The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983*

by

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Abstract

This paper examines the statistical properties of daily rates of change of of five foreign currencies from 1974 to 1983. The main purpose is to discriminate between two competing explanations for the observed heavy tails of the distribution: that the data are independently drawn from a heavy tail distribution which remains fixed over time, and that the data come from distributions which vary over time. Evidence point to the rejection of the first hypothesis. Further investigations show that the rejection can be attributed to changing means and variances in the data, which can be described by a simple statistical model.

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1. Introduction

This paper examines the statistical properties of daily rates of change of the prices of five foreign currencies from 1974 to 1983. Knowledge of this distribution has important economic applications: the effects of exchange rate movements on international trade and capital flows, mean-variance analysis of international asset portfolios, and the pricing of options on foreign currencies. In previous studies in the literature,[1] the consensus is that the distribution of these changes is unimodal and has fatter tails than the normal distribution. It is this second feature which has attracted the most attention. Two competing explanations have been put forth to account for the leptokurtosis. One view suggests that the data are independently drawn from a fat tail distribution that remains fixed over time, while the other proposes that the data come from distributions that vary over time.

There have been few attempts to determine which of the two explanations better characterizes the data. Friedman and Vandersteel (1982) considered the following three hypotheses: (a) The data are independent and identically distributed (iid), drawn from a stable symmetric paretian distribution. (b) The data are iid, drawn from a mixture of two normal distributions. (c) The data are independently drawn from a normal distribution whose mean and variance change over time. They found evidence in favor of the third hypothesis. Calderon-Rossell and Ben-Horim (1982) tested directly for the iid property without requiring distributional assumptions. They found that they can reject iid for most currencies. They attributed the rejection to shifts in the mean in 9 out of 13 currencies, and to changes in the skewness in the remaining cases.

This paper extends the previous work in several ways. We use ten full years of daily data to investigate in detail the cause of rejection of the iid hypothesis. We find that not only do the distributions of exchange rates differ across days of the week, but that their means and variances also shift over time. Furthermore, we are able to construct a simple statistical model to describe these time-varying means and variances.

2. The Data

The data consist of daily closing bid prices of foreign currencies (in terms of the U.S. dollar) from the interbank market provided by the University of Chicago Center for Research on Security Prices. Five major currencies are selected in this study: British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), Japanese Yen (JY), and Swiss Franc (SF). There are a total of 2510 daily observations, from January 2, 1974 to December 30, 1983.[2] We calculate the rate of change by taking the logarithmic difference between the close of two successive trading days. For the first year of our sample, the JY moved very little, if at all, which leads us to believe that the Japanese government might have pegged the exchange rate. We therefore calculate two sets of statistics for the JY, one using the entire sample of 2510 observations, the other using only the last 9 years of data (2262 observations).

Table 1 provides some summary statistics of the data. The means are quite small. The range of daily changes, however, is relatively high, the largest (in absolute value) being -7.0967% for the DM on November 1, 1978. The data are unimodal and approximately symmetric, with higher peaks and fatter tails than the normal distribution. (Frequency plots are available from the author upon request.) Non-normality is confirmed by the coefficients of kurtosis being larger than three for all five currencies.

3. Testing for Independence and Identical Distributions

Two explanations have been put forth to account for the fat tail distribution of exchange rate changes. One view is that the data are independent and identically distributed (iid), drawn from a non-normal distribution. A stable symmetric paretian distribution was used in Westerfield (1977), and the Student-t distribution in Rogalski and Vinso (1978). In addition to these distributions, Boothe and Glassman (1985) estimated a mixture of two normals with equal means but different variances. A second view is that the data are not iid, but drawn from distributions which change over time. Friedman and Vandersteel (1982) suggested a normal distribution with time varying means and variances.

In order to discriminate between these two explanations, we divide the data into two equal subsamples. If the data were iid, the two subsamples should have the same distribution. This can be tested by comparing their empirical densities. Partition the event space into k mutually exclusive and collectively exhaustive regions, $A_1, \ldots, A_k$. Define p_{i1} (p_{i2}) to be the probability of an observation in the first (second) subsample falling into region A_i. The null hypothesis of equal distribution is expressed as:

$$H_o: \; p_{i1} = p_{i2}, \quad \text{for} \; i=1, \ldots, k \tag{1}$$

This is a test of equality of two multinomial distributions, and can be accomplished by using a chi-square statistic.[3] [See Hogg and Craig (1970, p. 312).]

There is no clear cut rule on choosing the number of regions (k) in a partition. In order to apply the results from asymptotic distribution theory, the number of observations per region should increase with the sample size. On the other hand, the power of the test can be improved only by increasing the number of regions with the sample size. A possible candidate is $k = n^{1/2}$, where n is the number of observations. This leads to a choice of $k=50$ for our sample size. Kendall and Stuart (1967, vol. 2, p. 438) suggests $k = n^{2/5}$, which would be k=23. We tried values of 21, 31, 41, and 51, and found that the results are not sensitive to the choice of k. For a given value of k, we select the partition so as to equalize the number of observations (from the entire sample) in each region. [See Hogg and Craig (1970, p. 365-66).] The chi- square statistics for $k=21$ and 51 are presented in the first two rows of Table 2. They show very clearly that the distributions are not equal in the two subsamples for all five currencies.

It is possible that the rejection of the iid hypothesis results from halving the sample at 1979, which coincides with the change of the operating procedure of the U.S. Federal Reserve System. We therefore conduct the iid test using two year subperiods. Table 2 shows that out of the nine subsamples, the iid hypothesis is rejected at the 1 percent significance level 4 times for BP, once for CD, 4 times for DM, 5 times for JY, and 6 times for SF. These results indicate that the rejection of the iid hypothesis for the entire sample period is not related to splitting the sample at 1979.

4. Testing for Autocorrelation

The rejection of the iid hypothesis suggests that the data are either serially dependent or not identically distributed. In this section, we test for serial independence. Table 3 lists selective autocorrelation coefficients, $\rho(\tau)$, for the five currencies. No coefficient exceeds 0.1, implying that there is little serial correlation, which is in agreement with the literature.[4]

Under the null hypothesis that the data are white noise with a constant variance, the standard error for each sample autocorrelation coefficient is $\sqrt{1/n}$ or 0.01996 for this sample. This would indicate that several coefficients may be statistically different from zero. Furthermore, the null hypothesis implies that the sample autocorrelation coefficients are not correlated asymptotically. Thus the joint test that the frist K autocorrelation coefficients are zero can be conducted using the Box- Pierce Q statistic, $T \sum_{\tau=1}^{K} \rho(\tau)^2$, which is asymptotically a chi-square distribution with K degrees of freedom. In Table 3, we report the test for the first 50 autocorrelation coefficients. (The results are not sensitive to this particular choice of K.) The null hypothesis is rejected at the 1 percent significance level for CD and JY (and JY2), at the 5 percent level for SF, and not rejected for BP and DM at conventional significance levels.

These tests seem to indicate the presence of serial correlation in the data. We caution the reader in accepting this conclusion. Heteroscedasticity may cause the standard error of each sample autocorrelation coefficient to be underestimated by $\sqrt{1/n}$. Diebold (1986) gives a heteroscedasticity-consistent estimate of the standard error for the τ-th sample autocorrelation coefficient:

$$S(\tau) = \sqrt{(1/n)\ (1 + \gamma_{x^2}(\tau) / \sigma^4)} \tag{2}$$

where $\gamma_{x^2}(\tau)$ is the τ-th sample autocovariance of the squared data, and σ is the sample standard deviation of the data. These adjusted standard errors are reported in Table 3, and they show that few of the autocorrelation coefficients are statistically different from zero.

Moreover, Diebold (1986) shows that the adjusted Box- Pierce Q statistic, $\sum_{\tau=1}^{K} [\rho(\tau)/S(\tau)]^2$, is asymptotically a chi-square distribution with K degrees of freedom. This is also reported in Table 3. The statistics do not reject the null hypothesis of serial independence for the BP, CD, DM, and SF data. There appears to be some autocorrelation in JY and JY2, at the 5% but not the 1% significance level.

These results indicate that the rejections of serial independence using the standard testing procedure were caused by heteroscedasticity in the data. Furthermore, even if there is serial correlation in the data, it is of such a small magnitude that it cannot account for the strong rejections of the iid hypothesis.

5. Testing for the "Day of the Week" Effect

The results of the serial correlation test indicates that the rejection of the iid hypothesis cannot be caused by serial dependence. We therefore turn our attention to whether the rejection can be explained by changing distributions. One possibility is that the distribution of daily spot rate changes may be different between weekdays and weekends. In the stock market literature, the return to holding stocks between Friday's close and Monday's close is shown to be different from the returns on other days of the week.[5]

To check whether the distribution of exchange rates actually changes across days of the week, we divide the data into five subsamples according to the day of the week. Within each subsample, we assume that the data are iid, and test for equality of distribution across subsamples using the chi-square test. These pairwise tests are reported in the upper half of Table 4. Seven out of the 25 statistics are significant at the 1% level, with eight more significant at the 5% level. We therefore reject the null hypothesis that the distribution is iid for each day of the week and iid across different days of the week.

To check whether this "day of the week" effect is responsible for the rejection of the iid hypothesis, we split each subsample into two halves and perform the chi-square test. The results are in the lower half of Table 4. Fourteen out of the 25 statistics are significant at the 1% level, with five more significant at the 5% level. Thus, even though the distribution of exchange rates is different across the days of the week, it cannot account for the rejection of the iid hypothesis.

6. Testing For Time Varying Means and Variances

Another possibility of unequal distributions in the data is that means and variances change over time. This was offered by Friedman and Vandersteel (1982) as an explanation of the the fat tail distribution of spot rate changes. They never directly tested this hypothesis. Instead they used some moving statistics to summarize these changes in the data.

Calderon-Rossell and Ben-Horim (1982) checked whether changing means and variances could account for the rejection of the iid hypothesis. They divided their data into 3 subperiods of about 365 daily observations each. They assumed that the means and variances are constant within each subperiod, and tested for differences across subperiods. They found that shifts in mean accounted for 9 rejections out of 13 currencies, but shifts in variances accounted for none of the remaining 4 cases. They, too, never directly tested for changing means and variances.

Following the procedures in Calderon-Rossell and Ben-Horim (1982), we first checked whether changing means and variances can account for the rejection of the iid hypothesis. Rather than assuming constant means and variances over time intervals as lengthy as 18 months, we assume that means and variances are constant within each month. (An examination of the moving statistics in Friedman and Vandersteel reveals that means and variances can change substantially over time.) As a maintained hypothesis, we are assuming that the data are serially independent for the remainder of this section.

To check whether changes in monthly means can account for the rejection of iid, we center the data within a month by subtracting the monthly mean. We then recompute the chi-square test of iid using the centered data. As shown in the first row (Test 1) of Table 5, the statistics are still highly significant. Clearly, changes in monthly means alone cannot be responsible for the rejection of iid.

To check whether changes in monthly variances can account for the rejection of iid, we rescale the data within a month by dividing by the monthly standard deviation. The results of the chi-square tests are in the second row (Test 2) of Table 5. These statistics are much smaller than their counterparts in the preceeding row, although all but the CD are still significant at the 1% level. It appears that changes in monthly variances can partly account for the rejection of iid.

To check for the combined effects of changing means and variances, we standardize the data within a month by subtracting the monthly mean and dividing by the monthly standard deviation. The results of the chi-square tests are in the third row (Test 3) of Table 5. These statistics are dramatically lower than the two preceeding rows in some cases. The CD, DM, and SF are no longer significant, even at the 10% level. The BP and JY are still significant at the 1% level. It appears that changing means and variances across months can jointly account for the rejection of iid, at least for the CD, DM, and SF.

To verify that the means and variances actually change over time, we perform some direct statistical tests. Again, we assume that means and variances are constant within each month, and test whether they are equal across the 120 months of data.

The test of equality of means is best explained in a regression context. The daily rates of change are regressed on a constant term and 119 dummy variables, one for each of the 119 months after the first month. If the means are constant across months, then the coefficients for all 119 dummy variables should be zero. We test this with the Wald statistic, employing a covariance estimator which is consistent against a large class of heteroscedasticity.[6] The statistics are in the fourth row of Table 5. The equality of monthly means is rejected for the CD, DM, and JY at the 1% level, and for the BP at the 5% level. It is not rejected for the SF, even at the 10% level. This evidence show that means are indeed not constant over time.

The test for the equality of variances is more problematic, due to the heavy tails of the distribution. The Bartlett test, which is the standard test of homoscedasticity, is a likelihood ratio test for normally distributed data. It is given in the fifth row of Table 5. Since there is strong evidence against normality, we prefer to use tests that are robust against fat tail distributions, such as the modified Levene test described in Brown and Forsythe (1974). These are labelled "Levene I" and "Levene II" in the last two rows of Table 5. The first procedure centers the data within each month by the 47.5% trimmed mean, and the second procedure uses the 10% trimmed means. Brown and Forsythe (1974) show that these two procedures perform very well in Monte Carlo experiments under the normal distribution as well as fat tail distributions. Their results also indicate that the Bartlett test rejects homoscedasticity too frequently, whenever the distribution has fatter tails than the normal. As evident in Table 5, the equality of monthly variances is rejected at any conventional significance level for all currencies regardless of the test procedure.

7. A Statistical Model of Daily Exchange Rates

It would be of interest to try to describe the data with a statistical model. In particular, the change in variances appears to be the strongest characteristic in the data. A successful model of how variances change over time could have important applications in the pricing of foreign currency options, which depends critically on expectations of future variances.

To achieve this purpose, we adopt a version of the ARCH model in Engle (1982). This was used by Domowitz and Hakkio (1985) to model risk premia in the forward exchange market. Although Domowitz and Hakkio justified their model with theoretical considerations, it is not the case here. We choose this model for its computational tractability.

Our specification is as follows. Let $r_t = \log(S_t / S_{t-1})$ be the rate of change of spot rates between two trading days. Conditioning on all information at time t-1, we assume that r_t is normally distributed with mean μ_t and variance σ_t^2, where

$$\mu_t = C_o + C_M D_{Mt} + C_T D_{Tt} + C_W D_{Wt} + C_R D_{Rt} + C_H \, HOL_t + \sum_{i=1}^{5} a_i r_{t-i} + a_{10} \, r_{t-10} + a_{15} \, r_{t-15} + a_{16} \left(\sum_{i=1}^{P} r_{t-i} \right) \quad (3)$$

$$\sigma_t^2 = V_o + V_M D_{Mt} + V_T D_{Tt} + V_W D_{Wt} + V_R D_{Rt} + V_H HOL_t + b \left(\sum_{i=1}^{Q} r_{t-i}^2 / Q \right) \quad (4)$$

D_{Mt}, D_{Tt}, D_{Wt}, and D_{Rt} are dummy variables for Monday, Tuesday, Wednesday, and Thursday. HOL_t is the number of holidays (excluding weekends) between the (t-1)st and t-th trading day. The log likelihood of the data is:

$$\sum_{t=1}^{n} \left\{ -\frac{1}{2} \log \sigma_t^2 - \frac{1}{2} \log 2\pi - \frac{1}{2} \left[\frac{r_t - \mu_t}{\sigma_t} \right]^2 \right\} \quad (5)$$

The maximum likelihood estimates of the coefficients are obtained by using a numerical optimization procedure described in Berndt, Hall, Hall, and Hausman (1974).[7] The two lag lengths (P and Q) are chosen to maximize (5).

Table 6 contains the estimated parameters and the optimal choice of Q and P. Few parameters in the mean equations have any statistical significance. Hence the model is very poor in predicting the direction of exchange rate changes, even within sample. On the other hand, quite a number of parameters in the variance equation are statistically significant. The most important variable in the variance equation is the autoregressive term, b. In all five currencies, b is significantly different from zero at the 1% level. In the case of CD, DM, and SF, b is less than unity. This means that the effects of a large variance on future variances will eventually dampen out. In the case of BP and JY, b is greater than unity. This means that the effects of a large variance on future variances will be amplified, which is clearly undesirable. As we shall see later, this model fits the data for CD, DM, and SF, but not for BP and JY.

In addition, the coefficients for the Monday dummy (V_M) and the holiday dummy (V_H) in the variance equation are large and significant. The coefficients for the other daily dummies (V_T, V_W, and V_R) are typically smaller, although some are also significant. This means that the variances of daily exchange rates are larger whenever the trading days span a weekend or a holiday. Notice, however, that it is important to distinguish between the two effects. The increase in variance due to a weekend (which has 2 calendar days) is smaller than the increase in variance due to a holiday on a weekday (which typically has 1 calendar day), in the case of the CD, DM, and SF.

After estimation, we perform some diagnostics to check how well the model fits the data. We use the standardized residuals:

$$e_t = (r_t - \hat{\mu}_t)/\hat{\sigma}_t \tag{6}$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the estimated means and standard deviations. If the model is correctly specified, then e_t should be approximately iid, normal, with zero mean and unit variance. Table 7 presents some statistical properties of these standardized residuals. The means are close to zero, and the standard deviations are close to one. There is little evidence of serial correlation. But there is evidence of non-normality, as indicated by the coefficients of kurtosis being larger than three.

To test for iid of the residuals, we again employ the chi-square test described in section 3. The results in the first row of Table 8 show that the BP and JY reject iid at very high significance levels. The SF rejects iid at 2.74%, while the CD and DM do not reject at 10%. This test shows that the model does not fit the data for BP and JY.[8] If we test iid across days of the week and within each day of the week, we also find much lower rejection rates than the raw data.[9] However the rejection of iid does not come from shifts in means and variances. Test for the equality of monthly means and variances of the residuals are in Table 10. The equality of monthly means is rejected only for the CD at 3.26%. The equality of monthly variances using the modified Levene tests is rejected only for the BP at 4.34%.

8. Summary and Conclusions

In our investigation, we find that:

(1) Exchange rate changes are not independent and identically distributed.

(2) Each day of the week may have a different distribution, but this is not sufficient to explain the rejection of iid.

(3) There is little serial correlation in the data.

(4) Means and variances change over time. Jointly they may be able to explain the rejection of iid for at least three currencies --- CD, DM, and SF.

Furthermore, we constructed an ad hoc statistical model to capture these properties. In certain aspects, this attempt was successful. For all five currencies, the model was able to capture the change in means and variances over time. Based on the small number of statistically significant coefficients in the mean equation, we believe that the model would not be able to predict directions of exchange rate changes. However, based on the large number of statistically significant coefficients in the variance equation, we believe the model should be able to predict volatility. In other aspects, the model was less successful. It was able to account for the rejection of iid for CD, DM, and SF, but failed to do so for BP and JY. Furthermore, the standardized residuals for all five currencies are non-normal, even though they have less kurtosis than the raw data.[10] Time varying means and variances therefore are not sufficient to fully account for the leptokurtosis in exchange rate changes. The search for an appropriate distribution will be left for future research.

Table 1

Summary Statistics of Log Price Changes:
$r_t = \log(S_t/S_{t-1}) * 100$

Sample Period: January 2, 1974 - December 30, 1983

	BP	CD	DM	JY	SF	JY2**
Mean	-.0184	-.0089	.0005	.0077	.0171	.0116
Median	.0000	-.0098	.0000	.0000	.0000	.0000
Std. Dev.	.5921	.2234	.6372	.6260	.7889	.6307
Skewness	-.4136	-.3149	-.4249	-.2044	-.2835	.1670
	[0.0489]	[0.0489]	[0.0489]	[0.0489]	[0.0489]	[0.0515]
Kurtosis	8.8997	8.6144	12.7913	11.2712	10.2244	7.6320
	[0.0978]	[0.0978]	[0.0978]	[0.0978]	[0.0978]	[0.1030]
Maximum	3.7496	1.5492	3.6686	3.5703	4.4466	3.5703
Minimum	-4.6623	-1.8677	-7.0967	-6.2566	-7.0054	-5.2644
Studentized Range	14.2052	15.2892	16.8906	15.6948	14.5132	14.0053
NOBS	2510	2510	2510	2510	2510	2262

Standard errors in square brackets.

Note: The standard errors are computed as follows:
$\sqrt{6/NOBS}$ for the coefficient of skewness,
$\sqrt{24/NOBS}$ for the coefficient of kurtosis.

** January 2, 1975 to December 30, 1983

Table 2

Tests for Independence and Identical Distributions

Sample Period	BP	CD	DM	JY	SF	JY2**
74-83 (k=51)	308.79 (.0000)	183.19 (.0000)	110.80 (.0000)	456.01 (.0000)	103.35 (.0000)	324.58 (.0000)
74-83 (k=21)	259.69 (.0000)	68.72 (.0000)	84.28 (.0000)	392.42 (.0000)	77.93 (.0000)	263.10 (.0000)
Two year subsamples (k=21):						
74-75	15.84 (.7265)	26.84 (.1398)	36.34 (.0140)	49.17 (.0003)	53.21 (.0001)	
75-76	24.58 (.2180)	25.83 (.1715)	42.17 (.0026)	20.76 (.4114)	49.47 (.0003)	
76-77	111.32 (.0000)	35.13 (.0194)	33.52 (.0295)	90.64 (.0000)	32.67 (.0367)	
77-78	162.89 (.0000)	24.00 (.2424)	59.88 (.0000)	64.14 (.0000)	108.05 (.0000)	
78-79	18.86 (.5309)	20.92 (.4186)	49.65 (.0003)	21.75 (.3542)	59.46 (.0000)	
79-80	31.28 (.0516)	17.39 (.6275)	25.60 (.1794)	18.19 (.5749)	24.01 (.2420)	
80-81	52.63 (.0001)	24.83 (.2080)	48.42 (.0004)	40.67 (.0041)	42.67 (.0023)	
81-82	44.54 (.0013)	27.13 (.1316)	34.46 (.2318)	11.05 (.9449)	25.87 (.1709)	
82-83	26.37 (.1339)	97.44 (.0000)	16.89 (.6601)	40.69 (.0041)	37.92 (.0091)	

Marginal significance levels in parentheses
** January 2, 1975 to December 30, 1983.

Table 3

Autocorrelation Coefficients
[Heteroscedasticity-Consistent Standard Errors]

Lags	BP	CD	DM	JY	SF	JY2**
1	-.0216	.0406	-.0638*	-.0569	-.0410	-.0629
	[.0290]	[.0354]	[.0264]	[.0255]	[.0302]	[.0280]
2	-.0025	.0201	.0021	.0237	.0046	.0224
	[.0293]	[.0267]	[.0332]	[.0261]	[.0281]	[.0282]
3	-.0075	.0176	.0240	.0344	.0050	.0373
	[.0250]	[.0274]	[.0253]	[.0246]	[.0260]	[.0266]
4	-.0064	.0317	-.0244	.0055	-.0229	.0017
	[.0239]	[.0282]	[.0253]	[.0250]	[.0264]	[.0275]
5	.0224	.0740*	.0265	.0386	-.0074	.0410
	[.0279]	[.0254]	[.0277]	[.0267]	[.0283]	[.0293]
6	-.0021	.0212	.0373	.0140	.0561	.0135
	[.0256]	[.0254]	[.0243]	[.0241]	[.0247]	[.0261]
7	-.0124	.0306	.0113	-.0094	-.0070	-.0074
	[.0223]	[.0268]	[.0230]	[.0219]	[.0237]	[.0235]
8	-.0155	-.0065	.0128	.0093	-.0290	.0106
	[.0227]	[.0240]	[.0241]	[.0235]	[.0230]	[.0254]
9	.0586*	.0483	.0391	.0739*	.0385	.0796*
	[.0219]	[.0237]	[.0228]	[.0239]	[.0252]	[.0259]
10	.0017	.0266	.0333	.0715*	.0296	.0694*
	[.0255]	[.0221]	[.0221]	[.0246]	[.0267]	[.0256]
20	.0247	-.0238	-.0061	.0067	-.0099	.0085
	[.0215]	[.0216]	[.0233]	[.0253]	[.0237]	[.0272]
30	-.0033	-.0405	-.0203	.0095	-.0004	.0101
	[.0239]	[.0217]	[.0242]	[.0235]	[.0220]	[.0253]
40	.0197	-.0083	.0271	.0566*	-.0053	.0644*
	[.0237]	[.0239]	[.0218]	[.0231]	[.0230]	[.0248]
50	.0170	.0170	.0206	-.0267	.0029	-.0259
	[.0225]	[.0212]	[.0219]	[.0216]	[.0253]	[.0229]
Box-Pierce Q(50)	51.62 (.4103)	77.52 (.0076)	60.67 (.1434)	98.66 (.0000)	70.21 (.0312)	102.85 (.0000)
Adjusted Box-Pierce Q(50)	39.26 (.8631)	54.33 (.3130)	46.59 (.6110)	69.70 (.0342)	46.27 (.6238)	71.29 (.0256)

Marginal significance levels in parentheses.
* Significantly different from zero at the 1% level (one tailed test).
** January 2, 1975 to December 30, 1983.

Table 4

Test of the "Day of the Week" Effect

	BP	CD	DM	JY	SF	JY2**
Pairwise test for iid across days of the week (k=21)						
Monday vs. Tuesday	38.72 (.0072)	29.02 (.0874)	32.40 (.0392)	33.02 (.0336)	23.91 (.2463)	22.13 (.3335)
Tuesday vs. Wednesday	33.46 (.0300)	18.03 (.5854)	31.21 (.0526)	20.14 (.4492)	23.67 (.2571)	26.50 (.1499)
Wednesday vs. Thursday	51.80 (.0001)	36.26 (.0143)	38.43 (.0078)	24.43 (.2241)	33.15 (.0325)	35.50 (.0176)
Thursday vs. Friday	30.08 (.0686)	69.70 (.0000)	23.04 (.2868)	20.13 (.4498)	34.70 (.0218)	19.04 (.5192)
Friday vs. Monday	58.79 (.0000)	30.93 (.0294)	67.33 (.0000)	27.28 (.0385)	72.86 (.0000)	40.22 (.0047)
Test of iid within each subsample (k=21)						
Monday	68.18 (.0000)	17.12 (.0718)	23.08 (.0105)	75.08 (.0000)	39.57 (.0000)	76.49 (.0000)
Tuesday	47.52 (.0000)	21.76 (.0164)	22.95 (.0109)	86.32 (.0000)	13.58 (.1930)	65.61 (.0000)
Wednesday	51.37 (.0000)	11.30 (.3346)	16.23 (.0932)	54.11 (.0000)	13.17 (.2143)	54.71 (.0000)
Thursday	66.26 (.0000)	28.59 (.0015)	23.90 (.0787)	88.52 (.0000)	28.38 (.0022)	83.20 (.0000)
Friday	49.64 (.0000)	35.53 (.0001)	20.32 (.0264)	89.34 (.0000)	20.48 (.0250)	71.96 (.0000)

Marginal significance levels in parentheses.

** January 2, 1975 to December 30, 1983.

Table 5

Test of Equality of Monthly Means and Variances

	BP	CD	DM	JY	SF	JY2**
Equality of Distribution (k=21)						
Test 1	266.78	58.59	72.37	374.28	79.20	263.45
$\chi^2(20)$	(.0000)	(.0001)	(.0000)	(.0000)	(.0000)	(.0000)
Test 2	48.50	26.39	38.17	78.94	43.10	52.64
$\chi^2(20)$	(.0004)	(.1533)	(.0084)	(.0000)	(.0020)	(.0001)
Test 3	44.53	10.49	25.17	68.21	20.69	49.74
$\chi^2(20)$	(.0013)	(.9584)	(.1950)	(.0000)	(.4156)	(.0002)
Equality of Means Across Months						
Wald Test	154.86	199.75	159.09	181.46	136.29	165.35
$\chi^2(119)$	(.0151)	(.0000)	(.0083)	(.0002)	(.1327)	(.0032)
Equality of Variances Across Months						
Bartlett	1277.32	836.33	1074.01	1470.56	1137.85	1155.26
$\chi^2(119)$	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
Levene I	4.47	4.16	4.37	5.19	5.01	5.25
F(119,2390)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
Levene II	4.90	4.68	4.72	5.70	5.44	5.82
F(119,2390)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)

Marginal significance levels in parentheses.

** January 2, 1975 to December 30, 1983.

Notes:
Equality of Distribution:
Test 1: Data are centered by subtracting monthly means
Test 2: Data are scaled by dividing by monthly standard deviations.
Test 3: Data are standardized by monthly means and standard deviations.

Equality of Variances Across Months:
Levene I: Modified Levene Test, using 47.5% trimmed means.
Levene II: Modified Levene Test, using 10% trimmed means.

Table 6

Estimation Results

	BP	CD	DM	JY2**	SF
C_o	-.020290	-.037006	.001932	-.002249	-.013294
	[.010234]	[.007726]	[.018473]	[.016596]	[.021084]
C_M	.052891	.014197	-.008504	.051091	.042499
	[.032182]	[.013014]	[.034636]	[.027889]	[.038642]
C_T	.005181	.045150	.009708	.023805	.021745
	[.021865]	[.010982]*	[.027020]	[.028598]	[.032105]
C_W	.091246	.023447	.093639	.039944	.103481
	[.024432]*	[.010782]	[.028233]*	[.028028]	[.033793]*
C_R	-.055271	.073081	-.024701	-.022324	-.014051
	[.023314]	[.011676]*	[.028984]	[.026161]	[.034274]
C_H	-.007307	-.059211	-.097387	.000946	-.031798
	[.065290]	[.024250]	[.086744]	[.019812]	[.090907]
a_1	.004764	.059131	-.048588	.000017	-.038806
	[.021771]	[.20293]	[.022631]	[.025636]	[.020946]
a_2	.019793	.019626	.050761	.083986	.027966
	[.023698]	[.021563]	[.022873]	[.027405]*	[.021336]
a_3	-.002193	.016262	.055939	.051625	.024911
	[.025601]	[.022459]	[.022324]	[.226833]	[.022222]
a_4	-.012903	.055652	.027421	.046822	.005873
	[.025520]	[.021988]	[.023663]	[.026784]	[.022714]
a_5	.029473	.038800	.045566	.071051	.026312
	[.025129]	[.022022]	[.023748]	[.027202]*	[.023551]
a_{10}	.020278	.026861	.032815	.061387	.042042
	[.025155]	[.022606]	[.018994]	[.019455]*	[.022704]
a_{15}	.006846	.011142	-.009079	-.004614	.021596
	[.019380]	[.019114]	[.018208]	[.017976]	[.019414]
a_{16}	.449722	-.016022	.250812	-.033490	.260010
	[.119349]*	[.123667]	[.131808]	[.128168]	[.152302]

Table 6 (continue)

	BP	CD	DM	JY2**	SF
V_o	.001089 [.00058]*	.007245 [.001470]*	.022828 [.005664]*	.015024 [.002197]*	-.00013 [.006495]
V_M	.181191 [.008127]*	.013682 [.002358]*	.164185 [.020149]*	.034742 [.008387]*	.155586 [.023082]*
V_T	.024474 [.003832]*	-.001348 [.001740]	.017197 [.010015]	.047170 [.004878]*	.032349 [.014137]*
V_W	.033559 [.002635]*	.000117 [.001770]	.035359 [.009908]*	.018828 [.005106]*	.067677 [.014052]*
V_R	.023705 [.003234]*	.005401 [.001204]*	.033836 [.008231]*	.021669 [.004340]*	.066305 [.012812]*
V_H	.088363 [.024996]*	.019512 [.006038]*	.237628 [.043578]*	-.018355 [.004334]*	.327404 [.079402]*
b	1.060687 [.024884]*	.772861 [.037696]*	.871778 [.033574]*	1.163493 [.036254]*	.960309 [.029109]*
Q	11	12	8	7	14
P	40	60	60	50	60
log likelihood	-1920.23	517.38	-2029.79	-1814.29	-2570.37

Standard error in square brackets.

* Significantly different from zero at the 1% level (one tailed test).

** January 2, 1975 to December 30, 1983

Table 7

Statistical Properties of Standardized Residuals

	BP	CD	DM	JY2**	SF
Mean	-.0098	-.0003	-.0107	-.0089	-.0018
Median	.0078	.0097	-.0321	-.0511	-.0149
Std. Dev.	.9999	.9999	.9999	.9971	1.0000
Skewness	-.3142	-.0895	.1982	.2867	.1057
	[0.0489]	[0.0489]	[0.0489]	[0.0515]	[0.0489]
Kurtosis	9.6278	4.7239	5.4337	9.0555	5.9651
	[0.0978]	[0.0978]	[0.0978]	[0.1030]	[0.0978]
Maximum	7.6170	5.3362	6.1150	5.6662	5.4169
Minimum	-7.6430	-4.8340	-5.5605	-7.2946	-7.1274
Studentized Range	15.2587	10.1688	11.6742	12.9955	12.5421
Test of Serial Correlation					
Box-Pierce	35.25	39.42	37.27	47.06	39.33
Q(50)	(.9433)	(.8589)	(.9086)	(.5921)	(.8613)

Standard error in square brackets.
Marginal significance level in parentheses.

** January 2, 1975 to December 30, 1983.

Table 8

Test for Equality of Distribution of Standardized Residuals

	BP	CD	DM	JY2**	SF
Equality of Distribution (k=21)					
All days $\chi^2(20)$	98.55 (.0000)	24.43 (.2241)	28.18 (.1052)	91.84 (.0000)	33.81 (.0274)
Test of iid across weekdays (k=21)					
Monday vs Tuesday	14.77 (.7894)	23.82 (.2503)	20.47 (.4289)	16.38 (.6928)	18.38 (.5624)
Tuesday vs. Wednesday	26.04 (.1645)	28.22 (.1043)	11.92 (.9188)	17.35 (.6301)	40.91 (.0038)
Wednesday vs. Thursday	23.22 (.2781)	10.46 (.9590)	7.48 (.9948)	24.67 (.2144)	16.87 (.6614)
Thursday vs Friday	21.66 (.3592)	21.96 (.3427)	18.36 (.5637)	16.47 (.6871)	19.89 (.4648)
Friday vs Monday	21.61 (.3620)	19.88 (.4655)	13.51 (.8544)	20.32 (.4381)	23.46 (.2668)
Test of iid within weekdays (k=21)					
Mondays	54.30 (.0000)	19.88 (.0304)	22.56 (.0125)	35.59 (.0062)	31.16 (.0006)
Tuesdays	27.08 (.0025)	14.75 (.1414)	12.47 (.2548)	22.54 (.0126)	5.04 (.8885)
Wednesdays	13.17 (.2143)	5.68 (.8414)	12.49 (.2536)	6.80 (.7442)	9.93 (.4467)
Thursdays	29.74 (.0009)	14.78 (.1403)	14.26 (.1615)	32.04 (.0004)	19.48 (.0346)
Fridays	11.96 (.2877)	12.48 (.2542)	13.94 (.1757)	11.50 (.3199)	10.96 (.3606)

Marginal significance level in parenthesis.

*January 2, 1975 to December 30, 1983.

Table 9

Test of Equality of Monthly Means and Variances for Standardized Residuals

	BP	CD	DM	JY2**	SF
Test of Equality of Monthly Means					
	133.20	148.99	139.09	117.06	122.33
$\chi^2(119)$	(.1764)	(.0326)	(.1006)	(.5332)	(.3986)
Test of Equality of Monthly Variance					
Bartlett	422.11	197.34	216.55	212.19	236.08
$\chi^2(119)$	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
Levene I	1.12	0.95	1.01	0.79	1.06
F(119,2390)	(.1820)	(.6342)	(.4542)	(.9528)	(.3147)
Levene II	1.24	1.10	1.08	0.87	1.15
F(119,2390)	(.0434)	(.2213)	(.2656)	(.8380)	(.1325)

Marginal significance level in parentheses.

** January 2, 1975 to December 30, 1983.

Footnotes

1. See Burt, Kaen and Booth (1977), Westerfield (1977), Rogalaski and Vinso (1978).
2. The data actually begin on July 1, 1973. We start our analysis on January 2, 1974, because we need some pre-sample data in our statistical model in Section 7. A sample of one o'clock (New York) bid prices from 1977 to 1983 were also available. Analysis shows that the two data sets behaved in very similar manners.
3. Calderon-Rossell and Ben-Horim (1982) used the Mann-Whitney-Wilcoxon test. Let $F(x)$ be the distribution of X, and $G(y)$ the distribution of Y. The null hypothesis is $F(z) = G(z)$ for all z, and the alternative hypothesis is $F(z) > G(z)$ for all z, or $F(z) < G(z)$ for all z. (See Hogg and Craig, 1970, p. 371-2.) The alternative hypothesis in the Mann-Whitney-Wilcoxon test does not allow one distribution to be a mean preserving spread of the other. For the chi-square test in the current paper, the null hypothesis is $F(z) = G(z)$ for all z, and the alternative hypothesis is $F(z) \neq G(z)$ for some z. This allows one distribution to have heavier tails at both ends of the range. Since tail behavior is of primary interest in this paper, we deem the chi-square test to be more appropriate.
4. See Giddy and Dufey (1975), Burt, Kaen, and Booth (1977), Cornell (1977), Logue and Sweeney (1977), Logue, Sweeney, and Willett (1978), and Rogalski and Vinso (1982).
5. See French (1980), Gibbons and Hess (1981), and Keim and Stambaugh (1984).
6. See Hansen (1982) and Hsieh (1983) for more details.

7. Note that r_t will not be independent of past observations, since its mean and variance depends on them. Maximum likelihood estimation of the (unconditional) distribution of r_t is therefore not appropriate. On the other hand, the transformed series $(r_t - \mu_t)/\sigma_t$, is independent of each other, and hence maximum likelihood estimation is appropriate.

8. This was not surprising, in view of the results in the third row of Table 5.

9. We compute the marginal significance levels of the tests as if we knew the true parameter values. This is clearly not true, since we are using estimated parameter values. The adjustments, however, must be very small, because of the large number of degrees of freedom for our procedure.

10. The issue of misspecification arises when the distribution of $(r_t - \mu_t)/\sigma_t$ is not normal. In general, a misspecification of the distribution function may lead to inconsistent estimates of the parameters of interest. In the present context, however, such inconsistencies may not arise. The reason is that maximum likelihood using a normal distribution yields consistent estimates of means and variances, even if the true distribution is not normal, provided certain conditions are met. [See White (1982) and Gourieroux, Monfort, and Trognon (1984).]

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