Preliminary

Implications of Observed Properties of Daily Exchange Rate Movements

by

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Abstract

This paper discusses the implications of observed statistical properties of daily changes in foreign exchange rates. These properties are: negligible autocorrelations, fat tails, conditional heteroskedasticity, changing distributions, and nonlinearity. In the area of international economics, nonlinear models of exchange rates are needed to explain this short term phenomenon. In the area of international finance, mean-variance portfolios and standard option pricing models are not applicable. In the area of statistics and econometrics, maximum likelihood methods are not appropriate when they assume that exchange rate movements are independent and identically distributed. Exchange rate regressions should allow for conditional heteroskedasticity in the residuals. Also, event studies cannot assume homoskedasticity in calculating test statistics; rather, a bootstrap must be done to calculate its statistical significance.

1. Introduction

This paper discusses the implications of observed statistical properties of daily changes in foreign exchange rates. To fix the notation, let S_t denote the home currency price of one unit of a foreign currency at time t. Define $s_t = \log S_t$, where "log" means the natural logarithm, and $r_t = s_t - s_{t-1}$, the daily (continuously compounded) rate of change of the exchange rate.

The consensus in the literature is that r_t has little autocorrelation.[1] The presumption is that $s_t = \log S_t$ follows a random walk, which is equivalent to the restriction that r_t is independent and identically distributed (IID). This has spawned a number of papers. In international economics, models of exchange rates use rational expectations to explain the random walk nature of exchange rates. In international finance, a continuous time diffusion process (consistent with the discrete time random walk behavior) is used for mean-variance analysis of portfolio of international assets and the pricing of foreign currency options. In the econometrics, maximum likelihood methods have been used to estimate parameters of the distribution of exchange rates.

Recent evidence has cast doubt about the random walk hypothesis. Friedman and Vandersteel (1982) showed that the means and variances of r_t appear to change over time. Manas-Anton (1986) confirmed the violation of IID using the fact that both $|r_t|$ and r_t^2 exhibit substantial autocorrelation, while Hsieh (1989a) discovered other evidence of nonlinearity in r_t^2. To date, a few nonlinear models have been fit to the data; in particular, Bollerslev (1987), Milhøj (1987), Diebold (1988), and Hsieh (1989b) used Engle's (1982) autoregressive conditional heteroskedasticity (ARCH) model and Bollerslev's (1986) generalized ARCH (GARCH) model.

These observed properties of exchange rates forces some revisions of old theories. Section 2 deals with international economics. Section 3 deals with international finance. Section 4 deals with statistics and econometrics. A summary and conclusions are given in Section 5.

2. Implications for International Economics

The lack of autocorrelation in r_t has led to the conclusion that S_t follows a random walk. This spawned a number of economic models to explain this behavior. A typical monetary model is as follows:

$$(2.1) \qquad s_t = p_t - p_t^* + \epsilon_{1t}$$

$$(2.2) \qquad m_t - p_t = \alpha y_t - \beta i_t + \epsilon_{2t}$$

$$(2.3) \qquad m_t^* - p_t^* = \alpha y_t^* - \beta i_t^* + \epsilon_{2t}^*$$

$$(2.4) \qquad i_t - i_t^* = s_{t+1|t} - s_t$$

where s_t is the logarithm of S_t, p_t the logarithm of the domestic price level, m_t the logarithm of the domestic money supply, y_t the logarithm of the domestic output, i_t the domestic interest rate, and $s_{t+1|t}$ is the expected value of s_{t+1} given available information at time t. The corresponding foreign variables are starred, and ϵ_{1t}, ϵ_{2t}, and ϵ_{2t}^* are random shocks. The first equation is purchasing power parity, assumed to hold on average. The second and third equations describe money market equilibrium at home and abroad. The fourth equation is interest parity. One can then rearrangement terms to obtain the following:

$$(2.5) \qquad (1+\beta)s_t - \beta s_{t+1|t} = z_t + \epsilon_t,$$

where $z_t = [m_t - m_t^*] - \alpha[y_t - y_t^*]$ and $\epsilon_t = \epsilon_{1t} + \epsilon_{2t} - \epsilon_{2t}^*$.

Typically, m_t, m_t^*, y_t, and y_t^* (and hence z_t) are assumed to be independent of s_t, and ϵ_t is white noise. If expectations are rational in the sense of Muth (1960), a solution for s_t is obtained for a specific stochastic process of z_t. A general solution for s_t is:

$$(2.6) \qquad s_t = (1/[1+\beta]) \frac{1}{1+\beta} \Sigma_{i=0}^{\infty} \lambda^i z_{t+i|t} + \epsilon_t,$$

where $\lambda = \beta/[1+\beta] \frac{\beta}{1+\beta}$ and $z_{t+i|t}$ is the expection of z_{t+i} given information at time t. In the simplest case, if z_t is a random walk, $r_t = s_t - s_{t-1}$ is a moving average of order one. More generally, if z_t has a linear time series representation, so does r_t. Therefore, nonlinearity in the forcing variables $z_t = [m_t - m_t^*] - \alpha[y_t - y_t^*]$ (or in the disturbances ϵ_t) is required to explain the observed nonlinear behavior of r_t, such as conditional heteroskedasticity.

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Recent models of exchange rates have picked up this implication. Lai and Pauly (1988) showed that a model of Bayesian learning can lead to an autoregressive-moving average model with ARCH variances for r_t. Hsieh (1989c) constructed a central bank intervention rule which yields a stochastic and endogenous switching model for r_t. These models represent a step in the right direction to explain short term exchange rate movements, but they are still too simplistic to fully account for the observed behavior of exchange rate movements. More research is needed in this area.

3. Implications for International Finance

The following continuous time diffusion models of exchange rates has been used frequently in international finance:

(3.1) $dS/S = \mu dt + \sigma dz,$

where dz is a standard Weiner process. This model is consistent with the discrete time observation that exchange rates follow a random walk, i.e., r_t is IID. In addtion, (3.1) imposes the restriction that r_t is normally distributed. This contradicts the observed statistical property of daily exchange rate movements in two ways: r_t is neither normally distributed[2] nor IID. Since a number of models make use of the above Weiner process, their results must be viewed with suspicion.

One, the mean-variance approach to forming portfolios of international assets may not be applicable on a daily frequency. In a mean-variance framework, the returns of assets are assumed to have a normal distribution, which is completely described by a vector of means and a matrix of covariances. A given mix of assets will yield a portfolio mean and variance. Holding fixed the portfolio mean, an asset mix is found to minimize the portfolio variance. This traces out a mean-variance frontier for the portfolio return, and the investor can select a point on this frontier. In actual implementation, time series data are used to calculate sample means and sample covariances.

For portfolios of international assets, exchange rate fluctuations are inevitable. This arises because foreign assets and their payoffs are usually denominated in foreign currencies, and so exchange rates are needed to convert them into domestic currency. As exchange rates are highly volatile, their behavior can dominate the returns on foreign assets. This means that two basic assumptions of the mean-variance framework would not hold for portfolios of interational assets. Firstly, the observed leptokurtosis of exchange rates violates the normality assumption. Secondly, even if exchange rate changes are normally distributed, their means and variances change over time, which implies that the mean-variance frontier shifts from day to day. A complete description of these shifts requires elaborate models of the time-variation of means and variances. The simple minded rule of calculating sample means and covariances cannot work in this case.

Two, foreign currency option pricing models have relied on the continuous time diffusion model in (3.1).[3] In fact, Hsieh and Manas-Anton (1989) found systematic differences between the theoretical and observed prices for options on the Deutsche Mark futures contracts traded in the Chicago Mercantile Exchange. They conclude that these differences are consistent with an ARCH model of exchange rates. Recently, Hull and White (1987) have priced options when the underlying assets exhibit stochastic volatility. More work is definitely needed in this direction.

4. Implications for Econometric and Statistical Anslysis

The observed properties of exchange rates affect at least three important aspects of econometric and statistical analysis of exchange rate data. One, maximum likelihood methods are routinely used to estimate unknown parameters of the distribution of r_t.[4] Typically the researcher postulates a log likelihood function for r_t:

(4.1) $\ell(r_t;\theta)$,

where θ is a vector of unknown parameters. The maximum likelihood procedure assumes r_t to be IID, and finds the value of θ which maximizes the log likelihood for the entire sample:

(4.2) $\ell(\theta) = \Sigma_t \, \ell(r_t;\theta)$.

As a general rule, any violation of the assumptions of maximum likelihood will likely result in misspecification. The assumption of IID clearly does not hold in the daily (or even weekly) data, since there is strong evidence that the distribution of r_t changes over time in a systematic fashion.[5] So it is likely that model is misspecified.

One can still use a misspecified model in the context of the pseudo maximimum likelihood method of Gourieroux, Monfort, and Trognon (1984). However, it would clearly be better if misspecification can be eliminated. To do so, we must completely specify the complete nonlinear model of exchange rates. Suppose r_t follows a GARCH process, such that conditional on past information, r_t has mean zero and variance h_t:

(4.3) $$h_t = \beta_0 + \Sigma_i \beta_i r_{t-i}^2 + \Sigma_j \gamma_j h_{t-j}^2.$$

Let f( ) be the conditional density of $r_t/h_t^{\frac{1}{2}}$. The log likelihood can then be written as:

(4.4) $$\ell(r_t;\theta) = - \tfrac{1}{2} \log h_t - \log[\ f(r_t/h_t^{\frac{1}{2}})\ ],$$

where $\theta = [\beta_0\ \ \beta_1\ \ ...\ \gamma_1\ ...\ ]$. If this is the correct model, maximum likelihood estimation of the θ is appropriate.[6] Of course, (4.4) can be misspecified also. We seldom know the true model of exchange rate movements. Therefore diagnostics must be diligently carried out to check that the postulated model is not violated by the data.

Two, regressions of r_t on other variables must allow for conditional heteroskedasticity in the residuals. Consider the regression:

(4.5) $$r_t = x_t'\beta + \epsilon_t, \quad t = 1,...,T.$$

Write (4.5) more compactly as:

(4.6) $$y = X'\beta + \epsilon,$$

where $y=[r_t]$ is the vector of r_t, $X=[x_t']$ the matrix of regressors, β a vector of coefficients, and ϵ a vector of residuals. The standard way to calculate the covariance matrix of the least squares estimates of β is:

(4.7) $$V_{OLS} = s^2\,[X'X]^{-1},$$

where $s^2 = u'u/T$, u being the vector of regression residuals of (4.6).

Since y is known to be heteroskedastic, it is likely that ϵ is also heteroskedastic. In this case, the covariance matrix in (4.7) is incorrect, and may lead to incorrect inference. Instead, a heteroskasticity-consistent covariance matrix[7] should be used:

$$(4.8) \qquad V_{HC} = [X'X]^{-1} [\Sigma u_t^2 x_t x_t'][X'X]^{-1}.$$

This point has already been picked up in the literature.[8] But most researchers have not paid enough attention to this matter.

Three, statistical methods for event studies must be modified. Generally, an event study attempts to detect a change in the mean of exchange rates in response to some news event. Suppose the news event happened on date s. Let r_{s-i}, i=0,1,...,N, be the observed movements in exchange rates during the N days prior to the news event, and r_{s+i}, i=1,...,N, are the N days afterwards. If the news event has any impact on the mean of r_t, r_{s-i} and r_{s+i} will have different means. A test of this hypothesis is typically conducted by making the auxiliary assumption that the variances of r_{s-i} and r_{s+i} are the same. Heteroskedasticity of exchange rates may lead to an erroneous inference.

In order to deal with this problem, a methodology which is not sensitive to departures from homoskedasticity must be adopted. For example, a bootstrap can be used to generate an empirical distribution of the test statistic. Here is a simple suggestion. Draw randomly an r_t from the set of all available dates, whether a news event occurred or not. Compute the test statistic using the exchange rates N days before and N days after. Repeat this, say, 1000 times, to build up an empirical distribution for the test statistic, which can then be used to determine whether the original test statistic is a rare event or not.

5. Summary and Conclusion

This paper has reviewed the current literature which shows that exchange rate movements are not independent and identically distributed. Instead means and variances appear to be change over time. Conditional heteroskedasticity is perhaps the best characterization of the nonlinearity in the data. There are a number of implications. In the area of international economics, short term models for exchange rate must be developed to account for this nonlinearity. In the area of international finance, mean-variance portfolio selection rules and option pricing models are not applicable. In the area of econometric and statistical analysis, maximum likelihood estimation which assumes that r_t is IID can lead to misspecified models. Regressions of r_t should allow for heteroskedasticity in the residuals. And event studies must also use statistical procedures which take into account heteroskedasticity, perhaps by bootstraping the empirical distribution of any test statistic.

References

Biger N., and J. Hull (1983), "The Valuation of Currency Options," Financial Management, 12: 24-28.

Black, F. (1976), "The Pricing of Commodity Contracts," Journal of Financial Economics, 3: 167-179.

Black, F., and M. Scholes (1972), "The Valuation of Option Contracts and a Test of Market Efficiency," Journal of Finance, 27: 399-417.

Black, F., and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81: 637-654.

Bollerslev, T. (1986), "Generalized Autogressive Conditional Heteroskedasticity," Journal of Econometrics, 31: 307-327.

Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," Review of Economics and Statistics 9: 542-547.

Boothe, P. and D. Glassman (1987), "The Statistical Distribution of Exchange Rates: Empirical Evidence and Economic Implications," Journal of International Economics, 22: 297-319.

Borenstein, E.R., and M. P. Dooley (1987), "Options on Foreign Exchange and Exchange Rate Expectations," unpublished manuscript, International Monetary Fund.

Burt, J., F.R. Kaen and G.G. Booth (1977), "Foreign Market Efficiency Under Flexible Exchange Rates," Journal of Finance 32: 1325-1330.

Cornell, B. (1977), "Spot Rates, Forward Rates, and Market Efficiency," Journal of Financial Economics 5: 55-65.

Cumby, R.E., and M. Obstfeld (1984), "International Interest Rate and Price Level Linkages under Flexible Exchange Rates: A Review of the Evidence," in J.F.O. Bilson and R.C. Marston, eds., Exchange Rate Theory and Practice, Chicago: University of Chicago Press.

Diebold, F.X. (1988), Empirical Modeling of Exchange Rate Dynamics, New York: Springer-Verlag.

Domowitz, I. and C.S. Hakkio (1985), "Conditional Variance and the Risk Premium in the Foreign Exchange Market," Journal of International Economics 19: 47-66.

Engle, R.F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica 50: 987-1007.

Friedman, D. and S. Vandersteel (1982), "Short-Run Fluctuations in Foreign Exchange Rates," Journal of International Economics 13: 171-186.

Garman, M.B., and S.W. Kohlhagen (1983), "Foreign Currency Option Values," Journal of International Money and Finance, 2: 231-238.

Giddy, I. (1983), "Foreign Exchange Options," Journal of Futures Markets, 3: 143-166.

Giddy, I. and G. Dufey (1975), "The Random Behavior of Flexible Exchange Rates: Implications for Forecasting," Journal of International Business Studies 6: 1-32.

Gourieroux, C., A. Monfort, and A. Trognon (1984), "Pseudo Maximum Likelihood Methods: Theory," Econometrics 52: 681-700.

Grabbe, J.O. (1983), "The Pricing of Call and Put Options on Foreign Exchange," Journal of International Money and Finance, 2: 239-254.

Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moment Estimators," Econometrica 50: 1029-1054.

Hsieh, D.A. (1983), "A Heteroskedasticity-Consistent Covariance Matrix Estimator for Time Series Regressions," Journal of Econometrics 22: 281-290.

Hsieh, D.A. (1988), "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," Journal of International Economics 24: 129-145.

Hsieh, D.A. (1989a), "Testing for Nonlinear Dependence in Daily Foreign Exchange Rates," Journal of Business, forthcoming.

Hsieh, D.A. (1989b), "Modeling Heteroskedasticity in Daily Foreign Exchange Rates," Journal of Business and Economic Statistics, forthcoming.

Hsieh, D.A. (1989c), "A Nonlinear Stochastic Rational Expectations Model of Exchange Rates," unpublished manuscript, University of Chicago.

Hsieh, D.A., and L.A. Manas-Anton (1989), "Empirical Regularities in the Deutsche Mark Futures Options," Advances in Futures and Options Research 3: 183-208.

Hull, J. and A. White (1987), "The Pricing of Options on Assets with Stochastic Volatilities," Journal of Finance 42: 281-300.

Lai, K.S., and P. Pauly (1988), Time Series Properties of Foreign Exchange Rates Re-examined," unpublished manuscript, University of Pennsylvania.

Logue, D.E. and R.J. Sweeney (1977), "'White-Noise' in Imperfect Markets: The Case of the Franc/Dollar Exchange Rates," Journal of Finance 32: 761-768.

Logue, D.E., R.J. Sweeney and T.D. Willett (1978), "Speculative Behavior of Foreign Exchange Rates during the Current Float," Journal of Business Research 6: 159-174.

Manas-Anton, L.A. (1986), "Empirical Behavior of Flexible Exchange Rates: Statistical Analysis and Consistent Models," unpublished dissertation, University of Chicago.

Meese, R., and K. Rogoff (1983), "Empirical Exchange Rate Models of the Seventies: Are Any Fit to Survive?" Journal of International Economics 14: 3-24.

Mussa, M. (1979), "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," Carnegie-Rochester Series on Public Policy 11: 9-58.

Mustafa, C. (1987), "Option Valuation With Integrated GARCH Processes," unpublished manuscript, University of California at San Diego.

Rogalski, R.J. and J.D. Vinso (1978), "Empirical Properties of Foreign Exchange Rates," Journal of International Business Studies 9: 69-79.

Weiss, A. (1984), "ARMA Models with ARCH Errors," Journal of Time Series Analysis 5: 129-143.

Westerfield, J.M. (1977), "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," Journal of International Economics 7: 181-200.

White, H. (1980), "A Heteroscedasticity-Consistent Covariance Estimator and a Direct Test for Heteroscedasticity," Econometrica 48: 817-838.

Footnotes

1. See Giddy and Dufey (1975), Burt, Kaen and Booth (1977), Cornell (1977), Logue and Sweeney (1977), Logue, Sweeney and Willett (1978), Rogalski and Vinso (1982), Manas-Anton (1986), and Hsieh (1988).

2. Friedman and Vandersteel (1982) found that exchange rate changes become more normally distributed as the time between observations increases. Thus, this criticism applies only to daily and weekly movements in exchange rates, but not to monthly or quarterly movements.

3. See Giddy (1983), Garman and Kohlhagen (1983), and Grabbe (1983).

4. Westerfield (1977) estimated the parameters of a symmetric stable paretian distribution for weekly changes in exchange rates. Rogalski and Vinso (1978) used a Student-t distribution. Boothe and Glassman (1988) also tried a mixture of normals.

5. See Friedman and Vandersteel (1982) and Hsieh (1988).

6. Maximum likelihood is appropriate provided certain regularity conditions are met. See Weiss (1984) for details. only for ARCH

7. See White (1980), Hansen (1982), and Hsieh (1983).

8. See Cumby and Obstfeld (1984).