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System Identification in the Presence of Completely Unknown Periodic Disturbances

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A system identification method to extract the disturbance-free dynamics and the disturbance effect correctly despite the presence of unknown periodic disturbances is presented. The disturbance frequencies and waveforms can be completely unknown and arbitrary. Only measurements of the excitation input and the disturbance-contaminated response are used for identification. Initially, the disturbances are modeled *implicitly*. When the order of an assumed input-output model exceeds a certain minimum value, the disturbance information is completely absorbed in the identified model coefficients. A special interaction matrix explains the mechanism by which information about the system and the disturbances are intertwined and more importantly, how they can be separated uniquely and exactly for later use in identification and control. From the identified information a feedforward controller can be developed to reject the unwanted disturbances without requiring the measurement of a separate disturbance-correlated signal. The multi-input multi-output formulation is first derived in the deterministic setting for which the system and disturbance identification is exact. Extensions to handle noise-contaminated data are also provided. Experimental results illustrate the method on a flexible structure. A companion paper addresses the problem where the disturbance effect is modeled explicitly.

Introduction

TRADITIONAL system identification techniques,[1–3] and more recently developed methods,[4–6] can be used to identify the system dynamics from input and output data. These methods assume that all of the system inputs are known, or that any unknown inputs are white noise whose effects can be averaged out using sufficient data. Sometimes, however, not all of the system inputs are known, and it may not be possible to disable or eliminate these unknown inputs while data is collected for system identification. In addition, these inputs may be deterministic, for example, smooth or periodic functions, and they may be large enough to dominate the system response. There is no guarantee of perfect identification when unknown periodic disturbances are present in the system. In this case, the identified model may or may not be accurate enough for control design. In addition, these methods do not provide the information needed to calculate the feedforward control signal to cancel the effects of the unknown disturbances.

Reference 7 broadens the traditional approaches by considering system identification when unknown periodic inputs act on the system. When the common disturbance period is known, Ref. 7 shows that the disturbance effect and the system input–output dynamics can be identified exactly. The identification results are then used for feedback control design and to calculate a feedforward control that cancels the effect of the disturbance on the system response. The present paper extends the above method by considering periodic disturbance inputs where both the disturbance waveforms and periods are unknown. By removing the required knowledge of the common disturbance period, the paper seeks to answer the fundamental question of whether it is indeed possible to identify the system disturbance-free dynamics correctly in the presence of completely unknown periodic disturbances. Besides determining the conditions that ensure perfect identification, we also wish to extract the disturbance effect and use it to calculate a feedforward control signal that cancels the effect of the disturbance. For this purpose, we assume that the only information available are measurements of a control excitation signal and the system response. The system response is corrupted by unmeasurable periodic disturbances with unknown periods that enter the system at unknown locations. Unlike Ref. 7, there is no explicit model of the disturbance effect on the output in the current technique. Instead, the disturbance information is initially absorbed in the identified model coefficients from which the system disturbance-free dynamics and the disturbance effect are recovered.

We briefly mention the implication of the proposed identification technique on the disturbance-rejection control problem. In addition to the classical notch filter approach, the literature includes many methods for solving disturbance-rejection problems. These methods, some of which are capable of handling systems with unknown dynamics and disturbances, include state-space approaches based on disturbance-accommodation control (DAC),[8–11] transfer function approaches that use adaptive filtering techniques,[12–21] and techniques using neural networks.[22–24] Our primary goal is to address the system identification problem in the presence of unknown disturbance inputs. Once the system input–output dynamics and disturbance effect are identified, a disturbance-rejection controller can be designed using these results. In so doing, we avoid some of the assumptions inherent in the referenced techniques. For example, unlike DAC approaches, we assume no knowledge of the system dynamics. Also, rather than modeling and calculating the actual disturbances, we identify their combined effect on the system response instead. As a result, the number of disturbances or where they enter the system is unimportant. Unlike adaptive filtering or neural network approaches, we do not require a disturbance-correlated reference signal or need to determine the transfer function relating the disturbances to the system response.

This paper begins by determining the conditions for the existence of an exact model that relates the control excitation inputs to the system disturbance-contaminated outputs without an explicit disturbance model. The unknown periodic disturbances are entirely absorbed in the coefficients of this model that can be identified from input–output data. Next, from the identified model, we show that the system (disturbance-free) input–output dynamics can still be extracted exactly, as if the unknown disturbances were not present. We show how the disturbance effect can also be extracted and then eliminated by a feedforward control signal. Finally, we discuss issues related to system and disturbance identification using measurements that are contaminated by noise. Following the mathematical development, experimental results illustrate the theory in the identification of a flexible truss structure.

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Mathematical Formulation

We now develop an interaction matrix formulation that shows how information about the system and the disturbances are intertwined and how they can be uniquely separated.

Input–Output Models Relating Excitation Input to Disturbance-Corrupted Output Data

In this section we show that it is possible to construct a model that can exactly predict the current value of the response, given only past values of the response and control inputs, despite the system response data being corrupted by the effects of unknown periodic disturbance inputs. The main emphasis of this section is to determine the conditions under which such a model exists.

The system to be identified is assumed to be representable by a linear discrete-time state-space model

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k), \qquad y(k) = Cx(k) \quad (1)$$

where $x(k)$ is an $n \times 1$ state vector, $u(k)$ is the $m \times 1$ control vector, $y(k)$ is the $q \times 1$ output vector, and k is the time step. The vector $d(k)$ represents the unknown periodic disturbance inputs. The system matrices, A, B, and C have dimensions $n \times n$, $n \times m$, and $q \times n$. Both these matrices and the disturbance input matrix B_d are unknown. Only measurements of the input $u(k)$ and measurements of the disturbance-corrupted system response $y(k)$ are available for identification.

Equation (1) calculates a one-step-ahead state prediction $x(k+1)$ given $x(k)$, $u(k)$, and $d(k)$. By repeatedly evaluating $x(k+1)$, and then substituting it back into Eq. (1), we generate a p-step-ahead state vector prediction,

$$x(k+p) = A^p x(k) + \mathcal{C} u_p(k) + \mathcal{C}_d d_p(k) \quad (2)$$

where $u_p(k)$ and $d_p(k)$ are vectors of the control inputs and disturbances,

$$u_p(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+p-1) \end{bmatrix}, \qquad d_p(k) = \begin{bmatrix} d(k) \\ d(k+1) \\ \vdots \\ d(k+p-1) \end{bmatrix}$$

The matrices $\mathcal{C}$ and $\mathcal{C}_d$ are given by

$$\mathcal{C} = [A^{p-1}B, \ldots, AB, B], \qquad \mathcal{C}_d = [A^{p-1}B_d, \ldots, AB_d, B_d] \quad (3)$$

Note that $\mathcal{C}$ and $\mathcal{C}_d$ are controllability matrices associated with the control excitation and disturbance inputs, respectively. By repeated application of the output equation, the expression for the $pq \times 1$ vector of system outputs $y_p(k)$ is

$$y_p(k) = \mathcal{O} x(k) + \mathcal{T} u_p(k) + \mathcal{T}_d d_p(k) \quad (4)$$

where $y_p(k)$ and the matrices $\mathcal{O}$, $\mathcal{T}$, and $\mathcal{T}_d$ are given by

$$y_p(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+p-1) \end{bmatrix}, \qquad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-2} \\ CA^{p-1} \end{bmatrix}$$

$$\mathcal{T} = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ CB & 0 & \ddots & \ddots & \vdots \\ CAB & CB & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ CA^{p-2}B & \cdots & CAB & CB & 0 \end{bmatrix}$$

$$\mathcal{T}_d = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ CB_d & 0 & \ddots & \ddots & \vdots \\ CAB_d & CB_d & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ CA^{p-2}B_d & \cdots & CAB_d & CB_d & 0 \end{bmatrix} \quad (5)$$

Note that $\mathcal{O}$ is the $pq \times n$ system observability matrix. The matrix $\mathcal{T}$ is a $pq \times pm$ Toeplitz matrix made up of the $q \times m$ system Markov parameters $CB, CAB, \ldots, CA^{p-2}B$, whose elements are the system response to a unit pulse applied at each control input.

To eliminate the effects of the disturbances $d_p(k)$ and the initial state $x(k)$ on the system input–output mapping, additional degrees of freedom are now introduced into the model. This is accomplished by adding and subtracting $\boldsymbol{M} y_p(k)$ to the right-hand side of Eq. (2) to obtain

$$x(k+p) = A^p x(k) + \mathcal{C} u_p(k) + \mathcal{C}_d d_p(k) + M y_p(k) - M y_p(k) \quad (6)$$

where $\boldsymbol{M}$ is an arbitrary $n \times pq$ matrix. Next, substituting Eq. (4) for $y_p(k)$, expression (6) becomes

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{C} u_p(k) + \mathcal{C}_d d_p(k) \\ &\quad + M[\mathcal{O} x(k) + \mathcal{T} u_p(k) + \mathcal{T}_d d_p(k)] - M y_p(k) \\ &= (A^p + M\mathcal{O}) x(k) + (\mathcal{C} + M\mathcal{T}) u_p(k) \\ &\quad + (\mathcal{C}_d + M\mathcal{T}_d) d_p(k) - M y_p(k) \end{aligned} \quad (7)$$

As expected, Eq. (7) involves the system state $x(k)$ and the disturbance input $d_p(k)$, both of which are unknown. However, these terms can be eliminated from the equation if there exists an $n \times pq$ matrix M such that the following conditions are satisfied for all k:

$$A^p + M\mathcal{O} = 0 \quad (8)$$

$$(\mathcal{C}_d + M\mathcal{T}_d) d_p(k) = 0 \qquad \forall k \quad (9)$$

so that Eq. (7) becomes

$$x(k+p) = (\mathcal{C} + M\mathcal{T}) u_p(k) - M y_p(k) \quad (10)$$

Let us examine Eq. (9) more closely. For $k = 1, 2, \ldots, N, N+1, \ldots$, the constraint equations imposed by Eq. (9) can be grouped together as

$$(\mathcal{C}_d + \boldsymbol{M}\mathcal{T}_d)\mathcal{D} = 0 \quad (11)$$

where

$$\mathcal{D} = [d_p(1), d_p(2), \ldots, d_p(N), d_p(N+1), \ldots] \quad (12)$$

Although it may appear that Eq. (11) is a rather large set of constraints, not all of these constraint equations are linearly independent. The actual number of linearly independent constraint equations in Eq. (11) is $n\rho$, where n is the order of the system and ρ is the rank of $\mathcal{D}$. According to Eq. (12), the rows of $\mathcal{D}$ are time-shifted sampled histories of the periodic disturbances. Consequently, its row rank (for a sufficiently large N) is limited by the number of distinct frequencies present the disturbances. To see this, consider an example where the disturbance input is a sine wave function of a single frequency, for example, 1 Hz, and this signal is being sampled with a sampling interval of less than 0.5 s. Time shifting this 1-Hz signal by one time step will introduce a new (cosine) component to this signal, thus causing the time-shifted signal to be linearly independent from the original sampled signal. Additional shiftings will not introduce new linearly independent signals. Furthermore, these statements are valid even if the period of the signal is not an integer multiple of the sampling interval. If the sampling interval is exactly 0.5 s, then time shifting it will produce the same signal with the sign reversed, thus creating no new linearly independent signal. Thus, every distinct, zero-mean harmonic component of the disturbance contributes at most two linearly independent rows to

$\mathcal{D}$. If any one of these disturbances have nonzero mean, the rank of $\mathcal{D}$ will be increased further by one. In other words, if there are f distinct frequencies present in the disturbances, then $\rho = 2f$ or $2f + 1$, depending on whether any disturbance has nonzero mean. Let $\mathcal{D}_f$ be formed by ρ linearly independent columns of $\mathcal{D}$ (a row rank of a matrix is the same as its column rank), we can now write all of the equations that M must satisfy as follows:

$$M[\mathcal{O}, \mathcal{T}_d\mathcal{D}_f] = -[A^p, \mathcal{C}_d\mathcal{D}_f] \tag{13}$$

Equation (13) is a set of $n^2 + 2n\rho$ linear equations in $n \times qp = nqp$ unknowns in M. Thus, the existence of M is assured provided $[\mathcal{O}, \mathcal{T}_d\mathcal{D}_f]$ is full (column) rank and p is chosen such that $nqp \geq n^2 + \rho n$. When expressed in terms of the disturbance frequencies, we have the following condition for p:

$$pq \geq n + 2f + 1 \tag{14}$$

Equation (14) represents a safe lower bound for p because there are cases where the true lower bound for p is even smaller. First, Eq. (14) assumes the general case where the disturbances do not have zero mean. Otherwise, it becomes $pq \geq n + 2f$. Second, it also assumes that the disturbances have no frequency components at exactly the Nyquist frequency (half the sampling frequency). As illustrated in the preceding example, such sampled signals will contribute not two but only one linearly independent row to $\mathcal{D}$. Note that all of these exceptions will potentially cause the lower bound for p to be even smaller, indicating that Eq. (14) is a sufficient condition. It is important to realize that there is no need to select p corresponding to its truly lower bound because any larger p can be used. In so doing, one can avoid all of the subtleties discussed in the preceding explanation.

Therefore, for periodic disturbances, if p is selected to be large enough to satisfy Eq. (14), existence of M implies existence of an input–output model of the form

$$y(k + p) = C(\mathcal{C} + M\mathcal{T})u_p(k) - CMy_p(k) \tag{15}$$

Equation (15) is obtained by premultiplying both sides of Eq. (10) by C. Shifting the time index back by p steps, for $k \geq p$ this expression becomes

$$y(k) = C(\mathcal{C} + M\mathcal{T})u_p(k - p) - CMy_p(k - p) \tag{16}$$

where

$$u_p(k-p) = \begin{bmatrix} u(k-p) \\ u(k-p+1) \\ \vdots \\ u(k-1) \end{bmatrix}, \qquad y_p(k-p) = \begin{bmatrix} y(k-p) \\ y(k-p+1) \\ \vdots \\ y(k-1) \end{bmatrix}$$

Equation (16) shows that, even though the system response is corrupted by the unknown disturbance inputs, a model exists such that a one-step-ahead response prediction $y(k)$ can be calculated exactly from p past values of the inputs and outputs. The disturbance information is completely absorbed in such an input–output model, the coefficients of which are calculated as will be described. The special matrix M as derived here describes the mechanism by which the system and disturbance information is interrelated. For this reason we use the term interaction matrix to describe it.

Calculating Model Coefficients from Input–Output Data

Equation (16) expresses $y(k)$ in terms of p past values of the system response, $y(k-1), \ldots, y(k-p)$, and p past values of the control input, $u(k-1), \ldots, u(k-p)$. This expression has the same form as an autoregressive moving average model with exogenous input (ARX) model,

$$y(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \cdots + \alpha_p y(k-p) + \beta_1 u(k-1) + \beta_2 u(k-2) + \cdots + \beta_p u(k-p) \tag{17}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_p$ and $\beta_1, \beta_2, \ldots, \beta_p$ are the ARX model coefficients that are now known to be related to the matrices in Eq. (16) by

$$\begin{aligned} [\alpha_p, \alpha_{p-1}, \ldots, \alpha_1] &= -CM \\ [\beta_p, \beta_{p-1}, \ldots, \beta_1] &= C(\mathcal{C} + M\mathcal{T}) \end{aligned} \tag{18}$$

By the assumption that measurements of $u(k)$ and $y(k)$ are available for $k = 0, 1, \ldots, \ell$, the model coefficient matrices can be calculated from

$$[C(\mathcal{C} + M\mathcal{T}), -CM] = YV^T(VV^T)^+ \tag{19}$$

where Y and V are data matrices arranged in the form

$$Y = [y(p), y(p+1), \ldots, y(\ell)]$$
$$V = \begin{bmatrix} u_p(0) & u_p(1) & \cdots & u_p(\ell - p) \\ y_p(0) & y_p(1) & \cdots & y_p(\ell - p) \end{bmatrix} \tag{20}$$

The $+$ sign in Eq. (19) denotes the pseudoinverse operation that should be performed via the singular value decomposition to detect and eliminate numerical ill-conditioning issues if they arise. It is assumed that the control input is sufficiently rich, for example, random input, so that the rows containing the shifted sequences of $u(k)$ are linearly independent and that there are sufficient measurements available so that the number of equations is at least equal to or greater than the number of unknowns. As already mentioned, in the absence of noise, if p is selected to be large enough, Eq. (19) will determine a model that results in an exact fit to the data, despite the presence of the unknown periodic disturbance. As shown earlier, increasing p makes available additional degrees of freedom so that the disturbance effect can be completely absorbed into the model coefficients. In the following section we show that this absorption occurs in such a specific way that the system pulse response samples (Markov parameters) can still be recovered exactly from the altered coefficients without knowing the interaction matrix M. The system Markov parameters completely describe the system input–output dynamics.

Recovering System Markov Parameters from Model Coefficients

Once the model coefficients have been determined using Eq. (19), any number of system Markov parameters can be calculated as follows. The ARX model coefficients in Eq. (18) are related to one another by

$$\begin{aligned} [\beta_p, \beta_{p-1}, \ldots, \beta_1] &= C\mathcal{C} - [\alpha_p, \alpha_{p-1}, \ldots, \alpha_1]\mathcal{T} \\ &= [CA^{p-1}B, \ldots, CAB, CB] - [\alpha_p, \alpha_{p-1}, \ldots, \alpha_1] \\ &\quad \times \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ CB & 0 & \ddots & \ddots & \vdots \\ CAB & CB & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ CA^{p-2}B & \cdots & CAB & CB & 0 \end{bmatrix} \end{aligned} \tag{21}$$

By equating of the terms on each side of this expression, the first p system Markov parameters are

$$\begin{aligned} CB &= \beta_1 \\ CAB &= \beta_2 + \alpha_1 CB \\ CA^2B &= \beta_3 + \alpha_2 CB + \alpha_1 CAB \\ &\vdots \\ CA^{p-1}B &= \beta_p + \alpha_{p-1}CB + \alpha_{p-2}CAB + \cdots + \alpha_1 CA^{p-2}B \end{aligned} \tag{22}$$

To determine additional system Markov parameters, recall that by selection of the model order p to be large enough, a matrix $\boldsymbol{M}$ will exist so that Eq. (8) is satisfied. This condition can be expressed as

$$A^p + M_p C A^{p-1} + \cdots + M_2 C A + M_1 C = 0 \tag{23}$$

where $M_1, M_2, \ldots, M_p$ are the $n \times q$ partitions of $\boldsymbol{M}$, that is, $\boldsymbol{M} = [M_1, M_2, \ldots, M_p]$. Premultiplying Eq. (23) by C and postmultiplying it by B yield

$$CA^pB + (CM_p)CA^{p-1}B + \cdots + (CM_2)CAB + (CM_1)CB = 0 \tag{24}$$

By the recognition that $\alpha_1 = -CM_p$, $\alpha_2 = -CM_{p-1}, \ldots,$ and $\alpha_p = -CM_1$, the system Markov parameter CA^pB is given by

$$CA^pB = \alpha_1 CA^{p-1}B + \cdots + \alpha_{p-1}CAB + \alpha_p CB \tag{25}$$

Additional Markov parameters can be obtained analogously,

$$CA^{p+1}B = \alpha_1 CA^pB + \cdots + \alpha_{p-1}CA^2B + \alpha_p CAB$$
$$\vdots \tag{26}$$

Therefore, despite the unknown disturbances altering the ARX model coefficients, the system Markov parameters can still be recovered exactly as if the disturbances were not present. From these Markov parameters one can obtain a minimum-order state-space realization that can predict the disturbance-free response to arbitrary inputs. The following section will show how to construct this reduced-order model. Additionally, note that the formulas for the system Markov parameters given earlier turn out to be the same as those derived from the coefficients of an input–output transfer function identified from data that is not contaminated by any disturbances.[4–6] Therefore, the identified model whose coefficients are thought to be corrupted by the unknown disturbances is capable of predicting the system disturbance-free response exactly as well.

Determining a Minimum-Order State-Space Realization

Given the system Markov parameters, one can easily obtain a minimum-order state-space realization. For completeness we now provide the key equations involved in the realization by the eigensystem realization algorithm (ERA) that is described in detail in Ref. 4. One first forms the Hankel matrices $H(0)$ and $H(1)$ from the recovered system Markov parameters, $k = 0, 1$

$$H(k) = \begin{bmatrix} CA^kB & CA^{k+1}B & \cdots & CA^{k+s-1}B \\ CA^{k+1}B & CA^{k+2}B & \cdots & CA^{k+s}B \\ \vdots & \vdots & \cdots & \vdots \\ CA^{k+r-1}B & CA^{k+r}B & \cdots & CA^{k+s+r-2}B \end{bmatrix} \tag{27}$$

For sufficiently large values of r and s, the rank of the Hankel matrix is equal to the order n of the minimal realization. A minimum-order state-space realization is given by

$$\bar{A} = \Sigma_n^{-\frac{1}{2}} R_n^T H(1) S_n \Sigma_n^{-\frac{1}{2}}, \quad \bar{B} = \Sigma_n^{\frac{1}{2}} S_n^T E_m, \quad \bar{C} = E_q^T R_n \Sigma_n^{\frac{1}{2}} \tag{28}$$

where the $q \times rq$ matrix E_q^T and the $m \times sm$ matrix E_m^T are defined as $E_q^T = [I_{q\times q} \;\; O_{q\times(r-1)q}]$ and $E_m^T = [I_{m\times m} \;\; O_{m\times(s-1)m}]$. The singular value decomposition of $H(0)$ is $H(0) = R\Sigma S^T = R_n \Sigma_n S_n^T$ where n is the order of the minimum realization and only n nonzero singular values of the Hankel marix $H(0)$ are retained in Σ_n.

Identifying the Disturbance Effect from Input–Output Data

In the preceding sections the goal was to identify a model for the system where the explicit effect of the unknown disturbances on the input–output mapping was completely eliminated. By increasing the ARX model order, we showed that the disturbance effect was absorbed in the coefficients. Moreover, we showed that system Markov parameters could still be recovered correctly from the altered coefficients and be used to calculate a minimum-order state-space realization, as if there were no disturbances present. If the purpose of identification is only to produce a correct system model relating the control inputs to the system outputs, no further steps are necessary. The disturbance information, however, is contained in the identified model coefficients, and if extracted, it can be used to calculate a feedforward control that cancels the disturbance effect or to determine the steady-state effect of the disturbance on the system response. In the following we will show that, in addition to exact system identification, exact recovery of the disturbance information is also possible.

By the use of a similar argument as described in Eqs. (2–7), a ζ-step-ahead state prediction model corresponding to the identified minimum-order state-space model realization is

$$\bar{x}(k+\zeta) = (\bar{A}^\zeta + \bar{\boldsymbol{M}}\bar{\mathcal{O}})\bar{x}(k) + (\bar{\mathcal{C}} + \bar{\boldsymbol{M}}\bar{\mathcal{T}})u_\zeta(k)$$
$$+ (\bar{\mathcal{C}}_d + \bar{\boldsymbol{M}}\bar{\mathcal{T}}_d)d_\zeta(k) - \bar{\boldsymbol{M}}y_\zeta(k) \tag{29}$$

where $\bar{\mathcal{O}}$ is the $q\zeta \times n$ system observability matrix calculated from $\bar{A}$ and $\bar{C}$, $\bar{\mathcal{C}}$ is the $n \times m\zeta$ system controllability matrix calculated from $\bar{A}$ and $\bar{B}$, and $\bar{\mathcal{T}}$ is the $q\zeta \times m\zeta$ Toeplitz matrix calculated from Eq. (5) using the identified system Markov parameters. Note that Eq. (29) is valid for any value of ζ and any matrix $\bar{\boldsymbol{M}}$. The term on the right-hand side that contains $d_\zeta(k)$ is unknown, and in contrast to the previous development, we wish to extract this disturbance information rather than making this term vanish. Therefore, we seek only to eliminate $\bar{x}(k)$ from Eq. (29) by finding an $n \times q\zeta$ matrix $\bar{\boldsymbol{M}}$ that satisfies the condition

$$\bar{A}^\zeta + \bar{\boldsymbol{M}}\bar{\mathcal{O}} = 0 \tag{30}$$

Because $\bar{\mathcal{O}}$ is full column rank (only the system's observable subspace can be identified), such an $\bar{\boldsymbol{M}}$ can be found from

$$\bar{\boldsymbol{M}} = -(\bar{A})^\zeta(\bar{\mathcal{O}})^+ \tag{31}$$

if ζ is chosen such that $q\zeta \geq n$. We can even choose ζ to be p for convenience. This choice of $\bar{\boldsymbol{M}}$ will eliminate explicit dependence on the state variable $\bar{x}(k)$ in Eq. (29), but the disturbance effect will appear explicitly as an additive term in the model. Substituting $\bar{\boldsymbol{M}}$ given in Eq. (31) into Eq. (29) and reordering terms yield

$$\bar{x}(k+\zeta) = -\bar{\boldsymbol{M}}y_\zeta(k) + (\bar{\mathcal{C}} + \bar{\boldsymbol{M}}\bar{\mathcal{T}})u_\zeta(k) + (\bar{\mathcal{C}}_d + \bar{\boldsymbol{M}}\bar{\mathcal{T}}_d)d_\zeta(k) \tag{32}$$

Shifting the time index back by ζ time steps, and premultiplying Eq. (32) by $\bar{C}$ produce

$$y(k) = -\bar{C}\bar{\boldsymbol{M}}y_\zeta(k-\zeta) + \bar{C}(\bar{\mathcal{C}} + \bar{\boldsymbol{M}}\bar{\mathcal{T}})u_\zeta(k-\zeta)$$
$$+ \bar{C}(\bar{\mathcal{C}}_d + \bar{\boldsymbol{M}}\bar{\mathcal{T}}_d)d_\zeta(k-\zeta) \tag{33}$$

where

$$y_\zeta(k-\zeta) = \begin{bmatrix} y(k-\zeta) \\ y(k-\zeta+1) \\ \vdots \\ y(k-1) \end{bmatrix}, \qquad u_\zeta(k-\zeta) = \begin{bmatrix} u(k-\zeta) \\ u(k-\zeta+1) \\ \vdots \\ u(k-1) \end{bmatrix}$$

$$d_\zeta(k-\zeta) = \begin{bmatrix} d(k-\zeta) \\ d(k-\zeta+1) \\ \vdots \\ d(k-1) \end{bmatrix}$$

The last term on the right-hand side of Eq. (33) is a linear combination of ζ samples of the periodic disturbance and is, therefore, also periodic with a common period N. With this periodic term denoted as $\eta(k)$, Eq. (33) becomes

$$y(k) = -\bar{C}\bar{\boldsymbol{M}}y_\zeta(k-\zeta) + \bar{C}(\bar{\mathcal{C}} + \bar{\boldsymbol{M}}\bar{\mathcal{T}})u_\zeta(k-\zeta) + \eta(k) \tag{34}$$

In the form of an ARX model, we now have

$$y(k) = \bar{\alpha}_1 y(k-1) + \bar{\alpha}_2 y(k-2) + \cdots + \bar{\alpha}_\zeta y(k-\zeta) + \bar{\beta}_1 u(k-1) + \bar{\beta}_2 u(k-2) + \cdots + \bar{\beta}_\zeta u(k-\zeta) + \eta(k) \tag{35}$$

where the coefficients are given by

$$[\bar{\alpha}_\zeta, \bar{\alpha}_{\zeta-1}, \ldots, \bar{\alpha}_1] = -\bar{C}\bar{\boldsymbol{M}}$$
$$[\bar{\beta}_\zeta, \bar{\beta}_{\zeta-1}, \ldots, \bar{\beta}_1] = \bar{C}(\mathcal{C} + \bar{\boldsymbol{M}}\mathcal{T}) \tag{36}$$

It is important to realize that for periodic unknown disturbances both the model given by Eq. (17) and the one given by Eq. (35) are equally correct. Assuming noise-free measurements, both models will fit the response data perfectly. In the case of Eq. (17), the disturbance effect is entirely embedded in the model coefficients along with the system dynamics. In the case of Eq. (35), the model has been partitioned so that the disturbance effect appears as an additive periodic term $\eta(k)$, and the coefficients include only the system disturbance free dynamics. Once Eq. (36) is used to determine the minimum realization ARX model coefficients $\bar{\alpha}_i$ and $\bar{\beta}_i$, $i = 1, 2, \ldots, \zeta$, the disturbance effect $\eta(k)$ for $k \geq \zeta$ is given by

$$\eta(k) = y(k) - \bar{\alpha}_1 y(k-1) - \bar{\alpha}_2 y(k-2) - \cdots - \bar{\alpha}_\zeta y(k-\zeta) - \bar{\beta}_1 u(k-1) - \bar{\beta}_2 u(k-2) - \cdots - \bar{\beta}_\zeta u(k-\zeta) \tag{37}$$

Calculating the Steady-State Disturbance Response and Feedforward Control Signal

Once $\eta(k)$ and the ARX model coefficients $\bar{\alpha}_i$ and $\bar{\beta}_i$ have been determined, we have complete information to determine both a feedforward control signal that cancels the disturbance effect and the steady-state response of the system to the unknown disturbances. This is done as follows. With $u_f(k)$ denoted as the feedforward control to be applied to the system, the model of Eq. (35) becomes

$$y(k) = \bar{\alpha}_1 y(k-1) + \bar{\alpha}_2 y(k-2) + \cdots + \bar{\alpha}_\zeta y(k-\zeta) + \bar{\beta}_1 u_f(k-1) + \bar{\beta}_2 u_f(k-2) + \cdots + \bar{\beta}_\zeta u_f(k-\zeta) + \eta(k) \tag{38}$$

From this expression, the feedforward control signal that makes the steady-state system response equal zero must satisfy

$$\bar{\beta}_1 u_f(k-1) + \bar{\beta}_2 u_f(k-2) + \cdots + \bar{\beta}_\zeta u_f(k-\zeta) + \eta(k) = 0 \tag{39}$$

The needed feedforward control signal is simply the steady-state solution to Eq. (39). Various ways of calculating the feedforward control signal are discussed in Ref. 7. Whether a feedforward control signal exists that can exactly cancel the disturbances depends on several factors. If the disturbances cause the output to have a frequency component at which the control input has no influence (e.g., the input-output transfer function equals zero at this frequency), then this component cannot be canceled by feedforward control. In addition, provided the control can influence the system output, if the number of independent inputs is equal to or greater than the number of outputs, then a feedforward control exists that exactly cancels the disturbance response. Otherwise, a perfect cancellation is in general not possible.

From Eq. (35), in the absence of the input $u(k)$, the system response to the unknown disturbances $y_d(k)$ is given by

$$y_d(k) = \bar{\alpha}_1 y_d(k-1) + \bar{\alpha}_2 y_d(k-2) + \cdots + \bar{\alpha}_\zeta y_d(k-\zeta) + \eta(k) \tag{40}$$

Knowing $\eta(k)$ and $\bar{\alpha}_i$, $i = 1, 2, \ldots, \zeta$, the estimated disturbance response $y_d(k)$ can be solved for from Eq. (40), which will match the actual disturbance response in the steady state.

Analysis of Disturbance-Corrupted Model

In the absence of noise, the Hankel matrix $H(0)$ will have precisely n nonzero singular values, the same number as the true system order. In the presence of noise, however, model reduction by examining the singular values of the Hankel matrix $H(0)$ is generally difficult because $H(0)$ tends to appear full rank. This limitation can be overcome by recognizing that the identified model of Eq. (17) can be converted to a modal state-space form from which analysis can be carried out to determine each respective mode of the model. In general, they include true identifiable dynamic modes of the system, uncontrollable disturbance modes, and uncontrollable modes due to overparameterization. In the following we discuss ways by which one can distinguish these modes from the identified model.

The disturbance modes comprise the uncontrollable subspace of the ARX model that generates the disturbance effect $\eta(k)$. In the ideal noise-free case, these modes are easy to identify. They contribute to model's response prediction, but cannot be excited by the control input. In addition, because their contribution to the response prediction includes only sinusoidal components at the disturbance frequencies, these modes have zero damping. Identifying the disturbance modes allows the unknown disturbance frequencies to be determined. Over-parameterization modes result because in practice it is necessary to select a model order p much larger than that required by Eq. (14) for several reasons. This is because the true (or effective) order of the system is not known exactly and neither are the number of distinct disturbance frequencies. Therefore, the model order must be selected so that sufficient degrees of freedom are available to absorb the identifiable dynamics and disturbance effect given these uncertainties. Additionally, for accurate results, the model order must be further increased to reduce the effects of measurement and process noise. This situation will be illustrated later by an example.

In the presence of noise, to properly categorize the model's modes, it is necessary to obtain identification results for several values of p and, hence, for several different levels of model overparameterization. As the model order increases, the damping of the true system modes converge to nonzero positive values (for a stable system), whereas the disturbance mode damping drops to near zero. The disturbance mode damping may either be positive or negative with equal likelihood. Because of this, it is not unusual for the disturbance-corrupted model to contain unstable modes that restrict how it can be used without further processing. The overparameterization modes also have positive damping, and they generally constitute an easily recognizable band of modes distributed over the entire frequency range.

Besides modal damping, the other important discrimination criterion is the contribution of a mode to the model's input–output mapping or the system pulse response. Using the method to be described enables the modal pulse response contributions to be ranked in order from the most significant to least significant. Observing how the contribution factors and rankings change as the model order p increases provides a powerful tool for modal discrimination. If the system is identified when the disturbances are not present the situation is simple. The overparameterization modes have the smallest contribution factors, and the dominant dynamics have the largest. As p increases the rankings change little, except some weakly controllable or weakly observable dynamic modes may move up as they become better resolved. The overparameterization modes can be discarded if a reduced-order model is desired because they do not contribute significantly to the input–output mapping. Dynamic modes that are weakly controllable, weakly observable, or both have rankings in between and can also be discarded with minimal effect. When disturbances are present, the disturbance modes may appear to have large contribution factors. However, as p increases they diminish toward zero. In addition, the contribution factors for the dominant system dynamics will in general increase and ultimately converge to some constant value.

To compute the modal contribution factors, the identified ARX model is converted to a canonical state-space form and then to the modal form

$$w(k+1) = \Lambda w(k) + \Gamma u(k), \qquad y(k) = \Omega w(k) \tag{41}$$

where the dimension of the state vector $w(k)$ is $qp \times 1$, where q is the number of outputs and p is the assumed order of the input–output model. The system matrix Λ is a block diagonal matrix constructed from the system real and complex eigenvalues,

$$\Lambda = \mathrm{diag}\left\{\lambda_r^{(1)}, \lambda_r^{(2)}, \ldots, \begin{bmatrix} \sigma_c^{(1)} & -\omega_c^{(1)} \\ \omega_c^{(1)} & \sigma_c^{(1)} \end{bmatrix}, \begin{bmatrix} \sigma_c^{(2)} & -\omega_c^{(2)} \\ \omega_c^{(2)} & \sigma_c^{(2)} \end{bmatrix}, \ldots\right\} \tag{42}$$

and the output and input influence matrices are

$$\Omega = \left[c_r^{(1)}, c_r^{(2)}, \ldots, c_c^{(1)}, c_c^{(2)}, \ldots\right], \qquad \Gamma = \begin{bmatrix} b_r^{(1)} \\ b_r^{(2)} \\ \vdots \\ b_c^{(1)} \\ b_c^{(2)} \\ \vdots \end{bmatrix} \tag{43}$$

where $c_r^{(i)}$ and $c_c^{(i)}$ and $b_r^{(i)}$ and $b_c^{(i)}$ are the respective output and input influence coefficients associated with real eigenvalues $\lambda_r^{(i)}$ and the complex (conjugate) eigenvalue pairs $\sigma_c^{(i)} \pm j\omega_c^{(i)}$. The modal pulse responses are

$$P_r^{(i)}(k) = c_r^{(i)}\left(\lambda_r^{(i)}\right)^{k-1} b_r^{(i)}, \qquad P_c^{(i)}(k) = c_c^{(i)}\begin{bmatrix} \sigma_c^{(i)} & -\omega_c^{(i)} \\ \omega_c^{(i)} & \sigma_c^{(i)} \end{bmatrix}^{k-1} b_c^{(i)} \tag{44}$$

and the total system pulse response is

$$P_t(k) = \sum_{i=1}^{nr} P_r^{(i)}(k) + \sum_{i=1}^{nc} P_c^{(i)}(k) = \sum_{i=1}^{nm} P^{(i)}(k) \tag{45}$$

where n_r is the number of real eigenvalues and n_c is the number of complex-conjugate eigenvalue pairs. For simplicity the two summations are replaced with a single one, where $P^{(i)}(k)$ is the pulse response of the ith mode and $n_m = n_r + n_c$.

The matrix modal contribution factors are given by

$$S^{(i)} = \sum_{k=1}^{L} P_t(k) \cdot P^{(i)}(k) \tag{46}$$

where the $S^{(i)}$ are $q \times m$ matrices and L is the number of pulse response samples. The dot symbol denotes the product evaluated by multiplying the corresponding elements of each matrix. To simplify the discrimination process, the scalar modal contribution factors $s^{(i)}$ are determined by summing the elements of $S^{(i)}$,

$$s^{(i)} = \sum_{k=1}^{q} \sum_{l=1}^{m} S_{kl}^{(i)} \tag{47}$$

Equations (46) and (47) determine quantities that are both a measure of the correlation between the total and individual modal pulse responses and the norm of the individual modal pulse responses themselves. Therefore, modes that have a large effect on the response, and that are strongly correlated with the total pulse response, will have the largest contribution factors.

Once the disturbance modes have been identified based on their damping and contribution factors, then a disturbance-free model can be determined. The reduced-order state-space model is obtained by deleting the states corresponding to the disturbance modes, but retaining all other modes. This reduced-order model can then be used to determine the ARX model given in Eq. (35) and to extract the disturbance effect.

Finally, note that in the presence of noise, the method used to compute $\eta(k)$ can be modified to take advantage of the noise filtering effect of the identified model as follows. With noise, the identified model of Eq. (17) will not fit the data perfectly. The least-squares solution, however, minimizes the Euclidean norm of the error $\varepsilon(k)$ between the actual and predicted response over the data record,

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{48}$$

where

$$\hat{y}(k) = \alpha_1 y(k-1) + \alpha_2 y(k-2) + \cdots + \alpha_p y(k-p) + \beta_1 u(k-1) + \beta_2 u(k-2) + \cdots + \beta_p u(k-p) \tag{49}$$

The output $\hat{y}(k)$ is a filtered version of the original output $y(k)$. The filtering action is provided by the overparameterization of the one-step-ahead response model.[5,6] Therefore, to compute the disturbance effect $\eta(k)$, one should use

$$\eta(k) = \hat{y}(k) - \bar{\alpha}_1 y(k-1) - \bar{\alpha}_2 y(k-2) - \cdots - \bar{\alpha}_\zeta y(k-\zeta) - \bar{\beta}_1 u(k-1) - \bar{\beta}_2 u(k-2) - \cdots - \bar{\beta}_\zeta u(k-\zeta) \tag{50}$$

where $\hat{y}(k)$ is used in place of $y(k)$ and the uncontrollable modes due to overparameterization are retained in $\bar{\alpha}_i$ and $\bar{\beta}_i$ to filter the original data $y(k-1)$, $y(k-2)$, $u(k-1)$, $u(k-2)$, etc....

Experimental Results

The flexible lightly damped structure used for the experimental study is shown in Fig. 1 (Ref. 25). Two accelerometers at one end of the structure are used as output sensors. Located nearby is a proof-mass actuator acting as a disturbance source. Two other proof-mass actuators at the far end of the structure serve as excitation inputs. To collect data for system identification, random excitation are applied to the two input actuators, and the two accelerometer responses are recorded in the presence of a disturbance-containing frequency components at 13.6, 17, 20, and 24 Hz. To test the developed theory, we use only excitation input and disturbance-corrupted data in the

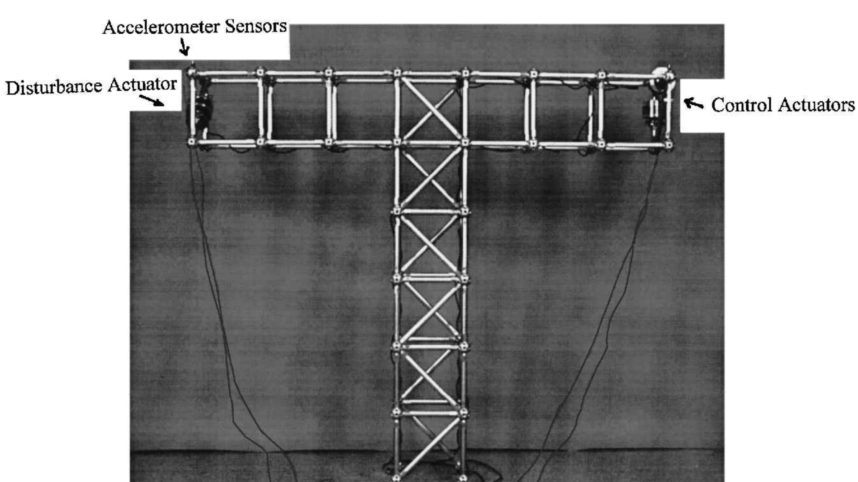

Fig. 1 Flexible truss structure.

Table 1 System and disturbance identification results

Reference values[a]			Model order 90			Model order 60		
Frequency, Hz	Damping, %	Contribution factor	Frequency, Hz	Damping, %	Contribution factor	Frequency, Hz	Damping, %	Contribution factor
				System dynamics[b]				
13.51	0.99	7.04	13.54	1.21	7.59	13.58	1.98	4.14
17.58	0.87	3.84	17.56	0.89	3.72	17.55	1.44	2.82
15.69	0.64	2.44	15.66	0.61	2.59	15.67	0.62	2.65
24.59	0.98	1.15	24.53	1.07	1.13	24.55	1.10	1.04
48.85	0.87	0.23	48.77	0.79	0.12	48.79	0.84	0.18
7.35	2.73	0.15	7.38	2.24	0.27	7.32	3.20	0.15
49.19	2.45	0.12	49.27	2.17	0.12	49.31	2.22	0.15
28.85	0.94	0.11	28.79	0.93	0.10	28.73	1.46	0.12
8.23	1.39	0.11	8.29	1.45	0.18	8.26	1.20	0.14
11.32	5.15	0.09	11.66	5.40	0.08	11.49	5.49	0.06
				Disturbance modes				
13.60	0.0	0.0	13.60	−0.0050	1.87	13.60	−0.0050	8.76
17.00	0.0	0.0	17.00	0.0013	0.19	16.99	−0.0006	3.17
20.00	0.0	0.0	20.00	−0.0012	0.001	20.00	0.0029	0.003
24.00	0.0	0.0	24.00	0.0020	0.003	24.00	0.0018	0.039

[a]System dynamics identified in the absence of the disturbances with model order 60. [b]Ten system modes with the largest pulse response contribution factors.

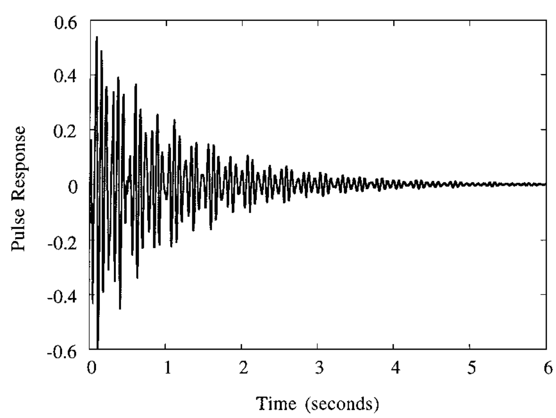

Fig. 2 Reference pulse response.

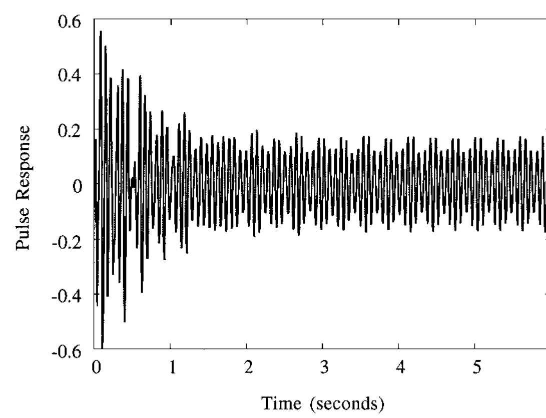

Fig. 3 Disturbance-corrupted pulse response, p = 60.

identification, and the disturbance input is assumed to be unknown. The disturbance components at 13.6 and 17 Hz are close to the truss structural modes, which makes the identification problem more difficult. Furthermore, the magnitude of the disturbance input is such that the disturbance-corrupted response is roughly six times larger than the response to the excitation input only so that we have a situation where the identification data is significantly dominated by the unknown disturbance.

Table 1 summarizes the identification results for assumed ARX model orders p of 60 and 90. For comparison, results are also given for a reference or truth model identified in the absence of the disturbance. In each case 3200 data samples (0.006-s sampling interval) are used for identification. Table 1 gives the frequencies, damping ratios, and pulse response contribution factors of the 10 most significant system modes (as ranked by their contribution factors). The same information is given for the modes associated with the unknown disturbance. Table 1 reveals among other things that with noise-contaminated measurements, models identified in the presence of disturbances can have serious defects that may render them unusable without additional processing. This is why the mode discrimination technique described in the preceding section is essential to producing high-quality models. To appreciate the problem, consider Fig. 2, which shows the reference model pulse response obtained with the disturbance turned off. Compare the reference pulse response to the one in Fig. 3 identified with disturbance-corrupted data and $p = 60$. In theory with noise-free measurements they should be identical, but in practice they are not. In fact, the pulse response obtained with disturbance-corrupted data is growing slowly due to the contribution of two identified disturbance modes that turn out to be slightly unstable. The appearance of possibly unstable disturbance modes should not be a surprise because the correct damping factor for a disturbance mode is zero, and with noise contaminated data, the identified damping ratio may turn out to be positive or negative for unbiased identification. Moreover, the disturbance mode at 13.6 Hz is not only unstable, it has the largest modal contribution factor. Recall that with noise-free measurements, the contribution factors for the disturbance modes are zero. Thus, the presence of these unstable modes renders this model unusable in its current form without additional processing.

The first step to produce a high-quality model is to discriminate the disturbance modes. This is best accomplished by examining the modal damping because disturbance modes will have very low (positive or negative) damping. The disparity between the disturbance mode damping and the damping of the other modes can be seen in Fig. 4 for the case $p = 90$. The four disturbance modes have damping ratios that are more than two orders of magnitude lower than those of the system dynamics modes and overparameterization modes (largest positive damping ratio of 0.002% for the disturbance at 24 Hz and model order 90). With increasing p, one obtains a reduction in the disturbance mode damping ratios with increasing model order whereas the system mode damping ratios converge to nonzero positive values as expected.

In the event that the actual system modes have near-zero damping, or if the data are very noisy, the disturbance modes can still be discriminated by examining the modal pulse response contribution factors. In the noise-free case, disturbance modes have zero contribution factors. This is because, in the input–output transfer functions, the disturbance modes appear along with zeros that cancel them perfectly. With noisy measurements, the pole-zero cancellations are imperfect, particularly if a disturbance's effect is large, that

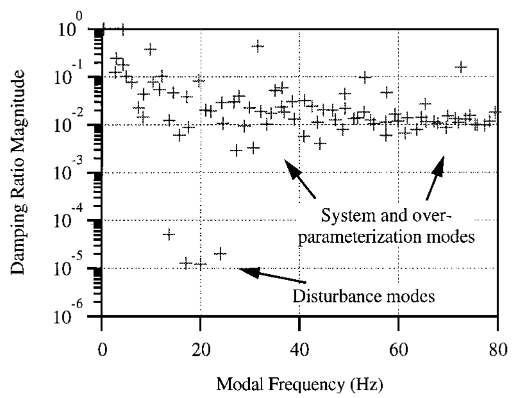

Fig. 4 Modal damping vs modal frequency, p = 90.

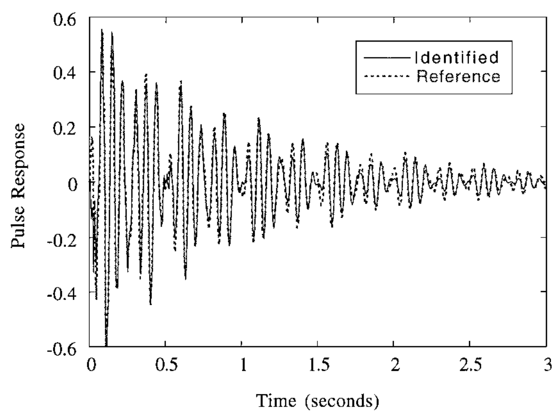

Fig. 5 Identified and reference pulse response, p = 90.

is, when a disturbance is coincident with a lightly damped system mode. Therefore, the disturbance mode may appear to have a large and, perhaps, dominant contribution to the system pulse response as illustrated in Fig. 3. The contribution factors for the disturbance modes will, however, decrease toward zero as the selected model order p increases. This is in contrast to the actual system dynamics, whose modal factors may increase and ultimately converge to some nonzero values with increasing model order. As seen in Table 1 the contribution factor for the disturbance mode at 13.6 Hz drops by almost a factor of five as the model order increases from 60 to 90. The contribution factor for the disturbance mode at 17 Hz drops by over an order of magnitude. In contrast, the contribution factor of the true system mode at 13.5 Hz nearly doubles. Overparameterization modes have inconsistent damping factors, pulse response contributing factors, and frequencies as p changes; therefore, they can be easily discriminated. Finally, Fig. 5 shows the pulse response identified with disturbance-corrupted data (with all disturbance modes identified and removed) is in close agreement with the reference pulse response identified with disturbance-free data.

Conclusions

We have developed an interaction matrix formulation that shows how it is possible to identify a system input–output dynamics correctly in the presence of unknown periodic disturbances. Provided the order of an assumed model is sufficiently large, the disturbance effect will be completely absorbed into its coefficients. Furthermore, the details of the absorption are such that the system input–output dynamics can still be recovered correctly from the altered coefficients as if there were no disturbances present. In addition to identifying the system disturbance-free dynamics correctly, the disturbance effect can also be extracted from the identified model coefficients. This information can be used to calculate a feedforward control signal and the steady-state response to the disturbances. The formulation only requires measurements of the control excitation inputs and the system outputs and is general enough to handle multi-input/multi-output systems with single or multiple disturbance sources. There is no need to measure the disturbances, or to know their periods or profiles, or to use steady-state data. In addition, the disturbance periods need not be integer multiples of the sampling interval. Required to be known, however, are upper bounds on the order of the system dynamics and the number of frequencies present in the disturbances. When implemented recursively, the developed method can be adapted to handle systems whose dynamics and disturbance frequencies are slowly time varying.

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References

[1]Ljung, L., and Söderström, T., *Theory and Practice of Recursive Identification*, MIT Press, Cambridge, MA, 1983, pp. 88–96.

[2]Johnson, C. R., Jr., "Recursive Least-Squares," *Lectures on Adaptive Parameter Estimation*, Prentice–Hall, Englewood Cliffs, NJ, 1988, pp. 24–31.

[3]Keller, B., "On-Line Physical Parameter Identification and Adaptive Control of a Launch Vehicle," Ph.D. Thesis, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, CA, March 1993, pp. 26–45.

[4]Phan, M., Horta, L. G., Juang, J.-N., and Longman, R. W., "Linear System Identification Via an Asymptotically Stable Observer," *Journal of Optimization Theory and Applications*, Vol. 79, No. 1, 1993, pp. 59–86.

[5]Juang, J.-N., Phan, M., Horta, L. G., and Longman, R. W., "Identification of Observer/Kalman Filter Markov Parameters: Theory and Experiments," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 320–329.

[6]Phan, M., Horta, L. G., Juang, J.-N., and Longman, R. W., "Improvement of Observer/Kalman Filter Identification by Residual Whitening," *Journal of Vibration and Acoustics*, Vol. 117, No. 2, 1995, pp. 232–239.

[7]Goodzeit, N. E., and Phan, M. Q., "System and Disturbance Identification for Feedforward and Feedback Control Applications," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 2, 2000, pp. 260–268; also AIAA Paper 97-0682.

[8]Addington, S., and Johnson, C. D., "Dual Mode Disturbance Accommodating Pointing Controller for Hubble Space Telescope," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 2, 1995, pp. 200–207.

[9]You, J. Z., and Johnson, C. D., "Indirect Adaptive Control Based on DAC Theory," *Proceedings of the 1990 American Control Conference*, Vol. 1, American Automatic Control Council, Green Vally, AZ, 1990, pp. 562–564.

[10]Sorrells, J. E., "Adaptive Control Law Design for Model Uncertainty Compensation Final Report," Dynetics, Inc., TR-89-WPAFB-3609-033, Huntsville, AL, June 1989.

[11]Johnson, C. D., "Linear Adaptive Control via Disturbance-Accommodation; Some Case Studies," *1986 American Control Conference*, Vol. 1, Inst. of Electrical and Electronics Engineers, New York, 1986, pp. 542–547.

[12]Widrow, B., and Stearns, S. D., *Adaptive Signal Processing*, Prentice–Hall, Englewood Cliffs, NJ, 1985, pp. 270–367.

[13]Sievers, L. A., and von Flotow, A. H., "Comparison and Extensions of Control Methods for Narrow-Band Disturbance Rejection," *IEEE Transactions on Signal Processing*, Vol. 40, No. 10, 1992, pp. 2377–2379.

[14]Clark, R. L., and Gibbs, G. P., "Implications of Noise on the Reference Signal in Feedforward Control of Harmonic Disturbances," *Journal of the Acoustical Society of America*, Vol. 96, No. 4, 1994, pp. 2585–2588.

[15]Hoefer, S., Kumaresan, R., Pandit, M., and Stollhof, T., "Adaptive FIR-Filter for Control Systems with Periodic Disturbances," *1991 International Conference on Acoustics, Speech, and Signal Processing*, Vol. 5, Inst. of Electrical and Electronics Engineers, New York, 1991, pp. 3241–3244.

[16]Wang, A. K., and Wei, R., "A New Indirect Adaptive Algorithm for Feedforward Control with Application to Active Noise Cancellation," *International Journal of Adaptive Control and Signal Processing*, Vol. 9, 1995, pp. 227–237.

[17]Hu, J., and Tomizuka, M., "A New Plug-In Adaptive Controller for Rejection of Periodic Disturbances," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 115, No. 3, 1993, pp. 543–546.

[18]Yang, W.-C., and Tomizuka, M., "Disturbance Rejection Through an External Model for Non-Minimum Phase Systems," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 116, No. 1, 1994, pp. 39–44.

[19]Woodard, S. E., and Nagchaudhuri, A., "Application of the Least Mean Square and Filtered-X Least Mean Square Algorithms to Spacecraft Vibration Compensation," Advances in the Astronautical Sciences, Vol. 89, Pt. 1, 1995, pp. 837–849.

[20] Hillerstrom, G., and Sternby, J., "Rejection of Periodic Disturbances with Unknown Period—A Frequency Domain Approach," *Proceedings of the 1994 American Control Conference*, Vol. 2, Inst. of Electrical and Electronics Engineers, New York, 1994, pp. 1626–1631.

[21] Sacks, A. H., Bodson, M., and Messner, W., "Advanced Methods for Repeatable Runout Compensation in Disk Drives," *IEEE Transactions on Magnetics*, Vol. 31, No. 2, 1995, pp. 1031–1036.

[22] Hyland, D. C., and Davis, L. D., "A Multiple-Input, Multiple-Output Neural Architecture for Suppression of a Multi-Tone Disturbance," 19th Annual AAS Guidance and Control Conf., American Astronautical Society, Paper AAS-96-067, Feb. 1996.

[23] Hyland, D. C., Davis, L., Das, A., and Yen, G., "Autonomous Neural Control for Structure Vibration Suppression," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Reston, VA, 1996.

[24] Snyder, S. D., and Tanaka, N., "A Neural Network for Feedforward Controlled Smart Structures," *Journal of Intelligent Material Systems and Structures*, Vol. 4, No. 3, 1993, pp. 373–378.

[25] Goodzeit, N. E., "System and Disturbance Identification for Adaptive Disturbance-Rejection Control," Ph.D. Dissertation, Dept. of Mechanical and Aerospace Engineering, Princeton Univ., Princeton, NJ, June 1998.