (Ticks)   | 6.14          | 4.31        | 0           | 0           | 0           |
| (Dollars)   | \$76.25       | \$53.83     | \$ 0        | \$ 0        | \$ 0        |

Table 4

Time Series Statistics of Implied Volatilities

| Strike Price                | Number of Observation | Mean x10 <sup>2</sup> | Max x10 <sup>2</sup> | Min x10 <sup>2</sup> | S.D. x10 <sup>2</sup> | $\rho_1$ | $\rho_2$ |
|-----------------------------|-----------------------|-----------------------|----------------------|----------------------|-----------------------|----------|----------|
| <u>Calls Maturing 8403:</u> |                       |                       |                      |                      |                       |          |          |
| 35                          | 8                     | .45                   | .50                  | .40                  | .05                   | .63      | .26      |
| 36                          | 21                    | .55                   | 1.30                 | .40                  | .24                   | .46      | .01      |
| 37                          | 24                    | .49                   | .57                  | .43                  | .03                   | .56      | .20      |
| 38                          | 20                    | .60                   | 1.41                 | .47                  | .20                   | .15      | .09      |
| 39                          | 13                    | .72                   | 1.10                 | .60                  | .12                   | .05      | -.01     |
| <u>Calls Maturing 8406:</u> |                       |                       |                      |                      |                       |          |          |
| 36                          | 44                    | .54                   | .76                  | .42                  | .071                  | .46      | .32      |
| 37                          | 90                    | .54                   | .74                  | .44                  | .056                  | .61      | .67      |
| 38                          | 91                    | .59                   | 1.50                 | .43                  | .137                  | .64      | .48      |
| 39                          | 72                    | .63                   | 2.36                 | .48                  | .245                  | .51      | .34      |
| 40                          | 58                    | .72                   | 3.61                 | .52                  | .441                  | .46      | .27      |
| 41                          | 37                    | .63                   | .70                  | .56                  | .037                  | .68      | .49      |
| 42                          | 19                    | .66                   | .73                  | .58                  | .049                  | .75      | .58      |
| <u>Calls Maturing 8409:</u> |                       |                       |                      |                      |                       |          |          |
| 34                          | 14                    | .70                   | 1.07                 | .52                  | .135                  | .09      | .13      |
| 35                          | 45                    | .64                   | .74                  | .51                  | .044                  | .56      | .48      |
| 36                          | 68                    | .65                   | .79                  | .52                  | .068                  | .90      | .82      |
| 37                          | 76                    | .64                   | .85                  | .51                  | .077                  | .90      | .86      |
| 38                          | 81                    | .64                   | .96                  | .53                  | .094                  | .87      | .78      |
| 39                          | 70                    | .64                   | .99                  | .52                  | .091                  | .85      | .69      |
| 40                          | 75                    | .67                   | 1.32                 | .53                  | .139                  | .82      | .62      |
| 41                          | 35                    | .66                   | 1.34                 | .55                  | .130                  | .26      | .16      |
| 42                          | 6                     | .69                   | .82                  | .58                  | .089                  | .40      | .03      |
| <u>Calls Maturing 8412:</u> |                       |                       |                      |                      |                       |          |          |
| 32                          | 23                    | .87                   | .93                  | .79                  | .039                  | .70      | .51      |
| 33                          | 28                    | .86                   | .92                  | .70                  | .060                  | .73      | .49      |
| 34                          | 40                    | .81                   | .96                  | .63                  | .115                  | .93      | .87      |
| 35                          | 60                    | .77                   | 1.00                 | .60                  | .138                  | .95      | .91      |
| 36                          | 74                    | .76                   | 1.06                 | .57                  | .152                  | .95      | .91      |
| 37                          | 74                    | .77                   | 1.09                 | .59                  | .156                  | .96      | .91      |
| 38                          | 57                    | .73                   | 1.05                 | .60                  | .116                  | .90      | .79      |
| 39                          | 39                    | .74                   | 1.13                 | .57                  | .133                  | .82      | .61      |
| 40                          | 19                    | .74                   | .82                  | .61                  | .061                  | .72      | .64      |

Table 4  
(continue)

Time Series Statistics of Implied Volatilities

| Strike Price               | Number of Observation | Mean x10 <sup>2</sup> | Max x10 <sup>2</sup> | Min x10 <sup>2</sup> | S.D. x10 <sup>2</sup> | $\rho_1$ | $\rho_2$ |
|----------------------------|-----------------------|-----------------------|----------------------|----------------------|-----------------------|----------|----------|
| <u>Puts Maturing 8403:</u> |                       |                       |                      |                      |                       |          |          |
| 35                         | 9                     | .49                   | .51                  | .46                  | .02                   | -.01     | -.06     |
| 36                         | 19                    | .49                   | .57                  | .42                  | .04                   | .70      | .53      |
| 37                         | 18                    | .51                   | .63                  | .40                  | .07                   | .73      | .53      |
| 38                         | 7                     | .63                   | .70                  | .53                  | .06                   | .43      | -.15     |
| 39                         | 6                     | .76                   | .94                  | .65                  | .11                   | -.31     | -.09     |
| <u>Puts Maturing 8406:</u> |                       |                       |                      |                      |                       |          |          |
| 35                         | 18                    | .55                   | .68                  | .44                  | .083                  | .76      | .60      |
| 36                         | 59                    | .58                   | 1.21                 | .44                  | .105                  | .39      | .34      |
| 37                         | 79                    | .57                   | .90                  | .44                  | .070                  | .66      | .47      |
| 38                         | 63                    | .57                   | .87                  | .47                  | .066                  | .46      | .40      |
| 39                         | 43                    | .58                   | .80                  | .49                  | .051                  | .28      | .24      |
| <u>Puts Maturing 8409:</u> |                       |                       |                      |                      |                       |          |          |
| 34                         | 40                    | .67                   | .85                  | .57                  | .060                  | .55      | .45      |
| 35                         | 74                    | .63                   | .82                  | .51                  | .048                  | .48      | .33      |
| 36                         | 66                    | .62                   | .83                  | .55                  | .052                  | .72      | .59      |
| 37                         | 58                    | .61                   | .87                  | .48                  | .067                  | .72      | .49      |
| 38                         | 26                    | .58                   | .66                  | .53                  | .035                  | .49      | .20      |
| 39                         | 4                     | .60                   | .64                  | .56                  | .060                  | .64      | .34      |
| <u>Puts Maturing 8412:</u> |                       |                       |                      |                      |                       |          |          |
| 30                         | 11                    | .95                   | 1.00                 | .88                  | .047                  | .41      | .28      |
| 31                         | 23                    | .91                   | .96                  | .81                  | .041                  | .66      | .43      |
| 32                         | 29                    | .87                   | .96                  | .71                  | .062                  | .72      | .57      |
| 33                         | 36                    | .82                   | .92                  | .64                  | .089                  | .89      | .83      |
| 34                         | 37                    | .78                   | .94                  | .64                  | .100                  | .90      | .83      |
| 35                         | 33                    | .70                   | .92                  | .58                  | .085                  | .84      | .69      |
| 36                         | 24                    | .65                   | .82                  | .57                  | .050                  | .36      | .07      |

Table 5

Comparison of Implied Volatilities Across Moneyness

| <u>Call Options:</u>    | <u>OM</u>          | <u>AM</u>          | <u>IM</u>          | <u>DIM</u>         |
|-------------------------|--------------------|--------------------|--------------------|--------------------|
| <u>DOM</u>              | -10.0<br>(23,451)* | -14.1<br>(16,570)* | -11.9<br>(13,192)* | -15.2<br>( 1, 24)* |
| <u>OM</u>               |                    | -5.9<br>(68,441)*  | -5.5<br>(35,169)*  | -8.5<br>( 9, 45)*  |
| <u>AM</u>               |                    |                    | -1.5<br>(68,182)*  | -0.8<br>(13, 44)*  |
| <u>IM</u>               |                    |                    |                    | -0.7<br>(13, 28)   |
| <br><u>Put Options:</u> | <br><u>OM</u>      | <br><u>AM</u>      | <br><u>IM</u>      | <br><u>DIM</u>     |
| <u>DOM</u>              | -4.0<br>(25,111)*  | -5.6<br>(19,134)*  | -4.8<br>(11, 49)*  | -2.0<br>( 5, 11)   |
| <u>OM</u>               |                    | -3.4<br>( 58,235)* | -0.4<br>(43, 93)   | 3.0<br>(11, 20)    |
| <u>AM</u>               |                    |                    | 2.4<br>(90,144)    | 6.8<br>(17, 24)    |
| <u>IM</u>               |                    |                    |                    | 2.4<br>( 7, 17)    |

Notes:

The numbers in the table are the averages of the logarithm of the implied volatility of the less out-of-the-money option minus that of the more out-of-the-money option. The first number in parentheses is the number of times the implied volatility of the less out-of-the-money option exceeds that of the more out-of-the-money option. The second number is the total number of such comparisons.

An asterisk (\*) denotes statistical significance at the 5% level of the one-tailed test, that the implied volatilities of the less out-of-the-money option is smaller than that of the more-out-of-the-money option, using the nonparametric test in Rubinstein [1986] .

Definition of moneyness:

| <u>Puts</u> | <u>Calls</u> |                                   |
|-------------|--------------|-----------------------------------|
| DIM:        | DOM:         | Futures/Strike $\leq$ .95         |
| IM:         | OM:          | .95 < Futures/Strike $\leq$ .98   |
| AM:         | AM:          | .98 < Futures/Strike $\leq$ 1.02  |
| OM:         | IM:          | 1.02 < Futures/Strike $\leq$ 1.05 |
| DOM:        | DIM:         | 1.05 < Futures/Strike             |

Table 6

Comparison of Implied Volatilities Between Across Maturity

| Money-<br>ness       | Weeks to Maturity of Shorter Option |                 |                 |                 |                 | <u>Total</u>     |
|----------------------|-------------------------------------|-----------------|-----------------|-----------------|-----------------|------------------|
|                      | <u>0-2</u>                          | <u>3-4</u>      | <u>5-6</u>      | <u>7-10</u>     | <u>10+</u>      |                  |
| <u>Call Options:</u> |                                     |                 |                 |                 |                 |                  |
| <u>DOM</u>           | 116.5<br>( 3, 3)                    | 26.0<br>(10,10) | 22.6<br>(21,21) | 11.7<br>(17,19) | 11.5<br>( 8, 8) | 24.6<br>(59, 61) |
| <u>OM</u>            | 57.6<br>( 8, 8)                     | 14.8<br>(21,21) | 5.6<br>(26,34)  | 0.7<br>(19,36)  | -2.4<br>(11,26) | 7.4<br>(85,125)  |
| <u>AM</u>            | 14.6<br>(26,36)                     | 2.6<br>(29,43)  | 0.4<br>(19,37)  | -4.4<br>( 7,39) | -4.9<br>( 3,16) | 2.3<br>(84,171)  |
| <u>IM</u>            | a<br>( 1, 3)                        | -2.1<br>( 2,10) | a<br>( 1, 3)    | a<br>( 1, 2)    | a<br>( 1, 2)    | -2.5<br>( 6, 20) |
| <u>DIM</u>           | -<br>( 0, 0)                        | a<br>( 2, 2)    | -<br>( 0, 0)    | -<br>( 0, 0)    | a<br>( 1, 1)    | 15.0<br>( 3, 3)  |
| <u>Total</u>         | 27.7<br>(38,50)                     | 9.3<br>(64,86)  | 7.1<br>(67,95)  | 0.7<br>(44,96)  | -1.3<br>(24,43) | 7.4<br>(237,380) |
| <u>Put Options:</u>  |                                     |                 |                 |                 |                 |                  |
| <u>DOM</u>           | -<br>( 0, 0)                        | -<br>( 0, 0)    | a<br>( 1, 1)    | -<br>( 0, 0)    | -<br>( 0, 0)    | a<br>( 1, 1)     |
| <u>OM</u>            | -<br>( 0, 0)                        | a<br>( 1, 1)    | 2.1<br>( 5, 8)  | -<br>( 0, 0)    | -<br>( 0, 0)    | 5.9<br>( 6, 9)   |
| <u>AM</u>            | 18.1<br>(24,27)                     | 2.1<br>(18,26)  | 0.1<br>(14,33)  | -3.2<br>( 4,14) | a<br>( 0, 2)    | 4.9<br>(60,102)  |
| <u>IM</u>            | 21.3<br>( 4, 5)                     | 5.8<br>(12,13)  | 1.3<br>( 7,10)  | a<br>( 2, 3)    | a<br>( 0, 1)    | 6.1<br>(25, 32)  |
| <u>DIM</u>           | -<br>( 0, 0)                        | a<br>( 1, 1)    | a<br>( 1, 1)    | -<br>( 0, 0)    | -<br>( 0, 0)    | 8.6<br>( 2, 2)   |
| <u>Total</u>         | 20.5<br>(28,32)                     | 13.7<br>(32,41) | 4.4<br>(28,53)  | -1.3<br>( 6,17) | a<br>( 0, 3)    | 6.0<br>(94,146)  |

The numbers are the averages of the logarithm of the implied volatility of the closer-to-maturity option minus that of the further-to-maturity option. The first number in parentheses is the number of times the implied volatility of the closer-to-maturity option is greater than that of the further-to-maturity option.

a Less than five comparisons.

Appendix A-1

Pairwise Comparison of Implied Volatilities Across Moneyness

Call Options With More Than 10 Weeks To Maturity:

|            | <u>OM</u>        | <u>AM</u>        | <u>IM</u>         | <u>DIM</u>        |
|------------|------------------|------------------|-------------------|-------------------|
| <u>DOM</u> | -7.1<br>(19,296) | -9.4<br>(15,361) | -10.8<br>(10,132) | -15.2<br>( 1, 24) |
| <u>OM</u>  |                  | -2.5<br>(62,255) | -5.1<br>(20,115)  | -8.5<br>( 9, 45)  |
| <u>AM</u>  |                  |                  | -2.6<br>(37,112)  | -7.4<br>( 9, 40)  |
| <u>IM</u>  |                  |                  |                   | -0.7<br>(13, 28)  |

Put Options With More Than 10 Weeks To Maturity:

|            | <u>OM</u>        | <u>AM</u>         | <u>IM</u>        | <u>DIM</u>       |
|------------|------------------|-------------------|------------------|------------------|
| <u>DOM</u> | -3.6<br>(20, 85) | -5.3<br>(17,100)  | -4.5<br>( 7, 29) | -2.2<br>( 4,  8) |
| <u>OM</u>  |                  | -2.0<br>( 39,123) | -1.0<br>(17, 37) | -0.4<br>( 4,  9) |
| <u>AM</u>  |                  |                   | 1.0<br>(28, 49)  | 0.4<br>( 6,  9)  |
| <u>IM</u>  |                  |                   |                  | 0.1<br>( 3,  8)  |

See notes in Table 5.

Appendix A-2

Pairwise Comparison of Implied Volatilities Across Moneyness

Call Options With 7 - 10 Weeks To Maturity:

|            | <u>OM</u>         | <u>AM</u>         | <u>IM</u>         | <u>DIM</u> |
|------------|-------------------|-------------------|-------------------|------------|
| <u>DOM</u> | -11.0<br>( 2,101) | -14.8<br>( 1,141) | -12.0<br>( 2, 44) | -          |
| <u>OM</u>  |                   | -5.2<br>( 5, 94)  | -3.4<br>( 8, 29)  | -          |
| <u>AM</u>  |                   |                   | 0.3<br>(19, 39)   | -          |
| <u>IM</u>  |                   |                   |                   | -          |

Put Options With 7 - 10 Weeks To Maturity:

|            | <u>OM</u>        | <u>AM</u>         | <u>IM</u>        | <u>DIM</u>   |
|------------|------------------|-------------------|------------------|--------------|
| <u>DOM</u> | -4.6<br>( 5, 23) | -6.9<br>( 2, 29)  | -4.9<br>( 4, 17) | a<br>( 0, 1) |
| <u>OM</u>  |                  | -2.6<br>( 12, 61) | -0.3<br>(16, 33) | a<br>( 0, 3) |
| <u>AM</u>  |                  |                   | 2.2<br>(34, 53)  | a<br>( 0, 3) |
| <u>IM</u>  |                  |                   |                  | a<br>( 0, 3) |

See notes in Table 5.

a Less than five comparisons.

Appendix A-3

Pairwise Comparison of Implied Volatilities Across Moneyness

Call Options With 5 - 6 Weeks To Maturity:

|            | <u>OM</u>         | <u>AM</u>         | <u>IM</u>         | <u>DIM</u> |
|------------|-------------------|-------------------|-------------------|------------|
| <u>DOM</u> | -21.7<br>( 0, 36) | -28.3<br>( 0, 42) | -27.9<br>( 0, 11) | -          |
| <u>OM</u>  |                   | -7.0<br>( 1, 41)  | -9.1<br>( 1, 10)  | -          |
| <u>AM</u>  |                   |                   | -5.2<br>( 2, 12)  | -          |
| <u>IM</u>  |                   |                   |                   | -          |

Put Options With 5 - 6 Weeks To Maturity:

|            | <u>OM</u>    | <u>AM</u>        | <u>IM</u>        | <u>DIM</u>       |
|------------|--------------|------------------|------------------|------------------|
| <u>DOM</u> | a<br>( 0, 3) | -4.9<br>( 0, 5)  | a<br>( 0, 3)     | a<br>( 1, 2)     |
| <u>OM</u>  |              | -6.3<br>( 6, 26) | -1.6<br>( 6, 16) | 12.4<br>( 6, 7)  |
| <u>AM</u>  |              |                  | 2.8<br>(17, 27)  | 16.3<br>(10, 11) |
| <u>IM</u>  |              |                  |                  | 13.9<br>( 4, 6)  |

See notes in Table 5.

a Less than five observations.



Appendix A-4

Pairwise Comparison of Implied Volatilities Across Moneyness

Call Options With 3 - 4 Weeks To Maturity:

|            | <u>OM</u>         | <u>AM</u>         | <u>IM</u>        | <u>DIM</u> |
|------------|-------------------|-------------------|------------------|------------|
| <u>DOM</u> | -23.1<br>( 2, 14) | -39.2<br>( 0, 21) | -4.2<br>( 1, 5)  | -          |
| <u>OM</u>  |                   | -14.5<br>( 0, 37) | -7.0<br>( 6, 14) | -          |
| <u>AM</u>  |                   |                   | 4.7<br>(10, 17)  | -          |
| <u>IM</u>  |                   |                   |                  | -          |

Put Options With 3 - 4 Weeks To Maturity:

|            | <u>OM</u> | <u>AM</u>        | <u>IM</u>       | <u>DIM</u>   |
|------------|-----------|------------------|-----------------|--------------|
| <u>DOM</u> | -         | -                | -               | -            |
| <u>OM</u>  |           | -6.5<br>( 1, 18) | -0.7<br>( 2, 5) | a<br>( 1, 1) |
| <u>AM</u>  |           |                  | 2.0<br>( 8, 12) | -            |
| <u>IM</u>  |           |                  |                 | -            |

See notes in Table 5.

a Less than five comparisons.

Appendix A-5

Pairwise Comparison of Implied Volatilities Across Moneyness

Call Options With 0 - 2 Weeks To Maturity:

|            | <u>OM</u>    | <u>AM</u>         | <u>IM</u>    | <u>DIM</u>   |
|------------|--------------|-------------------|--------------|--------------|
| <u>DOM</u> | a<br>( 0, 4) | -105.5<br>( 0, 5) | -            | -            |
| <u>OM</u>  |              | -47.1<br>( 0, 14) | a<br>( 0, 1) | -            |
| <u>AM</u>  |              |                   | a<br>( 0, 2) | a<br>( 4, 4) |
| <u>IM</u>  |              |                   |              | -            |

Put Options With 0 - 2 Weeks To Maturity:

|            | <u>OM</u> | <u>AM</u>        | <u>IM</u>    | <u>DIM</u> |
|------------|-----------|------------------|--------------|------------|
| <u>DOM</u> | -         | -                | -            | -          |
| <u>OM</u>  |           | -18.3<br>( 0, 7) | a<br>( 2, 2) | -          |
| <u>AM</u>  |           |                  | a<br>( 3, 3) | -          |
| <u>IM</u>  |           |                  |              | -          |

See notes in Table 5.

a Less than five comparisons.

# DEUTSCHE MARK SPOT 1984

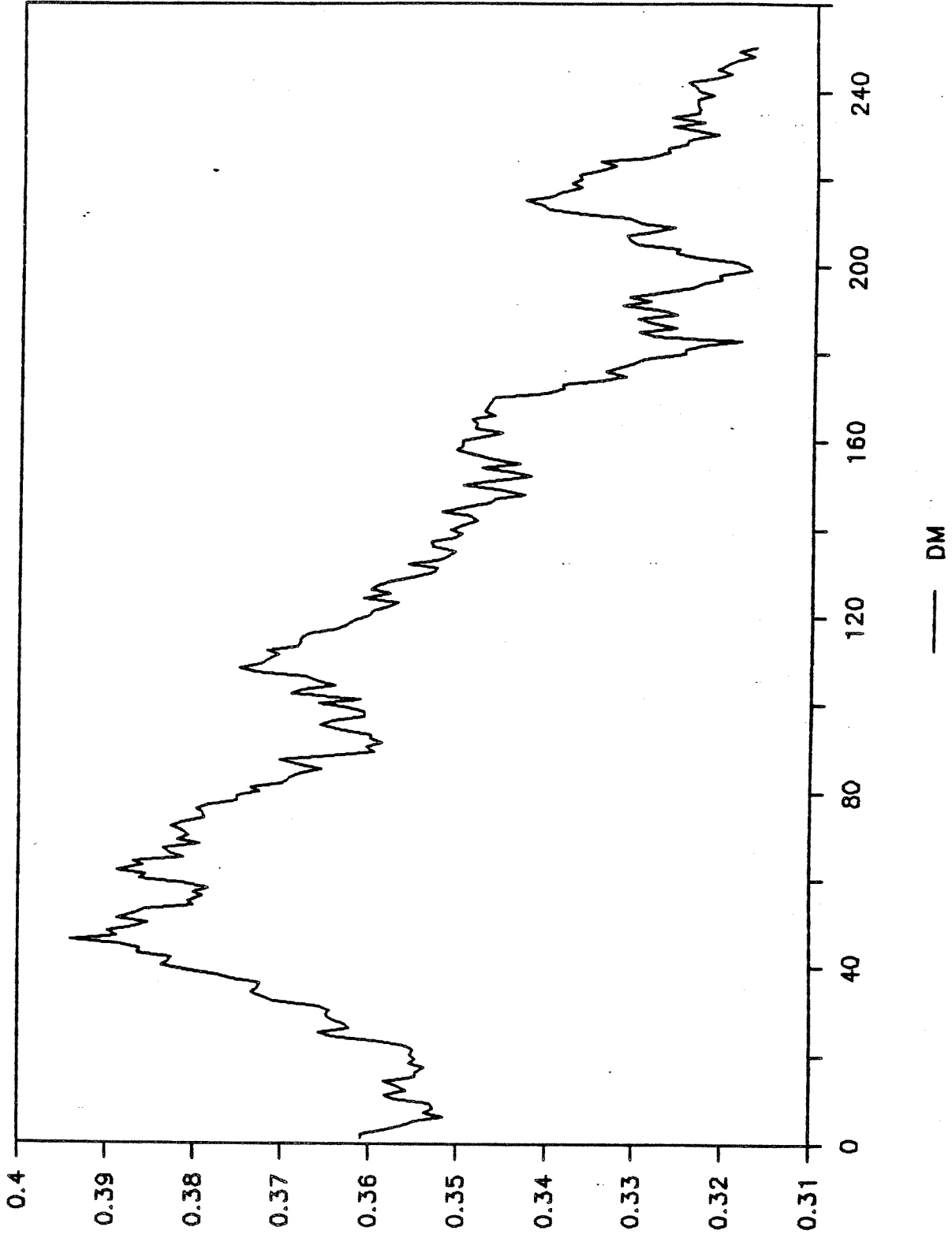


Figure 1

IMPLIED VOLATILITY FOR 8409 PUTS

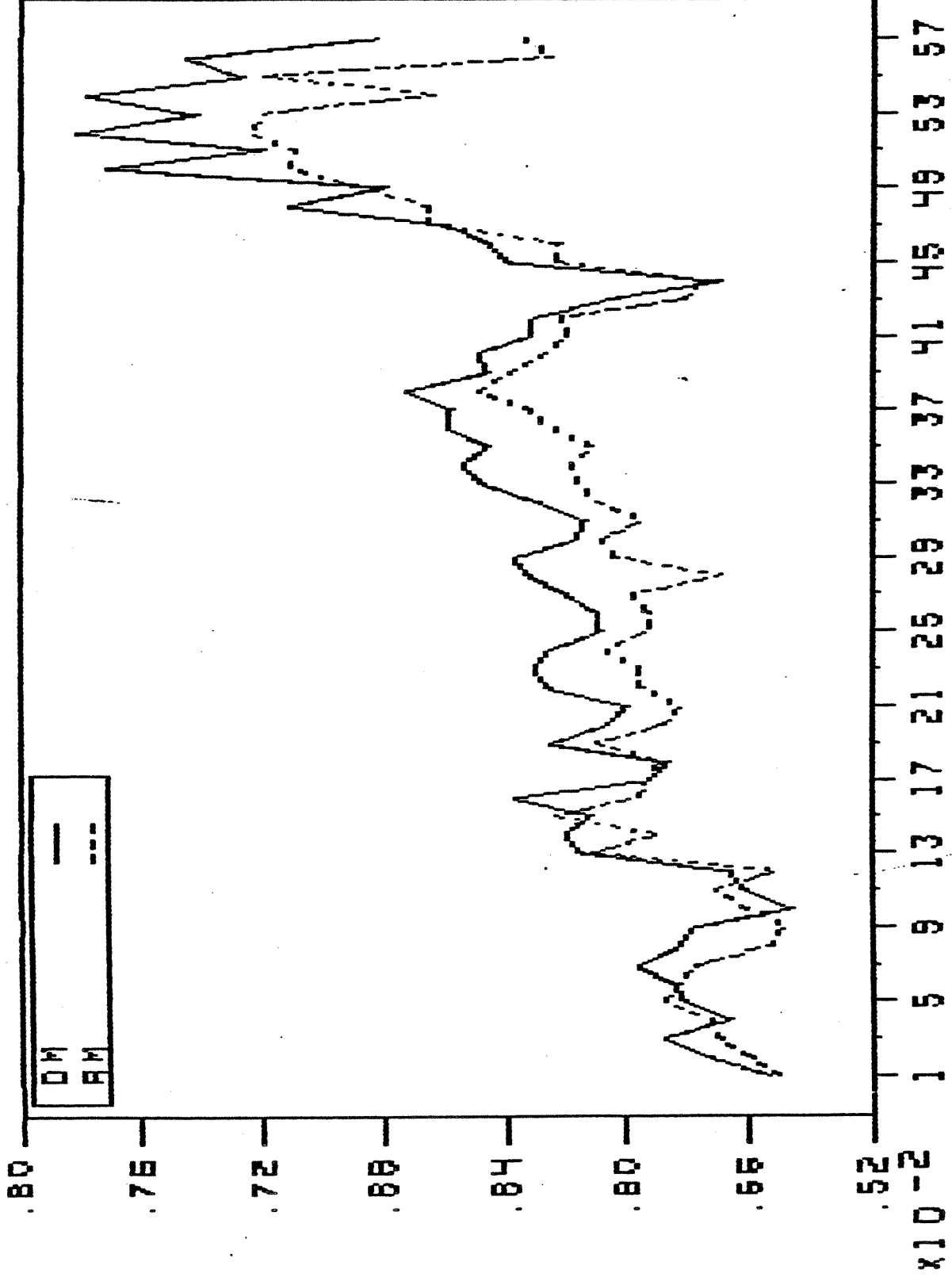


Figure 2

IMPLIED VOLATILITY FOR B409 CALLS

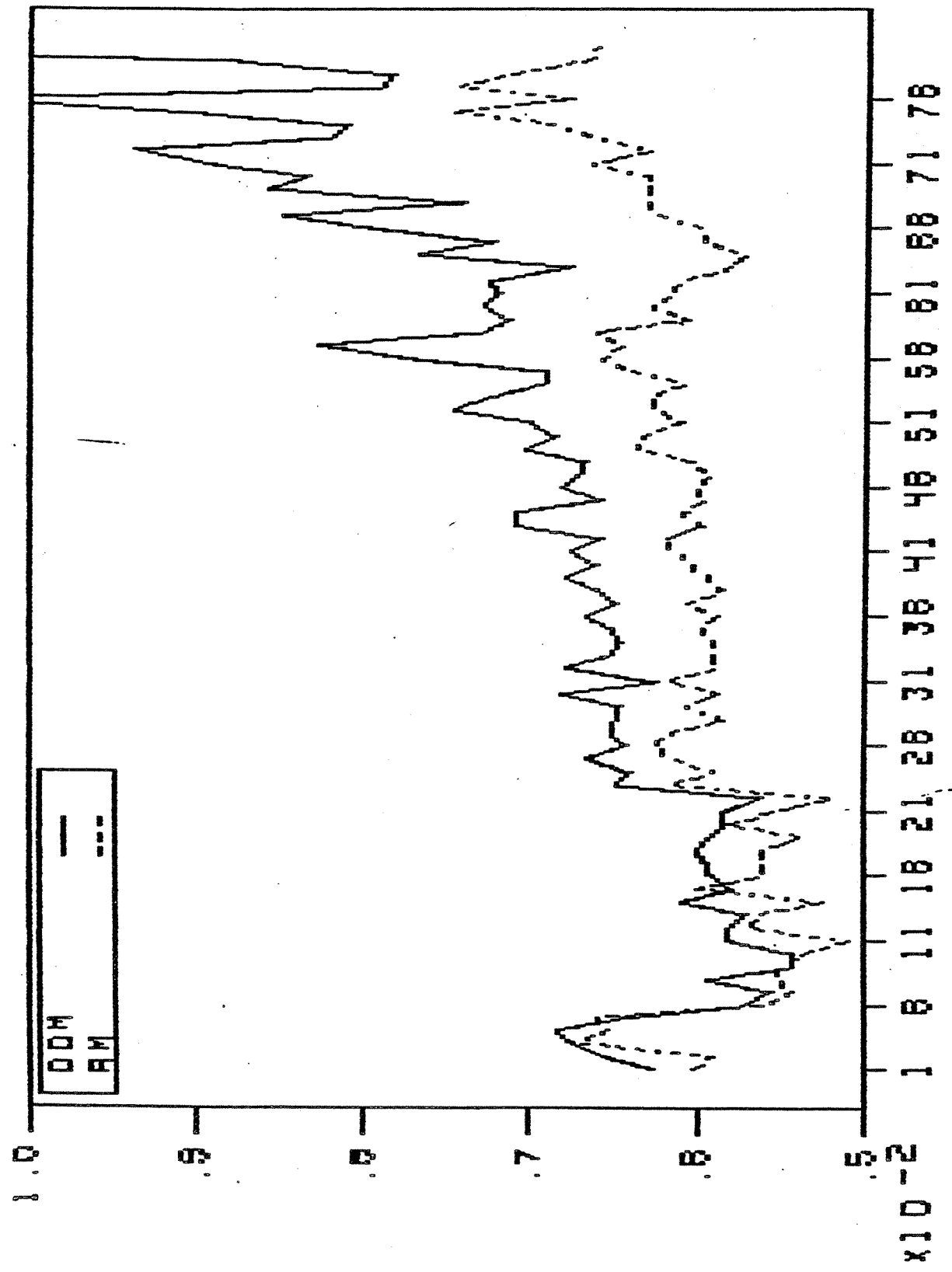


Figure 3

DM FUTURES DAILY HIGH/LOW

CONTRACT: 8403

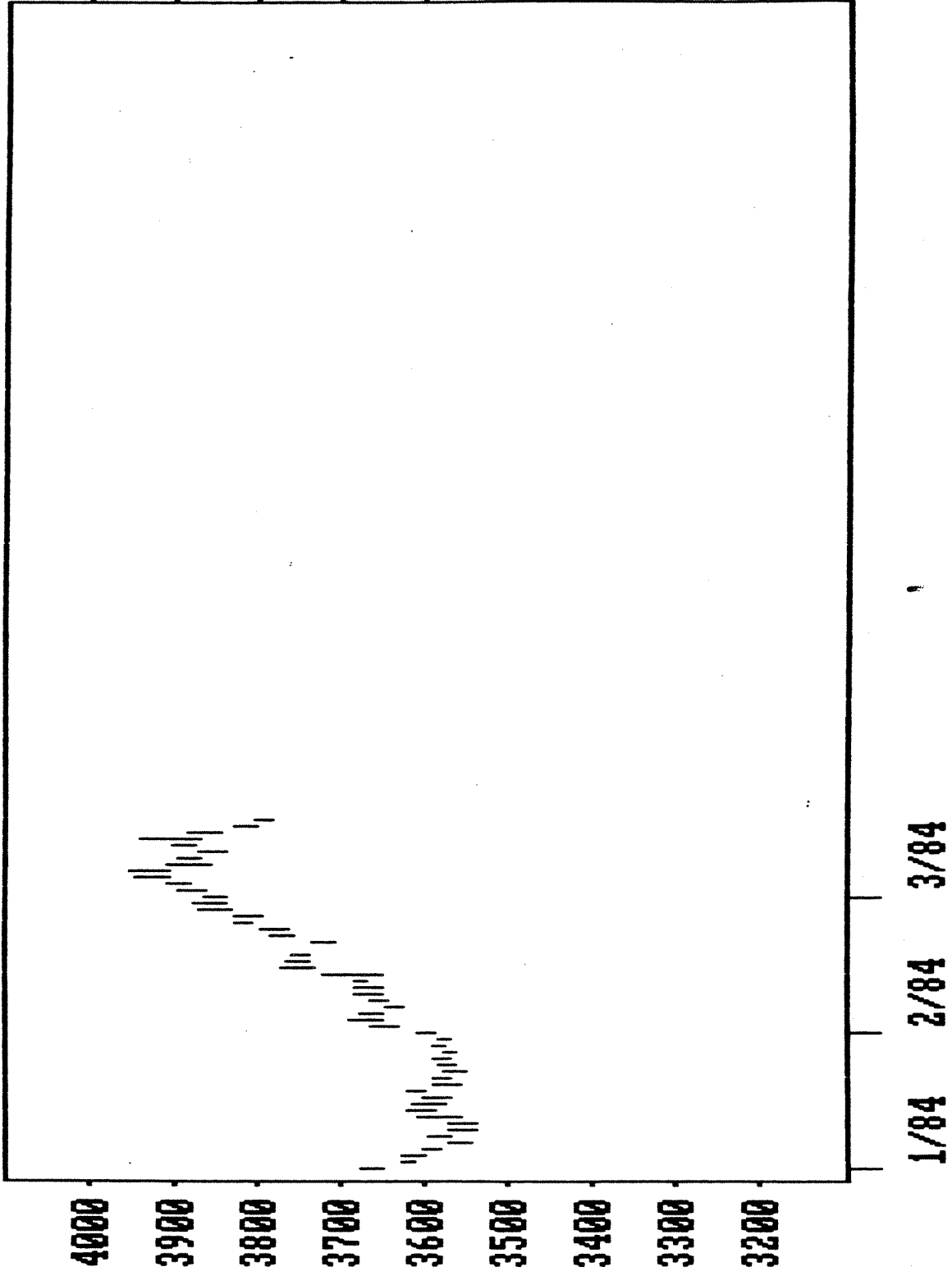


Figure 4

DM FUTURES DAILY HIGH/LOW

CONTRACT: 8409

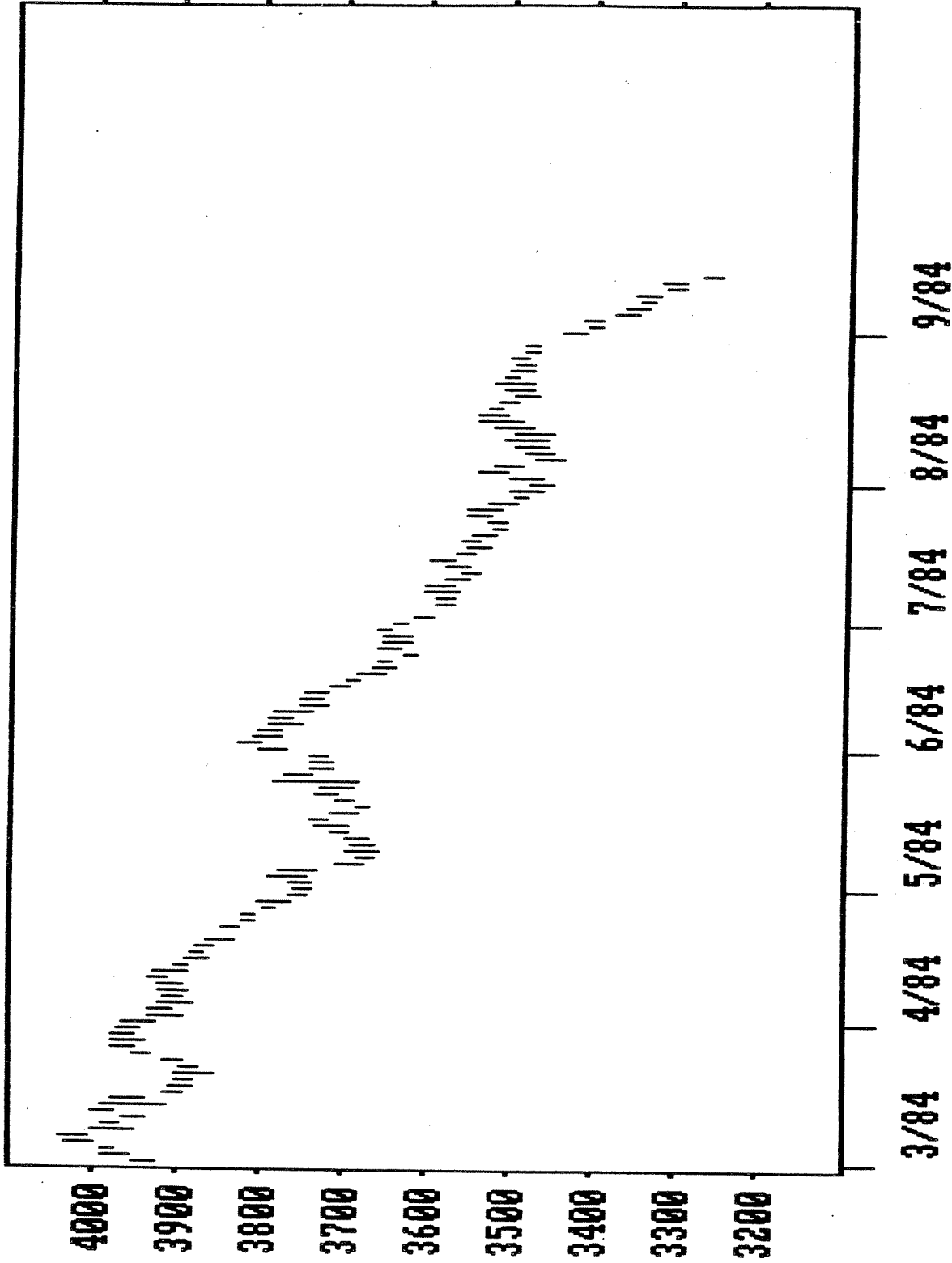


Figure 5

CONTRACT: 8486

DM FUTURES DAILY HIGH/LOW

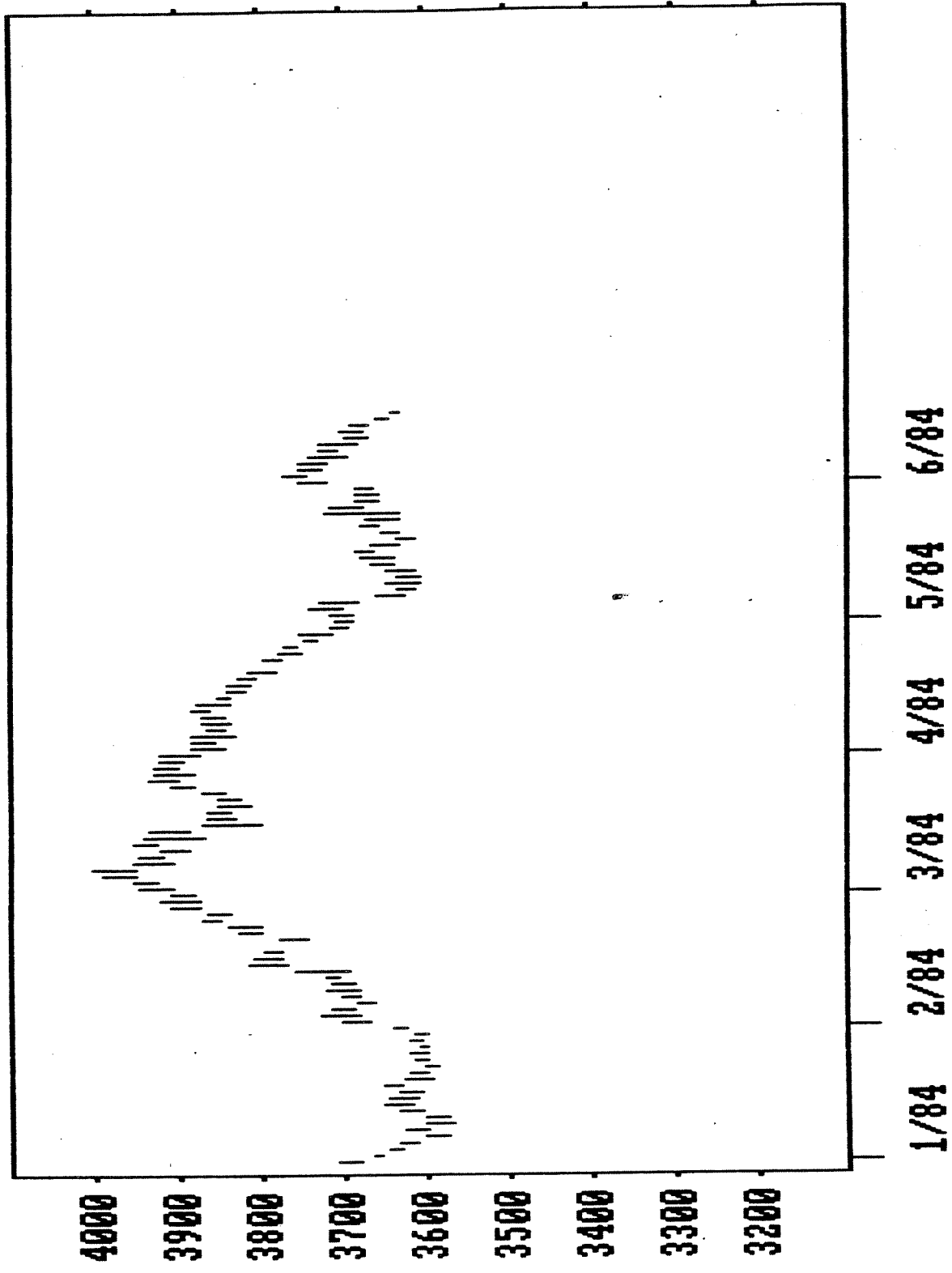


Figure 6



DM FUTURES DAILY HIGH/LOW

CONTRACT: 8412

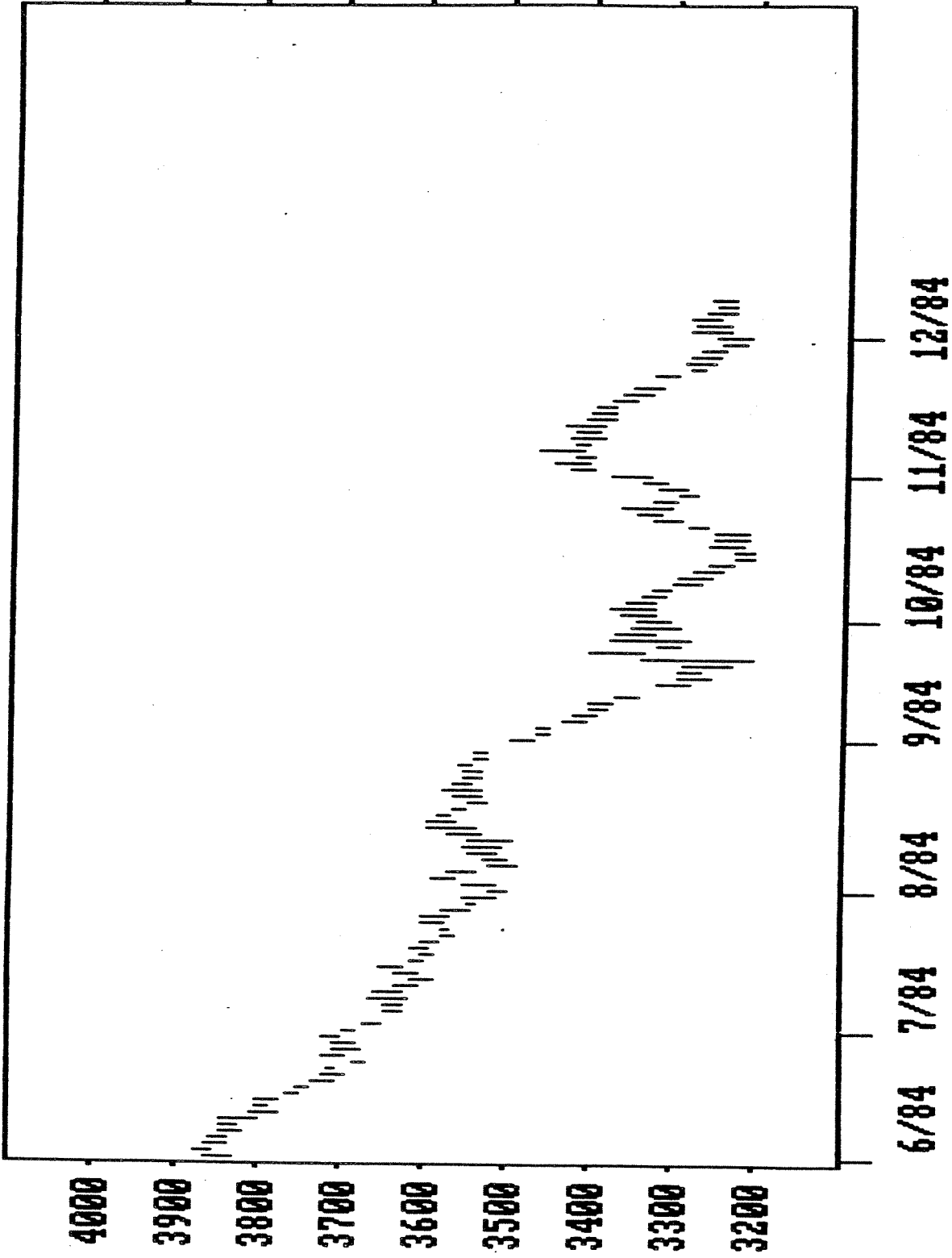


Figure 7