Observer-Kalman Identification (OKID) Example

CEE 699.04, ME 599.04 — System Identification — Fall, 2013

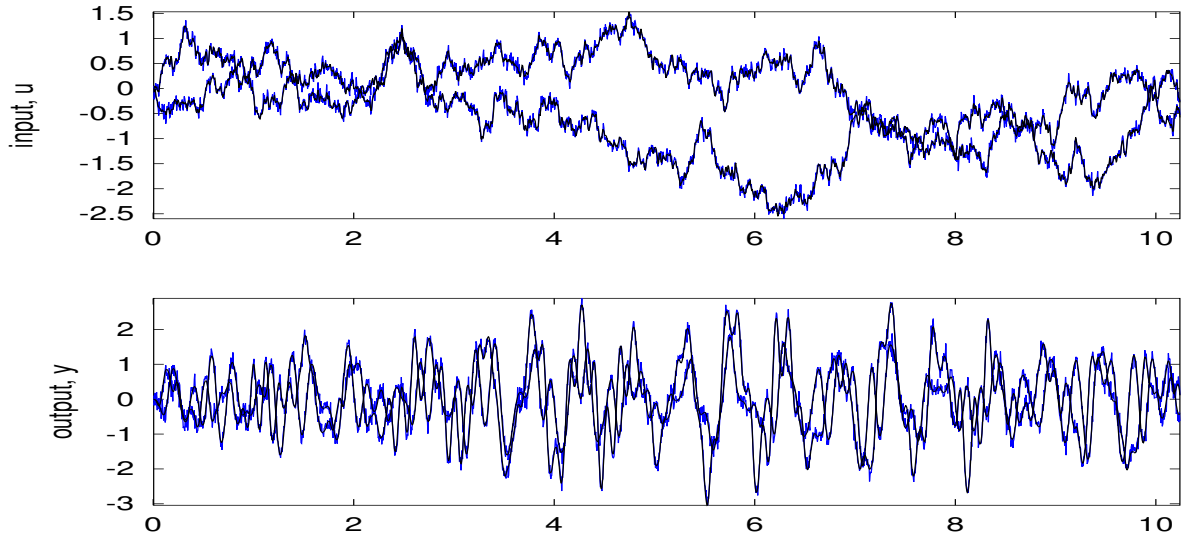
A continuous-time realization ($n = 8$, $r = 2$, $m = 2$).

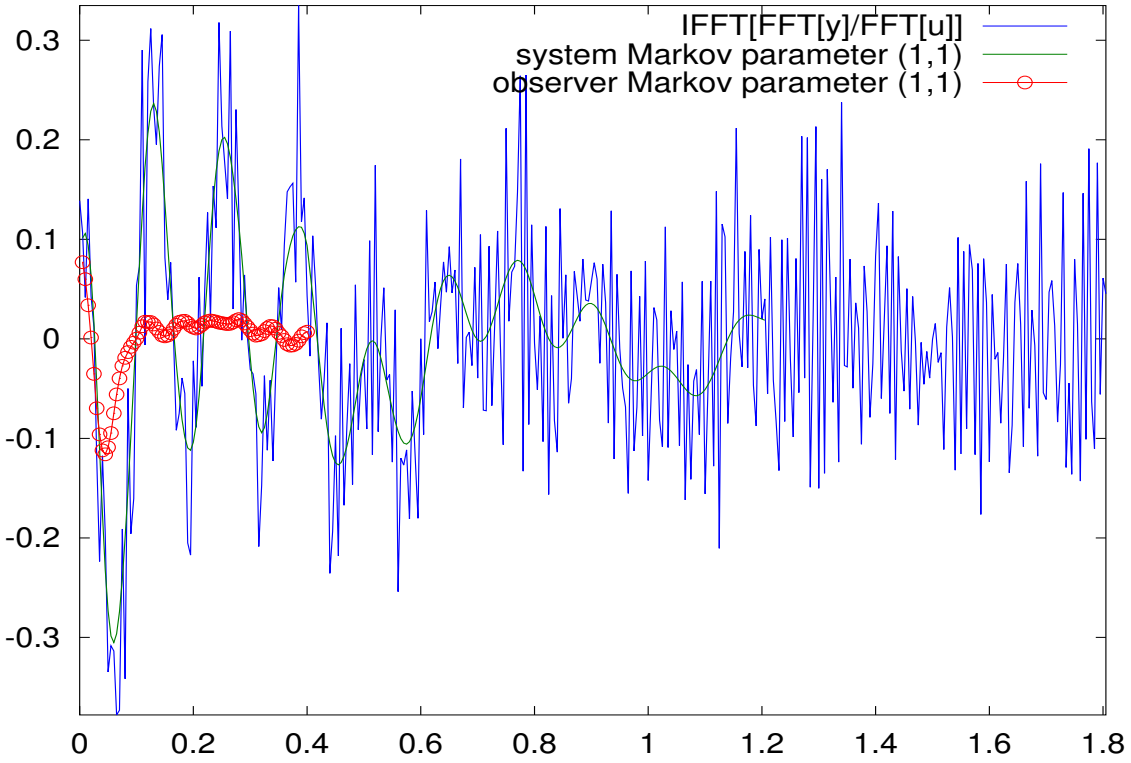
$$\left[\begin{array}{c|c} A_c & B_c \\ \hline C & D \end{array}\right] \sim \left[\begin{array}{c|c} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -2000 & 1000 & 0 & 0 & -5 & 1 & 0 & 0 \\ 500 & -1000 & 500 & 0 & 1 & -3 & 1 & 0 \\ 0 & 333 & -667 & 333 & 0 & 0 & -2 & 0 \\ 0 & 0 & 250 & -500 & 0 & 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -100 & 0 \\ -50 & 0 \\ -33 & -10 \\ -25 & 10 \end{bmatrix} \\ \hline \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}\right]$$

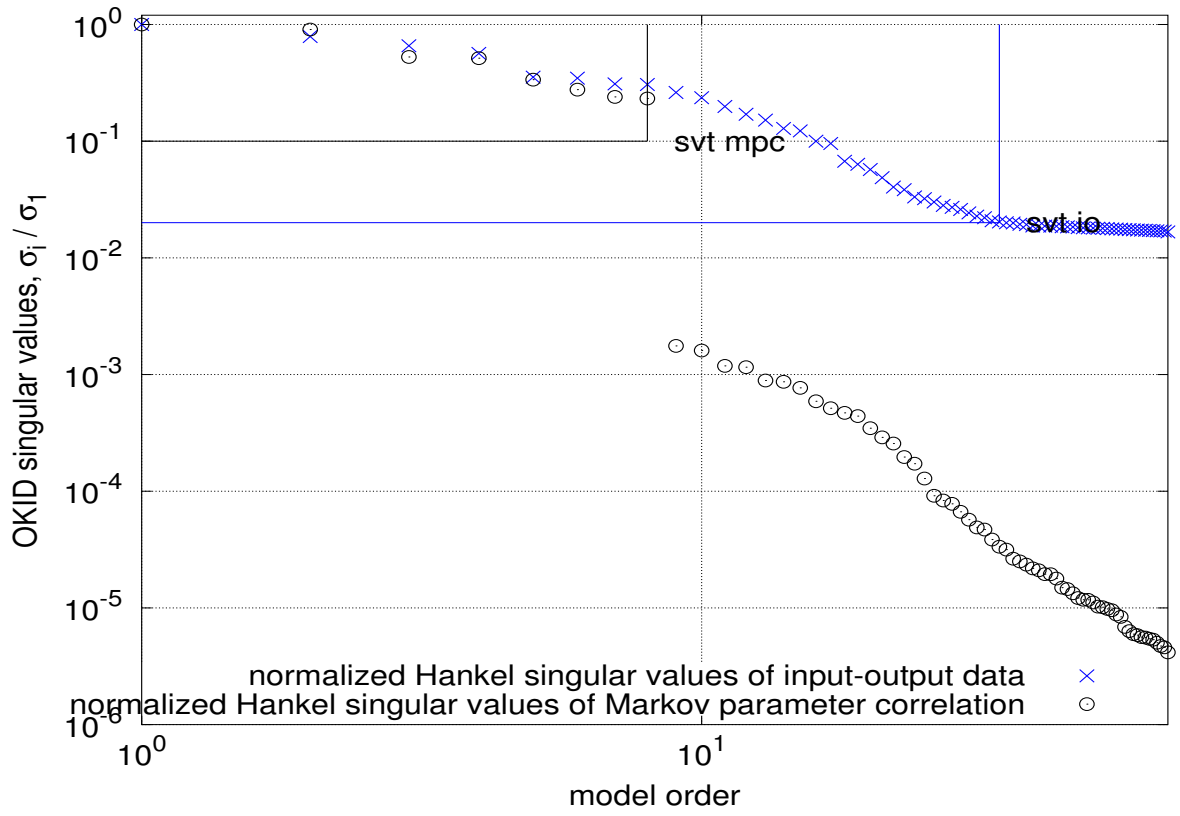
True model dynamics

Natural Frequency (cyc/sec)	Damping	Damped Frequency (cyc/sec)	Eigenvalue real	imag
1.92593	0.06539	1.92180	-0.79133	12.07504
3.74617	0.03368	3.74404	-0.79276	23.52451
5.22585	0.03658	5.22235	-1.20119	32.81299
7.77751	0.05555	7.76550	-2.71472	48.79207

$\Delta t = 0.005$s. input msmnt noise PSD = 0.005; output msmnt noise PSD = 0.010




IFFT[FFT[y]/FFT[u]]
system Markov parameter (1,1)
observer Markov parameter (1,1)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

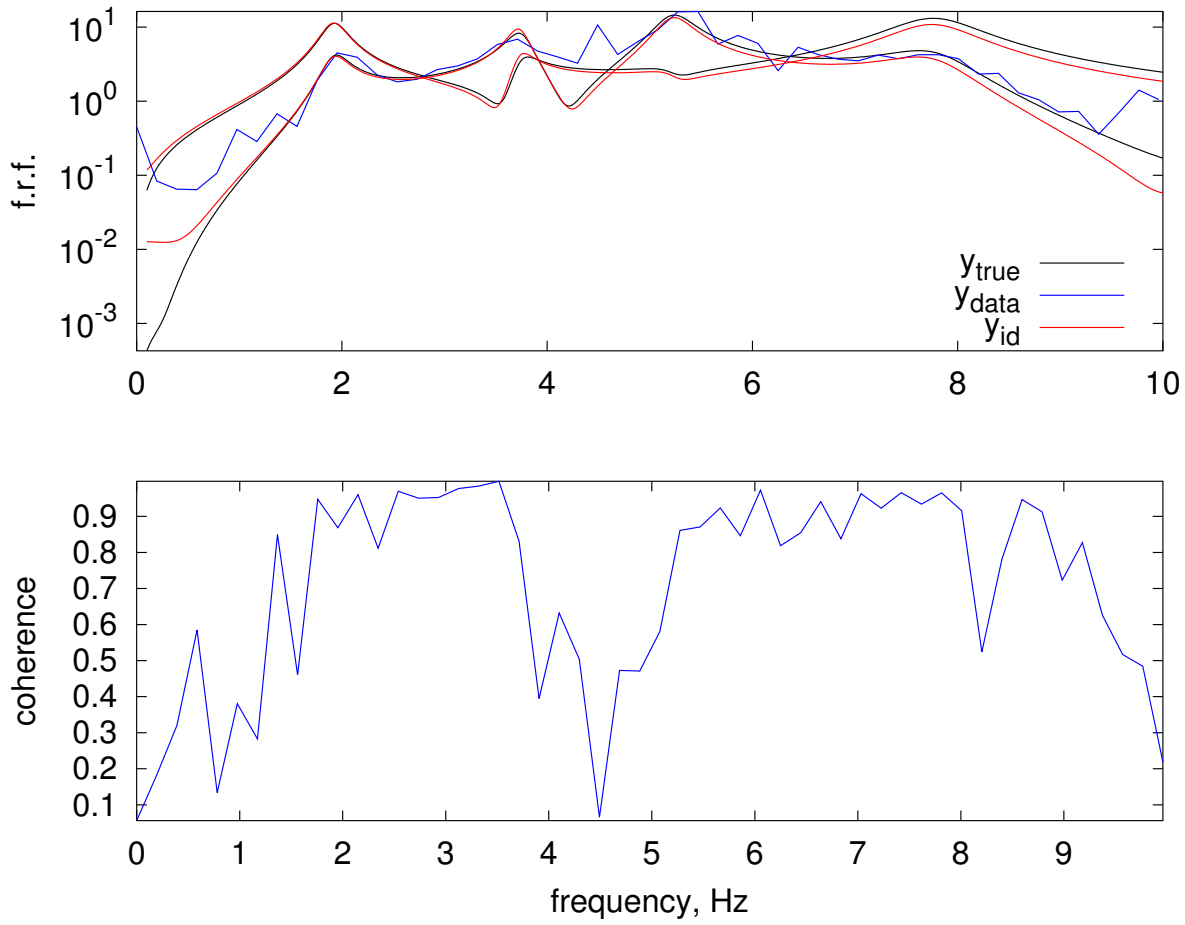

OKID singular values, σi / σ1
svt mpc
svt io
normalized Hankel singular values of input-output data
normalized Hankel singular values of Markov parameter correlation
model order
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^0
10^1

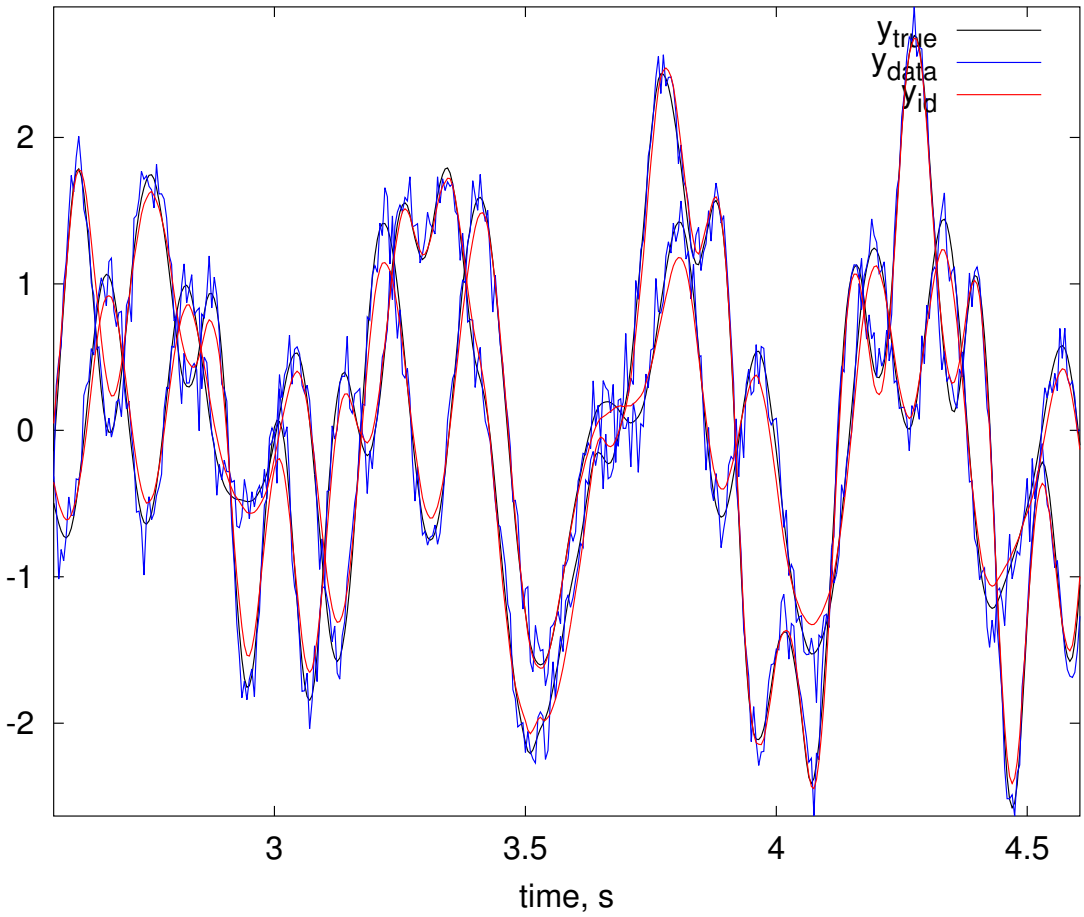
Identified model dynamics

```
p =  80            % number of observer Markov parameters
svt_io =   0.020   % singular value threshold for observer Markov parameters
svt_mpc =  0.100   % singular value threshold for identified model
mact   =   0.7     % mode amplitude coherence threshold
```

Natural Frequency (cyc/sec)	Damping	Damped Frequency (cyc/sec)	Eigenvalue real	imag
1.91804	0.07039	1.91328	-0.84833	12.02152
3.75263	0.03490	3.75034	-0.82296	23.56408
5.23069	0.03509	5.22747	-1.15314	32.84513
7.77205	0.04921	7.76264	-2.40286	48.77410

MAC =	0.9742	0.9524	0.9795	0.9940
MSV =	3.3917	2.9761	3.8665	3.5946






```
|y_id   - y_true|/|y_true|  . . .  0.147
|y_id   - y_meas|/|y_true|  . . .  0.222
|y_meas - y_true|/|y_true|  . . .  0.169
```

Recommendations for OKID parameters

- `p` : number of observer Markov parameters . . .
  between 0.5 and 1.0 of longest period in the system
- `svt_io` : s.v. threshold for computing observer Markov parameters from data
  Smaller is better in terms of accuracy of fit. It can be about 2 to 5 times min(s.v.)/max(s.v.).
- `svt_mpc` : s.v. threshold for computing system model from Hankel matrix
  should only contain the largest n singular values
- `mact` : modal amplitude coherence threshold to eliminate noise modes
  around 0.5 to 0.8, depending on the noise in the data.