Project 1. Generate and distribute electrical power to meet uncertain demand.

CEE 201L. Uncertainty, Design, and Optimization
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This project aims to provide an opportunity to explore linear programming to analyze a power supply system. This project provides the chance to analyze a power grid and to determine the optimal flow of electricity throughout. Additionally, it provides an opportunity to incorporate risk and resilience thinking into engineering design.

1 Case Studies
1.1 Case Study: The Valentine’s Day Texas Power Grid Emergency

In February of 2021, The American South, including all of Texas, was hit with anomalously cold winter weather. This climate crisis was made further by widespread blackouts to the Texas power grid, resulting in the Texas Power Grid Emergency. The majority of Texas is serviced by a single wide area interconnection (the technical term for a power grid). In general, a wide area interconnection is the solution to the number one problem surrounding supply and demand of electricity: volatility. Because electricity cannot be stored, must be produced, transmitted, and consumed in the same instant; this is why a large, robust power grid with exposure to a lot of different geographies, power generation types, etc. in aggregate creates resilience. It is very important that the frequency of the power grid remains within an acceptable range or the various elements connected to the grid risk being damaged. The frequency is determined by the supply-demand balance. Demand is changing all the time, so the party overseeing the grid must change the power generation at the same time in order to keep the supply and demand in sync. The Texas power grid is overseen by ERCOT, a non-profit corporation governed by a board of stakeholders from all parts of the grid. ERCOT doesn’t own or operate any part of the grid but rather it tells operators when to start and stop running in order to keep supply and demand in sync. It also manages scheduled outages and oversees the market by setting prices and managing transactions between buyers and sellers.

The week before the Texas Power Grid Emergency had already been a wintry one, and weather forecasts predicted extreme cold the next week. ERCOT anticipated the massive demand and cancelled planned maintenance outages in anticipation. Despite this planning, the extreme weather prevailed. The storm that eventually hit on Valentine’s day was so devastating for three reasons: 1) the extremely cold temperatures 2) its wide-ranging effects—it impacted the entire state and beyond 3) the duration of the frigid temperatures was so long. Texas reached its record peak winter electrical demand of 70,000 mW. The Texas Power Grid has a diverse portfolio of power generators by type, but this diversification did
not save the grid. Wind and natural gas made up the majority of the lost capacity. Natural gas alone lost 15,000 MW of capacity in the span of eight hours. The pipelines themselves are vulnerable to temperature, and when the pipelines froze, the roads to fix them were inaccessible. Even if natural gas power plants remained online, many were financially unable to operate because the wholesale price of natural gas spiked 100x during the period of the storm. Wind power shut off due to icing concerns, solar panels were covered in snow, and even one of Texas’ nuclear power plants was shut off for cold weather concerns surrounding a frozen water pipeline.

Compared to other power grids, Texas’ is relatively independent. This put it in less of a position to rely on other systems to help balance supply and demand. And while the Texas power grid does have a few connections to other power grids, these other grids (all geographically adjacent) were similarly affected by the wide breadth of the storm, leaving them without extra electricity to spare. In order to prevent permanent damage to the grid’s components, ERCOT had one choice to keep supply and demand in sync: “shed load” or turn off customers’ access to power. This left millions without power.

The energy market is an important part of the story. Unlike other markets that pay generators to secure capacity for the future, the wholesale electricity market in Texas is energy-only, meaning if you put power on the grid, you don’t get compensated for capacity when it isn’t needed. This means the generators do not have the same incentive to invest in the resiliency of the grid. [1]

1.2 Case Study: The 2003 Northeast Blackouts

Beginning on August 14, 2003, approximately 50M people from Ottawa, Canada to Cleveland, OH, to New York, NY were without power. All parts of life were disrupted. The most immediate concerns of emergency responders were the people stuck in elevators and underground subways. Attention quickly shifted to the water supply as the flow of clean water is dependent on electric pumps. Two hundred and sixty five power plants were affected. It took anywhere from two to 29 hours to restore power to the major cities; in some places, it took two weeks for the power to come back. [2] Overall, at least 11 deaths and $6.4B in economic losses are attributed to the blackout.

After the crisis was solved, engineers went to work to try to figure out what happened. Terrorism was quickly ruled out. It took engineers months to unravel the cause of the blackout, but it was ultimately traced back to a downed power line in Ohio. A power line in suburban Cleveland sagged due to excess current and intercepted a tree that should have been better trimmed. The initial problem was mostly the fault of FirstEnergy in Akron. [3] From there, a cascading effect of human and equipment failures meant that the problem wasn’t caught and lines continued overloading farther and farther away. An employee who worked for MISO - the overseeing agency - went to lunch and forgot to turn back on a tool that monitored grid problems. Eventually, the grid in Ohio got less and less stable and voltage became too much and plants farther and farther away began shutting down. The outage ended up taking down the entire Eastern Interconnection, one of three in the United States (Eastern, Western, and the aforementioned Texas). The highest impact outcome of the blackout was the making of industry standards mandatory and enforceable. [4]
2 Problem Statement

In this project, you will design the most economical distribution of electrical power within a power grid and will assess the robustness of the power grid to uncertain extreme electrical power demands. The topology of the power grid is shown in Figure 1.

![Power grid topology](image)

Figure 1. Power grid topology. Nodes A and B are power generating nodes and consume no power. Nodes 1 through 10 are power demand nodes (cities, factories, etc) and at any point in time demand a certain amount of power. The generating nodes and the demand nodes are interconnected as shown.

Optimization programs know nothing of the power system we are attempting to simulate. So we must define the problem as completely as we can. The system can ultimately be broken down into individual station-to-substation connections or substation-to-substation connections: there are 22 in all. Thus, there are 22 design variables. Any feasible power distribution must adhere to the constraints that:

1. The total power generated at a generating station can not exceed the power generating capacity of that station.
2. The total power delivered to a demand node must not be less than the power demanded at that node.
3. The power transmitted along a line must be within the power carrying capacity of the line.
4. Generating nodes must output power, they may not receive power.
The design variables in this problem are the set of 22 power flows from node $i$ to node $j$. Please use the following order of the 22 power flow design variables.

$$x = [P_{A,1}, P_{A,2}, P_{A,9}, P_{A,10}, P_{B,4}, P_{B,5}, P_{B,6}, P_{B,7}, \ldots, P_{1,2}, P_{1,3}, P_{1,10}, P_{2,3}, P_{3,4}, P_{3,5}, P_{4,5}, P_{5,6}, P_{6,7}, P_{6,8}, P_{7,8}, P_{8,9}, P_{8,10}, P_{9,10}]^T$$

The analysis of a power grid comes down to balancing the input power and output power of the grid. For example, at node 6, assume that power flows from node B to node 6, $P_{B,6}$, from node 5 to node 6, $P_{5,6}$, from node 6 to node 7, $P_{6,7}$, and from node 6 to node 8, $P_{6,8}$. And assume the power demanded by node 6 $D_6$ leaves node 6. Then the power balance on node 6 is:

$$P_{B,6} + P_{5,6} - P_{6,7} - P_{6,8} \geq D_6$$

Denote the power generating capacities of stations A and B as $G_A$ and $G_B$.

Denote the power transmission capacities of all lines as $T$. Note that $-T \leq P_{i,j} \leq +T$ except for at the power generating stations.

The cost to be minimized $f$, is the generating cost of the power generation station,

$$f(x) = c_A P_{A,1} + c_A P_{A,2} + c_A P_{A,9} + c_A P_{A,10} + c_B P_{B,4} + c_B P_{B,5} + c_B P_{B,6} + c_B P_{B,7}.$$
3 Tasks

The power grid is analyzed in Tasks 1, 2, 3, and 4; and the optimal power grid distribution is solved in Tasks 5, 6, and 7; and the risk of a blackout is assessed in Tasks 9 and 10.

1. Write the 12 inequalities corresponding to (a) and (b) above for nodes A, B, and 1 through 10.

2. Express these 12 inequalities in matrix form $A\mathbf{x} \leq \mathbf{b}$ in which the 12-by-22 matrix $A$ will contain values of 0, 1, and -1.

3. Express the objective function in vector form $f = \mathbf{c}^T \mathbf{x}$

4. Write lower bounds and upper bounds for the power transmission

5. Here is a general purpose .m-file for solving any LP by using SQPopt.

   Just type this into a .m-file called ... LP_analysis.m.

   ```matlab
   function [f,g] = LP_analysis(x, constants)
   % [f,g] = LP_analysis(x, constants)
   % analyze a trial solution x to a linear programming problem,
   % minimize f = c^T x such that g = A*x - b <= 0
   A = constants{1};  % constraint coefficient matrix (dimension m by n )
   b = constants{2};  % constraint vector (dimension m by 1 )
   c = constants{3};  % cost coefficient vector (dimension n by 1 )
   f = c'*x;          % the cost function
   g = A*x-b;         % the constraint inequalities, compared to zero
   ```

   Why are there curly brackets in this code?

6. Complete the .m-file shown on page 6. Use the variables provided in lines 1 to 6 (not the numerical values) when filling in matrix $A$, vectors $b$, $c$, $l_b$, $u_b$, and scalars $netGeneration$, and $netDemand$.

7. This code provides three numerical methods for solving the LP:

   • linprog using the dual-simplex algorithm
   • linprog using the interior-point algorithm
   • SQPopt

   Solve the problem with each of the three methods. For each of the three solutions

   i. Note the value of $f_{opt}$ and $x_{opt}$
   ii. Confirm that $A\mathbf{x} - \mathbf{b} \leq 0$
   iii. Confirm that $-T \leq P_{i,j} \leq T$.
   iv. Calculate the power shortfall ... $netDemand - netGeneration$

   (a) Which optimization method results in the lowest cost?
(b) Which optimization method results in the design that you think is best? Why do you think it is best? What you find depends on how you looked for it.

```
G = [100; 200]; % power generating capacities of plants A and B
ca = 20; % smaller plant, less capacity, less efficient
cb = 12; % larger plant, more capacity, more efficient
T = 48; % transmission line capacity
D = [30; 20; 35; 25; 20; 25; 20; 45; 30]; % ten power demands

A = [ ... ]; % matrix inequality coefficient matrix
b = [ ... ]; % matrix inequality vector b
c = [ ... ]; % cost function coefficients vector c
x_lb = [ ... ]; % lower bound of the solution, x
x_ub = [ ... ]; % upper bound of the solution, x

sumA = sum(A) % check columns of A sum to zero

% solve the problem using an LP solver ... matlab's linprog

% 
% disp('linprog ----------------------')
% use one of these two algorithm options
% linprog_options = optimoptions('linprog', 'Algorithm', 'dual-simplex');
% linprog_options = optimoptions('linprog', 'Algorithm', 'interior-point');

% [x_opt, f_opt, g_opt, y_opt] = ...% linprog( c, A, b, [], [], x_lb, x_ub, linprog_options);
x_opt
f_opt
g_opt = A*x_opt - b
y_opt = y_opt.ineqlin
netGeneration = ...
netDemand = ...% }

% solve the problem using an SQP solver ... hpg's SQPopt

disp('SQPopt ----------------------')
constants = {A, b, c}; % the set of constants in this problem
x_init = ... % initial guess for x ... start near zero for everything
rng(0)
options = [0 0.01 0.01 0.01 1e4];
[x_opt, f_opt, g_opt, cvg_hst, y_opt] = ...% SQPopt('LP_analysis', x_init, x_lb, x_ub, options, constants);

% y_opt = y_opt(1:length(b))
netGeneration = ...
netDemand = ...
```
8. Now consider the reliability of the power grid to uncertain extreme demand. Consider $D$ to be a vector of uncertain demand with median values as listed in the code above, and a coefficient of variation of 0.1 for all nodes. To start with assume that the variation in demand $D_i$ has no bearing on the variation of demand $D_j$ and vice-versa. In other words, $D_i$ and $D_j$ are not correlated. Download corr_logn_rnd.m

(a) Refer to sections 4, 5.1, 5.2, p. 13, sections 11.3, and 17 of Probability Distributions for background. Use the code below to generate a set of $m = 100$ observations of the $n = 10$ random demands, distributed as log-normal random variables with median values as listed on line 6 of the code above and coefficient of variation of 0.1 for all demand values.

```matlab
1 nd = 10; % number of random variables in D
2 md = 100; % number of observations of each rv
3 medD = D; % median value of demand
4 covD = 0.1.*ones(nd,1); % coefficient of variation of demand
5 R = eye(nd); % uncorrelated variability between node demands
6 Drand = corr_logn_rnd( medD , covD , R , nd , md ); % nd-by-md matrix of random D
```

(b) For each of the $m = 100$ observations in $D_{rand}$:

i. Use SQPopt to attempt to find a feasible solution to the power generation and distribution problem.

ii. Keep a record of the net power generated and the net power demanded for each set of the $m$ observations.

(c) Make a plot of the cumulative distribution of the sample one of the ten demands. Is the median value approximately correct?

(d) Make a scatter plot of the sample of the demand in one node vs the demand in another node. Do these demands appear uncorrelated?

(e) Plot the cumulative distribution of shortfall ... (net Demand - net Generated)

(f) For what fraction of this set is the shortfall greater than 0.1 MW?

(g) Is the unmet demand due to a bottleneck in the grid or insufficient generation capacity?

9. Repeat step 8 but with correlated demand. Correlated demand models the likely situation that higher (or lower) demand in one node affects the probability of higher (or lower) demand in other nodes. The probabilities are conditionally dependent. This is easily implemented in the code above by changing line 5 in 8.(a) to

```matlab
1 R = 0.8.*ones(nd) + 0.2.*eye(nd); % correlated variability between node demands
```

This means that the correlation in the demand variability between any pair of stations is 80 percent. If one node is experiencing higher demand it is more likely than not that the other nodes are too. Like in a summer heat wave.

Do correlations among uncertain system attributes affect the reliability of the system?
To hand in:

1. Your hand written work for tasks 1 through 4.
2. Your answer to the question in task 5.
4. Your values of $f_{\text{opt}}$ and $x_{\text{opt}}$ for each of the three numerical methods in task 7.
   Statements for parts ii., iii., and iv., in task 7. Written answers to 7.(a) and 7.(b).
5. Your code for tasks 8. and 9. These codes will differ by one line as indicated in task 9.
6. Your plots for 8.(c) 8.(d) and 8.(e) with axis labels. Your answers for 8.(f) and 8.(g).
7. Your plots for 9.(c) 9.(d) and 9.(e) with axis labels. Your answers for 9.(f) and 9.(g).
8. Your answer to the last question in task 9.

4 References

1. https://www.youtube.com/watch?v=08mwXICY4JM
2. https://www.youtube.com/watch?v=FMCPuE_7oE8
3. https://www.youtube.com/watch?v=REcl2ly34hg
5. https://www.youtube.com/watch?v=08mwXICY4JM