Base Isolation

CEE 541. Structural Dynamics
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1 An idealized linear base isolation system shaken by random loading

Consider a damped single-degree-of-freedom (SDOF) oscillator (with mass $m_{\rm s}$, stiffness $k_{\rm s}$ and damping $c_{\rm s}$ resting on a isolation pad with mass $m_{\rm i}$, stiffness $k_{\rm i}$ and damping $c_{\rm i}$. The system is shaken by support motions $z(t)$) The coupled equations of motion are

$$m_{\rm i}(\ddot r_{\rm i}(t) + \ddot z(t)) + c_{\rm i}\dot r_{\rm i}(t) + k_{\rm i} r_{\rm i}(t) - k_{\rm s}(r_{\rm s}(t) - r_{\rm i}(t)) - c_{\rm s}(\dot r_{\rm s}(t) - \dot r_{\rm i}(t)) = 0 \quad (1)$$

$$m_{\rm s}(\ddot r_{\rm s}(t) + \ddot z(t)) + k_{\rm s}(r_{\rm s}(t) - r_{\rm i}(t)) + c_{\rm s}(\dot r_{\rm s}(t) - \dot r_{\rm i}(t)) = 0 \quad (2)$$

The responses of interest in a base isolation problem are the displacement of the isolation system $r_{\rm i}(t)$, the deformation of the structure $r_{\rm s}(t) - r_{\rm i}(t)$, and the total acceleration of the structure $\ddot r_{\rm s}(t) + \ddot z(t)$. The input base acceleration, $\ddot z(t)$ may be related to the responses of interest through the state-space model

$$\frac{d}{dt}\left[\begin{array}{c} r_{\rm i} \\ r_{\rm s} \\ \dot r_{\rm i} \\ \dot r_{\rm s} \end{array}\right] = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (k_{\rm i}+k_{\rm s})/m_{\rm i} & -k_{\rm s}/m_{\rm i} & (c_{\rm i}+c_{\rm s})/m_{\rm i} & -c_{\rm s}/m_{\rm i} \\ -k_{\rm s}/m_{\rm s} & k_{\rm s}/m_{\rm s} & -c_{\rm s}/m_{\rm s} & c_{\rm s}/m_{\rm s} \end{array}\right] \left[\begin{array}{c} r_{\rm i} \\ r_{\rm s} \\ \dot r_{\rm i} \\ \dot r_{\rm s} \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ -1 \\ -1 \end{array}\right] \ddot z \quad (3)$$

$$\left[\begin{array}{c} \ddot r_{\rm s} + \ddot z \\ r_{\rm s} - r_{\rm i} \\ r_{\rm i} \end{array}\right] = \left[\begin{array}{cccc} -k_{\rm s}/m_{\rm s} & k_{\rm s}/m_{\rm s} & -c_{\rm s}/m_{\rm s} & c_{\rm s}/m_{\rm s} \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} r_{\rm i} \\ r_{\rm s} \\ \dot r_{\rm i} \\ \dot r_{\rm s} \end{array}\right] \quad (4)$$

or, more symbolically, $\dot x_{\rm bi} = A_{\rm bi} x_{\rm bi} + B_{\rm bi}\ddot z$, and $y_{\rm bi} = C_{\rm bi} x_{\rm bi}$.

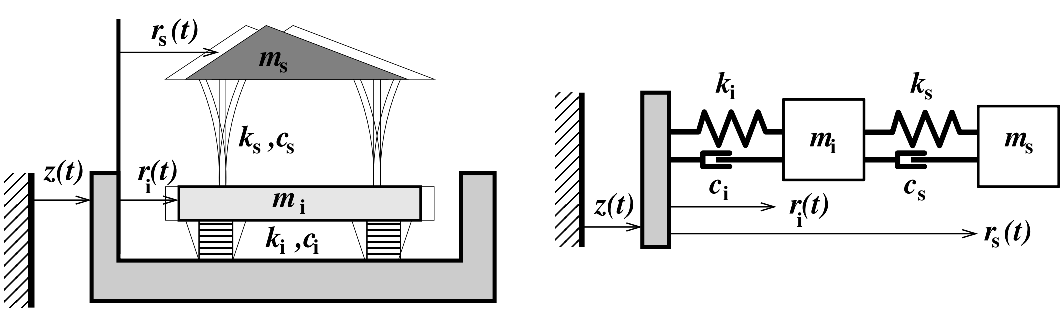

Figure 1. Displacement coordinates for a Base Isolation system

2 Ground motion model

The motion of the base is random, but not purely white noise. The base acceleration may be modeled as the output of another linear time-invariant system, forced by unit white noise, $u(t)$. The linear time-invariant system describing the relationship between the white noise process $u(t)$ and the assumed random input accelerations $\ddot{z}(t)$ is

$$\frac{d}{dt}\begin{bmatrix} r_{\rm g} \\ \dot{r}_{\rm g} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4\pi^2 f_{\rm g}^2 & -4\pi\zeta_{\rm g} f_{\rm g} \end{bmatrix}\begin{bmatrix} r_{\rm g} \\ \dot{r}_{\rm g} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma_{\ddot{z}}/\sqrt{2\pi f_{\rm g}\zeta_{\rm g}} \end{bmatrix} u(t) \quad (5)$$

$$\ddot{z} = (4\pi\zeta_{\rm g} f_{\rm g})\, \dot{r}_{\rm g} \quad (6)$$

or, more symbolically, $\dot{x}_{\rm g} = A_{\rm g}x_{\rm g} + B_{\rm g}u$, and $\ddot{z} = C_{\rm g}x_{\rm g}$. This model for the input accelerations is quantified in terms of the numerical values of three paramters: $f_{\rm g}$ is called the "ground frequency", $\zeta_{\rm g}$ is called the "ground damping ratio", and $\sigma_{\ddot{z}}$ is called the "root-mean-square ground acceleration amplitude." By changing these ground motion parameter values, ground motions corresponding to different geophysical settings may be simulated. Values for these parameters are given in the following table. The rise and decay parameters describe the increase and decrease of the ground motion over the course of the transient. The "envelope function" used to model this rise and decay is:

$$s(t) = (t/(a\tau))^a \ \exp(a - t/\tau) \ . \quad (7)$$

Table 1. Ground motion parameters for ATC-63 ground motion classes

ATC-63	root-mean-square ground acceleration $\sigma_{\ddot{z}}$	ground frequency $f_{\rm g}$	ground damping $\zeta_{\rm g}$	rise exponent a	decay time τ
Far-Field	0.6	1.5	0.9	4.0	2.0
Near-Fault, No Pulse	0.8	1.3	1.1	3.0	2.0
Near-Fault, Pulse	1.6	0.5	1.8	1.0	2.0
	m/s^2	Hz	.	.	s

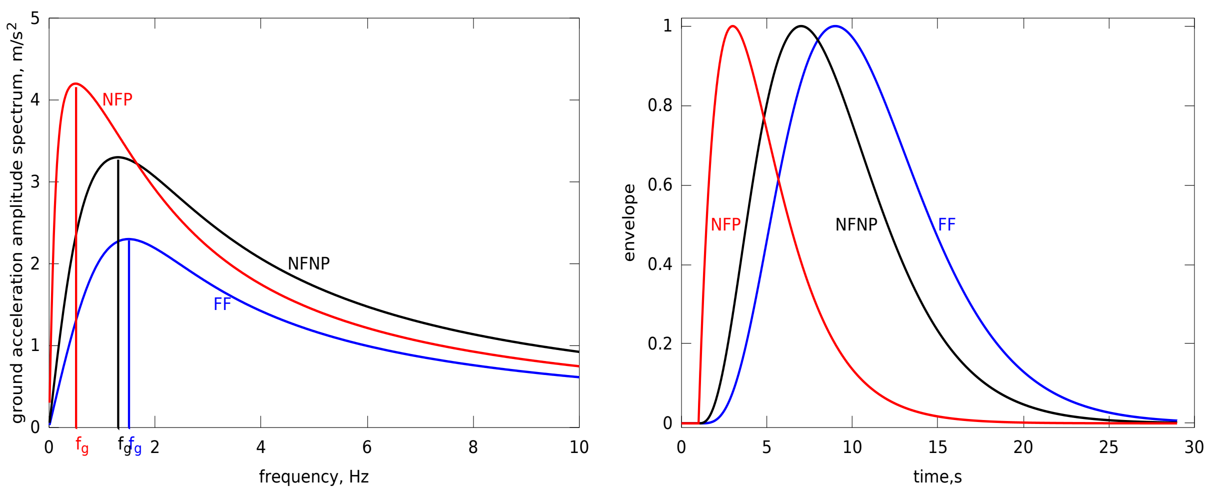

Figure 2. Power Spectral Densities and Envelope Functions for various earthquake ground motions

3 Cascaded ground motion and structural systems and r.m.s. performance

The cascade system of the ground motion model feeding into the base isolation model is

$$\frac{d}{dt}\begin{bmatrix} x_{\rm g} \\ x_{\rm bi} \end{bmatrix} = \begin{bmatrix} A_{\rm g} & 0_{2\times 4} \\ B_{\rm bi}C_{\rm g} & A_{\rm bi} \end{bmatrix}\begin{bmatrix} x_{\rm g} \\ x_{\rm bi} \end{bmatrix} + \begin{bmatrix} B_{\rm g} \\ 0_{4\times 1} \end{bmatrix} u \tag{8}$$

$$y_{\rm bi} = \begin{bmatrix} 0_{3\times 2} & C_{\rm bi} \end{bmatrix}\begin{bmatrix} x_{\rm g} \\ x_{\rm bi} \end{bmatrix} \tag{9}$$

or, more symbolically, $\dot{x}_{\rm c} = A_{\rm c}x_{\rm c} + B_{\rm c}u$, and $y = C_{\rm c}x_{\rm c}$. The mean-square responses of the cascade system are

$$\sigma_y^2 = \text{diag}\left(C_{\rm c}Q_{\rm c}C_{\rm c}^{\sf T}\right) \tag{10}$$

where $Q_{\rm c}$ is the controllability gramian which satisfies the right Liapunov equation

$$A_{\rm c}Q_{\rm c} + Q_{\rm c}A_{\rm c}^{\sf T} + B_{\rm c}B_{\rm c}^{\sf T} = 0 \tag{11}$$

4 Design optimization

Using this method to analyze the root-mean-square responses of the structural acceleration, the structural deformation and the isolation deformation for each of the three class of ground motion models, an optimization problem can be posed to determine values for:

- the isolation system period $T_{\rm i} = 2\pi\sqrt{(m_{\rm i} + m_{\rm s})/k_{\rm i}}$,
- the structural period $T_{\rm s} = 2\pi\sqrt{m_{\rm s}/k_{\rm s}}$,
- the isolation system damping $\zeta_{\rm i} = c_{\rm i}/\sqrt{2(m_{\rm i} + m_{\rm s})k_{\rm i}}$, and
- the structural damping $\zeta_{\rm s} = c_{\rm s}/\sqrt{2m_{\rm s}k_{\rm s}}$

that minimizes the root-mean-square of the structura acccleration $\sigma_{\ddot{r}_{\rm s}+\ddot{z}}$ such that:

- the root-mean-square isolator deformation $\sigma_{r_{\rm i}}$ and
- the structural deformation $\sigma_{r_{\rm s}-r_{\rm i}}$

are within prescribed bounds, and such that the design variables ($T_{\rm i}$, $T_{\rm s}$, $\zeta_{\rm i}$, and $\zeta_{\rm s}$) are bounded. Bounds used in this example are given in in the second and third columns of Table 2 Results of optimizations for the three types of ground motion are shown in columns 4, 5, and 6 of Table 2.

Figure 4 shows the amplitude spectra of the structural acceleration ($\sigma_{\ddot{r}_{\rm s}+\ddot{z}}$) and the deformations ($\sigma_{r_{\rm i}}$ and $\sigma_{r_{\rm s}-r_{\rm i}}$).

Table 2. bounds on design variables and root-mean-square deformation responses and optimized parameter values and performance metrics for Far Field, Near Field No Pulse, and Near Field Pulse ground motions

	min bound	max bound	FF	NFNP	NFP	
$T_{\rm i}$	1.0	3.0	3.0	3.0	3.0	s
$T_{\rm s}$	0.5	1.0	1.14	0.5	0.5	s
$\zeta_{\rm i}$	0.02	0.20	0.20	0.20	0.20	-
$\zeta_{\rm s}$	0.02	0.10	0.10	0.10	0.10	-
$\sigma_{\ddot{r}_{\rm s}+\ddot{z}}$			152	281	662	mm/s^2
$\sigma_{r_{\rm i}}$	0	200	30	58	140	mm
$\sigma_{r_{\rm s}-r_{\rm i}}$	0	5	5.0	1.8	4.2	mm

Figure 3. One of 45 seismic isolation bearings in the isolation galley of the Christchurch Women's Hospital, NZ, March 2013

References

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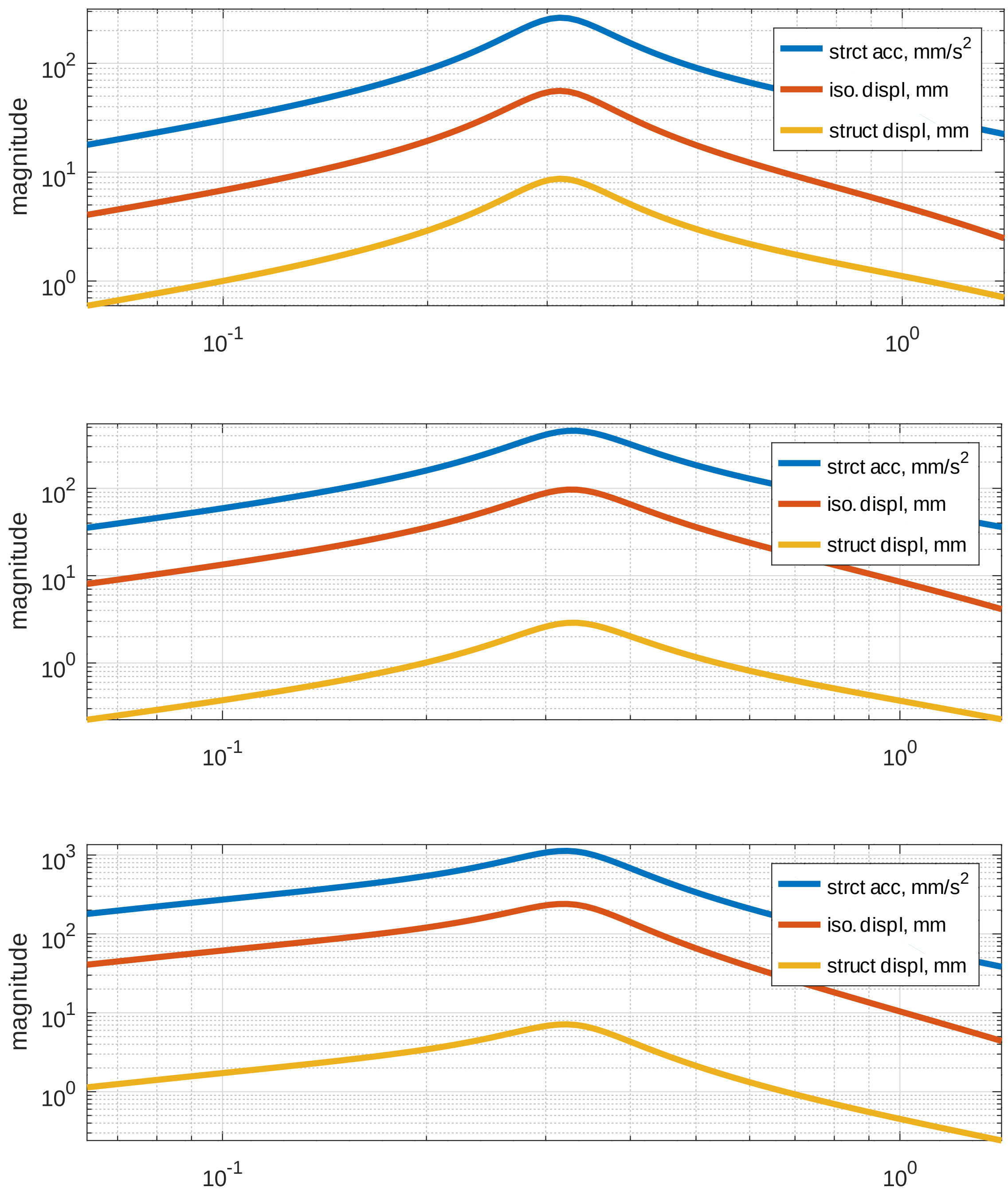

Figure 4. Frequency spectra of the response of base isolated structures to earthquake ground motions "optimized" for three types of ground motion (NF, NFNP, NFP).