

WP. No. 99

AN EXPLORATORY INVESTIGATION  
OF THE FIRM SIZE EFFECT

by

K.C. Chan  
Nai-Fu Chen  
David Hsieh

First Draft: November 1982  
Revised: August 1983  
Revised: December, 1983

Not for Quotation. Comments Welcomed

Acknowledgment. We thank Eugene Fama, Merton Miller and Myron Scholes for their many comments and suggestions, and Robert Ibbotson for providing us with some of the necessary data. We also benefited from discussions with John Abowd, Rolf Banz, Doug Breeden, George Constantinides, Wayne Ferson, Bob Hamada, Richard Leftwich, Richard Roll, Stephen Ross, Victor Zarnowitz, and workshop participants at Chicago, Dartmouth, Northwestern, Stanford, UCLA, and Yale.



#### ABSTRACT

We investigate the firm size effect in the period 1958 to 1977 in the framework of a multifactor pricing model. The risk adjusted difference in return between the top five percent and the bottom five percent of the NYSE firms is about one to two percent a year, a drop from about twelve percent per year before risk adjustment. The variables that appear to be most responsible for the adjustment are risk measures of the changing risk premium and the changing state of the economy.



## INTRODUCTION

The "firm size" effect was documented by Banz (1981) and Reinganum (1981). In their studies, small firms appeared to have higher average returns than large firms even after adjusting for risk via the Capital Asset Pricing Model (CAPM). Therefore, their results can be considered a rejection of the joint hypotheses that the CAPM is correct and that the market is efficient.

In a recent empirical study of the Arbitrage Pricing Model (APT), Chen (1981, 1983) found that the firm size effect is essentially captured by the factor loadings of the APT. Portfolios consisting of firms of different sizes do not have significantly different average returns after adjusting for factor risks. Chen's result is consistent with the hypothesis that risk is the explanation for the firm size effect while the market is efficient.

The Arbitrage Pricing Model, first formulated by Ross (1976),<sup>1</sup> posits that the realized return of an asset is a linear combination of the asset's expected return, the asset's responses to the fluctuations of pervasive forces in the economy, and the asset's own idiosyncratic fluctuations. Since idiosyncratic risks can be essentially diversified away in a portfolio, market equilibrium implies that only the risks corresponding to the pervasive forces are priced. The expected return is therefore a linear combination of the asset's sensitivities (i.e., the loadings) to the fluctuations of the

forces. With some additional assumptions, this multifactor pricing equation can be collapsed into a single factor--the market. In this case, we obtain the CAPM of Sharpe (1964), Lintner (1965), and Black (1972). Thus, the APT can be considered as a logical predecessor to the more advanced CAPM. However, the generality of the APT is bought with a heavy price. The model itself does not predict what the forces in the economy are. Even though the firm size effect is identified with risks in Chen's study, additional insights cannot be gained without the knowledge of the macrovariables that are correlated with assets' returns. In this study, we intend to remedy that deficiency. Our goal is to explore the economic interpretation of the firm size effect.

The paper is divided into five sections. In Section 1, we describe the variables to be included in the pricing of securities. Section 2 contains the cross-sectional results. Section 3 presents results containing firm size proxies. We reexamine the "January anomaly" in Section 4, and summarize our findings in Section 5.

We emphasize again the exploratory nature of our experiments. The validity of our interpretations is subject to the results of further independent studies and other experimental designs.

### **1. The Factors and the Macroeconomy**

Naturally, we would like to know what the factors are. An effort to link the factors to the macroeconomy is described in an empirical study by Chen, Roll, and Ross (1983). For completeness, we provide a brief intuitive discussion of the variables.

Let the current price of a financial asset be the sum of the risk adjusted discounted value of future expected cash flows. Consider the

variables that may influence the (related) numerators and denominators of the valuation formula.

The first variable concerns real activities in the economy. It is measured by the industrial production series.

- (i) (a)  $IP12_t = \log(IP_{t+12}/IP_t)$ , the growth rate of industrial production ending in month  $t + 12$ . This is the same variable Fama (1981)<sup>2</sup> used. On the other hand, the level of industrial production is often considered as an indicator of the current state of the economy. Therefore, we also define
- $IP1_t \equiv \log(IP_{t+1}/IP_t)$ , the monthly growth rate of industrial production, which can be regarded as the change in the state of the economy.

In an efficient financial market, we expect that new information concerning future real activities is quickly reflected in the aggregate return of the market. Thus in this study, we sometimes replace the industrial production variable with the more familiar stock market indices:

- (b) EWNY = equally weighted NYSE index,

or

- (c) VWNY = value weighted NYSE index.

Secondly, we consider the impact of inflation. As long as the effect of inflation is not perfectly cancelled out in the numerators and denominators of the valuation formula, the price of a financial asset will be influenced by inflation. The following two variables are related to inflation.

- (ii)  $DEI_t = EI_{t+1} - EI_t$ , the change in expected inflation.  
 Expected inflation is measured as in Fama-Gibbons (1982b) using the time series of one month T-Bill rates.

(iii)  $UITB_t = CPI_t - EI_t$ , the difference between the realized inflation rate as reported in the Consumer Price Index and the expected inflation rate.

Next, we consider interest rates. Interest rates are important because they represent opportunity costs. The one month T-Bill rate contains the expected real rate, and the expected inflation rate which we measured above. However, from the way that the expected inflation is extracted by Fama-Gibbons from the T-Bills, the unanticipated inflation rate  $UITB_t$  is perfectly negatively correlated with the change in the expected one month real rate, if we define the expected real rate as  $[T\text{-Bill rate}]_t - EI_t$ . Therefore, we do not include another variable for the real rate.

The current price of a financial asset also contains the discounted expected cash flows in the distant future, thus we include a measure of the change in the long term rate. Because of the linearity of the multifactor model we use, this variable is measured by returns rather than yields (which are in the denominators of the valuation formula).

(iv)  $TBD_t = LTGB_t - TB_t$ , the difference between the return of a portfolio of long term government bonds (from Ibbotson and Sinquefeld (1982)) and the T-Bill rate.

Finally, we observe that the discounted cash flow formula contains also an adjustment for risk--the risk premium. Among possibly other parameters, the risk premium is a function of the "price of risk" and "risk." Here, the price of risk refers to the marginal tradeoff between consumption and risky investments. It can vary conditional on the current state and expected future states of the economy. For example, if we are in a state where the level of consumption is low, we may require a higher expected return in exchange for the marginal unit of consumption. Similarly, if future real activities are



expected to be high, the induced market equilibrium expected returns may also be high.<sup>3</sup>

And as for risk, the conditional distribution of future cash flows from operations during an economic expansion is probably different from that during an economic contraction. Thus the business risk may be a function of the business cycles. Furthermore, since we are investigating returns to equity, the financial risk may also vary with the business cycles. Those firms who experience a drop in their stock prices due to disappointing current and future expected cash flows are likely to find that their stocks are riskier because of the leverage effect.

So, it seems reasonable to expect that economic conditions affect the risk premium. We shall measure the changing risk premium by measuring the behavior of bonds of different perceived riskiness.<sup>4</sup> Since the price of a bond summarizes its yield structure and our pricing equation is linear, we shall use returns rather than yields in the following. Our measure of the changing risk premium is

$$(v) \quad (a) \quad \text{PREM} = \text{"Under Baa" portfolio return}_t - \text{Aaa portfolio return}_t \text{ (Moody's rating)}$$

or

$$(b) \quad \text{PREMA} = \text{"Under Baa" portfolio return}_t - \text{long term government bond portfolio return}_t$$

The return series of these nonconvertible corporate bonds are obtained from R. G. Ibbotson & Company. A detailed description of the sample is in Ibbotson (1979). An interesting characteristic of the bond portfolios is that they are "value weighted by probability." From the universe of bonds that Ibbotson collected, he formed the portfolios by sampling. The probability of inclusion was directly proportional to the face value of the bond. Thus the

low grade bond portfolio<sup>5</sup> actually contained mostly bonds from relatively large corporations.<sup>6</sup>

The empirical results obtained from using PREM and PREMA are similar, with the PREMA results slightly stronger overall as expected because PREMA maximizes the perceived difference in riskiness. Only the PREMA results will be reported in this paper.

Tables 1 and 2 contain the correlation between the variables and the autocorrelation coefficients of each of the variables. The APT requires that we include in the pricing equation all the variables corresponding to pervasive forces, though ex-ante all except one may have a zero price (see Fama [1970], Merton [1973], Long [1974] and Chen [1981]). The results of Chen, Roll, and Ross would suggest including IP12 (or a measure of stock market index), PREMA (or PREM), and TBD, and maybe UITB. However, considering the potential "loss" associated with Type I and Type II errors,<sup>7</sup> all five are included in what follows. We are now ready to investigate whether these variables are priced, and if so, whether they are sufficient to explain the difference between the returns of small and of large firm portfolios.

## 2. Cross Sectional Results

### 2.1 The data

The availability of macroeconomic data limits our investigation to the time period 1953-1977. We shall divide these 25 years into 20 overlapping intervals, each consisting of 6 years. The first interval is January 1953 to December 1958, the second is January 1954 to December 1959, and so on, and the last one is January 1972 to December 1977. During each six-year interval, firms on the NYSE that exist at the beginning of the interval and have price information on December of the fifth year are chosen and ranked according to market value at the end of the fifth year. The firms are then put into one of

the 20 portfolios arranged in order of increasing size. Each portfolio contains roughly (i.e., within one) the same number of securities. In the sixth year, we conduct our testing. Thus, we have 20 nonoverlapping years of testing from 1958 to 1977.

## 2.2 The methodology

After the portfolios are ranked according to firm size, we shall use a variant of the Fama-MacBeth (1973) method<sup>8</sup> to test the firm size effect. To implement the Fama-MacBeth technique, we first regress each of the 20 portfolios on the macrovariables in the first five years to estimate the loadings. Then we perform cross sectional regressions of the 20 portfolios' returns on the obtained portfolios' loadings month by month in the sixth year for the 20 intervals. The result is an estimated time series of prices associated with the macrovariables, from which we can test the null hypothesis that they are zero. The cross-sectional regressions force the intercept term as well as the prices to be the same across the 20 portfolios. Hence, if a violation of the pricing equation occurs, it will be in the residuals. The cross-sectional sum of the residuals is zero for every month. However, if the firm size effect still persists, the time series mean of the residuals from small firms will be higher than the time series mean of the residuals from large firms. The residuals also have the natural interpretation of estimated risk adjusted returns (under the null hypothesis that the multifactor model is correct) since they are the difference of the realized returns and the estimated expected returns of the model.

Two types of tests are performed with the residuals.<sup>9</sup> The first is a multivariate test--the Hotelling  $T^2$ --to see if all residuals across time are the same. The null hypothesis is  $u_1 - u_2 = u_2 - u_3 = \dots = u_{19} - u_{20} = 0$ .

If the null hypothesis is not rejected, then, together with the restriction that the sum of the 20 residuals is zero, it implies that  $u_1 = u_2 = \dots = u_{20} = 0$ . A direct test of  $u_1 = u_2 = \dots = u_{20} = 0$  is not possible with the Hotelling  $T^2$  because of the restriction that the sum of all residuals is zero for each period. The time series of  $u_{20}$  is perfectly known once  $u_1, \dots, u_{19}$  are given, and hence the rank of the covariance matrix of the residuals is only 19. Of course, there is interdependence in  $u_1 - u_2, \dots, u_{19} - u_{20}$ , but this is taken care of in the covariance matrix in the Hotelling  $T^2$ . The Hotelling  $T^2$  is not very powerful in uncovering the firm size effect which is most pronounced in the difference between the extreme portfolios. In fact, the only equation the Hotelling  $T^2$  statistic can reject is the single market factor (equally weighted NYSE) model.

The second test is a paired  $t$  test using the two extreme portfolios. The restriction of the cross-sectional sum of residuals is implicitly accounted for by considering the residuals from the top and bottom portfolios as paired observations rather than as independent observations. The second test is more powerful than the Hotelling  $T^2$  if the firm size effect is monotonic. By maximizing the difference in firm size, we obtain a stronger test based on the residuals.

In this study, we do not pool time series and cross sectional data in a single regression. The result of the stacked regression would be the same as that for the month by month regression unless additional information about the residuals from the time series regression or restriction on the time series of the intercept and price series can be specified. Both of these are of dubious validity. Therefore we decided to use the Fama-MacBeth method.<sup>10</sup> The results are reported below.

### 2.3. Empirical results

The monthly average continuously compounded rates of return of the 20 portfolios are reported in table 3. P1 is the portfolio with the smallest five percent of firms in NYSE; P1-P20 is the difference between P1 and P20. Q1 and Q5 are the bottom and top quintile of firms; Q1 - Q5 is the difference. In addition to data from the overall period 1958-1977, data from two subperiods of 10 years, 1958-1967 and 1968-1977, and from a long subperiod of 15 years 1958-1972 are reported. Some selected firm size data of the 20 portfolios are given in table 4.

From the previous section, the five variables to be included in the pricing equation are: (1) change in expected real activities as measured by the growth rate of industrial production (IP12) or the return of a stock market index; (2) change in expected inflation (DEI); (3) unanticipated inflation (UITB); (4) a measure of the changing risk premium (PREMA); and (5) a measure of the change in the slope of the yield curve (TBD). Since the results of using the growth rate of industrial production and a stock market index return are similar, only results pertaining to the market index are reported here.<sup>11</sup> The equally weighted NYSE index (EWNY) is the index used in the following multifactor formulation:

$$r_i = \lambda_0 + \lambda_1 \hat{b}_i(\text{EWNY}) + \lambda_2 \hat{b}_i(\text{DEI}) + \lambda_3 \hat{b}_i(\text{UITB}) + \lambda_4 \hat{b}_i(\text{PREMA}) + \lambda_5 \hat{b}_i(\text{TBD}) + \varepsilon_i \quad (1)$$

Table 5A reports the cross sectional results of (1). For the overall period, the prices (i.e., the estimated  $\lambda$ 's) associated with EWNY and PREMA are positive, as expected. The negative prices associated with DEI, UITB, and TBD are plausible. If people prefer stocks whose returns are positively

correlated with inflation and if this is the determining factor, then the prices for the inflation risk variables would be negative. A possible explanation for the negative TBD price is that long term bonds provide a hedge against stochastic shifts in the interest rate (after controlling for the effect of inflation). The point estimate of the intercept term is .00386, which is slightly above the average one month T-Bill rate of .00352 for the same period. The point estimate for  $\lambda(\text{EWNY})$  is .00601, which is also slightly higher than the realized difference between the equally weighted NYSE index and the T-Bill rate of .00564 for that period.

Before looking at the  $t$  statistics, we recall that the cross-sectional regressions are run on portfolios ranked according to firm size. Thus it provides efficient estimates on only those prices most related to explaining the difference in return for firms of different sizes. The fact that the estimated  $\lambda$  associated with the EWNY is not significant does not necessarily mean it is not priced. If the effect of the market on firms of every size is roughly the same, our portfolio formation may not provide a good estimate for that price. However, since our goal here is to investigate the firm size effect, our procedure will naturally select those variables that are most responsible for the firm size effect.

Two prices--PREMA and UITB have significant  $t$  statistics for the overall period. It is interesting to note the contrast between the two ten-year subperiods. In the first subperiod, only PREMA is significant. In the second subperiod, when we started to have substantial inflation in the economy, the two inflation variables are significant.

Table 5B contains the average residuals for the 20 portfolios for the four time periods. The residuals of the smallest size portfolio are still positive, and those of the largest three portfolios are negative. The

Hotelling  $T^2$ 's are not significant, except for one subperiod. The difference between the residuals of the bottom and those of the top five percent of firms is given as  $U1 - U20$ . Although they are all insignificant at the 5% level, the overall  $t$  statistic is 1.73, which is marginal. The point estimate of 0.00169 translates to roughly a 2.0% per annum difference in the "risk adjusted" return.<sup>12</sup> This is considerably lower than the difference in the raw returns of .00956, or roughly 11.5% per annum.

Next, we compute the statistics of the top and bottom quintiles. While the difference here is typically smaller than the difference between the top and bottom portfolios, almost all the  $t$  statistics are larger. For the overall period, the quintile difference is .00067, or approximately .8% per annum. However, the .8% per annum carries a marginal  $t$  statistic of 1.86. Of course risk adjusted profit at this level will be totally eliminated by transaction costs if rebalancing is required once every several years.<sup>13</sup>

Thus, even though the residuals still exhibit some marginal firm size effect, the magnitude is so small that it is doubtful whether any risk adjusted profits can be realized in practice.

#### 2.4. Component contribution

We know that the prices associated with UITB and PREMA are significant statistically, but what is their economic significance in explaining the firm size effect? To assess the magnitude associated with each variable, we form the products of the prices and the loadings. Table 6 computes the monthly average of each component contribution. As noted earlier, the point estimates of the intercept term and the EWNV price look reasonable. This is most important since our present computation is based on the point estimates.

Perhaps the most interesting observation is that the EWNV component is higher in absolute value than any other component in each of the raw returns

of the 20 portfolios. However, when it comes to the difference between the top and bottom portfolio, the PREMA component is by far the largest. Of the difference of .956% per month, .587% can be ascribed to PREMA, .333% to EWNV, and .169% unexplained. Unfortunately, the DEI and the TBD components are negative. This makes a percentage contribution interpretation much harder. But it is obvious that the PREMA component explains most of what is left over from EWNV. Although UITB has a statistically significant price, it explains very little of the average return differences of the firm size portfolios.

#### 2.5. Some further evidence

We have shown above that a measure of the changing risk premium is responsible for a substantial portion of the firm size effect. We hypothesize that the risk premium may change as a result of changing business conditions. In this section, we shall attempt to duplicate the effect of PREMA directly with business cycle indicators. Among the indicators available, we choose for this purpose the series Net Business Formation and the Coincident Indicator. The data were collected from the Bureau of Economic Analysis (Handbook of Cyclical Indicators, Department of Commerce). The Net Business Formation series is a weighted average of two components: (a) the number of new incorporations minus the number of business failures, and (b) the change in the number of business telephone lines. It is designed to "measure the net entries to the business populations," and as such, it can be regarded as a gauge of the business conditions. The Coincident Indicator has four components,<sup>14</sup> and one of which is the level of industrial production. This indicator measures the current state of the economy, rather than the future state which the market index predicts.

The primary data sources of the series are quite divergent and the timing may not always correspond with the market returns for stocks. The series are



also seasonally adjusted. The readers should interpret the results with this caveat in mind.

We repeat the experiments replacing PREMA by the one month ahead growth rate of the Net Business Formation (BUSF),<sup>15</sup> using the Fama-MacBeth method. The results are reported in table 7A, first panel. Given the poor statistical properties of the BUSF, the similarity between table 7A, first panel and tables 5A and B (i.e., with PREMA rather than BUSF) is striking. In the absence of PREMA, it appears that BUSF can perform a function similar to PREMA in explaining the differences in returns among firms of different sizes.

Next, we include both BUSF and PREMA in the same experiment. A priori, three types of outcomes are possible: (i) the prices of both BUSF and PREMA are insignificant, which can happen if both variables are measuring the same thing and both are equally good, (ii) the price of one of them is significant while the other is not, which means that one is a better measure than the other, and (iii) both prices are significant, which means neither measure can dominate the other and each one is measuring something better or measuring something not reflected in the other. The results are reported in the second panel of table 7A. The price of PREMA remains significant while the price of BUSF has become insignificant. From the two results above, we conclude that the variable BUSF may replace PREMA, but it is not as good as PREMA.

The results of using the one month ahead growth rate of the Coincident Indicator are similar to those obtained with the BUSF and therefore not reported here. The only difference is that the price associated with the Coincident Indicator is still marginally significant ( $t = 1.86$ ) even when PREMA is included.

One of the components of the Coincident Indicator is the level of industrial production. Compared to other nonfinancial market data, the

production series data are relatively clean. The third panel of table 7A reports the results of using the one month ahead industrial production growth rate. Indeed, the price associated with the monthly growth rate of industrial production (IP1) has the highest overall  $t$  statistic and is strong in every subperiod. Table 7B reports the component contribution of each variable. PREMA remains to be the most the important variable explaining the return differences among portfolios, followed by IP1 and then EWNV. Comparing table 7B with table 6, we observe that IP1 has taken most of its explanatory power from PREMA.

Relating to the pricing of the Coincident Indicator and IP1, an interesting hypothesis is that those variables are also coincident with the level of consumption.<sup>16</sup> Since it is easier to measure industrial production than consumption, the results follow.

Whether or not the above hypothesis is true, the industrial production level is often regarded as the indicator of the current state of the economy and as such, it is a fundamental state variable on which conditional future expectations of many other economic variables are formed. Therefore, it is not surprising to find IP1 being priced. However, as long as IP1 is not a sufficient statistic for the current and expected future economic activities, variables such as PREMA, which we hypothesized to depend on the state of the economy among other things, can still retain explanatory power even when IP1 is included. Indeed, the evidence<sup>17</sup> suggests the following multifactor pricing equation:

$$r_i = \lambda_0 + \lambda_1 b(\text{EWNV}) + \lambda_2 b(\text{DEI}) + \lambda_3 b(\text{UITB}) + \lambda_4 b(\text{IP1})$$

(2)

$$+ \lambda_5 b(\text{PREMA}) + \lambda_6 b(\text{TBD}) + \epsilon_i .$$

Figure 1 plots the loadings of the 20 portfolios to each variable. It is interesting to observe how the PREMA loadings vary with size. The effect of PREMA, after controlling for the other variables, mainly concentrates on the smaller firms. This is consistent with the size effect documented by Banz (1982).

## 2.6. Summary

Let us summarize the results of this section. We started with the five variables in a time series cross-sectional experiment to examine the firm size effect, and found that the loadings of PREMA were able to explain most of the cross-sectional variations in average return. The series PREMA was used here as a measure of the market determined risk premium, which may fluctuate across time due to changes in the "price of risk" and changes in the perceived business and financial risk. The covariation between a portfolio return and PREMA thus measures the portfolio's risk exposure related to the changing risk premium. Since we have also included the equally weighted NYSE index (and other variables) in the time series cross-sectional regressions, the loadings of PREMA can be considered as measure of risk not fully reflected by the equally weighted NYSE index.<sup>18</sup>

As collaborating evidence with nonfinancial market data, we chose the series Net Business Formation (BUSF) to measure the business expansions and contractions (which presumably induce the fluctuations in PREMA). In the absence of PREMA, the loadings of BUSF were priced and could explain a substantial portion of the difference in cross-sectional returns. This is consistent with the scenario that during business contractions, small firms suffer a relatively high rate of failure.<sup>19</sup> This in turn is reflected in higher average returns to the bearer of such risk.

Finally, we chose the Coincident Indicator and the industrial production variable as indicators of the current state of the economy, and found that both of these variables had explanatory power. The loadings of the monthly growth rate of industrial production are priced even when PREMA is included in the same time series cross sectional regressions. We concluded that owners of smaller stocks derived higher average returns also from subjecting their wealth to higher covariations with the state of the economy.

So, the message we try to communicate in this section is actually quite simple, and is very much related to the intuition underlying most asset pricing models. The higher average returns of smaller firms are compensations for higher risks, and the most significant ones here are the covariation of portfolio returns with the risk premium and the state of the economy. The inability of the market betas to capture these risks<sup>20</sup> led us to analyze the size effect in a multifactor framework, and we found that the resultant pricing equation explained most of the size effect.

### 3. Regressions with Firm Size Proxies

The results of Section 2 demonstrate that (i) the variables PREMA, IP1, and UITB are priced; (ii) PREMA and IP1 are the most important variable explaining the cross-sectional difference in expected returns when portfolios are ranked by firms' equity; and (iii) the residuals, which may be interpreted as risk adjusted returns, are almost indistinguishable among portfolios of different firm size. Thus the firm size effect, which is manifested through the formation of the portfolios, is captured by an equation whose theoretical foundation is laid on the principles of portfolio theory and no arbitrage profit (or market equilibrium, to be exact).

The firm size effect is often called an anomaly because there is no theoretical reason why a firm size proxy should have any power explaining the

cross-sectional differences in asset returns, after controlling for risks. The fact that empirically a firm size proxy has explanatory power must have meant that firm sizes are proxying for some risks that were not measured, or not measured properly. Therefore, it would be interesting to reexamine the role of a firm size proxy in our multifactor pricing framework. Since no theory suggests which functional form the firm size should enter into the cross sectional return relation, we shall choose " $\ln MV$ "--the natural logarithm of the market value of a firm's equity--based on the work of Brown, Kleidon, and Marsh (1983), in which they observed that the relation between the realized "excess return" (from CAPM) and  $\ln MV$  is linear.

If we have indeed captured the size effect with the multifactor equation then  $\ln MV$  should have explanatory power on the estimated expected return generated by equation (2). Month by month cross-sectional regressions of the estimated expected return from (2) (i.e., the realized returns - the estimated residuals from (2)) on  $\ln MV$  indicate that indeed  $\ln MV$  has explanatory power and the average monthly unadjusted  $R^2$  is .487 (1958/1 to 1977/12) and the average adjusted  $R^2$  is .458. On the other hand, the month by month cross-sectional regressions of the estimated residuals on  $\ln MV$  indicate that  $\ln MV$  does not have explanatory power on the estimated residuals, and the average unadjusted  $R^2$  is .011 and the average adjusted  $R^2$  is -.044. This is consistent with the hypothesis that  $\ln MV$  is a good instrumental variable for the expected returns and that equation (2) has extracted the size effect that  $\ln MV$  is proxying for.

Nevertheless, it might still be informative to include the firm size proxy in the multifactor equation to examine the effect, even though such specification is not justified by any theory. Some symptoms of multicollinearity would probably surface using both the theoretical and the

instrumental variables. Whether the effect of an instrumental variable can be eliminated depends crucially on how correlated the realized return is with the variable. In the extreme case, if an instrumental variable were constructed ex post to fit realized returns perfectly, no theory would be able to eliminate its explanatory power. In general, when the correct variables are entered into a pricing equation, there should be some reduction in the significance of an instrumental variable.

Table 8 reports several variations of the multifactor pricing equation and the impact of  $\ln MV$  on them. Equation (i) is Black's version of CAPM. Observe that the market price is large (slightly more than 18% per annum) and significant when the list of macroeconomic variables is not included. In fact, the market price is so large that the intercept term becomes negative. This equation is rejected by both the Hotelling  $T^2$  (at .01 level) and the paired  $t$  statistic.

The second equation in Table 8 includes  $\ln MV$ . Indeed, the  $t$  statistic of the firm size coefficient is significant and the paired  $t$  statistic of the residuals is not. However, the disconcerting consequences of this equation are that (1) the estimated market premium becomes negative--a violation of our prior belief, (2) the intercept term is difficult to interpret, and (3) the residuals of portfolios have been twisted so much that the equation is rejected by the Hotelling  $T^2$ .

The negative sign of the estimated market premium is disturbing. Remember that small firms have higher market betas than large firms. When the estimated market premium becomes negative, the  $\ln MV$  variable not only has to explain the original return differences among size portfolios, but also explain the effect induced by the negative market premium. In other words, the size effect is amplified by the negative market premium and the entire

burden falls on  $\ln MV$  to explain. Consequently, the coefficient of  $\ln MV$  is larger in magnitude than what it should be. To find out if this is true, we run the same regression without the EWNY beta. Equation (iii) reports the result. As expected, without the negative estimated EWNY premium, the coefficient of  $\ln MV$  becomes smaller.

We have observed from the previous section that the variables PREMA and IP1 are responsible for most of the size effects not explained by the EWNY. For the sake of parsimony, if we are allowed only to add one or two variables to the EWNY beta in the pricing equation, how well behaved is the resultant equation? Table 8 reports the results of adding just the PREMA (eq. iv) and of adding IP1 and PREMA (eq. v) to the EWNY. We notice that the estimated intercepts and market premia look reasonable compared with the realized T-Bill rate of .00352 and realized excess return of the EWNY index over the T-Bill rate of .00564 during the same time period. If we compare (iv) and (v) with (ii), we notice that the signs of the estimated prices in (iv) and (v) are what we expect and the equations are not rejected by the paired  $t$  test nor the Hotelling  $T^2$  test. Thus, if we are interested in the minimal extension of the CAPM to capture the size effect, equations (iv) and (v) are good candidates. However, the specifications of (iv) and (v) are motivated by the empirical results in the previous section. Readers should interpret the results with this caveat in mind.

Equation (vi) reports the results of replacing EWNY by the value weighted NYSE index (VWNY) in equation (2) of the text. The undesirable result of (vi) is that the estimated VWNY premium is negative, which may impart upward biases in the prices of those variables that explain the size effect. Equation (vii) includes the  $\ln MV$  variable. The coefficient of  $\ln MV$  is insignificant. However, the inclusion of  $\ln MV$  causes the significance of many variables to

decrease. This is an indication that  $\ln MV$  and the other variables are measuring the same "risk," but multicollinearity renders them insignificant

Equation (viii) reports the result of adding  $\ln MV$  to equation (2) of the text. The surprising result of (viii) is that the  $\ln MV$  coefficient has a  $t$  of -2.20. The answer to this puzzle lies with the negative estimated EWN premium. As we discussed above in connection with equations (ii) and (iii), the negative estimated EWN premium<sup>21</sup> produces spuriously large statistics for those variables that explain the size effect. Equation (ix) discards the EWN betas in the cross-sectional regressions. Indeed, the  $t$  statistic associated with the  $\ln MV$  drops to -1.25. The  $t$  statistic of PREMA also drops to 1.95 from 2.10 of (viii) and from 3.01 of the original multifactor pricing equation without the  $\ln MV$ . The price of IP1, however, remains strong in almost every specification, with or without  $\ln MV$ .

In conclusion, the multifactor pricing equation, equation (2), has captured most of the size effect. The  $\ln MV$  variable does not add marginal explanatory power to the multifactor model in explaining the size effect.

#### 4. The January Anomaly

Keim (1982) points out that "more than fifty percent of the magnitude of the size effect is due to the anomalous January returns."<sup>22</sup> Since the results from Section 2 indicate that most of the size effect is captured by equations (2), the next question suggests itself. Is the January anomaly captured by (2) or is the magnitude of the January size effect essentially there in the residuals? If the latter is true, it would mean that large positive residuals from (2) in January are offset by large negative residuals in other months to make the overall average residual close to zero. This would be a violation of the joint hypotheses that markets are efficient or rational and that (2) is an equilibrium pricing model.



A careful examination of the average residuals from (2) for January does not reveal any particular pattern except for the smallest portfolio, which has a point estimate of .01203 ( $t = 3.76$ ). The January residual difference between the smallest and the largest portfolios is .01018 ( $t = 2.62$ ). This is significant and only slightly lower than the per annum difference of .01188 ( $t = 1.73$ ) annualized from the figure given in table 5B. Thus, most of the estimated risk adjusted difference in returns occurs in January. The difference is statistically significant, though the magnitude is not large.

There is another reason why we are interested in January. It turns out that PREMA, the difference in return between a low grade bond portfolio and a government bond portfolio, also has a strong January seasonal.<sup>23</sup> For the other variables, EWNY has a January seasonal, IP1 is seasonal but nothing extraordinary in January and the rest have no recognizable monthly patterns. Now, realizing the relation between the size effect and January, have we stumbled on a variable--PREMA (which ex-ante sounds like a reasonable candidate for explaining the size effect) that happens to have a January seasonal<sup>24</sup> and inadvertently explains the firm size effect? It is difficult to have an unambiguous test of this possibility. What we did was to run the model leaving out all Januarys in the time series (estimates of the sensitivities). The result for the entire period is (t-statistics in parentheses):

$$r_i = .00160 + .00817 \hat{b}_i(\text{EWNY}) - .00003 \hat{b}_i(\text{DEI}) + .01789 \hat{b}_i(\text{IP1}) \\ (.375) \quad (1.55) \quad (-.53) \quad (3.80) \\ -.00036 \hat{b}_i(\text{UITB}) + .00960 \hat{b}_i(\text{PREMA}) - .00536 \hat{b}_i(\text{TBD}) + \varepsilon_i \\ (-1.04) \quad (2.83) \quad (-1.97)$$

$$U1 - U20 = .00203, \quad \text{Hotelling} - T^2 = 1.30 \\ (1.94)$$

The point estimate for the PREMA price is almost the same as before (table 7A) and is still significant. The result above indicates that the explanatory power of PREMA does not come exclusively from its January seasonal.

## 5. Conclusions

We have explored the feasibility of a multifactor pricing equation based primarily on the APT as an explanation of the firm size effect. Among the economic variables included, a measure of the changing risk premium<sup>25</sup> and a measure of the changing state of the economy explained a large portion of the size effect. Based on the evidence we have gathered so far, we conclude that the firm size anomaly is essentially captured by a multifactor pricing model. The higher average returns of smaller firms are justified by the additional risks borne in an efficient market.

## FOOTNOTES

<sup>1</sup>See Ross (1976), Huberman (1982), and Connor (1982) for the formal development; see also Ingersoll (1982) and Chen and Ingersoll (1983).

<sup>2</sup>Similar results are obtained using the variable  $DEIP_t \equiv E_{t+1}(IP12_{t+1}) - E_t(IP12_t)$ , where the expectation is generated by a rolling one period ahead forecast model. The information set contains four lags of equally weighted NYSE stock index returns, four lags of the annual growth rate of industrial production, and four lags of the growth rate of the monetary base.

<sup>3</sup>This is similar to the arguments presented in Fama and Gibbons (1982a).

<sup>4</sup>See Van Horne (1976) for a description of the evidence relating the spread of bond yields to business cycles.

<sup>5</sup>The low grade bond portfolio also contains bonds that are unrated. Bonds are unrated because (i) they are too small, (ii) the issuing company does not agree with the rating agency (Moody) on the rating; or (iii) Moody, as a matter of policy, does not rate them. The most important class belonging to (iii) are financial bonds--Moody did not start rating financial bonds until around 1973. These bonds may have a rating above Ba after Moody started to rate them. Included are "firms" such as General Motors Acceptance Corporation, Household Finance Corp., North Carolina National Bank, and Family Finance Corp.

<sup>6</sup>In general, firms that have traded bonds are larger firms. Of the firms in the bottom decile in three selected years (1958, 1964, 1971) whose data can be found in Moody's handbooks, about 25% have bonds outstanding. Most of them are unrated or have no information on rating. Of those that have ratings, about 65% of them would have fallen into the low grade bond category. However, since our low grade bond portfolio is "value weighted by

probability," only one firm from the bottom decile is included in the portfolio from 1953 to 1958.

<sup>7</sup>Many researchers have documented the relation between stock returns and inflation. If our inclusion of the IP12 (or stock market index), PREMA, and TBD cannot completely eliminate the correlation between stock returns and inflations, then omitting the inflation variables in the time series regressions would lead to biased estimation of the loadings.

<sup>8</sup>A somewhat equivalent approach is the Gibbons (1982) method in which any unexplained size effect will show up in the intercepts of the estimated equations. However, at this stage, we only have the time series of the macroeconomic variables but not their mimicking portfolios. The estimated intercepts in this case are rather complicated and their statistical properties are not well understood. Therefore, we used the Fama-MacBeth method in this paper. In the Fama-MacBeth method, a multivariate extension of Shanken's (1982) or Gibbons' (1980) errors-in-variable analysis is applicable in certain experimental designs. The adjustment is dependent on the null hypothesis. In testing against the null hypothesis that  $\lambda = 0$ , the standard errors are exactly the same as estimated in the standard Fama-MacBeth procedure.

<sup>9</sup>The error structure of the estimated residual is analyzed in details in Shanken (1983). In that paper, Shanken used a similar  $T^2$  statistic to test the CAPM, with an errors-in-variable adjustment. Since the adjustment is a quadratic term in the denominator both for the univariate and multivariate case, the adjusted  $T^2$  is always lower. In this paper, almost all the Hotelling  $T^2$  statistics are insignificant, therefore, the adjustment becomes irrelevant. The only significant  $T^2$  statistics are those associated with equations where the market beta is the only estimated parameter. In that

case, the adjustment, which is proportional to the square of the ratio of excess market return to the standard deviation of the market return, is trivial.

<sup>10</sup>The Fama-MacBeth method also preserves the independence of residuals across time. In the cross sectional regression, we could have used the full estimated covariance matrix from the first step. This was not done because the estimated covariance matrix was so poorly estimated (due to lack of degrees of freedom) that the variance of the estimated residuals from the cross-sectional regressions became much higher. This has also been the experience of Black and Scholes (1974). Banz (1982) did not use the the full estimated covariance matrix in his study. (We thank Myron Scholes and Rolf Banz for a discussion on this point.) We have also tried putting in just the diagonal term from the estimated covariance matrix in the cross sectional regression. The results are essentially the same as reported in this section, except the estimated price are slightly flatter. This is to be expected when EWNV is included and the middle portfolios all have very small residual variance in the time series regression. However, since no claims of efficiency (which depends on the resultant eigenvalue system) have ever been made in that case, we decided to stay with the Fama-MacBeth method.

<sup>11</sup>Results not reported here may be obtained from the authors upon request.

<sup>12</sup>As a check of the quality of the estimated difference in residual, we regress U1-U20 on the time series of EWNV, PREMA, etc. For the "true" residuals, the regression coefficients should be insignificant. For our estimated U1-U20, only the coefficient associated with the EWNV is significant ( $t = 2.46$ ) with a point estimate of "beta" of 0.056. The statistical significance is not surprising since we are regressing residual

stock returns on a stock market index. The magnitude of "beta" indicates that the quality of our estimated difference is reasonable.

<sup>13</sup>See Stoll and Whaley (1983) for a discussion of the magnitude and differences in transaction costs associated with trading stocks of small and large firms. How differences in trading costs affect the size of the small firm effect is difficult to determine because differences in transaction costs will likely induce clientele effect: those who anticipate revising their holdings very often will choose to invest in securities with low transaction costs. Measuring its effect on the expected return would probably require some knowledge of the induced market equilibrium.

<sup>14</sup>The Coincident Indicator has four components: (1) Employees on nonagricultural payrolls, (2) Index of industrial production, (3) Personal income less transfer payment (1972 dollars), and (4) Manufacturing and trade sales (1972 dollars).

<sup>15</sup>Because of the timing problem with the data Net Business Formation, we try the one month ahead growth rate first because most of the information, such as the granting of the stock charters and posting of bankruptcy hearings, is known to the market before the month it is actually recorded. Subsequent to the experiments reported, we have tried the concurrent growth rate of Net Business Formation. The results are still significant but not as strong as with the one month ahead growth rate.

<sup>16</sup>We thank Doug Breeden for a discussion on this point. See Breeden (1979) for related issues.

<sup>17</sup>In our experiments, we have also tried (a) adding one-half of the estimated variance rate to the continuously compounded rates and (b) the simple arithmetic returns. The results were essentially the same.

<sup>18</sup>We have also run the cross-sectional regressions replacing the loadings of the EWNV by the market betas estimated directly from a single factor market model. The results are qualitatively the same as reported in table 5A

<sup>19</sup>It is interesting to consider the impact of a disproportionately high rate of failure among small firms on our interpretation of the firm size effect. It is intuitive to ascribe the higher average return of small firms to this differential risk when the loadings of BUSF are priced cross sectionally. The economic reasoning is also straightforward. The series BUSF, being an indicator of the business cycles, varies directly with economic expansions and contractions. During a downturn, the decrease in current and future expected cash flows is felt more heavily with smaller firms, and thus simultaneously causes a high failure rate among smaller firms and large negative returns for the smaller firm portfolios. As a result, the BUSF loadings are higher for smaller firms, and when the loadings are priced, the smaller firm portfolios have higher estimated expected returns. Note that the fact that both the regressor (portfolio returns) and the regressand (new incorporations - failures + changes in business telephone lines) are somehow related to small firms when the BUSF loadings were computed could not have in itself explained the firm size effect. After all, it is the inability of the market betas to explain the cross sectional average returns that created the "anomaly," despite the fact that market betas were computed by regressing portfolio returns on portfolio returns, and the equally weighted market index can typically explain about 90% of the time series variations of a small firm portfolio. Also, the mere monotonicity of the BUSF loadings from small to large firm portfolios in itself could not be responsible for the pricing since we can see that market betas as well as other loadings are also approximately monotonic from small to large firms. In order for the BUSF loadings to be

priced, the covariation between BUSF with both small and large firm portfolios must have such a pattern that it is statistically related to the pattern of average returns after controlling for other loadings and market betas. While the statistical relation may still occur by chance, a more likely explanation is that we have found a good proxy for one of the factors (state variables) responsible for the size effect in a multifactor asset pricing model

<sup>20</sup>When we regress a small firm portfolio returns on the five variables across time, both the loadings of EWNY and PREM (and some other variables) are significantly different from zero. Therefore, we can say that PREM is measuring some fluctuations of the portfolio not captured by the market betas. There are at least two possibilities. One hypothesis is related to the leverage effect. The returns to equity, especially for smaller firms, are nonlinearly related to the market returns. A further pursuit of this hypothesis is undertaken by one of the authors. Another hypothesis is related to the different timing of EWNY and PREM relative to the returns of smaller firms. There is evidence indicating that while the EWNY forecasts future economic conditions, the PREM is more closely related to the Coincident Indicator of the business cycles. In fact, PREM is much better than EWNY as a predictor of the Coincident Indicator. If the viability of many smaller firms is not resolved until the actual growth rate of the economy is known with higher precision, then the PREM may be just measuring this effect. Evidence of the EWNY and PREM as predictor of the Coincident Indicator may be obtained from the authors upon request.

<sup>21</sup>The products of the negative market premium with the higher betas for smaller firms make the small and large firm average return difference bigger after adjusting for the market risk.



<sup>22</sup>Many other authors have since then looked at the January anomaly; see, e.g., Roll (1983). The January returns for the first seventeen portfolios are significantly positive. There is no obvious pattern for returns from February to November for most portfolios except that most of them are insignificant. The December returns from the tenth portfolio on are significantly positive (except portfolio 15). The first three months of the smallest three portfolios and the last three months of the largest three portfolios usually have large positive returns.

<sup>23</sup>The  $t$  statistics for the January return of PREMA is 4.24. In no other months is the return of PREMA significant.

<sup>24</sup>We thank George Constantinides for this suggestion.

<sup>25</sup>We have also investigated the change in the expected market premium ( $\equiv$  EWNY - T-Bill rate). We found that the expected market premium is significantly positively correlated with expected (i.e., using only past information) future industrial production and negatively correlated with past growth rate of industrial production. See also Merton (1980) for other related evidence.

## REFERENCES

- Banz, Rolf, 1981, "The Relationship between Return and Market Value of Common Stocks," Journal of Financial Economics 9, 3-18.
- Black, Fischer, 1972, "Capital Market Equilibrium with Restricted Borrowing," Journal of Business 45, 444-454.
- Black, F., M. Jensen and M. Scholes, 1972, "The Capital Asset Pricing Model: Some Empirical Results," in Michael Jensen (ed.), Studies in the Theory of Capital Market, Praeger.
- Black, F. and M. Scholes, 1974, "The Effects of Dividend Yield and Dividend Policy on Common Stock Prices and Returns," Journal of Financial Economics 1, 1-22.
- Breeden, Douglas, 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics 7, 265-296.
- Brown, P., A. Kleidon and T. Marsh, 1983, "New Evidence on the Nature of Size Related Anomalies in Stock Prices," Journal of Financial Economics 12, 33-56.
- Chen, N., 1981, "Arbitrage Asset Pricing: Theory and Evidence," unpublished Ph.D. Dissertation, UCLA.
- Chen, N., 1983, "Some Empirical Tests of the Theory of Arbitrage Pricing," forthcoming in the Journal of Finance.
- Chen, N. and J. Ingersoll, 1983, "Exact Pricing in Linear Factor Models with Finitely Many Assets: A Note," Journal of Finance 38, 985-988.
- Chen, N., R. Roll and S. Ross, 1983, "Economic Forces and the Stock Market," manuscript, Yale.
- Connor, G., 1982, "A Factor Pricing Theory for Capital Assets," manuscript, Northwestern University.

- Fama, E., 1970, "Multiperiod Consumption-Investment Decisions," American Economic Review 60, 163-174.
- Fama, E. and J. MacBeth, 1973, "Risk, Return and Equilibrium: Empirical Tests," Journal of Political Economy 81, 607-636.
- Fama, E., 1976, "Foundations of Finance," Basic Books, New York.
- Fama, E., 1981, "Stock Returns, Real Activity, Inflation and Money," American Economic Review 71, 545-565.
- Fama, E. and M. Gibbons, 1982a, "Inflation, Real Returns and Capital Investment," Journal of Monetary Economics 8, 297-324.
- Fama, E. and M. Gibbons, 1982b, "A Comparison of Inflation Forecasts," CRSP Working Paper No. 86, University of Chicago.
- Gibbons, M., 1980, "Estimating the Parameters of the Capital Asset Pricing Model: A Minimum Expected Loss Approach," unpublished manuscript, Graduate School of Business, Stanford University.
- Gibbons, M., 1982, "Multivariate Tests of Financial Models: A New Approach," Journal of Financial Economics 10, 3-27.
- Huberman, G., 1982, "Arbitrage Pricing Theory: A Simple Approach," Journal of Economic Theory 28, 183-191.
- Ibbotson, R., 1979, "The Corporate Bond Market: Structure and Returns," manuscript, University of Chicago.
- Ibbotson, R. and Sinquefield, 1982, "Stocks, Bonds, Bills and Inflation: The Past and the Future," The Financial Analysts Research Foundation.
- Ingersoll, J., 1982, "Some Results in the Theory of Arbitrage Pricing," CRSP Working Paper No. 67, University of Chicago.
- Keim, D., 1983, "Size Related Anomalies and Stock Return Seasonality: Empirical Evidence," Journal of Financial Economics 12, 13-32.

- Lintner, J., 1965, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics 47, 13-37.
- Long, J., 1974, "Stock Prices, Inflation and the Term Structure of Interest Rates," Journal of Financial Economics 1, 131-170.
- Merton, R., 1973, "An Intertemporal Capital Asset Pricing Model," Econometrica 41, 867-887.
- Merton, R., 1980, "On Estimating the Expected Return on the Market: An Exploratory Investigation," Journal of Financial Economics 8, 323-361.
- Reinganum, M., 1981, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings' Yields and Market Values," Journal of Financial Economics 9, 19-46.
- Roll, R., 1983, "The Turn-of-the-Year Effect and the Return Premia of Small Firms," Journal of Portfolio Management.
- Ross, S., 1976, "The Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory 13, 341-360.
- Shanken, Jay, 1982, "An Asymptotic Analysis of the Traditional Risk-Return Model," unpublished manuscript, University of California, Berkeley.
- Shanken, Jay, 1983, "Multivariate Tests of the Zero-Beta CAPM," unpublished manuscript, University of California, Berkeley.
- Sharpe, W., 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance 19, 425-442.
- Stoll, Hans and Robert Whaley, 1983, "Transaction Costs and the Small Firm Effect," Journal of Financial Economics 12, 57-79.
- Van Horne, J., 1973, Financial Market Rates and Flows. Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

Table 1

## Correlations among the Macro Variables (1953/1-1977/11)

	EWNY	PREM	UITB	DEI	TBD	IP12	PREMA
EWNY							
PREM	.3709						
UITB	-.1937	-.0509					
DEI	-.2120	-.0280	.4333				
TBD	.1399	-.1340	-.0251	-.2268			
IP12	.2862	.1542	-.1289	-.0552	.0921		
PREMA	.3325	.7202	-.0962	.0357	-.5813	.1478	

EWNY = Equally weighted NYSE stock index

PREM = Difference in return of "Under BAA" bond portfolio and "AAA" bond portfolio (rated by Moody). Data were obtained from R. G. Ibbotson & Co., Chicago.

UITB = Unanticipated inflation, defined as CPI - expected inflation

DEI = Change in expected inflation

TBD = Difference in return of long term government bond portfolio and the one month T-bill

IP12 = Growth rate of industrial production of the next twelve months.

PREMA = Difference in return of "Under BAA" bond portfolio and Long Term Government Bond Portfolio. The Government Bond Data were taken from Ibbotson and Sinquefeld (1982)

TABLE 2

Autocorrelations of the Variables<sup>a</sup>  
(1953/1-1977/11)

	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\rho_6$	$\rho_7$	$\rho_8$	$\rho_9$	$\rho_{10}$	$\rho_{11}$	$\rho_{12}$	Adjusted Box Pierce (24)	Monthly <sup>b</sup> Mean	Variance
EWNY	.154	.009	.053	.051	.021	-.019	-.098	-.118	.022	-.027	.032	.112	36.73	.909	.239
PREM	-.058	.036	.020	.071	-.026	.036	-.066	-.028	-.187	.005	-.186	.106	43.96	.085	.022
UITB	.033	.100	-.037	-.077	-.052	-.051	-.074	-.029	.090	.082	.006	.132	33.45	.008	.001
DEI	-.221	.063	.040	.023	.036	-.157	-.016	.060	.103	-.093	.000	.115	55.01	.002	.00002
TDD	-.013	.008	-.102	.151	-.028	-.002	-.090	-.001	.063	-.094	.038	.008	29.18	-.064	.036
IP12	.955	.874	.769	.652	.527	.406	.285	.165	.053	-.051	-.146	-.225	1289.	3.880	.406
PREMA	-.152	.011	-.009	.114	-.009	-.022	-.135	-.019	-.046	-.098	-.104	.113	44.89	.106	.041

<sup>a</sup>The variables are defined in Table 1.<sup>b</sup>Multipplied by 100.

TABLE 3

Monthly Continuous Compounded Returns of the Twenty Portfolios<sup>a</sup>

	1958-1977	1958-1972	1958-1967	1968-1977
P1	.01513	.01508	.01895	.01131
P2	.01248	.01278	.01662	.00834
P3	.01265	.01346	.01839	.00691
P4	.01153	.01227	.01723	.00583
P5	.01155	.01201	.01630	.00679
P6	.01099	.01186	.01665	.00533
P7	.01143	.01212	.01633	.00653
P8	.01123	.01176	.01364	.00882
P9	.00906	.00932	.01190	.00622
P10	.00975	.01096	.01428	.00522
P11	.00900	.00996	.01314	.00486
P12	.00872	.01013	.01229	.00516
P13	.00907	.01023	.01350	.00465
P14	.00965	.01127	.01280	.00650
P15	.00825	.00934	.01244	.00406
P16	.00751	.00942	.01196	.00305
P17	.00759	.00925	.01148	.00370
P18	.00624	.00847	.00983	.00265
P19	.00514	.00665	.00798	.00229
P20	.00558	.00830	.00924	.00191
P1-P20	.00956 (2.4692)	.00678 (1.9373)	.00971 (2.4902)	.00940 (1.4022)
Q1	.01306 (3.21)	.01349 (3.40)	.01788 (4.16)	.00823 (1.19)
Q5	.00618 (2.31)	.00820 (2.99)	.00966 (3.21)	.00270 (0.61)
Q1-Q5	.00687 (2.6422)	.00528 (2.2041)	.00822 (3.1401)	.00552 (1.2265)

<sup>a</sup>P1, ..., P20 are the twenty portfolios in increasing equity size. P1-P20 is the difference between the two extreme portfolios. Q1, Q5, Q1-Q5 are respectively the smallest quintile, the largest quintile and the difference. t statistics in parentheses.

TABLE 4

AVERAGE FIRM SIZE OF THE TWENTY PORTFOLIOS  
(in millions)

	Average (Dec. '57-Dec. '76)	Dec. '57	Dec. '76
P1	8.8	2.6	10.8
P2	15.9	5.1	18.2
P3	22.3	7.2	26.3
P4	29.3	9.5	34.8
P5	37.8	12.1	46.4
P6	46.9	15.4	57.8
P7	57.9	19.6	72.6
P8	71.7	24.4	91.5
P9	87.7	30.9	113.7
P10	106.9	39.4	138.9
P11	131.1	48.3	172.4
P12	162.6	60.9	220.2
P13	203.4	73.1	275.1
P14	259.3	90.8	351.6
P15	336.1	121.8	469.0
P16	432.0	169.9	614.6
P17	552.0	230.1	773.9
P18	742.9	330.6	987.0
P19	1122.6	496.0	1537.2
P20	4938.9	2212.0	6300.8



TABLE 5A

Cross Sectional Regressions of Stock Returns on the  
Macro Variable Loadings<sup>a</sup>

$$r_i = \lambda_0 + \lambda_1 \hat{b}_i(\text{EWNY}) + \lambda_2 \hat{b}_i(\text{DEI}) + \lambda_3 \hat{b}_i(\text{UITB}) + \lambda_4 \hat{b}_i(\text{PREMA}) + \lambda_5 \hat{b}_i(\text{TBD}) + \varepsilon_i$$

	$\bar{\lambda}_0$	$\bar{\lambda}_1(\text{EWNY})$	$\bar{\lambda}_2(\text{DEI})$	$\bar{\lambda}_3(\text{UITB})$	$\bar{\lambda}_4(\text{PREMA})$	$\bar{\lambda}_5(\text{TBD})$
1958-1977	.00386 (0.93)	.00601 (1.13)	-.00007 (-1.31)	-.00085 (-2.39)	.01061 (3.25)	-.00471 (-1.76)
1958-1972	.00640 (1.58)	.00429 (0.85)	-.00003 (-0.44)	-.00055 (-1.51)	.01367 (4.50)	-.00515 (-1.97)
1958-1967	.00544 (1.08)	.00839 (1.39)	.00006 (0.70)	-.00011 (-0.23)	.01072 (3.10)	-.00136 (-0.62)
1968-1977	.00229 (0.34)	.00364 (0.41)	-.00020 (-2.54)	-.00159 (-3.11)	.01050 (1.89)	-.00806 (-1.65)

<sup>a</sup> t statistics in parentheses. They are computed directly from the time series of estimated  $\lambda$  from the monthly cross-sectional regressions. Average adjusted  $R^2$  (1958-1977) is 0.39, average unadjusted  $R^2$  is 0.53. The variables are defined in Table 1.

Table 5B  
Average Residuals<sup>a</sup>

	<u>1958-1977</u>	<u>1958-1972</u>	<u>1958-1967</u>	<u>1968-1977</u>
U1	.00106	.00068	.00018	.00194
U2	-.00044	-.00024	-.00122	.00035
U3	-.00073	-.00057	.00025	-.00172
U4	.00013	-.00006	.00076	-.00050
U5	.00020	.00051	.00055	-.00015
U6	.00033	-.00006	.00056	.00009
U7	.00011	.00009	.00046	-.00024
U8	.00038	.00071	.00022	.00054
U9	-.00152	-.00181	-.00232	-.00071
U10	.00059	.00053	.00074	.00044
U11	-.00059	-.00066	.00046	-.00163
U12	.00028	.00053	-.00015	.00071
U13	.00077	.00069	.00113	.00040
U14	.00125	.00157	.00008	.00242
U15	.00019	-.00026	.00071	-.00032
U16	.00057	.00148	.00128	-.00016
U17	.00043	.00019	.00095	-.00009
U18	-.00124	-.00085	-.00154	-.00094
U19	-.00113	-.00142	-.00224	-.00001
U20	-.00063	-.00103	-.00087	-.00040
U1-U20	.00169 (1.73)	.00171 (1.50)	.00106 (0.83)	.00233 (1.57)
Hotelling T <sup>2</sup>	1.45	1.42	1.91*	0.79
UQ1	.00006	.00000	.00004	.00008
UQ5	-.00061	-.00075	-.00090	-.00032
UQ1-UQ5	.00067 (1.86)	.00076 (1.91)	.00094 (1.84)	.00040 (0.78)

<sup>a</sup>U1, ..., U20 are the estimated residuals for the twenty portfolios, U1-U20 is the difference. The Hotelling T<sup>2</sup> is the test that U1-U2, ..., U19-U20 are jointly zero. The critical values at the .05 level are 1.63, 1.65, 1.69 for the 20 years, 15 years and 10 years periods respectively. UQ1, UQ5, UQ1-UQ5 are respectively the residuals for the smallest quintile, the largest quintile and the difference.

Table 6

Average Monthly Component Contributions  $(\hat{\lambda}b)$   
(in percentage per month)  
1958-1977<sup>a</sup>

	<u>Total Return</u>	<u>EWNY</u>	<u>DEI</u>	<u>UITB</u>	<u>PREMA</u>	<u>TBD</u>	<u>Constant</u>	<u>Unexplained</u>
P1	1.513	.736	-.077	-.008	.385	-.015	.386	.106
P2	1.248	.728	-.087	-.010	.385	-.110	.386	-.044
P3	1.265	.692	.020	-.016	.405	-.149	.386	-.073
P4	1.153	.656	-.024	-.080	.262	-.061	.386	.013
P5	1.155	.702	.033	-.029	.086	-.044	.386	.020
P6	1.099	.657	-.001	.046	.062	-.084	.386	.033
P7	1.143	.700	.053	-.004	.079	-.082	.386	.011
P8	1.123	.671	.014	.012	-.014	.016	.386	.038
P9	0.906	.564	-.028	.045	.026	.065	.386	-.152
P10	0.975	.550	.017	.003	-.074	.033	.386	.059
P11	0.900	.585	.016	.052	-.070	-.010	.386	-.059
P12	0.872	.549	-.035	.049	-.173	.068	.386	.028
P13	0.907	.542	.002	.020	-.167	.047	.386	.077
P14	0.965	.605	.052	-.048	-.175	.020	.386	.125
P15	0.825	.446	.076	-.003	-.103	.004	.386	.019
P16	0.751	.459	-.021	-.008	-.169	.048	.386	.057
P17	0.759	.442	-.003	.010	-.148	.029	.386	.043
P18	0.624	.469	.082	-.052	-.219	.082	.386	-.124
P19	0.514	.471	-.027	-.074	-.242	.112	.386	-.113
P20	0.558	.403	.027	-.069	-.202	.075	.386	-.063
P1-P20	0.956	.333	-.104	.061	.587	-.090	0	.169
Q1-Q5	0.687	.257	-.062	.018	.562	-.157	0	.067

<sup>a</sup>EWNY is the equally weighted NYSE index. DEI is the difference in expected inflation. UITB is the unanticipated inflation. PREMA is the difference in return between low grade bond and government bond portfolios. TBD is the difference in return between long term government bonds and T-bill.

TABLE 7A

Cross Sectional Regressions of Stock Returns on Macro Variable  
and Business Indicator Loadings<sup>a</sup>

$$r_{it} = \lambda_0 + \lambda_1 \hat{b}_i(\text{EWNY}) + \lambda_2 \hat{b}_i(\text{DEI}) + \lambda_3 \hat{b}_i(\text{UITB}) + \lambda_4 \hat{b}_i(\text{BUSF}) + \lambda_5 \hat{b}_i(\text{TBD}) + \varepsilon_{it}$$

	$\bar{\lambda}_0$	$\bar{\lambda}_1(\text{EWNY})$	$\bar{\lambda}_2(\text{DEI})$	$\bar{\lambda}_3(\text{UITB})$	$\bar{\lambda}_4(\text{BUSF})$	$\bar{\lambda}_5(\text{TBD})$
'58-'77	.00585 (1.41)	.00396 (.77)	-.00010 (-1.78)	-.00106 (-2.94)	.00638 (2.91)	-.00566 (-2.12)
'58-'72	.01140 (2.89)	-.00075 (-.16)	-.00004 (-.66)	-.00094 (-2.44)	.00844 (3.19)	-.00576 (-2.15)
'58-'67	.01161 (2.33)	.00214 (.38)	.00008 (1.01)	-.00069 (-1.30)	.00793 (2.63)	-.00173 (-.79)
'68-'77	.00009 (.01)	.00578 (.67)	-.00029 (-3.40)	-.00144 (-2.90)	.00484 (1.51)	-.00959 (-1.97)
	<u>U1-U20</u>		<u>UQ1-UQ5</u>		<u>Hotelling T<sup>2</sup></u>	
'58-'77	.00198 (1.97)		.00113 (2.78)		1.61	
'58-'72	.00160 (1.43)		.00140 (2.93)		1.58	
'58-'67	.00156 (1.18)		.00177 (2.86)		1.92*	
'68-'77	.00240 (1.57)		.00049 (.93)		0.90	

$$r_{it} = \lambda_0 + \lambda_1 \hat{b}_i(\text{EWNY}) + \lambda_2 \hat{b}_i(\text{DEI}) + \lambda_3 \hat{b}_i(\text{UITB}) + \lambda_4 \hat{b}_i(\text{BUSF}) + \lambda_5 \hat{b}_i(\text{PREMA}) + \lambda_6 \hat{b}_i(\text{TBD}) + \varepsilon_{it}$$

	$\bar{\lambda}_0$	$\bar{\lambda}_1(\text{EWNY})$	$\bar{\lambda}_2(\text{DEI})$	$\bar{\lambda}_3(\text{UITB})$	$\bar{\lambda}_4(\text{BUSF})$	$\bar{\lambda}_5(\text{PREMA})$	$\bar{\lambda}_6(\text{TBD})$
'58-'77	.00463 (1.14)	.00523 (1.02)	-.00011 (-1.92)	-.00104 (-2.80)	.00276 (1.36)	.00993 (3.09)	-.00468 (-1.73)
'58-'72	.00789 (2.12)	.00280 (.59)	-.00005 (-.83)	-.00076 (-1.93)	.00346 (1.45)	.01298 (4.45)	-.00477 (-1.78)
'58-'67	.00629 (1.37)	.00757 (1.39)	.00005 (.61)	-.00038 (-.71)	.00216 (.73)	.01044 (3.19)	-.00118 (-.50)
'68-'77	.00297 (.44)	.00298 (0.33)	-.00026 (-3.40)	-.00170 (-3.33)	.00336 (1.19)	.00942 (1.69)	-.00818 (-1.68)
	<u>U1-U20</u>		<u>UQ1-UQ5</u>		<u>Hotelling T<sup>2</sup></u>		
'58-'77	.00133 (1.55)		.00055 (1.73)		1.59		
'58-'72	.00114 (1.15)		.00066 (1.91)		1.61		
'58-'67	.00052 (.47)		.00092 (2.00)		1.92*		
'68-'77	.00214 (1.62)		.00018 (.40)		0.98		

<sup>a</sup>BUSF and IP1 are respectively the one month ahead growth rate of the Net Business Formation and the Industrial Production. The other variables are defined in Table 1. The t statistics are in parentheses. They are computed from the time series of estimated  $\lambda$ . U1-U20, UQ1-UQ5, and Hotelling T<sup>2</sup> are defined in Table 5B.

TABLE 7A (Continued)

$$r_{it} = \lambda_0 + \lambda_1 \hat{b}_i(\text{EWNY}) + \lambda_2 \hat{b}_i(\text{DEI}) + \lambda_3 \hat{b}_i(\text{UITB}) + \lambda_4 \hat{b}_i(\text{IP1}) + \lambda_5 \hat{b}_i(\text{PREMA}) + \lambda_6 \hat{b}_i(\text{TBD}) + \varepsilon_{it}$$

	$\hat{\lambda}_0$	$\hat{\lambda}_1(\text{EWNY})$	$\hat{\lambda}_2(\text{DEI})$	$\hat{\lambda}_3(\text{UITB})$	$\hat{\lambda}_4(\text{IP1})$	$\hat{\lambda}_5(\text{PREMA})$	$\hat{\lambda}_6(\text{TBD})$
'58-'77	.00522 (1.28)	.00466 (.89)	-.00010 (-1.92)	-.00094 (-2.64)	.01669 (3.74)	.00929 (3.01)	-.00407 (-1.49)
'58-'72	.00798 (2.02)	.00272 (.56)	-.00006 (-.99)	-.00056 (-1.53)	.01650 (3.34)	.01179 (4.07)	-.00524 (-1.99)
'58-'67	.00721 (1.52)	.00670 (1.18)	00001 (.18)	-.00013 (-.26)	.01592 (2.61)	.00791 (2.47)	-.00068 (-.30)
'68-'77	.00324 (.48)	.00262 (.30)	-.00023 (-3.01)	-.00175 (-3.46)	.01746 (2.66)	.01066 (2.01)	-.00746 (-1.50)
		<u>U1-U20</u>	<u>UQ1-UQ5</u>	<u>Hotelling T<sup>2</sup></u>			
'58-'77		.00099 (1.11)	.00051 (1.46)	1.48			
'58-'72		.00112 (1.06)	.00062 (1.65)	1.22			
'58-'67		.00030 (.26)	.00073 (1.54)	1.81*			
'68-'77		.00167 (1.23)	.00030 (.57)	0.90			

TABLE 7B

Average Monthly Component Contributions ( $\hat{\lambda}b$ ) with the Monthly  
Growth Rate of Industrial Production<sup>a</sup>  
(in percentage per month)  
1958-1977

	<u>Total Return</u>	<u>EWNY</u>	<u>DEI</u>	<u>UITB</u>	<u>IP1</u>	<u>PREMA</u>	<u>TBD</u>	<u>Constant</u>	<u>Unexplained</u>
P1	1.513	.574	-.099	-.019	.242	.266	-.011	.522	.040
P2	1.248	.574	-.101	-.027	.120	.316	-.125	.522	-.030
P3	1.265	.541	.014	-.030	.118	.310	-.131	.522	-.078
P4	1.153	.516	-.041	-.097	.104	.199	-.056	.522	.007
P5	1.155	.546	.039	-.045	.035	.068	-.053	.522	.044
P6	1.099	.508	.002	.034	-.018	.048	-.080	.522	.083
P7	1.143	.548	.043	-.026	.041	.070	-.084	.522	.028
P8	1.123	.525	.020	.015	.037	-.022	.006	.522	.021
P9	0.906	.419	-.046	.056	-.029	.036	.048	.522	-.101
P10	0.975	.417	.026	-.006	.036	-.050	.025	.522	.004
P11	0.900	.453	.026	.044	.008	-.070	.004	.522	-.087
P12	0.872	.408	-.034	.043	-.031	-.133	.082	.522	.014
P13	0.907	.394	.004	.025	-.062	-.114	.049	.522	.089
P14	0.965	.472	.058	-.036	-.069	-.127	.020	.522	.125
P15	0.825	.337	.078	.015	-.084	-.082	.012	.522	.027
P16	0.751	.339	-.003	-.005	-.112	-.105	.041	.522	.073
P17	0.759	.323	.003	.014	-.028	-.121	.042	.522	.003
P18	0.624	.366	.100	-.040	-.121	-.165	.082	.522	-.119
P19	0.514	.356	-.019	-.057	-.143	-.178	.119	.522	-.085
P20	0.558	.288	.029	-.070	-.087	-.126	.061	.522	-.059
P1-P20	0.957	.286	-.128	.051	.329	.392	-.072	0	.099
Q1-Q5	0.687	.218	-.085	-.005	.241	.421	-.157	0	.051

<sup>a</sup>IP1 is the one month ahead growth rate of industrial production. The other variables are defined in Table 1.

Table 8

Cross Sectional Regressions of Returns on Macro Variable Loadings and Firm Size Proxies (1958-1977)<sup>a</sup>  
 (t statistics in parentheses)

	$\bar{\lambda}_0$	$\bar{\lambda}(EWNV)$	$\bar{\lambda}(IP1)$	$\bar{\lambda}(VWNY)$	$\bar{\lambda}(\ln MV)$	$\bar{\lambda}(DEI)$	$\bar{\lambda}(UITB)$	$\bar{\lambda}(PREMA)$	$\bar{\lambda}(TBD)$	U1- U20	Hotelling <sup>b</sup> $T^2$
(i)	-.00550 (-1.15)	.01545 (2.59)								-.00401 (2.56)	2.03**
(ii)	-.03997 (4.41)	-.00642 (-1.61)			-.00207 (-3.70)					-.00049 (-0.46)	2.09**
(iii)	.02651 (2.96)				-.00147 (-2.53)					-.00019 (-.18)	1.34
(iv)	.00282 (0.66)	.00711 (1.27)						-.00925 (2.98)		-.00188 (1.37)	1.27
(v)	.00446 (1.15)	.00543 (1.10)	.01507 (3.19)					.00689 (2.39)		.00118 (0.99)	1.44
(vi)	.00816 (1.74)		.01397 (2.86)	-.00105 (-0.22)		-.00012 (-2.12)	-.00084 (-2.32)	.00984 (3.31)	-.00436 (-1.56)	.00075 (0.88)	1.50
(vii)	.02317 (2.56)		.01095 (2.50)	-.00617 (-1.39)	-.00074 (-1.29)	-.00004 (-0.64)	-.00053 (-1.43)	.00630 (2.06)	-.00444 (-1.49)	-.00017 (-0.26)	1.13
(viii)	.02903 (3.08)	-.00512 (-1.08)	.01263 (2.89)		-.00124 (-2.20)	-.00005 (-0.85)	-.00056 (-1.52)	.00663 (2.10)	-.00462 (-1.61)	-.00027 (-.42)	1.28
(ix)	.01822 (1.95)		.01268 (2.94)		-.00077 (-1.25)	-.00005 (-0.91)	-.00063 (-1.80)	.00576 (1.95)	-.00319 (-1.19)	-.00001 (-.02)	0.91

<sup>a</sup>EWNV is the equally weighted NYSE index. VWNY is the value weighted NYSE index. IP1 is the growth rate of industrial production in the next month.  $\ln MV$  is the natural logarithm of a firm's equity value. DEI is the difference in expected inflation. UITB is the unexpected inflation. PREMA is the difference in return between low grade bonds and government bonds. TBD is the difference in return between long term government bonds and T-bill. U1-U20 and Hotelling  $T^2$  are defined in Table 5B.

<sup>b</sup>The critical value at the .01 and .05 level are respectively 1.99 and 1.69.

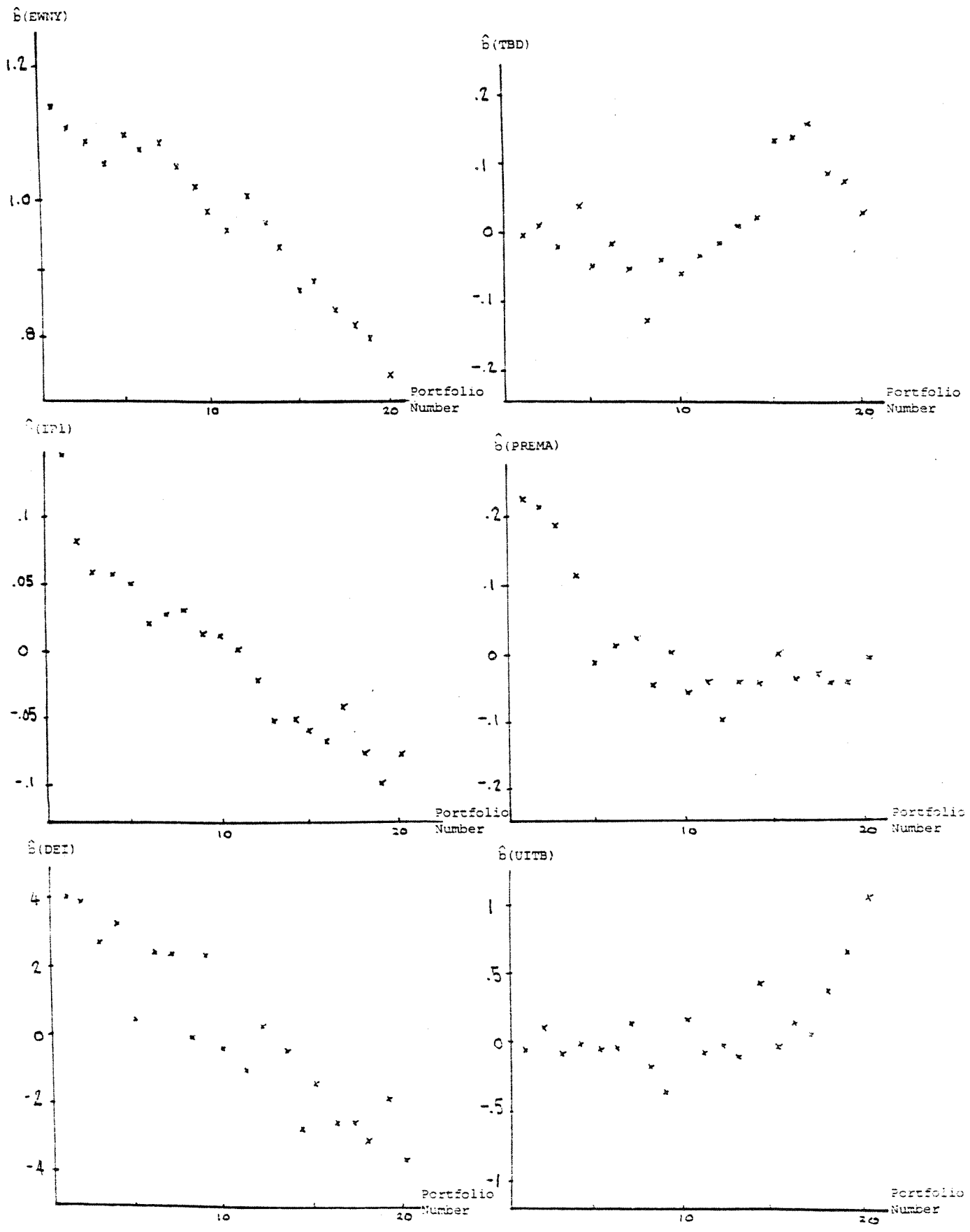


Fig. 1. The average estimated loadings corresponding to the economic variables. EWNY is the equally weighted NYSE stock index. TBD is the difference in return between long term government bonds and T-Bill. IPL is the one month ahead monthly growth rate of industrial production. PREMA is the difference in return between low grade bonds and government bonds. DEI is the difference in expected inflation. UITB is the unanticipated inflation.