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**INTERNATIONAL RISK SHARING AND THE
CHOICE OF EXCHANGE RATE REGIME***

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Abstract

This paper examines the argument that the fixed exchange rate regime should be preferred to the flexible rate regime because the former allows risk sharing across countries while the latter does not. The analysis is performed in a two-country overlapping generations model, where markets are incomplete under all exchange regimes. It is shown that risks are pooled across countries when the equilibrium exchange rate is constant across states of nature, which arises under the fixed rate regime with or without capital restriction, and under the flexible rate regime without capital restriction. Risks are not pooled across countries when the equilibrium exchange rate is different across states of nature, which arises under the flexible rate regime with capital restriction. But in a model with incomplete markets, the ability to share risk across countries in the regimes with constant exchange rates ^{does} not necessarily lead to higher welfare than the inability to share risk in the regime with random exchange rates.

1. INTRODUCTION

One argument which has been advanced in favor of the fixed exchange rate regime over the flexible rate regime is that the former allows risk sharing across countries while the latter does not. (See for example Mundell [1973].) A maintained hypothesis in this line of argument must be that there is a missing insurance market in both regimes. The fixed rate regime is superior, it is argued, because its risk sharing characteristic is "closer" to the optimal allocations if the insurance market had been operative.

This paper examines these arguments in the context of a two-country overlapping generation model similar to that in Karekan and Wallace [1978], and outlined in Section 2. Individuals in one country face an exogenous uncertainty in their endowments, while individuals in the other country receive a nonrandom endowment. They are unable to pool their risks optimally because of a missing insurance market.

We analyze the stationary equilibria under four different exchange rate regimes: flexible rate with and without capital restriction, and fixed rate with and without capital restriction. Section 3 shows that under the flexible rate regime with capital restriction, the equilibrium exchange rate depends on the state of nature; hence it is random. Agents cannot share risk across countries. Sections 4 and 5 show that under the other three regimes the exchange rate is constant over state of nature. Residents in the country with nonrandom endowments must share the risk of residents in the country with random endowments but not in an optimal manner. Explicit welfare comparisons in Section 6 shows that the choice of exchange rate regime rests on the degree of relative risk aversion. Some concluding remarks are offered in the final section.

2. The Model and Notation

There are two countries, denoted A and B, and one nonstorable consumption good.¹ All agents are identical, live for two periods, and receive an endowment of the good only in the first period. Each agent is indexed by a vector (t, h), where t is the current date, and h his age. A young person is denoted by h = 1, and an old person by h = 2. A superscript indicates the country of residence. There are N^a agents in each generation residing in country A, and N^b in country B.

When young, an agent of generation t in country i receives wⁱ(t) units of the consumption good, which depends on the state of nature z(t). For simplicity, there are two states of nature, denoted by "one" and "two", occurring with probability q and (1 - q) respectively. Draws of z(t) are stochastically independent. An A-young gets w₁^a units in state one, and w₂^a units in state two. Without loss of generality, assume w₂^a > w₁^a. Each B-young gets the same endowment in both states, i.e., w₁^b = w₂^b.

Each agent consumes cⁱ(t, 1) units when young, selling the remaining portion of his endowment for mⁱ(t) units of real balance. When old, he spends this money to buy cⁱ(t + 1, 2) units of consumption, leaving no bequest. Money is the only store of value. Each country issues its own currency. The nominal quantities of each are fixed at Mⁱ. At the outset, an agent is allowed to hold only money of his own country.

The problem faced by each young person is to maximize his expected lifetime utility, which is separable in the two periods:

(1) maximize $U(c^i(t, 1)) + E[V(c^i(t + 1, 2))]$

subject to $c^i(t, 1) = w^i(t) - m^i(t)$

$$c^i(t + 1, 2) = m^i(t)p^i(t)/p^i(t + 1)$$

$$c^i(t, 1), c^i(t + 1, 2), m^i(t) \geq 0.$$

$U(\cdot)$ is the first period utility and $V(\cdot)$ the second period utility. We assume that $U' > 0$, $V' > 0$, $U'' < 0$, $V'' < 0$, $U'(0) = \infty$, and $V'(0) = \infty$. The agent knows the current prices, $p^a(t)$ and $p^b(t)$, and the distribution of prices in the next period. $E[\cdot]$ is the expectations operator.

We consider actions of agents only in stationary equilibria. A stationary equilibrium is one in which the same prices, p_z^a and p_z^b , occur whenever the state of nature is z . It is completely characterized by the ratios of prices in the two states, $\phi^a = p_1^a/p_2^a$ and $\phi^b = p_1^b/p_2^b$.

An individual born in state "one" will solve the first-order condition:

$$(2) \quad U'(w_1^i - m_1^i) = qV'(m_1^i) + (1 - q)V'(m_1^i\phi^i)\phi^i, \quad i = a, b.$$

It is straightforward to show that the optimal real balance, $m_1^i(\phi^i)$, is a decreasing function if $R < 1$, a constant function if $R = 1$, and an increasing function if $R > 1$, where R is the degree of relative risk aversion of the second period utility:

$$(3) \quad R(x) = -xV''(x)/V'(x)$$

as defined in Pratt [1964]. The expected lifetime welfare is:

$$(4) \quad w_1^i(\phi^i) = U(w_1^i - m_1^i(\phi^i)) + qV(m_1^i(\phi^i)) + (1 - q)V(m_1^i(\phi^i)\phi^i), \quad i = a, b.$$

An individual born in state "two" will solve the first-order condition:

$$(5) \quad U'(w_2^i - m_2^i\phi^i) = qV'(m_2^i/\phi^i) + (1 - q)V'(m_2^i)\phi^i, \quad i = a, b.$$

The optimal real balance, $m_2^i(\phi^i)$, is an increasing function if $R < 1$, a constant function if $R = 1$, and a decreasing function if $R > 1$. His expected lifetime welfare is:

$$(6) \quad w_2^i(\phi^i) = U(w_2^i - m_2^i(\phi^i)) + qV(m_2^i(\phi^i)/\phi^i) + (1 - q)V(m_2^i(\phi^i)), \quad i = a, b.$$

From (4) and (6), we can calculate the expected lifetime welfare of an unborn individual:

$$(7) \quad w^i(\phi^i) = qW_1^i(\phi^i) + (1 - q)W_2^i(\phi^i).$$

$w^i(\phi^i)$ is a convex function when $R < 1$, a constant function when $R = 1$, and a concave function when $R > 1$. This fact is crucial in the welfare comparisons in Section 6.

3. Equilibrium under the Flexible Rate Regime with Capital Restriction

Each country has its own central bank. In the flexible rate regime, the sole function of the central banks is to distribute money to the old at time $t = 0$. They do not intervene in the exchange market. The trade balance in the two countries must therefore be zero, since agents are not allowed to hold the other country's money. Hence prices are random in A, and nonrandom in B.

In country A, the goods market at time t clears when the money market clears:

$$(8) \quad N^a p^a(t) m^a(t) = N^a p^a(t-1) m^a(t-1) = M^a.$$

There are possibly many equilibrium price paths. We are, however, interested only in the stationary equilibrium because this greatly simplifies the welfare calculations in Section 6. In such an equilibrium, we observe the same real and nominal variables whenever the same state of nature occurs. These can be found by a fixed point argument. Let p_1^a and p_2^a be the prices observed in states "one" and "two" respectively. A-residents take these as given, and calculate their optimal holdings of real balances, m_1^a and m_2^a , according to the first order conditions in (2) and (5). Under the market clearing condition (8), the real balance in each state should yield the same prices p_1^a and p_2^a :

$$(9) \quad p_z^a = M^a / [N^a m_z^a], \quad z = 1, 2,$$

In other words, p_1^a and p_2^a is a fixed point in the space of discrete probability distributions over prices.

Our assumptions about $U(\cdot)$ and $V(\cdot)$ guarantee the existence and uniqueness of this fixed point. Furthermore, we can show that A-residents born in state "two" will save more and consume more when young, i.e.,

$$m_2^a > m_1^a \quad \text{and} \quad w_2^a - m_2^a > w_1^a - m_1^a. \quad \text{This yields } p_2^a < p_1^a, \text{ and so}$$

$$\phi^a = p_1^a/p_2^a > 1.$$

In country B, there is no uncertainty in endowments. Therefore prices and real balances in the stationary equilibrium are constant:

$$(10) \quad m_1^b = m_2^b$$

$$(11) \quad p_1^b = p_2^b = M^b/[N^b m_1^b].$$

The equilibrium price ratio is:

$$(12) \quad \phi^b = 1.$$

We can calculate the equilibrium exchange rate in each state by the "law of one price":³

$$(13) \quad s_z = p_z^a/p_z^b, \quad z = 1, 2,$$

which is the B-currency price in terms of A-currency. The crucial result is that the exchange rate is different across states of nature:

$$(14) \quad s_1/s_2 = \phi^a > 1.$$

4. Equilibrium under the Fixed Rate Regime with Capital Restriction

In the fixed rate regime, the central banks cooperatively fix the exchange rate at S forever, and are willing to trade any amount of currencies at this rate. In this regime, there is essentially only one currency. We choose the A-currency to be the numeraire. The unification of the two currencies fixes the world private stock of money at $M = M^a + SM^b$. The world goods market clears at time t when the money market clears:

$$(15) \quad \pi(t)[N^a m^a(t) + N^b m^b(t)] = \pi(t-1)[N^a m^a(t-1) + N^b m^b(t-1)] = M,$$

$\pi(t)$ is the price of consumption in units of A-currency. The terms in the square brackets represent the total world demand for real balances.

To find the stationary equilibrium, we must solve for prices π_1 and π_2 , in states "one" and "two" respectively, satisfying:

$$(16) \quad \pi_z = M/[N^a \mu_z^a + N^b \mu_z^b], \quad z = 1, 2,$$

where μ_1^a and μ_2^a are the optimal real balances held by A-young in the two states, and μ_1^b and μ_2^b are the optimal real balances held by B-young in the two states. By a fixed point argument similar to the one used in Section 3, we can show that π_1 and π_2 exist uniquely.

The reader can verify that π_1 is greater than π_2 . The reason is that the total world endowment is high in state "two", and so the total world demand for real balance is also high. We can also show that $\phi^* = \pi_1/\pi_2$ is less than ϕ^a . (The proof is outlined in Appendix 1.)

There are several interesting points about this equilibrium. First, the real balances and the price ratio are independent of the money supplies and the (fixed) exchange rate. This is not surprising due to the separation of the real and nominal economies. Second, the larger the population of A relative to that of B, the closer ϕ^* is to $\phi^a > 1$; conversely, the smaller the population of A relative to that of B, the closer ϕ^* is to $\phi^b = 1$. Third, A-young born in state "two" desires to hold a higher real balance than those born in state "one". In fact, they hold a higher nominal balance: $\pi_1 \mu_2^a > \pi_1 \mu_1^a$. (A proof is furnished in Appendix 1.)

Fourth, the nominal trade balance is zero on average, while the real trade balance is positive (negative) for country A (B) on average. This comes from the following observation. There are three cases to consider: (a) If the state of nature was "one" last period and "two" this period, the current

A-young will want to accumulate a higher nominal balance than the savings of current A-old. They sell some of their endowment to B-old, running a trade surplus. (b) Conversely, if the state of nature was "two" last period and "one" this period, A will run a trade deficit of an equal nominal amount. (c) When the same state of nature occurs consecutively, the trade balance in the second period is zero. Since the events (a) and (b) occur with equal probability, the nominal trade balance is zero on average. But the real trade balance is nonzero, since A tends to run a surplus when the price of consumption is low, and a deficit when it is high, which implies that A expects to run a real trade surplus.

Intuitively, the fixed regime unifies the currency and goods markets, forcing B-residents to share the endowment risks of A-residents. In a good state, i.e., $z = 2$, A "gives" B consumption by running a trade surplus. In a bad state, i.e., $z = 1$, A "takes" consumption from B by running a trade deficit. This risk sharing arrangement increases the mean consumption in B, i.e., A expects to run a real trade surplus.

5. Fixed and Flexible Rate Regimes without Capital Restrictions

Under the fixed rate regime, removal of the capital restriction does not alter the stationary equilibrium. The reason is that there is essentially one currency, because the central banks guarantee that any two currencies trade at the fixed exchange rate. There is no additional advantage for agents to hold more than one currency. The fixed rate regime without capital restriction is indistinguishable from the fixed rate regime with capital restriction.

Under the flexible rate regime, the removal of capital restriction has important consequences. In the stationary equilibrium, the exchange rate must be the same across states of nature. The argument goes as follows. Suppose

S_1 and S_2 are the two exchange rates under states "one" and "two" respectively, and $S_1 > S_2$. Let today's state be "one", i.e., the current exchange rate is S_1 . Everyone knows tomorrow's state is either "one" or "two". Tomorrow's exchange rate is either the same as today's (i.e. S_1) or it appreciates to S_2 . Therefore everyone wants to hold A-currency today. No one wants to hold B-currency today. We can construct a similar argument if $S_1 < S_2$. Hence in equilibrium, $S_1 = S_2$.⁴

This result is similar to the findings in Karekan and Wallace [1978]. They show that in a two country model with no randomness and no growth in the two money supplies, the flexible rate regimes (with or without capital restrictions) are indistinguishable from the fixed rate regimes. When we introduce randomness in endowments in our model, the flexible rate regime with capital restriction has a random exchange rate distinguishable from the fixed rate regimes. However, the flexible rate regime without capital restriction remains indistinguishable from the fixed rate regimes.

6. Comparison of Exchange Rate Regimes

For simplicity, we use the term "regime with random exchange rates" to denote the flexible rate regime with capital restrictions, and the term "regime with constant exchange rates" to denote the flexible rate regime without capital restrictions, as well as the fixed rate regimes with or without capital restrictions.

First we examine the distribution of consumption and prices under the different exchange rate regimes. In Section 4, we have already shown that the mean of consumption in A (B) is lower (higher) in the regime with random exchange rates. The reader can also verify that the variances of consumption and prices in A (B) are lower (higher) in the regime with random exchange

rates. These results agree with those in Fischer [1977].

Second we compare welfare between the different regimes. In general, welfare comparisons are carried out using a Pareto criterion. An allocation is optimal whenever there is no other reallocation which can benefit at least one generation but does not harm any other generation. (See Karekan and Wallace [1978].) The Pareto criterion, however, is not suitable in this paper, because there is a missing insurance market. A-young would like to buy insurance on their second period consumption. But this market is not open. Some allocations which may be Pareto optimal are not feasible, given the structure of incomplete markets. In this paper, we choose to rank welfare by comparing the expected lifetime utility of an unborn individual, $W^i(\phi)$, across different exchange rate regimes. For example A-residents prefer the regime with random exchange rates if $W^a(\phi^a) > W^a(\phi^*)$. B-residents prefer the same regime if $W^b(1) > W^b(\phi^*)$, since $\phi^b = 1$.

As noted in Section 2, $W^i(\phi)$ depends on the degree of relative risk aversion, R , of the second period utility V . The various cases are exhibited in Figure 1. In the upper panel, we plot $W^b(\phi)$ as a function of ϕ . In the lower panel, we plot $W^a(\phi)$ as a function of ϕ . We can summarize these figures as follows. (The proofs are outlined in Appendix 2.)

Case 1. R is between 0 and 1 everywhere. Agents in both countries are not very risk averse. We perform the welfare analysis by starting in the regime with random exchange rates, and then moving to the regime with constant exchange rates. Under the former regime, A-residents bear all the risk and B-residents bear none. If there had been an insurance market, A-residents would like to reduce the variance of consumption by offering some expected consumption. B-residents would like to increase the variance of consumption by accepting some expected consumption. The insurance market would have

provided the appropriate incentives so that both sides would benefit in terms of expected utility. Since this market does not exist, the question of interest is whether the regime with constant exchange rates can duplicate the optimal allocation. The answer turns out to be negative.

B-residents are unambiguously better off under the regime with constant exchange rate. They are willing to accept some variance in consumption, so long as they are compensated by an increase in expected level of consumption. This is indicated by the graph of $W^b(\phi)$, which reaches a global minimum at $\phi = \phi^b = 1$.

A-residents have an ambiguous choice. They are willing to accept a lower expected level of consumption, only if they can reduce the variance of consumption by a significant amount, because they are not very risk averse. This is indicated by the graph of $w^a(\phi)$, which reaches a global minimum at a point ψ between 1 and ϕ^a . We can show that $w^a(1) < w^a(\phi^a)$. But we cannot unambiguously rank $w^a(\phi^*)$ and $w^a(\phi^a)$. We know that ϕ^* is between 1 and ϕ^a . If A is large relative to B, ϕ^* is close to ϕ^a . By moving to the regime with constant exchange rates, A-residents are giving up expected consumption in return for a small reduction in variance of consumption, and so they are worse off. If A is small relative to B, ϕ^* is close to $\phi^b = 1$. By moving to the regime with constant exchange risk, A-residents are giving up expected consumption in return for a large reduction in variance, and so they are better off.

Case 2. R is equal to 1 everywhere. This is a very special case. The second period utility, $V(\cdot)$, is logarithmic. This function is the separation point between utility functions which are highly risk averse and those which are not very risk averse. It turns out that agents are indifferent between the various regimes. This is indicated by the constant graphs of

$W^a(\phi)$ and $W^b(\phi)$.

Case 3. R is greater than 1 everywhere. Agents in both countries are highly risk averse. We again perform the welfare analysis by starting in the regime with constant exchange rates. Under the former regime, B-residents bear no risk. When we move to the latter regime, B-residents have a larger variance of consumption as well as a higher expected level of consumption. They are however, worse off because the increase in expected consumption does not compensate them adequately since they are highly risk averse. This is indicated by the graph of $W^b(\phi)$, which reaches a global maximum at $\phi = \phi^b = 1$. Under the former regime, A-residents bear all the risk. When we move to the latter regime, A-residents have a lower variance of consumption as well as a lower expected level of consumption. They are unambiguously better off, because the decrease in expected consumption is more than adequately compensated by the decrease in variance of consumption, since they are highly risk averse. This is indicated by the graph of $W^a(\phi)$, which reaches a global maximum at a point ψ between 1 and ϕ^a . We know that $W^a(1) > W^a(\phi^a)$, and so $W^a(\phi^*) > W^a(\phi^a)$.

Thus far, our analysis has assumed that preferences are identical across countries. All results will obtain, when we allow preferences to differ across countries but remain identical within each country. Table 1 gives the welfare rankings for various assumptions about preferences. R^a and R^b are the relative risk aversion parameters for the second period utilities of A- and B-residents respectively. Each row represents the three cases:

$R^a > 1$, $R^a = 1$, $R^a < 1$. Each column represents also three cases: $R^b > 1$, $R^b = 1$, $R^b < 1$. The welfare rankings are given by an ordered pair, the first (second) element indicating the preference in A (B). A "+" ("-") means the country prefers the regime with constant (random) exchange rates. A "0" means

the country is indifferent between exchange rates. In the case where $R^a < 1$, there is some ambiguity between the exchange rate regimes for A-residents, as indicated by a "?" This ambiguity can be removed if A is large relative to B (in row "**"), or if A is small relative to B (in row "***").)

We can see in Table 1 that there are cases in which both countries agree on the same regime. For example when $R^a > 1$ and $R^b < 1$ (i.e. the upper right hand corner), both countries prefer the regime with constant exchange rate. When $R^a < 1$ and $R^b > 1$, and when A is large, both countries prefer the regime with random exchange rates. There are also cases in which the two countries disagree -- when $R^a > 1$ and $R^b < 1$, or when $R^a < 1$ and $R^b > 1$, with A large compared to B.

Our results are slightly different from those in Lapan and Enders [1980]. They also use an overlapping generations model, but in a one country setting. Not all of their results extend to the two-country setting. For example, Lapan and Enders find that a small country with a real internal disturbance will always prefer the regimes with constant exchange rates. This result does not carry over to the two-country model, and may be reversed if the country is actually large. (For example, see Table 1, row "**"). This discrepancy between the results in the one- and two-country models has an intuitive explanation. Take the case of country A being small relative to B. In the limit, actions of country A does not affect prices in B. In this case, A prefers a constant price to a random price of goods. However, this is not true when A is a large country, since its action does affect prices in B. The comparison between the two regimes involves both the mean and the variance of consumption, and so depends on the degree of relative risk aversion.

Helpman and Razin [1979] point out that capital restriction biases the welfare comparison of flexible versus fixed rate regimes in favor of the latter. They argue that in a fixed rate regime, central bank borrowings and lendings effectively allow residents to circumvent the capital restriction. In a flexible rate regime, the central bank does not act, and so residents cannot circumvent the capital restriction. We do not dispute their arguments. But we show in our model that there are still circumstances in which both countries prefer the flexible rate regime with capital restriction to the fixed rate regime with or without capital restriction (e.g. $R^b > 1$, $R^a < 1$, and country A is large.)

7. Concluding Remarks

In our model, there is a missing insurance market. A-residents would like to buy insurance for their second period consumption, but they cannot do so. In the regime with random exchange rates A-residents bear all the risk, and B residents none. In the regimes with constant exchange rates, both A- and B-residents bear some risk. However, the amount of risk bearing is not optimally allocated. We show that the ability to share risk across countries in the latter case does not necessarily improve welfare over the inability to share risk across countries in the former case. This is a standard result in a second best world, and is likely to be robust to modifications of the model as long as the insurance market remains missing in all regimes. This is the main message of the paper.

There is one caveat which we must point out. In our model, the flexible rate regime with capital restriction completely isolates country B from the random disturbances in country A. This comes from the one-good model, and is not a general result. If there were more than one good, then random terms of trade changes will be transmitted across countries, regardless of the exchange rate regime, so long as trade is allowed between countries.

between 1 and ϕ^a such that $\psi m_1^a(\psi) = m_2^a(\psi)/\psi$. Hence $g_a'(\psi) = 0$. Suppose $\phi < \psi$. Then $\phi m_1^a(\phi) < m_2^a(\phi)/\phi$, by Lemma 3. Suppose $R > 1$. Then $\phi m_1^a(\phi) V'(\phi m_1^a(\phi)) > [m_2^a(\phi)/\phi] V'(m_2^a(\phi)/\phi)$, by Lemma 4. So, $g_a'(\phi) > 0$ for $\phi < \tilde{\phi}$. Similarly, $g_a'(\phi) < 0$ for $\phi > \tilde{\phi}$. This means $E[w^a(\phi)]$ is decreasing over $(0, \psi)$, reaching a minimum at $\phi = \psi$, and increasing over (ψ, ∞) . The other cases for A are shown analogously.

Appendix 1

Lemma 1. $\phi m_1^b(\phi) \leq m_2^b(\phi)/\phi$ if and only if $\phi \leq 1$.

Proof. Define $h(\phi) = \phi m_1^b(\phi) - m_2^b(\phi)/\phi$. Dropping the superscripts to save notation, we have:

$$h' = \left[1 + \phi \frac{m_1'}{m_1} \right] m_1 + \left[1 - \phi \frac{m_2'}{m_2} \right] \left[\frac{m_2}{\phi^2} \right]$$

It is easy to show that $\left[1 + \phi m_1'/m_1 \right] > 0$, and $\left[1 - \phi m_2'/m_2 \right] > 0$. Thus, $h' > 0$. Now $h(1) = 0$. Thus $h(\phi) > 0$ for $\phi > 1$, $h(\phi) < 0$ for $\phi < 1$. Q.E.D.

Corollary 1. $\phi m_1^b(\phi) > m_2^b(\phi)$ when $\phi > 1$.

Lemma 2. $\phi m_1^a(\phi) \leq m_2^a(\phi)/\phi$ if and only if $\phi \leq \psi$, for some ψ in $(1, \phi^a)$.

Proof. Define $h(\phi) = \phi m_1^a(\phi) - m_2^a(\phi)/\phi$. Dropping superscripts to save notation, we have:

$$h' = \left[1 + \phi m_1^a/m_1 \right] m_1 + \left[1 - \phi m_2^a/m_2 \right] \left[m_2/\phi^2 \right]$$

Clearly, $h' > 0$, for the same reasons as in Lemma 1. Note that $h(1) < 0$, since $m_1^a(1) < m_2^a(1)$.

Also $h(\phi^a) > 0$, since $\phi^a m_1^a(\phi^a) = m_2^a(\phi^a) > m_2^a(\phi^a)/\phi^a$. Therefore, there exists a unique ψ between 1 and ϕ^a , such that $h(\psi) = 0$.

THEOREM 1. $1 < \phi^* < \phi^a$.

Proof: Define

$$f(\phi) = \frac{N m_2^a(\phi) + N m_2^b(\phi)}{N m_1^a(\phi) + N m_1^b(\phi)}$$

Clearly, $f(1) > 1$, since $m_1^a(1) > m_1^b(1)$, and $m_2^b(1) = m_1^b(1)$. Also,

$f(\phi^a) < \phi^a$. This is shown as follows: $m_2^a(\phi^a) = \phi^a m_1^a(\phi^a)$. Hence $f(\phi^a) < \phi^a$ if and only if $m_2^b(\phi^a) < \phi^a m_1^b(\phi^a)$, which is true by Corollary 1 (since $\phi^a > 1$).

By continuity, there exists ϕ^* such that

$$\phi^* = f(\phi^*), \text{ and } \phi^b = 1 < \phi^* < \phi^a.$$

Corollary 2: $\mu_2^a = m_2^a(\phi^*) > \phi^* m_1^a(\phi^*) = \phi^* \mu_1^a$.

Proof: $\phi^* [N^a \mu_1^a + N^b \mu_1^b] = N^a \mu_2^a + N^b \mu_2^b$. So $N^b [\phi^* \mu_1^b - \mu_2^b] = N^a [\mu_2^a - \phi^* \mu_1^a]$.

Since $\phi^* > 1$, we know $\phi^* \mu_1^b > \mu_2^b$, from Corollary 1. Hence $\mu_2^a > \phi^* \mu_1^a$.

Appendix 2

Lemma 3: Let $f(x) = xV'(x)$, where $V(\cdot)$ is increasing and concave.

Then $f'(x) > 0$ if $R(x) < 1$, $f'(x) = 0$ if $R(x) = 1$, and $f'(x) < 0$ if $R(x) > 1$.

Proof: $f'(x) = V'(x) + xV''(x) = V'(x) \left[1 + \frac{xV''(x)}{V'(x)} \right] = V'(x) [1 - R(x)]$. Q.E.D.

Now, define

$$\begin{aligned} g_i(\phi) = E[w^i(\phi)] &= qw_1^i(\phi) + (1-q)w_2^i(\phi) = qU(w_1^i - m_1^i(\phi)) + (1-q)U(w_2^i - m_2^i(\phi)) \\ &+ q^2V(m_1^i(\phi)) + q(1-q)V(\phi m_1^i(\phi)) + q(1-q)V(m_2^i(\phi)/\phi) \\ &+ (1-q)^2V(m_2^i(\phi)), \quad \text{for } i = a, b. \end{aligned}$$

Note that $g_i'(\phi) = q(1-q)\{\phi m_1^i(\phi)V'(\phi m_1^i(\phi)) - [m_2^i(\phi)/\phi]V'(m_2^i(\phi)/\phi)\}$.

Consider the case for $i = b$. At $\phi = 1$, $g_b'(1) = 0$. Suppose $\phi < 1$. Then

$\phi m_1^b(\phi) < m_2^b(\phi)/\phi$, by Lemma 2. Suppose $R < 1$. Then $\phi m_1^b(\phi)V'(\phi m_1^b(\phi))$

$< [m_2^b(\phi)/\phi]V'(m_2^b(\phi)/\phi)$, by Lemma 4. In other words, $g_b'(\phi) < 0$ for $\phi < 1$.

Similarly, $g_b'(\phi) > 0$ for $\phi > 1$. This means that $E[w^b(\phi)]$ is increasing

over $(0, 1)$, and decreasing over $(1, \infty)$, reaching a maximum at $\phi = 1$.

The other cases for B are shown analogously.

Now consider the case for $i = a$. By Lemma 2 we know there exists ψ

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Footnotes

1. There are no terms of trade effects in the model.
2. ϕ^i is a measure of uncertainty of prices. A rise in ϕ^i increases the mean and variances of prices.
3. We assume that there are no barriers to trade, i.e., no transport cost, tariffs, quotas, etc. Therefore goods arbitrage ensures the validity of the "law of one price."
4. The same argument holds for any finite number of states of nature.

Table 1

Welfare Comparison Between Exchange Regimes

Degree of Relative Risk Aversion in

Country B

Degree of Relative Risk Aversion in <u>Country A</u>		<u>Country B</u>		
		$R^b > 1$	$R^b = 1$	$R^b < 1$
$R^a > 1$		(+, -)	(+, 0)	(+, +)
$R^a = 1$		(0, -)	(0, 0)	(0, +)
$R^a < 1$		(?, -)	(?, 0)	(?, +)
	*	(-, -)	(-, 0)	(-, +)
	**	(+, -)	(+, 0)	(+, +)

The first entry of the ordered pair pertains to country A, the second entry pertains to B. The following conventions are used:

+: prefer regime with constant exchange rates

0: indifferent between the two regimes

-: prefer regime with random exchange rates

?: ambiguous, depending on the size of A relative to B

Note: * $R^a < 1$ and A is large relative to B.

** $R^a < 1$ and A is small relative to B.

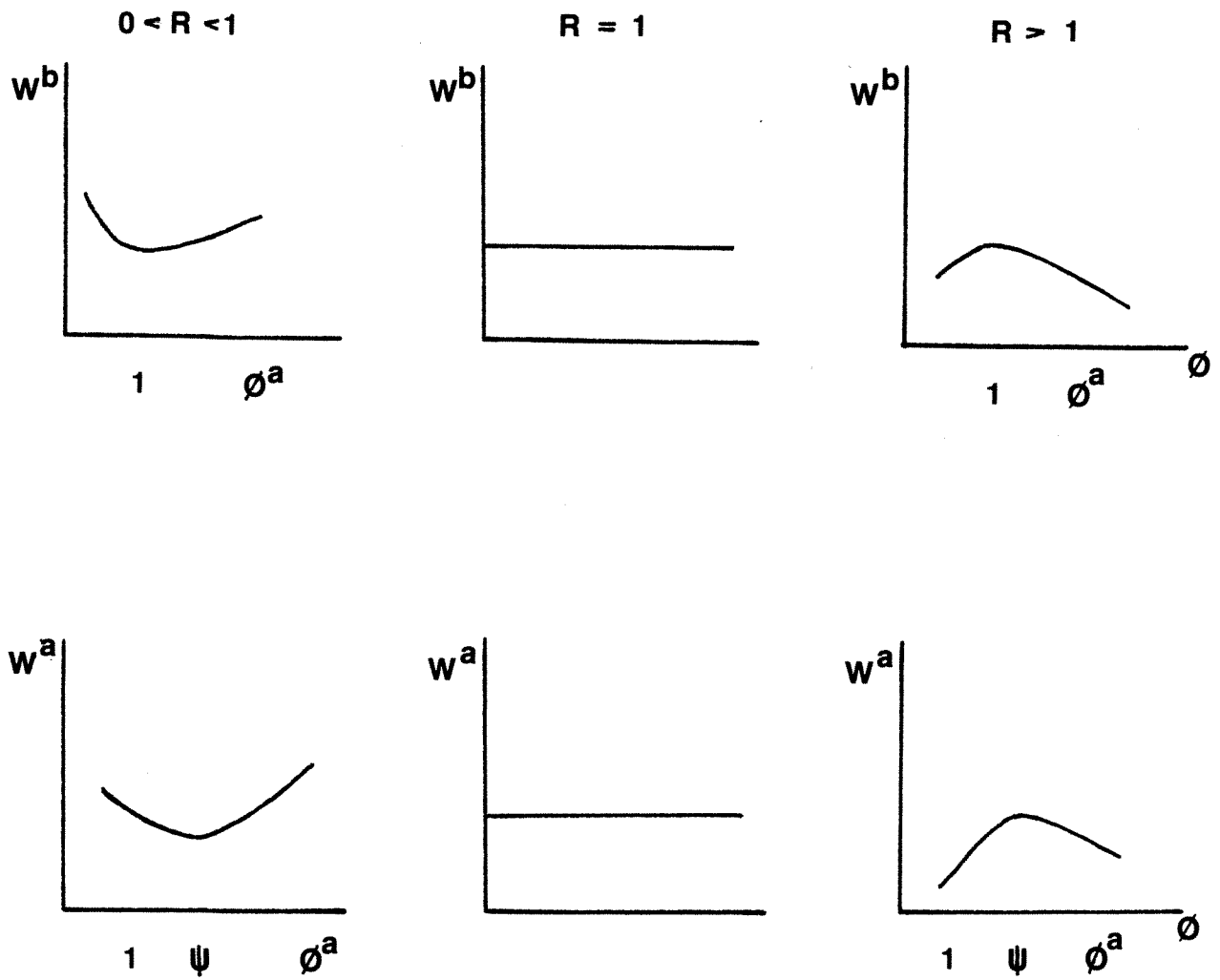


Figure 1
 Expected Lifetime Utility Function

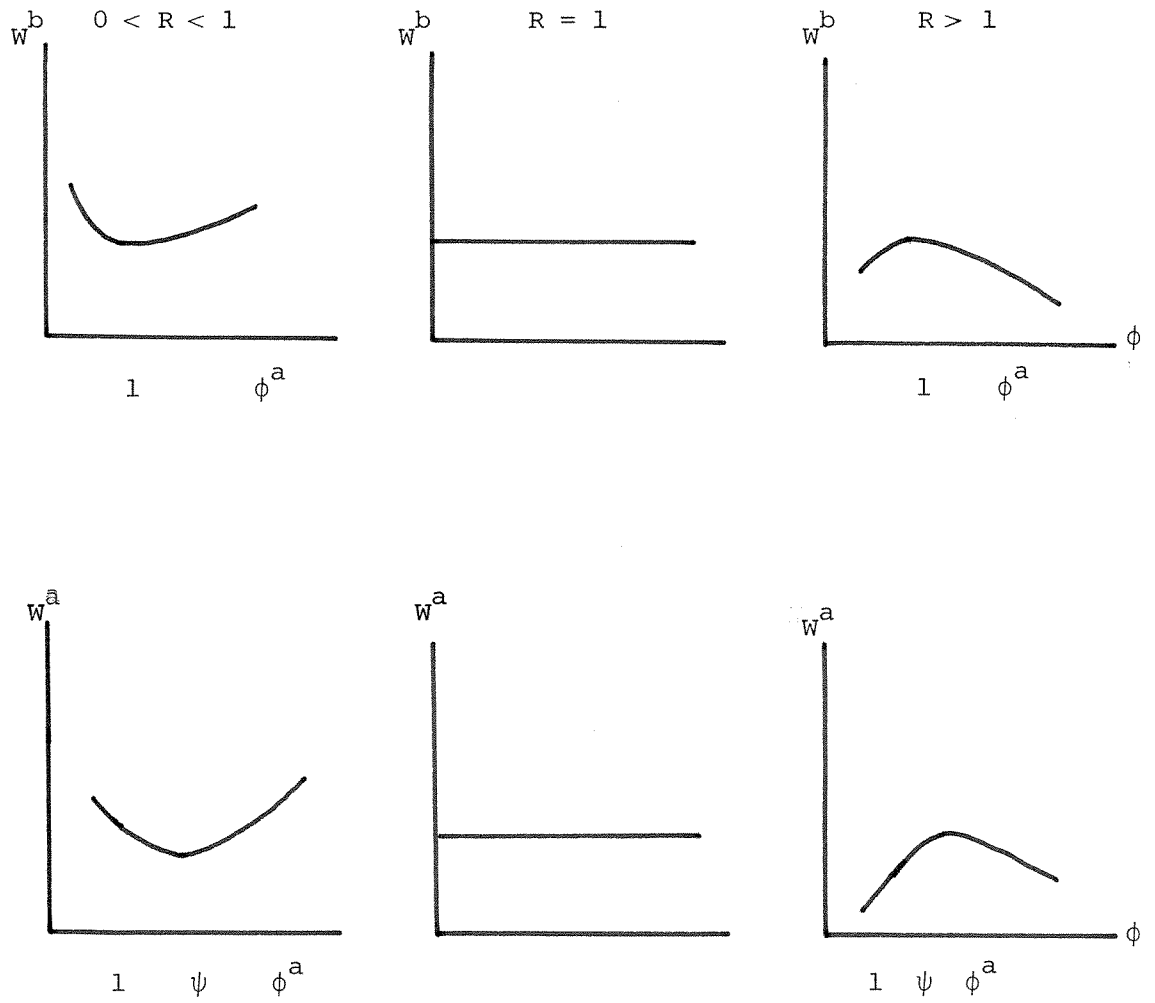


Figure 1

Expected Lifetime Utility Function